



Assignment

Definition, Equation of the Circle

Basic Level

- The two points A and B in a plane such that for all points P lies on circle satisfied $\frac{PA}{PB} = k$, then k will not be equal to [IIT 1982]
 - 0
 - 1
 - 2
 - None of these
- Locus of a point which moves such that sum of the squares of its distances from the sides of a square of side unity is 9, is [IIT 1976]
 - Straight line
 - Circle
 - Parabola
 - None of these
- The equation of the circle which touches both the axes and whose radius is a , is [MP PET 1984]
 - $x^2 + y^2 - 2ax - 2ay + a^2 = 0$
 - $x^2 + y^2 + ax + ay - a^2 = 0$
 - $x^2 + y^2 + 2ax + 2ay - a^2 = 0$
 - $x^2 + y^2 - ax - ay + a^2 = 0$
- $ABCD$ is a square the length of whose side is a . Taking AB and AD as the coordinate axes, the equation of the circle passing through the vertices of the square, is [MP PET 2003]
 - $x^2 + y^2 + ax + ay = 0$
 - $x^2 + y^2 - ax - ay = 0$
 - $x^2 + y^2 + 2ax + 2ay = 0$
 - $x^2 + y^2 - 2ax - 2ay = 0$
- The equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin is [Rajasthan PET 1991; MP PET 1987, 1989]
 - $x^2 + y^2 - 2x - 2y + 1 = 0$
 - $x^2 + y^2 - 2x - 2y - 1 = 0$
 - $x^2 + y^2 - 2x - 2y = 0$
 - None of these
- The equation of the circle which touches both axes and whose centre is (x_1, y_1) , is [MP PET 1988]
 - $x^2 + y^2 + 2x_1(x + y) + x_1^2 = 0$
 - $x^2 + y^2 - 2x_1(x + y) + x_1^2 = 0$
 - $x^2 + y^2 = x_1^2 + y_1^2$
 - $x^2 + y^2 + 2xx_1 + 2yy_1 = 0$
- The equation of the circle which touches x -axis and whose centre is $(1, 2)$, is [MP PET 1984]
 - $x^2 + y^2 - 2x + 4y + 1 = 0$
 - $x^2 + y^2 - 2x - 4y + 1 = 0$
 - $x^2 + y^2 + 2x + 4y + 1 = 0$
 - $x^2 + y^2 + 4x + 2y + 4 = 0$
- The equation of the circle having centre $(1, -2)$ and passing through the point of intersection of lines $3x + y = 14$, $2x + 5y = 18$ is [MP PET 1990]
 - $x^2 + y^2 - 2x + 4y - 20 = 0$
 - $x^2 + y^2 - 2x - 4y - 20 = 0$
 - $x^2 + y^2 + 2x - 4y - 20 = 0$
 - $x^2 + y^2 + 2x + 4y - 20 = 0$
- The equation of the circle passing through $(4, 5)$ and having the centre at $(2, 2)$, is [MNR 1986; MP PET 1984; UPSEAT 2000]
 - $x^2 + y^2 + 4x + 4y - 5 = 0$
 - $x^2 + y^2 - 4x - 4y - 5 = 0$
 - $x^2 + y^2 - 4x = 13$
 - $x^2 + y^2 - 4x - 4y + 5 = 0$
- The equation of the circle which passes through the points $(2, 3)$ and $(4, 5)$ and the centre lies on the straight line $y - 4x + 3 = 0$, is [Rajasthan PET 1985; MP PET 1989]
 - $x^2 + y^2 + 4x - 10y + 25 = 0$
 - $x^2 + y^2 - 4x - 10y + 25 = 0$
 - $x^2 + y^2 - 4x - 10y + 16 = 0$
 - $x^2 + y^2 - 14y + 8 = 0$
- The equation of the circle passing through the points $(0, 0)$, $(0, b)$ and (a, b) is [AMU 1978]

- (a) $x^2 + y^2 + ax + by = 0$ (b) $x^2 + y^2 - ax + by = 0$ (c) $x^2 + y^2 - ax - by = 0$ (d) $x^2 + y^2 + ax - by = 0$
12. The equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ will represent a circle, if [MNR 1979; MP PET 1988; Rajasthan PET 1997, 2003]
 (a) $a = b = 0$ and $c = 0$ (b) $f = g$ and $h = 0$ (c) $a = b \neq 0$ and $h = 0$ (d) $f = g$ and $c = 0$
13. The equation of the circle whose diameters have the end points $(a, 0)$, $(0, b)$ is given by [MP PET 1993]
 (a) $x^2 + y^2 - ax - by = 0$ (b) $x^2 + y^2 + ax - by = 0$ (c) $x^2 + y^2 - ax + by = 0$ (d) $x^2 + y^2 + ax + by = 0$
14. The equation of the circle which touches x -axis at $(3, 0)$ and passes through $(1, 4)$ is given by [MP PET 1993]
 (a) $x^2 + y^2 - 6x - 5y + 9 = 0$ (b) $x^2 + y^2 + 6x + 5y - 9 = 0$
 (c) $x^2 + y^2 - 6x + 5y - 9 = 0$ (d) $x^2 + y^2 + 6x - 5y + 9 = 0$
15. From three non-collinear points we can draw [MP PET 1984; BIT Ranchi 1990]
 (a) Only one circle (b) Three circle (c) Infinite circles (d) No circle
16. Equation of a circle whose centre is origin and radius is equal to the distance between the lines $x = 1$ and $x = -1$ is [MP PET 1984]
 (a) $x^2 + y^2 = 1$ (b) $x^2 + y^2 = \sqrt{2}$ (c) $x^2 + y^2 = 4$ (d) $x^2 + y^2 = -4$
17. If the centre of a circle is $(2, 3)$ and a tangent is $x + y = 1$, then the equation of this circle is [Rajasthan PET 1985, 1989]
 (a) $(x - 2)^2 + (y - 3)^2 = 8$ (b) $(x - 2)^2 + (y - 3)^2 = 3$ (c) $(x + 2)^2 + (y + 3)^2 = 2\sqrt{2}$ (d) $(x - 2)^2 + (y - 3)^2 = 2\sqrt{2}$
18. $ax^2 + 2y^2 + 2bxy + 2x - y + c = 0$ represents a circle through the origin, if [MP PET 1984]
 (a) $a = 0, b = 0, c = 2$ (b) $a = 1, b = 0, c = 0$ (c) $a = 2, b = 2, c = 0$ (d) $a = 2, b = 0, c = 0$
19. If the equation $\frac{K(x+1)^2}{3} + \frac{(y+2)^2}{4} = 1$ represents a circle, then $K =$ [MP PET 1994]
 (a) $3/4$ (b) 1 (c) $4/3$ (d) 12
20. A circle has radius 3 units and its centre lies on the line $y = x - 1$. Then the equation of this circle if it passes through point $(7, 3)$, is [Roorkee 1988]
 (a) $x^2 + y^2 - 8x - 6y + 16 = 0$ (b) $x^2 + y^2 + 8x + 6y + 16 = 0$
 (c) $x^2 + y^2 - 8x - 6y - 16 = 0$ (d) None of these
21. The equation of circle whose diameter is the line joining the points $(-4, 3)$ and $(12, -1)$ is [IIT 1971; Rajasthan PET 1984, 87, 89; MP PET 1984; Roorkee 1969; AMU 1979]
 (a) $x^2 + y^2 + 8x + 2y + 51 = 0$ (b) $x^2 + y^2 + 8x - 2y - 51 = 0$
 (c) $x^2 + y^2 + 8x + 2y - 51 = 0$ (d) $x^2 + y^2 - 8x - 2y - 51 = 0$
22. The equation of the circle which passes through the points $(3, -2)$ and $(-2, 0)$ and centre lies on the line $2x - y = 3$, is [Roorkee 1971]
 (a) $x^2 + y^2 - 3x - 12y + 2 = 0$ (b) $x^2 + y^2 - 3x + 12y + 2 = 0$
 (c) $x^2 + y^2 + 3x + 12y + 2 = 0$ (d) None of these
23. For $ax^2 + 2hxy + 3y^2 + 4x + 8y - 6 = 0$ to represent a circle, one must have
 (a) $a = 3, h = 0$ (b) $a = 1, h = 0$ (c) $a = h = 3$ (d) $a = h = 0$
24. The equation of the circle in the first quadrant which touches each axis at a distance 5 from the origin is [MP PET 1997]
 (a) $x^2 + y^2 + 5x + 5y + 25 = 0$ (b) $x^2 + y^2 - 10x - 10y + 25 = 0$
 (c) $x^2 + y^2 - 5x - 5y + 25 = 0$ (d) $x^2 + y^2 + 10x + 10y + 25 = 0$
25. If (α, β) is the centre of a circle passing through the origin, then its equation is [MP PET 1999]
 (a) $x^2 + y^2 - \alpha x - \beta y = 0$ (b) $x^2 + y^2 + 2\alpha x + 2\beta y = 0$ (c) $x^2 + y^2 - 2\alpha x - 2\beta y = 0$ (d) $x^2 + y^2 + \alpha x + \beta y = 0$
26. The equation of the circle whose diameter lies on $2x + 3y = 3$ and $16x - y = 4$ and which passes through $(4, 6)$ is [Kurukshetra CEE 1998]
 (a) $5(x^2 + y^2) - 3x - 8y = 200$ (b) $x^2 + y^2 - 4x - 8y = 200$
 (c) $5(x^2 + y^2) - 4x = 200$ (d) $x^2 + y^2 = 40$
27. The equation of the circle of radius 5 and touching the coordinate axes in third quadrant is [EAMCET 2002]

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- (a) $(x-5)^2 + (y+5)^2 = 25$ (b) $(x+4)^2 + (y+4)^2 = 25$ (c) $(x+6)^2 + (y+6)^2 = 25$ (d) $(x+5)^2 + (y+5)^2 = 25$
28. The centre of a circle is $(2, -3)$ and the circumference is 10π . Then the equation of the circle is [Kerala (Engg.) 2002]
- (a) $x^2 + y^2 + 4x + 6y + 12 = 0$ (b) $x^2 + y^2 - 4x + 6y + 12 = 0$
 (c) $x^2 + y^2 - 4x + 6y - 12 = 0$ (d) $x^2 + y^2 - 4x - 6y - 12 = 0$
29. The circle described on the line joining the points $(0, 1)$, (a, b) as diameter cuts the x -axis in points whose abscissae are roots of the equation
- (a) $x^2 + ax + b = 0$ (b) $x^2 - ax + b = 0$ (c) $x^2 + ax - b = 0$ (d) $x^2 - ax - b = 0$
30. Four distinct points $(2k, 3k)$, $(1, 0)$, $(0, 1)$ and $(0, 0)$ lie on a circle for
- (a) All integral values of k (b) $0 < k < 1$ (c) $k < 0$ (d) For two values of k
31. The equations of the circles which touch both the axes and the line $x = a$ are
- (a) $x^2 + y^2 \pm ax \pm ay + \frac{a^2}{4} = 0$ (b) $x^2 + y^2 + ax \pm ay + \frac{a^2}{4} = 0$
 (c) $x^2 + y^2 - ax \pm ay + \frac{a^2}{4} = 0$ (d) None of these.
32. The equation of the unit circle concentric with $x^2 + y^2 + 8x + 4y - 8 = 0$ is [EAMCET 1991]
- (a) $x^2 + y^2 - 8x + 4y - 8 = 0$ (b) $x^2 + y^2 - 8x + 4y + 8 = 0$
 (c) $x^2 + y^2 - 8x + 4y - 28 = 0$ (d) $x^2 + y^2 - 8x + 4y + 19 = 0$
33. A circle of radius 2 touches the coordinate axes in the first quadrant. If the circle makes a complete rotation on the x -axis along the positive direction of the x -axis then the equation of the circle in the new position is
- (a) $x^2 + y^2 - 4(x+y) - 8\pi x + (2+4\pi)^2 = 0$ (b) $x^2 + y^2 - 4x - 4y + (2+4\pi)^2 = 0$
 (c) $x^2 + y^2 - 8\pi x - 4y + (2+4\pi)^2 = 0$ (d) None of these
34. A circle which touches the axes and whose centre is at distance $2\sqrt{2}$ from the origin, has the equation
- (a) $x^2 + y^2 - 4x + 4y + 4 = 0$ (b) $x^2 + y^2 + 4x - 4y + 4 = 0$
 (c) $x^2 + y^2 + 4x + 4y + 4 = 0$ (d) None of these
35. If $(-1, 4)$ and $(3, -2)$ are end points of a diameter of a circle, then the equation of this circle is [Rajasthan PET 1987, 89]
- (a) $(x-1)^2 + (y-1)^2 = 13$ (b) $(x+1)^2 + (y+1)^2 = 13$ (c) $(x-1)^2 + (y+1)^2 = 13$ (d) $(x+1)^2 + (y-1)^2 = 13$
36. The equation of the circle concentric with the circle $x^2 + y^2 - 3x + 4y - c = 0$ and passing through the point $(-1, -2)$ is [Rajasthan PET 1984, 92]
- (a) $x^2 + y^2 - 3x + 4y - 1 = 0$ (b) $x^2 + y^2 - 3x + 4y = 0$
 (c) $x^2 + y^2 - 3x + 4y + 2 = 0$ (d) None of these
37. If $(-3, 2)$ lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which is concentric with $x^2 + y^2 + 6x + 8y - 5 = 0$, then c is equal to [Rajasthan PET 1986]
- (a) -11 (b) 11 (c) -24 (d) 24
38. Equation $x^2 + y^2 + 4x + 6y + 13 = 0$ represents [Roorkee 1990]
- (a) A circle (b) A pair of two different lines (c) A pair of coincident lines (d) A point
39. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π , then the equation of the circle is [AIEEE 2004]
- (a) $x^2 + y^2 + 2x - 2y - 23 = 0$ (b) $x^2 + y^2 - 2x - 2y - 23 = 0$
 (c) $x^2 + y^2 + 2x + 2y - 23 = 0$ (d) $x^2 + y^2 - 2x + 2y - 23 = 0$

40. $y = mx$ is a chord of a circle of radius a and the diameter of the circle lies along x -axis and one end of this chord is origin. The equation of the circle described on this chord as diameter is [MP PET 1990]
- (a) $(1 + m^2)(x^2 + y^2) - 2ax = 0$ (b) $(1 + m^2)(x^2 + y^2) - 2a(x + my) = 0$
 (c) $(1 + m^2)(x^2 + y^2) + 2a(x + my) = 0$ (d) $(1 + m^2)(x^2 + y^2) - 2a(x - my) = 0$
41. If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, then the equation of the circle of which this chord is a diameter, is [Rajasthan PET 1988]
- (a) $x^2 + y^2 - 2x + 4y = 0$ (b) $x^2 + y^2 + 2x + 4y = 0$ (c) $x^2 + y^2 + 2x - 4y = 0$ (d) $x^2 + y^2 - 2x - 4y = 0$
42. The circle on the chord $x \cos \alpha + y \sin \alpha = p$ of the circle $x^2 + y^2 = a^2$ as diameter has the equation [Roorkee 1967; MP PET 1993]
- (a) $x^2 + y^2 - a^2 - 2p(x \cos \alpha + y \sin \alpha - p) = 0$ (b) $x^2 + y^2 + a^2 + 2p(x \cos \alpha - y \sin \alpha + p) = 0$
 (c) $x^2 + y^2 - a^2 + 2p(x \cos \alpha + y \sin \alpha + p) = 0$ (d) $x^2 + y^2 - a^2 - 2p(x \cos \alpha - y \sin \alpha - p) = 0$
43. The equation of circle which touches the axes of coordinates and the line $\frac{x}{3} + \frac{y}{4} = 1$ and whose centre lies in the first quadrant is $x^2 + y^2 - 2cx - 2cy + c^2 = 0$, where c is [Ranchi BIT 1986; Kurukshetra CEE 1996]
- (a) 1 (b) 2 (c) 3 (d) 6
44. The equation of a circle which touches both axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant is [MP PET 1986]
- (a) $x^2 + y^2 - 4x + 4y - 4 = 0$ (b) $x^2 + y^2 - 4x + 4y + 4 = 0$
 (c) $x^2 + y^2 + 4x + 4y + 4 = 0$ (d) $x^2 + y^2 - 4x - 4y - 4 = 0$
45. Equation of the circle which touches the lines $x = 0$, $y = 0$ and $3x + 4y = 4$ is [MP PET 1991]
- (a) $x^2 - 4x + y^2 + 4y + 4 = 0$ (b) $x^2 - 4x + y^2 - 4y + 4 = 0$
 (c) $x^2 + 4x + y^2 + 4y + 4 = 0$ (d) $x^2 + 4x + y^2 - 4y + 4 = 0$
46. The equation of the circumcircle of the triangle formed by the lines $y + \sqrt{3}x = 6$, $y - \sqrt{3}x = 6$ and $y = 0$, is [EAMCET 1982]
- (a) $x^2 + y^2 - 4y = 0$ (b) $x^2 + y^2 + 4x = 0$ (c) $x^2 + y^2 - 4y = 12$ (d) $x^2 + y^2 + 4x = 12$
47. A variable circle passes through the fixed point $A(p, q)$ and touches x -axis. The locus of the other end of the diameter through A is [AIEEE 2004]
- (a) $(y - q)^2 = 4px$ (b) $(x - q)^2 = 4py$ (c) $(y - p)^2 = 4qx$ (d) $(x - p)^2 = 4qy$
48. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of λ is [IIT 1991]
- (a) 1 (b) 2 (c) 3 (d) 4
49. Equation to the circles which touch the lines $3x - 4y + 1 = 0$, $4x + 3y - 7 = 0$ and pass through $(2, 3)$ are [EAMCET 1989]
- (a) $(x - 2)^2 + (y - 8)^2 = 25$ (b) $5x^2 + 5y^2 - 12x - 24y + 31 = 0$
 (c) Both (a) and (b) (d) None of these
50. The equation of the circle which passes through $(1, 0)$ and $(0, 1)$ and has its radius as small as possible, is
- (a) $x^2 + y^2 - 2x - 2y + 1 = 0$ (b) $x^2 + y^2 - x - y = 0$
 (c) $2x^2 + 2y^2 - 3x - 3y + 1 = 0$ (d) $x^2 + y^2 - 3x - 3y + 2 = 0$
51. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is [AIEEE 2002]
- (a) $4 \leq x^2 + y^2 \leq 64$ (b) $x^2 + y^2 \leq 25$ (c) $x^2 + y^2 \geq 25$ (d) $3 \leq x^2 + y^2 \leq 9$
52. The equation of the circle which touches both the axes and the straight line $4x + 3y = 6$ in the first quadrant and lies below it is [Roorkee 1992]
- (a) $4x^2 + 4y^2 - 4x - 4y + 1 = 0$ (b) $x^2 + y^2 - 6x - 6y + 9 = 0$

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- (c) $x^2 + y^2 - 6x - y + 9 = 0$ (d) $4(x^2 + y^2 - x - 6y) + 1 = 0$
53. Three sides of a triangle have the equations $L_r \equiv y - m_r x - c_r = 0$; $r = 1, 2, 3$. Then $\lambda L_2 L_3 + \mu L_3 L_1 + \nu L_1 L_2 = 0$, where $\lambda \neq 0$, $\mu \neq 0$, $\nu \neq 0$, is the equation of the circumcircle of the triangle, if
- (a) $\lambda(m_2 + m_3) + \mu(m_3 + m_1) + \nu(m_1 + m_2) = 0$ (b) $\lambda(m_2 m_3 - 1) + \mu(m_3 m_1 - 1) + \nu(m_1 m_2 - 1) = 0$
 (c) Both (a) and (b) hold together (d) None of these
54. The equation of the circle passing through the point (1, 1) and having two diameters along the pair of lines $x^2 - y^2 - 2x + 4y - 3 = 0$ is
- (a) $x^2 + y^2 - 2x - 4y + 4 = 0$ (b) $x^2 + y^2 + 2x + 4y - 4 = 0$
 (c) $x^2 + y^2 - 2x + 4y + 4 = 0$ (d) None of these
55. The equation of a circle which touches x -axis and the line $4x - 3y + 4 = 0$, its centre lying in the third quadrant and lies on the line $x - y - 1 = 0$, is
- (a) $9(x^2 + y^2) + 6x + 24y + 1 = 0$ (b) $9(x^2 + y^2) - 6x - 24y + 1 = 0$
 (c) $9(x^2 + y^2) - 6x + 2y + 1 = 0$ (d) None of these
56. Two vertices of an equilateral triangle are $(-1, 0)$ and $(1, 0)$ and its third vertex lies above the x -axis. The equation of the circumcircle of the triangle is
- (a) $x^2 + y^2 = 1$ (b) $\sqrt{3}(x^2 + y^2) + 2y - \sqrt{3} = 0$ (c) $\sqrt{3}(x^2 + y^2) - 2y - \sqrt{3} = 0$ (d) None of these
57. A triangle is formed by the lines whose combined equation is given by $(x + y - 4)(xy - 2x - y + 2) = 0$. The equation of its circumcircle is
- (a) $x^2 + y^2 - 5x - 3y + 8 = 0$ (b) $x^2 + y^2 - 3x - 5y + 8 = 0$
 (c) $x^2 + y^2 - 3x - 5y - 8 = 0$ (d) None of these
58. If the centroid of an equilateral triangle is (1, 1) and its one vertex is $(-1, 2)$ then the equation of its circumcircle is
- (a) $x^2 + y^2 - 2x - 2y - 3 = 0$ (b) $x^2 + y^2 + 2x - 2y - 3 = 0$
 (c) $x^2 + y^2 + 2x + 2y - 3 = 0$ (d) None of these
59. The equation of the circle whose one diameter is PQ , where the ordinates of P, Q are the roots of the equation $x^2 + 2x - 3 = 0$ and the abscissae are the roots of the equation $y^2 + 4y - 12 = 0$, is
- (a) $x^2 + y^2 + 2x + 4y - 15 = 0$ (b) $x^2 + y^2 - 4x - 2y - 15 = 0$
 (c) $x^2 + y^2 + 4x + 2y - 15 = 0$ (d) None of these
60. The equation of the circumcircle of an equilateral triangle is $x^2 + y^2 + 2gx + 2fy + c = 0$ and one vertex of the triangle is (1, 1). The equation of incircle of the triangle is
- (a) $4(x^2 + y^2) = g^2 + f^2$ (b) $4(x^2 + y^2) + 8gx + 8fy = (1 - g)(1 + 3g) + (1 - f)(1 + 3f)$
 (c) $4(x^2 + y^2) + 8gx + 8fy = g^2 + f^2$ (d) None of these
61. The equation of the circle of radius $2\sqrt{2}$ whose centre lies on the line $x - y = 0$ and which touches the line $x + y = 4$, and whose centre's coordinates satisfy the inequality $x + y > 4$ is
- (a) $x^2 + y^2 - 8x - 8y + 24 = 0$ (b) $x^2 + y^2 = 8$
 (c) $x^2 + y^2 - 8x + 8y = 24$ (d) None of these
62. The circumcircle of the quadrilateral formed by the lines $x = a, x = 2a, y = -a, y = \sqrt{2}a$ is
- (a) $x^2 + y^2 + 3ax + a^2 = 0$ (b) $x^2 + y^2 - 3ax - a^2 = 0$ (c) $x^2 + y^2 - 3ax + 2a^2 = 0$ (d) $x^2 + y^2 + 3ax - a^2 = 0$
63. Equation of a circle $S(x, y) = 0$, $S(2, 3) = 16$, which touches the line $3x + 4y - 7 = 0$ at (1, 1) is given by
- (a) $x^2 + y^2 + x + 2y - 5 = 0$ (b) $x^2 + y^2 + 2x + 2y - 6 = 0$ (c) $x^2 + y^2 + 4x - 6y = 0$ (d) None of these

64. The area of the circle whose centre is at (1, 2) and which passes through the point (4, 6) is [MNR 1982; IIT 1980; Karnataka CET 1999; MP PET 2002; DCE 2000]
 (a) 5π (b) 10π (c) 25π (d) None of these
65. The centres of the circles $x^2 + y^2 = 1$, $x^2 + y^2 + 6x - 2y = 1$ and $x^2 + y^2 - 12x + 4y = 1$ are [MP PET 1986]
 (a) Same (b) Collinear (c) Non-collinear (d) None of these
66. If a circle passes through the point (0, 0), (a, 0), (0, b), then its centre is [MNR 1975]
 (a) (a, b) (b) (b, a) (c) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (d) $\left(\frac{b}{2}, -\frac{a}{2}\right)$
67. If the radius of the circle $x^2 + y^2 - 18x + 12y + k = 0$ be 11, then $k =$ [MP PET 1987]
 (a) 347 (b) 4 (c) -4 (d) 49
68. The centre and radius of the circle $2x^2 + 2y^2 - x = 0$ are [MP PET 1984, 87]
 (a) $\left(\frac{1}{4}, 0\right)$ and $\frac{1}{4}$ (b) $\left(-\frac{1}{2}, 0\right)$ and $\frac{1}{2}$ (c) $\left(\frac{1}{2}, 0\right)$ and $\frac{1}{2}$ (d) $\left(0, -\frac{1}{4}\right)$ and $\frac{1}{4}$
69. Centre of the circle $(x-3)^2 + (y-4)^2 = 5$ is [MP PET 1988]
 (a) (3, 4) (b) (-3, -4) (c) (4, 3) (d) (-4, -3)
70. A circle has its equation in the form $x^2 + y^2 + 2x + 4y + 1 = 0$. Choose the correct coordinates of its centre and the right value of its radius from the following [MP PET 1982]
 (a) Centre (-1, -2), radius = 2 (b) Centre (2, 1), radius = 1
 (c) Centre (1, 2), radius = 3 (d) Centre (-1, 2), radius = 2
71. A circle touches the axes at the points (3, 0) and (0, -3). The centre of the circle is [MP PET 1992]
 (a) (3, -3) (b) (0, 0) (c) (-3, 0) (d) (6, -6)
72. Radius of the circle $x^2 + y^2 + 2x \cos \theta + 2y \sin \theta - 8 = 0$, is [MNR 1974]
 (a) 1 (b) 3 (c) $2\sqrt{3}$ (d) $\sqrt{10}$
73. The area of a circle whose centre is (h, k) and radius a is [MP PET 1994]
 (a) $\pi(h^2 + k^2 - a^2)$ (b) $\pi a^2 hk$ (c) πa^2 (d) None of these
74. If the coordinates of one end of the diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (-3, 2), then the coordinates of other end are [Roorkee 1995]
 (a) (5, 3) (b) (6, 2) (c) (1, -8) (d) (11, 2)
75. The centre of the circle $x = -1 + 2 \cos \theta$, $y = 3 + 2 \sin \theta$, is [MP PET 1995]
 (a) (1, -3) (b) (-1, 3) (c) (1, 3) (d) None of these
76. If $g^2 + f^2 = c$, then the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ will represent [MP PET 2003]
 (a) A circle of radius g (b) A circle of radius f (c) A circle of diameter \sqrt{c} (d) A circle of radius 0
77. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$, is [IIT Screening 2003]
 (a) (4, 7) (b) (7, 4) (c) (9, 4) (d) (4, 9)
78. The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ will represent a real circle if
 (a) $g^2 + f^2 - c < 0$ (b) $g^2 + f^2 - c \geq 0$ (c) Always (d) None of these
79. One of the diameters of the circle $x^2 + y^2 - 12x + 4y + 6 = 0$ is given by
 (a) $x + y = 0$ (b) $x + 3y = 0$ (c) $x = y$ (d) $3x + 2y = 0$
80. The radius of the circle passing through the point (6, 2) two of whose diameters are $x + y = 6$ and $x + 2y = 4$ is [BIT Ranchi 1993]
 (a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4
81. If the equation of a circle is $ax^2 + (2a-3)y^2 - 4x - 1 = 0$ then its centre is
 (a) (2, 0) (b) (2/3, 0) (c) (-2/3, 0) (d) None of these
82. If $2(x^2 + y^2) + 4\lambda x + \lambda^2 = 0$ represents a circle of meaningful radius then the range of real values of λ is

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- (a) R (b) $(0, +\infty)$ (c) $(-\infty, 0)$ (d) None of these
83. The locus of the centres of the circles for which one end of a diameter is $(1, 1)$ while the other end is on the line $x + y = 3$ is
 (a) $x + y = 1$ (b) $2(x - y) = 5$ (c) $2x + 2y = 5$ (d) None of these
84. If A and B are two points on the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ which are farthest and nearest respectively from the point $(7, 2)$ then
 (a) $A = (2 - 2\sqrt{2}, -3 - 2\sqrt{2})$ (b) $B = (2 + 2\sqrt{2}, -3 + 2\sqrt{2})$
 (c) $A = (2 + 2\sqrt{2}, -3 + 2\sqrt{2})$ (d) $B = (2 - 2\sqrt{2}, -3 + 2\sqrt{2})$
85. The radius of the circle passing through the point $(5, 4)$ and concentric to the circle $x^2 + y^2 - 8x - 12y + 15 = 0$ is
 (a) 5 (b) $\sqrt{5}$ (c) 10 (d) $\sqrt{10}$
86. The length of the radius of the circle $x^2 + y^2 + 4x - 6y = 0$ is [Rajasthan PET 1995]
 (a) $\sqrt{11}$ (b) 12 (c) $\sqrt{13}$ (d) $\sqrt{14}$
87. $(2, y)$ is the centre of a circle. If $(x, 3)$ and $(3, 5)$ are end points of a diameter of this circle, then [Roorkee 1986]
 (a) $x = 1, y = 4$ (b) $x = 4, y = 1$ (c) $x = 8, y = 2$ (d) None of these
88. The greatest distance of the point $P(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is
 (a) 5 (b) 15 (c) 10 (d) None of these
89. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ be $(3, 4)$, then the other end is [MP PET 1986; BIT Ranchi 1991]
 (a) $(0, 0)$ (b) $(1, 1)$ (c) $(1, 2)$ (d) $(2, 1)$

Advance Level

90. If $2x - 4y = 9$ and $6x - 12y + 7 = 0$ are the tangents of same circle, then its radius will be [Roorkee 1995]
 (a) $\frac{\sqrt{3}}{5}$ (b) $\frac{17}{6\sqrt{5}}$ (c) $\frac{2\sqrt{5}}{3}$ (d) $\frac{17}{3\sqrt{5}}$
91. If $5x - 12y + 10 = 0$ and $12y - 5x + 16 = 0$ are two tangents to a circle, then the radius of the circle is [EAMCET 2003]
 (a) 1 (b) 2 (c) 4 (d) 6
92. If $2x^2 + \lambda xy + 2y^2 + (\lambda - 4)x + 6y - 5 = 0$ is the equation of a circle then its radius is
 (a) $3\sqrt{2}$ (b) $2\sqrt{3}$ (c) $2\sqrt{2}$ (d) None of these
93. C_1 is a circle of radius 1 touching the x -axis and the y -axis. C_2 is another circle of radius >1 and touching the axes as well as the circle C_1 . Then the radius of C_2 is
 (a) $3 - 2\sqrt{2}$ (b) $3 + 2\sqrt{2}$ (c) $3 + 2\sqrt{3}$ (d) None of these
94. If p and q be the longest distance and the shortest distance respectively of the point $(-7, 2)$ from any point (α, β) on the curve whose equation is $x^2 + y^2 - 10x - 14y - 51 = 0$ then GM of p and q is equal to
 (a) $2\sqrt{11}$ (b) $5\sqrt{5}$ (c) 13 (d) None of these
95. The equation of a circle is $x^2 + y^2 = 4$. The centre of the smallest circle touching this circle and the line $x + y = 5\sqrt{2}$ has the coordinates
 (a) $\left(\frac{7}{2\sqrt{2}}, \frac{7}{2\sqrt{2}}\right)$ (b) $\left(\frac{3}{2}, \frac{3}{2}\right)$ (c) $\left(-\frac{7}{2\sqrt{2}}, -\frac{7}{2\sqrt{2}}\right)$ (d) None of these
96. A circle touches the line $2x - y - 1 = 0$ at the point $(3, 5)$. If its centre lies on the line $x + y = 5$ then the centre of that circle is [Rajasthan PET 1992]
 (a) $(3, 2)$ (b) $(-3, 8)$ (c) $(4, 1)$ (d) $(8, -3)$

97. The locus of the centre of the circle $(x \cos \theta + y \sin \theta - a)^2 + (x \sin \theta - y \cos \theta + a)^2 = a^2$ is
 (a) $x^2 + y^2 = a^2$ (b) $x^2 + y^2 = 2a^2$ (c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 - 2ax - 2ay + a^2 = 0$
98. If a circle $S(x, y) = 0$ touches at the point $(2, 3)$ of the line $x + y = 5$ and $S(1, 2) = 0$, then radius of such circle
 (a) 2 units (b) 4 units (c) $\frac{1}{2}$ units (d) $\frac{1}{\sqrt{2}}$ units

Intersection of a Line and a Circle

Basic Level

99. A circle touches the y -axis at the point $(0, 4)$ and cuts the x -axis in a chord of length 6 units. The radius of the circle is [MP PET 1992]
 (a) 3 (b) 4 (c) 5 (d) 6
100. The radius of a circle which touches y -axis at $(0, 3)$ and cuts intercept of 8 units with x -axis, is [IIT 1972]
 (a) 3 (b) 2 (c) 5 (d) 8
101. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB . Equation of the circle with AB as a diameter is [IIT 1996]
 (a) $x^2 + y^2 - x - y = 0$ (b) $x^2 + y^2 - 2x - y = 0$ (c) $x^2 + y^2 - x + y = 0$ (d) $x^2 + y^2 + x - y = 0$
102. The circle $x^2 + y^2 - 3x - 4y + 2 = 0$ cuts x -axis at [Karnataka CET 2001]
 (a) $(2, 0), (-3, 0)$ (b) $(3, 0), (4, 0)$ (c) $(1, 0), (-1, 0)$ (d) $(1, 0), (2, 0)$
103. If the line $y = x + 3$ meets the circle $x^2 + y^2 = a^2$ at A and B , then the equation of the circle having AB as a diameter will be [Rajasthan PET 1988]
 (a) $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$ (b) $x^2 + y^2 - 3x + 3y - a^2 + 9 = 0$
 (c) $x^2 + y^2 + 3x + 3y - a^2 + 9 = 0$ (d) None of these
104. If the circle $x^2 + y^2 + 2ax + 8y + 16 = 0$ touches x -axis, then the value of a is [Rajasthan PET 1994]
 (a) ± 16 (b) ± 4 (c) ± 8 (d) ± 1
105. The length of the intercept made by the circle $x^2 + y^2 = 1$ on the line $x + y = 1$ is
 (a) 2 (b) $\sqrt{2}$ (c) $1/\sqrt{2}$ (d) $2\sqrt{2}$
106. The AM of the abscissae of points of intersection of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ with x -axis is
 (a) g (b) $-g$ (c) f (d) $-f$
107. The straight line $(x - 2) + (y + 3) = 0$ cuts the circle $(x - 2)^2 + (y - 3)^2 = 11$ at [MNR 1975]
 (a) No points (b) One point (c) Two points (d) None of these
108. The equation of a circle whose centre is $(3, -1)$ and which cuts off a chord of length 6 on the line $2x - 5y + 18 = 0$ [Roorkee 1977]
 (a) $(x - 3)^2 + (y + 1)^2 = 38$ (b) $(x + 3)^2 + (y - 1)^2 = 38$ (c) $(x - 3)^2 + (y + 1)^2 = \sqrt{38}$ (d) None of these
109. The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are [IIT 1983]
 (a) $(-2, -6), (4, 2)$ (b) $(2, 6), (-4, -2)$ (c) $(-2, 6), (-4, 2)$ (d) None of these
110. The line $y = mx + c$ intersects the circle $x^2 + y^2 = r^2$ at two real distinct points, if
 (a) $-r\sqrt{1+m^2} < c \leq 0$ (b) $0 \leq c < r\sqrt{1+m^2}$ (c) (a) and (b) both (d) $-c\sqrt{1+m^2} < r$
111. A line through $(0, 0)$ cuts the circle $x^2 + y^2 - 2ax = 0$ at A and B , then locus of the centre of the circle drawn AB as diameter is [Rajasthan PET 2002]
 (a) $x^2 + y^2 - 2ay = 0$ (b) $x^2 + y^2 + ay = 0$ (c) $x^2 + y^2 + ax = 0$ (d) $x^2 + y^2 - ax = 0$
112. If the line $y - 1 = m(x - 1)$ cuts the circle $x^2 + y^2 = 4$ at two real points then the number of possible values of m is

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- (a) 1 (b) 2 (c) Infinite (d) None of these
113. The GM of the abscissae of the points of intersection of the circle $x^2 + y^2 - 4x - 6y + 7 = 0$ and the line $y = 1$ is
 (a) $\sqrt{7}$ (b) $\sqrt{2}$ (c) $\sqrt{14}$ (d) 1
114. The equation(s) of the tangent at the point (0, 0) to the circle, making intercepts of length $2a$ and $2b$ units on the coordinate axes, is (are)
 (a) $ax + by = 0$ (b) $ax - by = 0$ (c) $x = y$ (d) None of these
- Advance Level**
115. A circle which passes through origin and cuts intercepts on axes a and b , the equation of circle is [Rajasthan PET 1991]
 (a) $x^2 + y^2 - ax - by = 0$ (b) $x^2 + y^2 + ax + by = 0$ (c) $x^2 + y^2 - ax + by = 0$ (d) $x^2 + y^2 + ax - by = 0$
116. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 [IIT 1999]
 (a) $x + y = 0$ (b) $x - y = 0$ (c) $x + 7y = 0$ (d) $x - 7y = 0$
117. The two lines through (2, 3) from which the circle $x^2 + y^2 = 25$ intercepts chords of length 8 units have equations
 (a) $2x + 3y = 13$, $x + 5y = 17$ (b) $y = 3$, $12x + 5y = 39$
 (c) $x = 2$, $9x - 11y = 51$ (d) None of these
118. Circles are drawn through the point (2, 0) to cut intercepts of length 5 units on the x -axis. If their centres lie in the first quadrant, then their equation is [Roorkee 1992]
 (a) $x^2 + y^2 - 9x + 2ky + 14 = 0$ (b) $3x^2 + 3y^2 + 27x - 2ky + 42 = 0$
 (c) $x^2 + y^2 - 9x - 2ky + 14 = 0$ (d) $x^2 + y^2 - 2kx - 9y + 14 = 0$
119. A circle touches the y -axis at (0, 2) and has an intercept of 4 units on the positive side of the x -axis. Then the equation of the circle is [IIT 1995]
 (a) $x^2 + y^2 - 4(\sqrt{2}x + y) + 4 = 0$ (b) $x^2 + y^2 - 4(x + \sqrt{2}y) + 4 = 0$
 (c) $x^2 + y^2 - 2(\sqrt{2}x + y) + 4 = 0$ (d) None of these
120. Circles are drawn through the point (3, 0) to cut an intercept of length 6 units on the negative direction of the x -axis. The equation of the locus of their centres is
 (a) The x -axis (b) $x - y = 0$ (c) The y -axis (d) $x + y = 0$
121. Circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 8x + 11 = 0$ cut off equal intercepts on a line through the point $\left(-2, \frac{1}{2}\right)$. The slope of the line is
 (a) $\frac{-1 + \sqrt{29}}{14}$ (b) $\frac{1 + \sqrt{7}}{4}$ (c) $\frac{-1 - \sqrt{29}}{14}$ (d) None of these
122. If $2l$ be the length of the intercept made by the circle $x^2 + y^2 = a^2$ on the line $y = mx + c$, then c^2 is equal to
 (a) $(1 + m^2)(a^2 + l^2)$ (b) $(1 + m^2)(a^2 - l^2)$ (c) $(1 - m^2)(a^2 + l^2)$ (d) $(1 - m^2)(a^2 - l^2)$
123. For the circle $x^2 + y^2 + 4x - 7y + 12 = 0$ the following statement is true
 (a) The length of tangent from (1, 2) is 7 (b) Intercept on y -axis is 2
 (c) Intercept on x -axis is $2 - \sqrt{2}$ (d) None of these
124. The length of the chord joining the points in which the straight line $\frac{x}{3} + \frac{y}{4} = 1$ cuts the circle $x^2 + y^2 = \frac{169}{25}$ is [Orissa JEE 2003]
 (a) 1 (b) 2 (c) 4 (d) 8
125. A line is drawn through a fixed point $P(\alpha, \beta)$ to cut the circle $x^2 + y^2 = r^2$ at A and B . Then $PA \cdot PB$ is equal to
 (a) $(\alpha + \beta)^2 - r^2$ (b) $\alpha^2 + \beta^2 - r^2$ (c) $(\alpha - \beta)^2 + r^2$ (d) None of these

126. The range of values of m for which the line $y = mx + 2$ cuts the circle $x^2 + y^2 = 1$ at distinct or coincident points is
 (a) $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty)$ (b) $[-\sqrt{3}, \sqrt{3}]$ (c) $[\sqrt{3}, +\infty)$ (d) None of these

Position of a point w.r.t. a Circle**Basic Level**

127. A point inside the circle $x^2 + y^2 + 3x - 3y + 2 = 0$ is [MP PET 1988]
 (a) $(-1, 3)$ (b) $(-2, 1)$ (c) $(2, 1)$ (d) $(-3, 2)$
128. Position of the point $(1, 1)$ with respect to the circle $x^2 + y^2 - x + y - 1 = 0$ is [MP PET 1986, 1990]
 (a) Outside the circle (b) Upon the circle (c) Inside the circle (d) None of these
129. The number of tangents that can be drawn from $(0, 0)$ to the circle $x^2 + y^2 + 2x + 6y - 15 = 0$ is [MP PET 1992]
 (a) None (b) One (c) Two (d) Infinite
130. The number of tangents which can be drawn from the point $(-1, 2)$ to the circle $x^2 + y^2 + 2x - 4y + 4 = 0$ is [BIT Ranchi 1991]
 (a) 1 (b) 2 (c) 3 (d) 0
131. The point $(0.1, 3.1)$ with respect to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$, is [MNR 1980]
 (a) At the centre of the circle (b) Inside the circle but not at the centre
 (c) On the circle (d) Outside the circle
132. The number of the tangents that can be drawn from $(1, 2)$ to $x^2 + y^2 = 5$ is
 (a) 1 (b) 2 (c) 3 (d) 0
133. The number of points on the circle $2x^2 + 2y^2 - 3x = 0$ which are at a distance 2 from the point $(-2, 1)$ is
 (a) 2 (b) 0 (c) 1 (d) None of these
134. If $x^2 + y^2 - 6x + 8y - 11 = 0$ is a given circle and $(0, 0)$, $(1, 8)$ are two points, then
 (a) Both the points are inside the circle (b) Both the points are outside the circle
 (c) One point is on the circle another is outside the circle (d) One point is inside and another is outside the circle

Advance Level

135. A region in the x - y plane is bounded by the curve $y = \sqrt{25 - x^2}$ and the line $y = 0$. If the point $(a, a + 1)$ lies in the interior of the region, then
 (a) $a \in (-4, 3)$ (b) $a \in (-\infty, -1) \cup (3, +\infty)$ (c) $a \in (-1, 3)$ (d) None of these
136. If $(2, 4)$ is a point interior to the circle $x^2 + y^2 - 6x - 10y + \lambda = 0$ and the circle does not cut the axes at any point, then λ belongs to the interval
 (a) $(25, 32)$ (b) $(9, 32)$ (c) $(32, +\infty)$ (d) None of these
137. The range of values of $\theta \in [0, 2\pi]$ for which $(1 + \cos \theta, \sin \theta)$ is an interior point of the circle $x^2 + y^2 = 1$ is
 (a) $(\pi/6, 5\pi/6)$ (b) $(2\pi/3, 5\pi/3)$ (c) $(\pi/6, 7\pi/6)$ (d) $(2\pi/3, 4\pi/3)$
138. The range of values of r for which the point $\left(-5 + \frac{r}{\sqrt{2}}, -3 + \frac{r}{\sqrt{2}}\right)$ is an interior point of the major segment of the circle $x^2 + y^2 = 16$, cut off by the line $x + y = 2$ is
 (a) $(-\infty, 5\sqrt{2})$ (b) $(4\sqrt{2} - \sqrt{14}, 5\sqrt{2})$ (c) $(4\sqrt{2} - \sqrt{14}, 4\sqrt{2} + \sqrt{14})$ (d) None of these
139. If $P(2, 8)$ is an interior point of a circle $x^2 + y^2 - 2x + 4y - p = 0$ which neither touches nor intersects the axes, then set for p is
 (a) $p < -1$ (b) $p < -4$ (c) $p > 96$ (d) ϕ

Equation of Tangent, Condition for Tangency and the points of Contact**Basic Level**

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140. The equation of the tangent to the circle $x^2 + y^2 = r^2$ at (a, b) is $ax + by - \lambda = 0$, where λ is
 (a) a^2 (b) b^2 (c) r^2 (d) None of these
141. $x = 7$ touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$, then the coordinates of the point of contact are [MP PET 1996]
 (a) (7, 3) (b) (7, 4) (c) (7, 8) (d) (7, 2)
142. A circle with centre (a, b) passes through the origin. The equation of the tangent to the circle at the origin is [Rajasthan PET 2000]
 (a) $ax - by = 0$ (b) $ax + by = 0$ (c) $bx - ay = 0$ (d) $bx + ay = 0$
143. If the tangent at a point $P(x, y)$ of a curve is perpendicular to the line that joins origin with the point P , then the curve is [MP PET 1998]
 (a) Circle (b) Parabola (c) Ellipse (d) Straight line
144. The circle $x^2 + y^2 - 8x + 4y + 4 = 0$ touches [Karnataka CET 1999]
 (a) x -axis only (b) y -axis only (c) Both x and y -axis (d) Does not touch any axis
145. The condition that the line $x \cos \alpha + y \sin \alpha = p$ may touch the circle $x^2 + y^2 = a^2$ is [AMU 1999]
 (a) $p = a \cos \alpha$ (b) $p = a \tan \alpha$ (c) $p^2 = a^2$ (d) $p \sin \alpha = a$
146. The equation of circle with centre (1, 2) and tangent $x + y - 5 = 0$ is [MP PET 2001]
 (a) $x^2 + y^2 + 2x - 4y + 6 = 0$ (b) $x^2 + y^2 - 2x - 4y + 3 = 0$
 (c) $x^2 + y^2 - 2x + 4y + 8 = 0$ (d) $x^2 + y^2 - 2x - 4y + 8 = 0$
147. The equation of tangent to the circle $x^2 + y^2 = a^2$ parallel to $y = mx + c$ is [Rajasthan PET 2001]
 (a) $y = mx \pm \sqrt{1 + m^2}$ (b) $y = mx \pm a\sqrt{1 + m^2}$ (c) $x = my \pm a\sqrt{1 + m^2}$ (d) None of these
148. The line $3x - 2y = k$ meets the circle $x^2 + y^2 = 4r^2$ at only one point, if $k^2 =$ [Karnataka CET 2003]
 (a) $20r^2$ (b) $52r^2$ (c) $\frac{52}{9}r^2$ (d) $\frac{20}{9}r^2$
149. The line $lx + my + n = 0$ will be a tangent to the circle $x^2 + y^2 = a^2$ if [MNR 1974; AMU 1981]
 (a) $n^2(l^2 + m^2) = a^2$ (b) $a^2(l^2 + m^2) = n^2$ (c) $n(l + m) = a$ (d) $a(l + m) = n$
150. The circle $x^2 + y^2 + 4x - 4y + 4 = 0$ touches [MP PET 1988]
 (a) x -axis (b) y -axis (c) x -axis and y -axis (d) None of these
151. If the line $lx + my = 1$ be a tangent to the circle $x^2 + y^2 = a^2$, then the locus of the point (l, m) is [MNR 1978; Rajasthan PET 1997]
 (a) A straight line (b) A circle (c) A parabola (d) An ellipse
152. The straight line $x - y - 3 = 0$ touches the circle $x^2 + y^2 - 4x + 6y + 11 = 0$ at the point whose coordinates are [MP PET 1993]
 (a) (1, -2) (b) (1, 2) (c) (-1, 2) (d) (-1, -2)
153. If the straight line $y = mx + c$ touches the circle $x^2 + y^2 - 4y = 0$, then the value of c will be [Rajasthan PET 1988]
 (a) $1 + \sqrt{1 + m^2}$ (b) $1 - \sqrt{m^2 + 1}$ (c) $2(1 + \sqrt{1 + m^2})$ (d) $2 + \sqrt{1 + m^2}$
154. At which point on y -axis the line $x = 0$ is a tangent to circle $x^2 + y^2 - 2x - 6y + 9 = 0$ [Rajasthan PET 1984]
 (a) (0, 1) (b) (0, 2) (c) (0, 3) (d) (0, 4)
155. At which point the line $y = x + \sqrt{2}a$ touches to the circle $x^2 + y^2 = a^2$
 or
 Line $y = x + a\sqrt{2}$ is a tangent to the circle $x^2 + y^2 = a^2$ at [Rajasthan PET 1991; MP PET 1999]
 (a) $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$ (b) $\left(-\frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}}\right)$ (c) $\left(\frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}}\right)$ (d) $\left(-\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$
156. If the line $3x + 4y - 1 = 0$ touches the circle $(x - 1)^2 + (y - 2)^2 = r^2$, then the value of r will be [Rajasthan PET 1986]

- (a) 2 (b) 5 (c) $\frac{12}{5}$ (d) $\frac{2}{5}$
157. If the centre of a circle is $(-6, 8)$ and it passes through the origin, then equation to its tangent at the origin, is [MNR 1976]
 (a) $2y = x$ (b) $4y = 3x$ (c) $3y = 4x$ (d) $3x + 4y = 0$
158. If the line $3x - 4y = \lambda$ touches the circle $x^2 + y^2 - 4x - 8y - 5 = 0$, then λ is equal to [Roorkee 1972; Kurukshetra CEE 1996]
 (a) $-35, -15$ (b) $-35, 15$ (c) $35, 15$ (d) $35, -15$
159. The tangent to $x^2 + y^2 = 9$ which is parallel to y-axis and does not lie in the third quadrant touches the circle at the point
 (a) $(3, 0)$ (b) $(-3, 0)$ (c) $(0, 3)$ (d) $(0, -3)$
160. The points of contact of tangents to the circle $x^2 + y^2 = 25$ which are inclined at an angle of 30° to the x-axis are
 (a) $\left(\pm \frac{5}{2}, \pm \frac{1}{2}\right)$ (b) $\left(\pm \frac{1}{2}, \pm \frac{5}{2}\right)$ (c) $\left(\mp \frac{5}{2}, \mp \frac{1}{2}\right)$ (d) None of these.
161. If the line $hx + ky = 1$ touches $x^2 + y^2 = a^2$, then the locus of the point (h, k) is a circle of radius
 (a) a (b) $\frac{1}{a}$ (c) \sqrt{a} (d) $\frac{1}{\sqrt{a}}$
162. The slope of the tangent at the point (h, h) of the circle $x^2 + y^2 = a^2$ is [Roorkee 1993]
 (a) 0 (b) 1 (c) -1 (d) Depends on h .
163. The line $y = mx + \sqrt{4 + 4m^2}$, $m \in R$, is a tangent to the circle
 (a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 4$ (c) $x^2 + y^2 = 1$ (d) None of these
164. The point of contact of a tangent from the point $(1, 2)$ to the circle $x^2 + y^2 = 1$ has the coordinates
 (a) $\left(\frac{1 - 2\sqrt{19}}{5}, \frac{2 + \sqrt{19}}{5}\right)$ (b) $\left(\frac{1 - 2\sqrt{19}}{5}, \frac{2 - \sqrt{19}}{5}\right)$ (c) $\left(\frac{1 + 2\sqrt{19}}{5}, \frac{2 + \sqrt{19}}{5}\right)$ (d) $\left(\frac{1 + 2\sqrt{19}}{5}, \frac{2 - \sqrt{19}}{5}\right)$
165. If the line $x + y = 1$ is a tangent to a circle with centre $(2, 3)$, then its equation will be [Rajasthan PET 1985, 89]
 (a) $x^2 + y^2 - 4x - 6y + 4 = 0$ (b) $x^2 + y^2 - 4x - 6y + 5 = 0$
 (c) $x^2 + y^2 - 4x - 6y - 5 = 0$ (d) None of these
166. A tangent to the circle $x^2 + y^2 = a^2$ meets the axes at points A and B . The locus of the mid point of AB is
 (a) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{a^2}$ (b) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{a^2}$ (c) $\frac{1}{x^2} + \frac{1}{y^2} = 4a^2$ (d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{a^2}{4}$
167. If the tangent to the circle $x^2 + y^2 = 5$ at point $(1, -2)$ touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$, then its point of contact is [IIT 1989]
 (a) $(-1, -3)$ (b) $(3, -1)$ (c) $(-2, 1)$ (d) $(5, 0)$
168. The equation of the tangent to the circle $x^2 + y^2 = 25$ which is inclined at 60° angle with x-axis, will be
 (a) $y = \sqrt{3}x \pm 10$ (b) $\sqrt{3}y = x \pm 10$ (c) $y = \sqrt{3}x \pm 2$ (d) None of these
169. If $y = c$ is a tangent to the circle $x^2 + y^2 = 4$, then
 (a) $|c| > 2$ (b) $|c| < 2$ (c) $|c| = 2$ (d) $|c| = 0$

Advance Level

170. If the circle $(x - h)^2 + (y - k)^2 = r^2$ is a tangent to the curve $y = x^2 + 1$ at a point $(1, 2)$, then the possible location of the points (h, k) are given by [AMU 2000]
 (a) $hk = 5/2$ (b) $h + 2k = 5$ (c) $h^2 - 4k^2 = 5$ (d) $k^2 = h^2 + 1$
171. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is [IIT Screening 2002]
 (a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$

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172. The tangents to $x^2 + y^2 = a^2$ having inclinations α and β intersect at P . If $\cot \alpha + \cot \beta = 0$, then the locus of P is
 (a) $x + y = 0$ (b) $x - y = 0$ (c) $xy = 0$ (d) None of these
173. If the points $A(1, 4)$ and B are symmetrical about the tangent to the circle $x^2 + y^2 - x + y = 0$ at the origin then coordinates of B are
 (a) $(1, 2)$ (b) $(\sqrt{2}, 1)$ (c) $(4, 1)$ (d) None of these
174. A line parallel to the line $x - 3y = 2$ touches the circle $x^2 + y^2 - 4x + 2y - 5 = 0$ at the point
 (a) $(1, -4)$ (b) $(1, 2)$ (c) $(3, -4)$ (d) $(3, 2)$
175. The possible values of p for which the line $x \cos \alpha + y \sin \alpha = p$ is a tangent to the circle $x^2 + y^2 - 2qx \cos \alpha - 2qy \sin \alpha = 0$ is/are
 (a) 0 and q (b) q and $2q$ (c) 0 and $2q$ (d) q [SCRA, 1999]
176. A circle passes through $(0, 0)$ and $(1, 0)$ and touches to the circle $x^2 + y^2 = 9$, then the centre of circle is
 (a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \pm\sqrt{2}\right)$ [IIT 1992]

Length of Tangent

Basic Level

177. The length of tangent from the point $(5, 1)$ to the circle $x^2 + y^2 + 6x - 4y - 3 = 0$, is
 (a) 81 (b) 29 (c) 7 (d) 21 [MNR 1981]
178. Length of the tangent from (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, is
 (a) $(x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c)^{1/2}$ (b) $(x_1^2 + y_1^2)^{1/2}$
 (c) $[(x_1 + g)^2 + (y_1 + f)^2]^{1/2}$ (d) None of these [EAMCET 1980]
179. The length of the tangent from the point $(4, 5)$ to the circle $x^2 + y^2 + 2x - 6y = 6$ is
 (a) $\sqrt{13}$ (b) $\sqrt{38}$ (c) $2\sqrt{2}$ (d) $2\sqrt{13}$ [DCE 1999]
180. The square of the length of the tangent from $(3, -4)$ on the circle $x^2 + y^2 - 4x - 6y + 3 = 0$ is
 (a) 20 (b) 30 (c) 40 (d) 50 [MP PET 2000]
181. The length of the tangent from $(0, 0)$ to the circle $2(x^2 + y^2) + x - y + 5 = 0$ is
 (a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\sqrt{2}$ (d) $\sqrt{\frac{5}{2}}$ [EAMCET 1994]
182. The length of the tangent to the circle $x^2 + y^2 - 2x - y - 7 = 0$ from $(-1, -3)$ is
 (a) 2 (b) $2\sqrt{2}$ (c) 4 (d) 8 [Karnataka CET 1994]
183. A tangent is drawn to the circle $2(x^2 + y^2) - 3x + 4y = 0$ and it touches the circle at point A . The tangent passes through the point $P(2, 1)$. Then PA is equal to
 (a) 4 (b) 2 (c) $2\sqrt{2}$ (d) None of these
184. Lines are drawn through the point $P(-2, -3)$ to meet the circle $x^2 + y^2 - 2x - 10y + 1 = 0$. The length of the line segment PA , A being the point on the circle where the line meets the circle at coincident points, is
 (a) 16 (b) $4\sqrt{3}$ (c) 48 (d) None of these

Advance Level

185. The coordinates of the point from where the tangents are drawn to the circles $x^2 + y^2 = 1$, $x^2 + y^2 + 8x + 15 = 0$ and $x^2 + y^2 + 10y + 24 = 0$ are of same length, are
 (a) $\left(2, \frac{5}{2}\right)$ (b) $\left(-2, -\frac{5}{2}\right)$ (c) $\left(-2, \frac{5}{2}\right)$ (d) $\left(2, -\frac{5}{2}\right)$ [Roorkee 1982]
186. Length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + c_1 = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

[Kerala (Engg.) 2002]

- (a) $\sqrt{c_1 - c}$ (b) $\sqrt{c - c_1}$ (c) $\sqrt{c_1 + c}$ (d) None of these

187. If P is a point such that the ratio of the squares of the lengths of the tangents from P to the circles $x^2 + y^2 + 2x - 4y - 20 = 0$ and $x^2 + y^2 - 4x + 2y - 44 = 0$ is $2 : 3$, then the locus of P is a circle with centre [EAMCET 2003]

- (a) $(7, -8)$ (b) $(-7, 8)$ (c) $(7, 8)$ (d) $(-7, -8)$

188. The lengths of the tangents from any point on the circle $15x^2 + 15y^2 - 48x + 64y = 0$ to the two circles $5x^2 + 5y^2 - 24x + 32y + 75 = 0$, $5x^2 + 5y^2 - 48x + 64y + 300 = 0$ are in the ratio

- (a) $1 : 2$ (b) $2 : 3$ (c) $3 : 4$ (d) None of these

189. If the squares of the lengths of the tangents from a point P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ are in A.P., then

- (a) a, b, c are in G.P. (b) a, b, c are in A.P. (c) a^2, b^2, c^2 are in A.P. (d) a^2, b^2, c^2 are in G.P.

Pair of Tangents to a Circle

Basic Level

190. A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. The equation of the pair of tangents is

[MP PET 1990]

- (a) $x^2 + y^2 + 10xy = 0$ (b) $x^2 + y^2 + 5xy = 0$ (c) $2x^2 + 2y^2 + 5xy = 0$ (d) $2x^2 + 2y^2 - 5xy = 0$

191. The equations of the tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are

[Roorkee 1989; IIT 1988; Rajasthan PET 1996]

- (a) $x = 0, y = 0$ (b) $(h^2 - r^2)x - 2rhy = 0, x = 0$ (c) $y = 0, x = 4$ (d) $(h^2 - r^2)x + 2rhy = 0, x = 0$

192. The equations of the tangents drawn from the point $(0, 1)$ to the circle $x^2 + y^2 - 2x + 4y = 0$ are

[Roorkee 1979]

- (a) $2x - y + 1 = 0, x + 2y - 2 = 0$ (b) $2x - y + 1 = 0, x + 2y + 2 = 0$
(c) $2x - y - 1 = 0, x + 2y - 2 = 0$ (d) $2x - y - 1 = 0, x + 2y + 2 = 0$

193. The two tangents to a circle from an external point are always

[MP PET 1986]

- (a) Equal (b) Perpendicular to each other (c) Parallel to each other (d) None of these

194. The equation of pair of tangents to the circle $x^2 + y^2 - 2x + 4y + 3 = 0$ from $(6, -5)$, is

[AMU 1980]

- (a) $7x^2 + 23y^2 + 30xy + 66x + 50y - 73 = 0$ (b) $7x^2 + 23y^2 + 30xy - 66x - 50y - 73 = 0$
(c) $7x^2 + 23y^2 - 30xy - 66x - 50y + 73 = 0$ (d) None of these

195. Tangents drawn from origin to the circle $x^2 + y^2 - 2ax - 2by + b^2 = 0$ are perpendicular to each other, if

[MP PET 1995]

- (a) $a - b = 1$ (b) $a + b = 1$ (c) $a^2 = b^2$ (d) $a^2 + b^2 = 1$

196. The equation to the tangents to the circle $x^2 + y^2 = 4$, which are parallel to $x + 2y + 3 = 0$, are

[MP PET 2003]

- (a) $x - 2y = 2$ (b) $x + 2y = \pm 2\sqrt{3}$ (c) $x + 2y = \pm 2\sqrt{5}$ (d) $x - 2y = \pm 2\sqrt{5}$

197. If $3x + y = 0$ is a tangent to the circle with centre at the point $(2, -1)$, then the equation of the other tangent to the circle from the origin is [MNR 1990]

- (a) $x - 3y = 0$ (b) $x + 3y = 0$ (c) $3x - y = 0$ (d) $2x + y = 0$

198. The equation of a tangent to the circle $x^2 + y^2 = 25$ passing through $(-2, 11)$ is

- (a) $4x + 3y = 25$ (b) $3x + 4y = 38$ (c) $24x - 7y + 125 = 0$ (d) $7x + 24y = 230$

199. Tangents drawn from the point $(4, 3)$ to the circle $x^2 + y^2 - 2x - 4y = 0$ are inclined at an angle

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

200. The angle between the pair of tangents from the point $(1, 1/2)$ to the circle $x^2 + y^2 + 4x + 2y - 4 = 0$ is

- (a) $\cos^{-1} \frac{4}{5}$ (b) $\sin^{-1} \frac{4}{5}$ (c) $\sin^{-1} \frac{3}{5}$ (d) None of these

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201. The equation of the pair of tangents drawn from the point $(0, 1)$ to the circle $x^2 + y^2 = 1/4$ is [Rajasthan PET 1998]
 (a) $x^2 - 3y^2 + y + 1 = 0$ (b) $x^2 - 3y^2 - y - 1 = 0$ (c) $3x^2 - y^2 + 2y + 1 = 0$ (d) $3x^2 - y^2 + 2y - 1 = 0$

Advance Level

202. The angle between the two tangents from the origin to the circle $(x-7)^2 + (y+1)^2 = 25$ is [MNR 1990; Rajasthan PET 1997; DCE 2000]
 (a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$
203. Tangents are drawn from the point $(4, 3)$ to the circle $x^2 + y^2 = 9$. The area of the triangle formed by them and the line joining their points of contact is [MP PET 1991; IIT 1981, 1987]
 (a) $\frac{24}{25}$ (b) $\frac{64}{25}$ (c) $\frac{192}{25}$ (d) $\frac{192}{5}$
204. An infinite number of tangents can be drawn from $(1, 2)$ to the circle $x^2 + y^2 - 2x - 4y + \lambda = 0$, then $\lambda =$ [MP PET 1989]
 (a) -20 (b) 0 (c) 5 (d) Cannot be determined
205. The area of the triangle formed by the tangents from the points (h, k) to the circle $x^2 + y^2 = a^2$ and the line joining their points of contact is [MNR 1990]
 (a) $a \frac{(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$ (b) $a \frac{(h^2 + k^2 - a^2)^{1/2}}{h^2 + k^2}$ (c) $\frac{(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$ (d) $\frac{(h^2 + k^2 - a^2)^{1/2}}{h^2 + k^2}$
206. Two tangents PQ and PR drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ from point $P(16, 7)$. If the centre of the circle is C then the area of quadrilateral $PQCR$ will be [IIT 1981; MP PET 1994]
 (a) 75 sq. units (b) 150 sq. units (c) 15 sq. units (d) None of these
207. The tangents are drawn from the point $(4, 5)$ to the circle $x^2 + y^2 - 4x - 2y - 11 = 0$. The area of quadrilateral formed by these tangents and radii, is [IIT 1985]
 (a) 15 sq. units (b) 75 sq. units (c) 8 sq. units (d) 4 sq. units
208. Tangents are drawn to the circle $x^2 + y^2 = 50$ from a point ' P ' lying on the x -axis. These tangents meet the y -axis at points ' P_1 ' and ' P_2 '. Possible coordinates of ' P ' so that area of triangle PP_1P_2 is minimum, is /are
 (a) $(10, 0)$ (b) $(10\sqrt{2}, 0)$ (c) $(-10, 0)$ (d) $(-10\sqrt{2}, 0)$
209. The angle between the tangents from α, β to the circle $x^2 + y^2 = a^2$ is, (where $S_1 = \alpha^2 + \beta^2 - a^2$)
 (a) $\tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$ (b) $2 \tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$ (c) $2 \tan^{-1}\left(\frac{\sqrt{S_1}}{a}\right)$ (d) None of these

Normal and Condition of Normality

Basic Level

210. The normal to the circle $x^2 + y^2 - 3x - 6y - 10 = 0$ at the point $(-3, 4)$, is [Rajasthan PET 1986, 89]
 (a) $2x + 9y - 30 = 0$ (b) $9x - 2y + 35 = 0$ (c) $2x - 9y + 30 = 0$ (d) $2x - 9y - 30 = 0$
211. The equation of normal to the circle $2x^2 + 2y^2 - 2x - 5y + 3 = 0$ at $(1, 1)$ is [MP PET 2001]
 (a) $2x + y = 3$ (b) $x - 2y = 3$ (c) $x + 2y = 3$ (d) None of these
212. The normal at the point $(3, 4)$ on a circle cuts the circle at the point $(-1, -2)$. Then the equation of the circle is [Orissa JEE 2002]
 (a) $x^2 + y^2 + 2x - 2y - 13 = 0$ (b) $x^2 + y^2 - 2x - 2y - 11 = 0$
 (c) $x^2 + y^2 - 2x + 2y + 12 = 0$ (d) $x^2 + y^2 - 2x - 2y + 14 = 0$
213. The line $\lambda x + \mu y = 1$ is a normal to the circle $2x^2 + 2y^2 - 5x + 6y - 1 = 0$ if

- (a) $5\lambda - 6\mu = 2$ (b) $4 + 5\mu = 6\lambda$ (c) $4 + 6\mu = 5\lambda$ (d) None of these
- 214.** The equation of a normal to the circle $x^2 + y^2 + 6x + 8y + 1 = 0$ passing through $(0, 0)$ is [Rajasthan PET 1986]
 (a) $3x + 4y = 0$ (b) $3x - 4y = 0$ (c) $4x - 3y = 0$ (d) $4x + 3y = 0$
- 215.** The equation of the normal at the point $(4, -1)$ of the circle $x^2 + y^2 - 40x + 10y = 153$ is [Rajasthan PET 1989]
 (a) $x + 4y = 0$ (b) $x - 4y = 0$ (c) $4x + y = 3$ (d) $4x - y = 0$
- 216.** The equation of the normal to the circle $x^2 + y^2 - 4x + 6y = 0$ at $(0, 0)$ is [Rajasthan PET 1992]
 (a) $3x - 2y = 0$ (b) $2x - 3y = 0$ (c) $3x + 2y = 0$ (d) $2x + 3y = 0$

Advance Level

- 217.** The area of triangle formed by the tangent, normal drawn at $(1, \sqrt{3})$ to the circle $x^2 + y^2 = 4$ and positive x -axis, is [IIT 1989; Rajasthan PET 1997, 99; Kurukshetra CEE 1998]
 (a) $2\sqrt{3}$ (b) $\sqrt{3}$ (c) $4\sqrt{3}$ (d) None of these
- 218.** $y - x + 3 = 0$ is the equation of normal at $\left(3 + \frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ to which of the following circles [Roorkee 1990]
 (a) $\left(x - 3 - \frac{3}{\sqrt{2}}\right)^2 + \left(y - \frac{\sqrt{3}}{2}\right)^2 = 9$ (b) $\left(x - 3 - \frac{3}{\sqrt{2}}\right)^2 + y^2 = 6$
 (c) $(x - 3)^2 + y^2 = 9$ (d) $(x - 3)^2 + (y - 3)^2 = 9$
- 219.** The line $ax + by + c = 0$ is normal to the circle $x^2 + y^2 = r^2$. The portion of the line $ax + by + c = 0$ intercepted by this circle is of length
 (a) r (b) r^2 (c) $2r$ (d) \sqrt{r}
- 220.** If the straight line $ax + by = 2$; $a, b \neq 0$ touches the circle $x^2 + y^2 - 2x = 3$ and is normal to the circle $x^2 + y^2 - 4y = 6$ then the values of a and b are respectively [Roorkee 2000]
 (a) $1, -1$ (b) $1, 2$ (c) $-\frac{4}{3}, 1$ (d) $2, 1$
- 221.** The number of feet of normals from the point $(7, -4)$ to the circle $x^2 + y^2 = 5$ is
 (a) 1 (b) 2 (c) 3 (d) 4

Basic Level

222. If (a, b) is a point on the chord AB of the circle, where the ends of the chord are $A = (2, -3)$ and $B = (3, 2)$ then
 (a) $a \in [-3, 2], b \in [2, 3]$ (b) $a \in [2, 3], b \in [-3, 2]$ (c) $a \in [-2, 2], b \in [-3, 3]$ (d) None of these
223. The equation of the circle with the chord $y = 2x$ of the circle $x^2 + y^2 - 10x = 0$ as its diameter is
 (a) $x^2 + y^2 - 2x - 4y - 5 = 0$ (b) $x^2 + y^2 = 2x + 4y$
 (c) $x^2 + y^2 = 4x + 2y$ (d) None of these
224. The radius of the circle, having centre at $(2, 1)$, whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$
 (a) 1 (b) 2 (c) 3 (d) $\sqrt{3}$ [IIT Screening 2004]

Advance Level

225. The equation of the chord of the circle $x^2 + y^2 = 25$ of length 8 that passes through the point $(2\sqrt{3}, 2)$ and makes an acute angle with the positive direction of the x -axis is
 (a) $(4\sqrt{3} - 3\sqrt{7})x + 3y = 18 - 6\sqrt{21}$ (b) $(4\sqrt{3} + 3\sqrt{7})x - 3y = 18 + 6\sqrt{21}$
 (c) $(4\sqrt{3} + 3\sqrt{7})x - 3y + 18 + 6\sqrt{21} = 0$ (d) None of these
226. $P(\sqrt{2}, \sqrt{2})$ is a point on the circle $x^2 + y^2 = 4$ and Q is another point on the circle such that arc $PQ = \frac{1}{4} \times \text{circumference}$. The coordinates of Q are
 (a) $(-\sqrt{2}, -\sqrt{2})$ (b) $(\sqrt{2}, -\sqrt{2})$ (c) $(-\sqrt{2}, \sqrt{2})$ (d) None of these
227. If a line passing through the point $(-\sqrt{8}, \sqrt{8})$ and making an angle 135° with x -axis cuts the circle $x = 5 \cos \theta, y = 5 \sin \theta$ at points A and B , then length of the chord AB is [Bihar CEE 1999]
 (a) 10 (b) 20 (c) 5 (d) $2\sqrt{5}$
228. Equation of chord AB of circle $x^2 + y^2 = 2$ passing through $P(2, 2)$ such that $PB/PA = 3$, is given by
 (a) $x = 3y$ (b) $x = y$ (c) $y - 2 = \sqrt{3}(x - 2)$ (d) None of these
229. If a chord of the circle $x^2 + y^2 = 8$ makes equal intercepts of length a on the coordinate axes, then
 (a) $|a| < 8$ (b) $|a| < 4\sqrt{2}$ (c) $|a| < 4$ (d) $|a| > 4$

Basic Level

230. The distance between the chords of contact of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and the point (g, f) is
 (a) $g^2 + f^2$ (b) $\frac{1}{2}(g^2 + f^2 + c)$ (c) $\frac{1}{2} \cdot \frac{g^2 + f^2 + c}{\sqrt{g^2 + f^2}}$ (d) $\frac{1}{2} \cdot \frac{g^2 + f^2 - c}{\sqrt{g^2 + f^2}}$
231. If the straight line $x - 2y + 1 = 0$ intersects the circle $x^2 + y^2 = 25$ in points P and Q , then the coordinates of the point of intersection of tangents drawn at P and Q to the circle $x^2 + y^2 = 25$ are
 (a) $(25, 50)$ (b) $(-25, -50)$ (c) $(-25, 50)$ (d) $(25, -50)$

232. If the chord of contact of tangents drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$ subtends a right angle at the centre, then
 (a) $h^2 + k^2 = a^2$ (b) $2(h^2 + k^2) = a^2$ (c) $h^2 - k^2 = a^2$ (d) $h^2 + k^2 = 2a^2$
233. The chord of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point [IIT 1997]
 (a) $(1/2, -1/4)$ (b) $(1/2, 1/4)$ (c) $(-1/2, 1/4)$ (d) $(-1/2, -1/4)$

Advance Level

234. If the tangents are drawn to the circle $x^2 + y^2 = 12$ at the point where it meets the circle $x^2 + y^2 - 5x + 3y - 2 = 0$, then the point of intersection of these tangents is
 (a) $(6, -6)$ (b) $(6, 18/5)$ (c) $(6, -18/5)$ (d) None of these
235. A tangent to the circle $x^2 + y^2 = 1$ through the point $(0, 5)$ cuts the circle $x^2 + y^2 = 4$ at A and B . The tangents to the circle $x^2 + y^2 = 4$ at A and B meet at C . The coordinates of C are
 (a) $\left(\frac{8}{5}\sqrt{6}, \frac{4}{5}\right)$ (b) $\left(\frac{8}{5}\sqrt{6}, -\frac{4}{5}\right)$ (c) $\left(-\frac{8}{5}\sqrt{6}, -\frac{4}{5}\right)$ (d) None of these
236. Tangents drawn from $(2, 0)$ to the circle $x^2 + y^2 = 1$ touch the circle at A and B . Then
 (a) $A = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), B = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ (b) $A = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right), B = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
 (c) $A = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), B = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ (d) $A = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), B = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Equation of a Chord whose Middle point is given

Basic Level

237. The equation of the chord of the circle $x^2 + y^2 = a^2$ having (x_1, y_1) as its mid-point is [IIT 1983; MP PET 1986]
 (a) $xy_1 + yx_1 = a^2$ (b) $x_1 + y_1 = a$ (c) $xx_1 + yy_1 = x_1^2 + y_1^2$ (d) $xx_1 + yy_1 = a^2$
238. From the origin chords are drawn to the circle $(x-1)^2 + y^2 = 1$. The equation of the locus of the middle points of these chords is [IIT 1985; EAMCET 1991]
 (a) $x^2 + y^2 - 3x = 0$ (b) $x^2 + y^2 - 3y = 0$ (c) $x^2 + y^2 - x = 0$ (d) $x^2 + y^2 - y = 0$
239. The equation to the chord of the circle $x^2 + y^2 = 9$ whose middle point is $(1, -2)$ is [Roorkee 1989]
 (a) $x - 2y = 9$ (b) $x - 2y - 4 = 0$ (c) $x - 2y - 5 = 0$ (d) $x - 2y + 5 = 0$
240. The locus of the middle point of chords of the circle $x^2 + y^2 = a^2$ which pass through the fixed point (h, k) is
 (a) $x^2 + y^2 - hx - ky = 0$ (b) $x^2 + y^2 + hx + ky = 0$ (c) $x^2 + y^2 - 2hx - 2ky = 0$ (d) $x^2 + y^2 + 2hx + 2ky = 0$
241. Equation of the chord of the circle $x^2 + y^2 - 4x = 0$ whose mid point is $(1, 0)$ is
 (a) $y = 2$ (b) $y = 1$ (c) $x = 2$ (d) $x = 1$
242. The equation of a chord of the circle $x^2 + y^2 - 4x = 0$ which is bisected at the point $(1, 1)$ is
 (a) $x + y = 2$ (b) $3x - y = 2$ (c) $x - 2y + 1 = 0$ (d) $x - y = 0$
243. The locus of the mid points of the chords of the circle $x^2 + y^2 - 2y = 0$ which are drawn from the origin, is [EAMCET 1991]
 (a) $x^2 + y^2 - y = 0$ (b) $x^2 + y^2 - x = 0$ (c) $x^2 + y^2 - 2x = 0$ (d) $x^2 + y^2 - x - y = 0$

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- 244.** The locus of the middle points of chords of the circle $x^2 + y^2 - 2x - 6y - 10 = 0$ which passes through the origin, is [Roorkee 1989]
 (a) $x^2 + y^2 + x + 3y = 0$ (b) $x^2 + y^2 - x + 3y = 0$ (c) $x^2 + y^2 + x - 3y = 0$ (d) $x^2 + y^2 - x - 3y = 0$
- 245.** The locus of mid-point of the chords of the circle $x^2 + y^2 - 2x - 2y - 2 = 0$ which makes an angle of 120° at the centre is [MNR 1994]
 (a) $x^2 + y^2 - 2x - 2y + 1 = 0$ (b) $x^2 + y^2 + x + y - 1 = 0$
 (c) $x^2 + y^2 - 2x - 2y - 1 = 0$ (d) None of these
- 246.** If the equation of a given circle is $x^2 + y^2 = 36$, then the length of the chord which lies along the line $3x + 4y - 15 = 0$ is
 (a) $3\sqrt{6}$ (b) $2\sqrt{3}$ (c) $6\sqrt{3}$ (d) None of these
- 247.** The locus of the mid-points of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is
 (a) $x + y = 2$ (b) $x^2 + y^2 = 1$ (c) $x^2 + y^2 = 2$ (d) $x + y = 1$
- 248.** The equation of the locus of the middle point of a chord of the circle $x^2 + y^2 = 2(x + y)$ such that the pair of lines joining the origin to the point of intersection of the chord and the circle are equally inclined to the x-axis is
 (a) $x + y = 2$ (b) $x - y = 2$ (c) $2x - y = 1$ (d) None of these
- 249.** The locus of the mid-point of chords of length $2l$ of the circle $x^2 + y^2 = a^2$ is [Rajasthan PET 1998]
 (a) $x^2 + y^2 = l^2 - a^2$ (b) $x^2 + y^2 = l^2 + a^2$ (c) $x^2 + y^2 = a^2 - 2l^2$ (d) $x^2 + y^2 = a^2 - l^2$

Diameter of a Circle and Director Circle

Basic Level

- 250.** The equation of the director circle of the circle $x^2 + y^2 = 16$ is
 (a) $x^2 + y^2 = 8$ (b) $x^2 + y^2 = 32$ (c) $x^2 + y^2 = 64$ (d) $x^2 + y^2 = 4$
- 251.** If $y = 2x + k$ is a diameter of the circle $2(x^2 + y^2) + 3x + 4y - 1 = 0$, then the value of k is
 (a) $1/2$ (b) $-1/2$ (c) 1 (d) -1
- 252.** The equation of the diameter of the circle $x^2 + y^2 - 2x + 4y = 0$ passing through the origin is [Rajasthan PET 1991]
 (a) $x + 2y = 0$ (b) $x - 2y = 0$ (c) $2x + y = 0$ (d) $2x - y = 0$
- 253.** The locus of the point of intersection of perpendicular tangents to the circle $x^2 + y^2 = a^2$ is [MNR 1987]
 (a) A circle passing through origin (b) A circle of radius $2a$
 (c) A concentric circle of radius $\sqrt{2}a$ (d) None of these
- 254.** The equation of director circle of the circle $x^2 + y^2 = a^2$, is [Ranchi BIT 1990]
 (a) $x^2 + y^2 = 4a^2$ (b) $x^2 + y^2 = \sqrt{2}a^2$ (c) $x^2 + y^2 - 2a^2 = 0$ (d) None of these

Advance Level

- 255.** The equation of the diameter of the circle $3(x^2 + y^2) - 2x + 6y - 9 = 0$ which is perpendicular to the line $2x + 3y = 12$ is
 (a) $3x - 2y = 3$ (b) $3x - 2y + 1 = 0$ (c) $3x - 2y = 9$ (d) None of these
- 256.** A point on the line $x = 3$ from which the tangents drawn to the circle $x^2 + y^2 = 8$ are at right angles is
 (a) $(3, -\sqrt{7})$ (b) $(3, \sqrt{23})$ (c) $(3, \sqrt{7})$ (d) $(3, -\sqrt{23})$

Basic Level

257. The coordinates of pole of line $lx + my + n = 0$ with respect to circle $x^2 + y^2 = 1$, is [Rajasthan PET 1987]
 (a) $\left(\frac{l}{n}, \frac{m}{n}\right)$ (b) $\left(-\frac{l}{n}, -\frac{m}{n}\right)$ (c) $\left(\frac{l}{n}, -\frac{m}{n}\right)$ (d) $\left(-\frac{l}{n}, \frac{m}{n}\right)$
258. The equation of polar of the point (1, 2) with respect to the circle $x^2 + y^2 = 7$, is [MNR 1973; Rajasthan PET 1983, 84]
 (a) $x - 2y - 7 = 0$ (b) $x + 2y - 7 = 0$ (c) $x - 2y = 0$ (d) $x + 2y = 0$
259. If polar of a circle $x^2 + y^2 = a^2$ with respect to (x', y') is $Ax + By + C = 0$, then its pole will be [Rajasthan PET 1984, 95]
 (a) $\left(\frac{a^2 A}{-C}, \frac{a^2 B}{-C}\right)$ (b) $\left(\frac{a^2 A}{C}, \frac{a^2 B}{C}\right)$ (c) $\left(\frac{a^2 C}{A}, \frac{a^2 C}{B}\right)$ (d) $\left(\frac{a^2 C}{-A}, \frac{a^2 C}{-B}\right)$
260. Polar of origin (0, 0) with respect to the circle $x^2 + y^2 + 2\lambda x + 2\mu y + c = 0$ touches circle $x^2 + y^2 = r^2$ if [Rajasthan PET 1990]
 (a) $c = r(\lambda^2 + \mu^2)$ (b) $r = c(\lambda^2 + \mu^2)$ (c) $c^2 = r^2(\lambda^2 + \mu^2)$ (d) $r^2 = c^2(\lambda^2 + \mu^2)$
261. The polar of the point $(5, -1/2)$ w.r.t. circle $(x - 2)^2 + y^2 = 4$ is [Rajasthan PET 1996]
 (a) $5x - 10y + 2 = 0$ (b) $6x - y - 20 = 0$ (c) $10x - y - 10 = 0$ (d) $x - 10y - 2 = 0$
262. The pole of the line $2x + 3y = 4$ w.r.t. circle $x^2 + y^2 = 64$ is [Rajasthan PET 1996]
 (a) (32, 48) (b) (48, 32) (c) (-32, 48) (d) (48, -32)
263. The pole of the straight line $x + 2y = 1$ with respect to the circle $x^2 + y^2 = 5$ is [Rajasthan PET 2000, 01]
 (a) (5, 5) (b) (5, 10) (c) (10, 5) (d) (10, 10)
264. The polars drawn from $(-1, 2)$ to the circles $S_1 \equiv x^2 + y^2 + 6y + 7 = 0$ and $S_2 \equiv x^2 + y^2 + 6x + 1 = 0$, are [Rajasthan PET 2000]
 (a) Parallel (b) Equal (c) Perpendicular (d) Intersect at a point
265. Let the equation of a circle be $x^2 + y^2 = a^2$. If $h^2 + k^2 - a^2 < 0$ then the line $hx + ky = a^2$ is the
 (a) Polar line of the point (h, k) with respect to the circle (b) Real chord of contact of the tangents from (h, k) to the circle
 (c) Equation of a tangent to the circle from the point (h, k) (d) None of these
266. The pole of the line $4x + 3y = 50$ with respect to the circle $x^2 + y^2 = 200$ is [Rajasthan PET 1993]
 (a) (16, 12) (b) (-16, 12) (c) (12, 16) (d) (-16, -12)
267. The equation of the polar of the point (4, 4) with respect to the circle $(x - 1)^2 + (y - 2)^2 = 0$ is
 (a) $3x + 2y = 7$ (b) $3x + 2y + 8 = 0$ (c) $3x - 2y = 8$ (d) $7x + 5y = 8$
268. The chord of contact and polar of a circle with respect to a point are coincident iff [MP PET 1984; BIT Ranchi 1990]
 (a) The point is inside the circle (b) The point is outside the circle
 (c) The point is not inside the circle (d) Never
269. The pole of the line $9x + 4y = 28$ with respect to the circle $x^2 + y^2 = 16$ is [Rajasthan PET 1994]
 (a) $(36/7, 9/7)$ (b) $(36/7, 16/7)$ (c) $(16/7, 36/7)$ (d) None of these
270. The polar of the point $(-2, 3)$ w.r.t. the circle $x^2 + y^2 - 4x - 6y + 5 = 0$ is [Rajasthan PET 1996; EAMCET 1996]
 (a) $x = 0$ (b) $y = 0$ (c) $x = 1$ (d) $y = 1$
271. If the pole of a line w.r.t. the circle $x^2 + y^2 = c^2$ lies on the circle $x^2 + y^2 = 9c^2$, then that line will touch [Rajasthan PET 1996]
 (a) $x^2 + y^2 = 4c^2$ (b) $x^2 + y^2 = c^2/9$ (c) $x^2 + y^2 = c^2/4$ (d) $x^2 + y^2 = 2c^2$

Advance Level

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- 272.** If the polar of a point (p, q) with respect to the circle $x^2 + y^2 = a^2$ touches the circle $(x - c)^2 + (y - d)^2 = b^2$, then
- (a) $b^2(p^2 + q^2) = (a^2 - cp - dq)^2$ (b) $b^2(p^2 + q^2) = (a^2 - cq - dp)^2$
(c) $a^2(p^2 + q^2) = (b^2 - cp - dq)^2$ (d) None of these
- 273.** The equation of a circle is $x^2 + y^2 - 4x + 2y - 4 = 0$. With respect to the circle
- (a) The pole of the line $x - 2y + 5 = 0$ is $(1, 1)$
(b) The chord of contact of real tangents from $(1, 1)$ is the line $x - 2y + 5 = 0$
(c) The polar of the point $(1, 1)$ is $x - 2y + 5 = 0$
(d) None of these

System of Cricles

Basic Level

- 274.** If d is the distance between the centres of two circles, r_1, r_2 are their radii and $d = r_1 + r_2$, then [MP PET 1986]
- (a) The circles touch each other externally (b) The circles touch each other internally
(c) The circles cut each other (d) The circles are disjoint
- 275.** The points of intersection of the circles $x^2 + y^2 = 25$ and $x^2 + y^2 - 8x + 7 = 0$ are [MP PET 1988]
- (a) $(4, 3)$ and $(4, -3)$ (b) $(4, -3)$ and $(-4, -3)$ (c) $(-4, 3)$ and $(4, 3)$ (d) $(4, 3)$ and $(3, 4)$
- 276.** Circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$ [MP PET 1990]
- (a) Touch internally (b) Touch externally
(c) Intersect each other at two distinct points (d) Do not intersect each other at any point
- 277.** For the given circles $x^2 + y^2 - 6x - 2y + 1 = 0$ and $x^2 + y^2 + 2x - 8y + 13 = 0$, which of the following is true [MP PET 1989]
- (a) One circle lies inside the other (b) One circle lies completely outside the other
(c) Two circle intersect in two points (d) They touch each other
- 278.** The two circles $x^2 + y^2 - 4y = 0$ and $x^2 + y^2 - 8y = 0$ [Ranchi BIT 1985]
- (a) Touch each other internally (b) Touch each other externally
(c) Do not touch each other (d) None of these
- 279.** Circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$ [IIT 1973]
- (a) Touch each other internally (b) Touch each other externally
(c) Cuts each other at two points (d) None of these
- 280.** A tangent to the circle $x^2 + y^2 = 5$ at the point $(1, -2)$ to the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ [IIT 1975]
- (a) Touches (b) Cuts at real points (c) Cuts at imaginary points (d) None of these
- 281.** If the circles $x^2 + y^2 - 9 = 0$ and $x^2 + y^2 + 2ax + 2y + 1 = 0$ touch each other, then $a =$ [Roorkee 1998]
- (a) $-4/3$ (b) 0 (c) 1 (d) $4/3$
- 282.** The equation of the circle through the point of intersection of the circles $x^2 + y^2 - 8x - 2y + 7 = 0$, $x^2 + y^2 - 4x + 10y + 8 = 0$ and $(3, -3)$ is [AI CBSE 1981]
- (a) $23x^2 + 23y^2 - 156x + 38y + 168 = 0$ (b) $23x^2 + 23y^2 + 156x + 38y + 168 = 0$
(c) $x^2 + y^2 + 156x + 38y + 168 = 0$ (d) None of these

- 283.** The locus of the centre of a circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y -axis is given by the equation [IIT 1993; DCE 2000]
 (a) $x^2 - 6x - 10y + 14 = 0$ (b) $x^2 - 10x - 6y + 14 = 0$ (c) $y^2 - 6x - 10y + 14 = 0$ (d) $y^2 - 10x - 6y + 14 = 0$
- 284.** Circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch externally, if [MP PET 1994; Karnataka CET 2003]
 (a) $f'g = g'f$ (b) $fg = f'g'$ (c) $f'g' + fg = 0$ (d) $f'g + g'f = 0$
- 285.** The circle passing through point of intersection of the circle $S = 0$ and the line $P = 0$ is [Rajasthan PET 1995]
 (a) $S + \lambda P = 0$ (b) $S - \lambda P = 0$ (c) $\lambda S + P = 0$ (d) $P - \lambda S = 0$
- 286.** The two circles $x^2 + y^2 - 2x - 3 = 0$ and $x^2 + y^2 - 4x - 6y - 8 = 0$ are such that [MNR 1995]
 (a) They touch each other (b) They intersect each other (c) One lies inside the other (d) One lies outside the other
- 287.** Consider the circles $x^2 + (y - 1)^2 = 9$, $(x - 1)^2 + y^2 = 25$. They are such that [EAMCET 1994]
 (a) These circles touch each other (b) One of these circles lies entirely inside the other
 (c) Each of these circles lies outside the other (d) They intersect in two points
- 288.** Find the equation of the circle passing through the point $(-2, 4)$ and through the points of intersection of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ and the line $3x + 2y - 5 = 0$ [Rajasthan PET 1996]
 (a) $x^2 + y^2 + 2x - 4y - 4 = 0$ (b) $x^2 + y^2 + 4x - 2y - 4 = 0$
 (c) $x^2 + y^2 - 3x - 4y = 0$ (d) $x^2 + y^2 - 4x - 2y = 0$
- 289.** If the circles $x^2 + y^2 = 4$, $x^2 + y^2 - 10x + \lambda = 0$ touch externally, then λ is equal to [AMU 1999]
 (a) -16 (b) 9 (c) 16 (d) 25
- 290.** The condition that the circle $(x - 3)^2 + (y - 4)^2 = r^2$ lies entirely within the circle $x^2 + y^2 = R^2$, is [AMU 1999]
 (a) $R + r \leq 7$ (b) $R^2 + r^2 < 49$ (c) $R^2 - r^2 < 25$ (d) $R - r > 5$
- 291.** If the centre of a circle which passing through the points of intersection of the circles $x^2 + y^2 - 6x + 2y + 4 = 0$ and $x^2 + y^2 + 2x - 4y - 6 = 0$ is on the line $y = x$, then the equation of the circle is [Rajasthan PET 1991; Roorkee 1989]
 (a) $7x^2 + 7y^2 - 10x + 10y - 11 = 0$ (b) $7x^2 + 7y^2 + 10x - 10y - 12 = 0$
 (c) $7x^2 + 7y^2 - 10x - 10y - 12 = 0$ (d) $7x^2 + 7y^2 - 10x - 12 = 0$
- 292.** The equation of a circle passing through points of intersection of the circles $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ and point $(1, 1)$, is [Rajasthan PET 1988, 89; IIT 1983]
 (a) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$ (b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
 (c) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$ (d) None of these
- 293.** The equation of circle passes through the points of intersection of circles $x^2 + y^2 - 6x + 8 = 0$ and $x^2 + y^2 = 6$ and point $(1, 1)$ is [Rajasthan PET 1988; IIT 1980; MP PET 2002]
 (a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$ (c) $x^2 + y^2 - 4y + 2 = 0$ (d) None of these
- 294.** The equation of the circle having its centre on the line $x + 2y - 3 = 0$ and passing through the points of intersection of the circles $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 - 4x - 2y + 4 = 0$, is [MNR 1992]
 (a) $x^2 + y^2 - 6x + 7 = 0$ (b) $x^2 + y^2 - 3y + 4 = 0$ (c) $x^2 + y^2 - 2x - 2y + 1 = 0$ (d) $x^2 + y^2 + 2x - 4y + 4 = 0$
- 295.** A circle of radius 5 touches another circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at $(5, 5)$, then its equation is [IIT 1979]
 (a) $x^2 + y^2 + 18x + 16y + 120 = 0$ (b) $x^2 + y^2 - 18x - 16y + 120 = 0$
 (c) $x^2 + y^2 - 18x + 16y + 120 = 0$ (d) None of these

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- 296.** The points of intersection of circles $x^2 + y^2 = 2ax$ and $x^2 + y^2 = 2by$ are [AMU 2000]
(a) (0, 0), (a, b) (b) (0, 0), $\left(\frac{2ab^2}{a^2 + b^2}, \frac{2ba^2}{a^2 + b^2}\right)$ (c) (0, 0), $\left(\frac{a^2 + b^2}{a^2}, \frac{a^2 + b^2}{b^2}\right)$ (d) None of these
- 297.** The equation of the circle which passes through the intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ and whose centre lies on $13x + 30y = 0$ [DCE 2001]
(a) $x^2 + y^2 + 30x - 13y - 25 = 0$ (b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
(c) $2x^2 + 2y^2 + 30x - 13y - 25 = 0$ (d) $x^2 + y^2 + 30x - 13y + 25 = 0$
- 298.** The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and $x^2 + y^2 - 5x + 6y + 15 = 0$ [Karnataka CET 2001]
(a) Intersect (b) Are concentric (c) Touch internally (d) Touch externally
- 299.** The equation of the circle passing through (1, -3) and the points common to the two circles $x^2 + y^2 - 6x + 8y - 16 = 0$, $x^2 + y^2 + 4x - 2y - 8 = 0$ is
(a) $x^2 + y^2 - 4x + 6y + 24 = 0$ (b) $2x^2 + 2y^2 + 3x + y - 20 = 0$
(c) $3x^2 + 3y^2 - 5x - 7y - 19 = 0$ (d) None of these
- 300.** The circles whose equations are $x^2 + y^2 + c^2 = 2ax$ and $x^2 + y^2 + c^2 = 2by$ will touch one another externally if
(a) $\frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a^2}$ (b) $\frac{1}{c^2} + \frac{1}{a^2} = \frac{1}{b^2}$ (c) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ (d) None of these
- 301.** The equation of the circle and its chord are respectively $x^2 + y^2 = a^2$ and $x \cos \alpha + y \sin \alpha = p$. The equation of the circle of which this chord is a diameter is
(a) $x^2 + y^2 - 2px \cos \alpha - 2py \sin \alpha + 2p^2 - a^2 = 0$ (b) $x^2 + y^2 - 2px \cos \alpha - 2py \sin \alpha + p^2 - a^2 = 0$
(c) $x^2 + y^2 + 2px \cos \alpha + 2py \sin \alpha + 2p^2 - a^2 = 0$ (d) None of these
- 302.** The two circles $x^2 + y^2 - 5 = 0$ and $x^2 + y^2 - 2x - 4y - 15 = 0$
(a) Touch each other externally (b) Touch each other internally
(c) Cut each other orthogonally (d) Do not intersect
- 303.** The circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 4x + 6y + 4 = 0$ [EAMCET 1991]
(a) Touch externally (b) Touch internally (c) Intersect at two points (d) Do not intersect
- 304.** The equations of two circles are $x^2 + y^2 - 26y + 25 = 0$ and $x^2 + y^2 = 25$ then
(a) They touch each other (b) They cut each other orthogonally
(c) One circle is inside the other circle (d) None of these
- 305.** The equation of a circle C_1 is $x^2 + y^2 - 4x - 2y - 11 = 0$. A circle C_2 of radius 1 rolls on the outside of the circle C_1 . The locus of the centre of C_2 has the equation [MP PET 2003]
(a) $x^2 + y^2 - 4x - 2y - 20 = 0$ (b) $x^2 + y^2 + 4x + 2y - 20 = 0$
(c) $x^2 + y^2 - 3x - y - 11 = 0$ (d) None of these
- 306.** The locus of the centres of the circles passing through the intersection of the circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x + y = 0$ is
(a) A line whose equation is $x + 2y = 0$ (b) A line whose equation is $2x - y = 1$
(c) A circle (d) A pair of lines
- 307.** If circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 8y + c = 0$ touch each other, then c is equal to [Rajasthan PET 1994]
(a) 15 (b) -15 (c) 16 (d) -16

308. The locus of the centre of the circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y -axis, is

[IIT 1993]

- (a) $x^2 - 6x - 10y + 14 = 0$ (b) $x^2 - 10x - 6y + 14 = 0$ (c) $y^2 - 6x - 10y + 14 = 0$ (d) $y^2 - 10x - 6y + 14 = 0$

309. The circle S_1 with centre $C_1(a_1, b_1)$ and radius r_1 touches externally the circle S_2 with centre $C_2(a_2, b_2)$ and radius r_2 . If the tangent at their common point passes through the origin, then

- (a) $(a_1^2 + a_2^2) + (b_1^2 + b_2^2) = r_1^2 + r_2^2$ (b) $(a_1^2 - a_2^2) + (b_1^2 - b_2^2) = r_1^2 - r_2^2$
 (c) $(a_1^2 - b_2^2) + (a_2^2 + b_2^2) = r_1^2 + r_2^2$ (d) $(a_1^2 - b_1^2) + (a_2^2 + b_2^2) = r_1^2 + r_2^2$

Advance Level

310. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points if [IIT 1994]

- (a) $r < 2$ (b) $r > 8$ (c) $2 < r < 8$ (d) $2 \leq r \leq 8$

311. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is [AIEEE 2002]

- (a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, -\sqrt{2}\right)$ (c) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{3}{2}\right)$

312. The locus of the centre of the circles which touch both the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 4ax$ externally has the equation

- (a) $12(x - a)^2 - 4y^2 = 3a^2$ (b) $9(x - a)^2 - 5y^2 = 2a^2$ (c) $8x^2 - 3(y - a)^2 = 9a^2$ (d) None of these

313. Tangents OP and OQ are drawn from the origin O to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Then the equation of the circumcircle of the triangle OPQ is

- (a) $x^2 + y^2 + 2gx + 2fy = 0$ (b) $x^2 + y^2 + gx + fy = 0$ (c) $x^2 + y^2 - gx - fy = 0$ (d) $x^2 + y^2 - 2gx - 2fy = 0$

314. If the circle $x^2 + y^2 + 2x + 3y + 1 = 0$ cuts $x^2 + y^2 + 4x + 3y + 2 = 0$ in A and B , then the equation of the circle on AB as diameter is

- (a) $x^2 + y^2 + x + 3y + 3 = 0$ (b) $2x^2 + 2y^2 + 2x + 6y + 1 = 0$ (c) $x^2 + y^2 + x + 6y + 1 = 0$ (d) None of these

315. The equation of the smallest circle passing through the intersection of the line $x + y = 1$ and the circle $x^2 + y^2 = 9$ is

- (a) $x^2 + y^2 + x + y - 8 = 0$ (b) $x^2 + y^2 - x - y - 8 = 0$ (c) $x^2 + y^2 - x + y - 8 = 0$ (d) None of these

316. $x^2 + y^2 + 2(2k + 3)x - 2ky + (2k + 3)^2 + k^2 - r^2 = 0$ represents the family of circles with centres on the line [SCRA 1999]

- (a) $x - 2y - 3 = 0$ (b) $x + 2y - 3 = 0$ (c) $x - 2y + 3 = 0$ (d) $x + 2y + 3 = 0$

Common Tangents to Two Circles

Basic Level

317. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ is [MP PET 1995]

- (a) 1 (b) 2 (c) 3 (d) 4

318. The number of common tangents to two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x + 12 = 0$ is [EAMCET 1990]

- (a) 1 (b) 2 (c) 3 (d) 4

319. The number of common tangents to the circles $x^2 + y^2 - x = 0$, $x^2 + y^2 + x = 0$ is [EAMCET 1994]

- (a) 2 (b) 1 (c) 4 (d) 3

320. The circles $x^2 + y^2 = 9$ and $x^2 + y^2 - 12y + 27 = 0$ touch each other. The equation of their common tangent is [MP PET 1995]

- (a) $4y = 9$ (b) $y = 3$ (c) $y = -3$ (d) $x = 3$

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- 321.** The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and $x^2 + y^2 - 5x + 6y + 15 = 0$ touch each other. The equation of their common tangent is [KCET 1993; DCE 1999]
 (a) $x = 3$ (b) $y = 6$ (c) $7x - 12y - 21 = 0$ (d) $7x + 12y + 21 = 0$
- 322.** The number of common tangents to the circle $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 19 = 0$ is
 (a) 1 (b) 2 (c) 3 (d) 4

Advance Level

- 323.** If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x-a)^2 + y^2 = b^2$, is [IIT Screening 2002]
 (a) $\frac{2b}{\sqrt{a^2 - 4b^2}}$ (b) $\frac{\sqrt{a^2 - 4b^2}}{2b}$ (c) $\frac{2b}{a - 2b}$ (d) $\frac{b}{a - 2b}$
- 324.** Two circles, each of radius 5, have a common tangent at $(1, 1)$ whose equation is $3x + 4y - 7 = 0$. Then their centres are
 (a) $(4, -5), (-2, 3)$ (b) $(4, -3), (-2, 5)$ (c) $(4, 5), (-2, -3)$ (d) None of these
- 325.** The number of common tangents to the circles one of which passes through the origin and cuts off intercepts 2 from each of the axes, and the other circle has the line segment joining the origin and the point $(1, 1)$ as a diameter, is
 (a) 0 (b) 1 (c) 3 (d) 2
- 326.** The range of values of λ for which the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 4\lambda x + 9 = 0$ have two common tangents, is
 (a) $\lambda \in \left[-\frac{13}{8}, \frac{13}{8}\right]$ (b) $\lambda > \frac{13}{8}$ or $\lambda < -\frac{13}{8}$ (c) $1 < \lambda < \frac{13}{8}$ (d) None of these
- 327.** Two circles with radii ' r_1 ' and ' r_2 ', $r_1 > r_2 \geq 2$, touch each other externally, if ' θ ' be the angle between the direct common tangents, then
 (a) $\theta = \sin^{-1}\left(\frac{r_1 + r_2}{r_1 - r_2}\right)$ (b) $\theta = 2 \sin^{-1}\left(\frac{r_1 - r_2}{r_1 + r_2}\right)$ (c) $\theta = \sin^{-1}\left(\frac{r_1 - r_2}{r_1 + r_2}\right)$ (d) None of these.

Common Chord of Two Circles

Basic Level

- 328.** The common chord of the circle $x^2 + y^2 + 4x + 1 = 0$ and $x^2 + y^2 + 6x + 2y + 3 = 0$ is [MP PET 1991]
 (a) $x + y + 1 = 0$ (b) $5x + y + 2 = 0$ (c) $2x + 2y + 5 = 0$ (d) $3x + y + 3 = 0$
- 329.** The equation of line passing through the points of intersection of the circles $3x^2 + 3y^2 - 2x + 12y - 9 = 0$ and $x^2 + y^2 + 6x + 2y - 15 = 0$, is [IIT 1986; UPSEAT 1999]
 (a) $10x - 3y - 18 = 0$ (b) $10x + 3y - 18 = 0$ (c) $10x + 3y + 18 = 0$ (d) None of these
- 330.** Length of the common chord of the circles $x^2 + y^2 + 5x + 7y + 9 = 0$ and $x^2 + y^2 + 7x + 5y + 9 = 0$ is [Kurukshetra CEE 1996]
 (a) 9 (b) 8 (c) 7 (d) 6
- 331.** The length of the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ is [MP PET 2000]
 (a) $9/2$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $3/2$
- 332.** If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of the circle $x^2 + y^2 + 2g'x + 2f'y + c' = 0$, then
 (a) $2g(g - g') + 2f(f - f') = c - c'$ (b) $2g'(g - g') + 2f'(f - f') = c' - c$
 (c) $2g'(g - g') + 2f'(f - f') = c - c'$ (d) $2g(g - g') + 2f(f - f') = c' - c$
- 333.** If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of the circle $x^2 + y^2 + 2g'x + 2f'y + c' = 0$, then the length of the common chord of these two circles is

- (a) $2\sqrt{g^2 + f^2 - c}$ (b) $2\sqrt{g'^2 + f'^2 - c'}$ (c) $2\sqrt{g^2 + f^2 + c}$ (d) $2\sqrt{g'^2 + f'^2 + c'}$

334. The equation of the circle described on the common chord of the circles $x^2 + y^2 + 2x = 0$ and $x^2 + y^2 + 2y = 0$ as diameter is

[EAMCET 1994]

- (a) $x^2 + y^2 + x - y = 0$ (b) $x^2 + y^2 - x - y = 0$ (c) $x^2 + y^2 - x + y = 0$ (d) $x^2 + y^2 + x + y = 0$

335. The distance of the point (1, 2) from the common chord of circles $x^2 + y^2 - 2x + 3y - 5 = 0$ and $x^2 + y^2 + 10x + 8y - 1 = 0$ is

[EAMCET 1990]

- (a) 2 units (b) 3 units (c) 4 units (d) None of these

Advance Level

336. The length of common chord of the circles $(x - a)^2 + y^2 = a^2$ and $x^2 + (y - b)^2 = b^2$ is [MP PET 1989]

- (a) $2\sqrt{a^2 + b^2}$ (b) $\frac{ab}{\sqrt{a^2 + b^2}}$ (c) $\frac{2ab}{\sqrt{a^2 + b^2}}$ (d) None of these

337. The length of common chord of the circles $x^2 + y^2 = 12$ and $x^2 + y^2 - 4x + 3y - 2 = 0$, is [Rajasthan PET 1990, 99]

- (a) $4\sqrt{2}$ (b) $5\sqrt{2}$ (c) $2\sqrt{2}$ (d) $6\sqrt{2}$

338. The line L passes through the points of intersection of the circles $x^2 + y^2 = 25$ and $x^2 + y^2 - 8x + 7 = 0$. The length of perpendicular from centre of second circle onto the line L , is [Bihar CEE 1994]

- (a) 4 (b) 3 (c) 1 (d) 0

339. The common chord of $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends at the origin an angle equal to

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

340. The length of the common chord of the circles $(x - a)^2 + (y - b)^2 = c^2$ and $(x - b)^2 + (y - a)^2 = c^2$ is

- (a) $\sqrt{c^2 - (a - b)^2}$ (b) $\sqrt{4c^2 - 2(a - b)^2}$ (c) $\sqrt{2c^2 - (a - b)^2}$ (d) $\sqrt{4c^2 + (a - b)^2}$

341. If the circles $(x - a)^2 + (y - b)^2 = c^2$ and $(x - b)^2 + (y - a)^2 = c^2$ touch each other, then

- (a) $a = b \pm 2c$ (b) $a = b \pm \sqrt{2}c$ (c) $a = b \pm c$ (d) None of these

342. If the circle $c_1: x^2 + y^2 = 16$ intersects another circle c_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to $3/4$, the coordinates of the centre of c_2 are [IIT 1988]

- (a) $\left(-\frac{9}{5}, \frac{12}{5}\right), \left(\frac{9}{5}, -\frac{12}{5}\right)$ (b) $\left(-\frac{9}{5}, -\frac{12}{5}\right), \left(\frac{9}{5}, \frac{12}{5}\right)$ (c) $\left(\frac{12}{5}, -\frac{9}{5}\right), \left(-\frac{12}{5}, \frac{9}{5}\right)$ (d) None of these

343. The common chord of the circle $x^2 + y^2 + 6x + 8y - 7 = 0$ and a circle passing through the origin, and touching the line $y = x$, always passes through the point

- (a) $(-1/2, 1/2)$ (b) $(1, 1)$ (c) $(1/2, 1/2)$ (d) None of these

344. The equation of the circle drawn on the common chord of circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ as a diameter is

[Rajasthan PET 1998]

- (a) $x^2 + y^2 + \frac{2ab^2}{a^2 + b^2}x + \frac{2a^2b}{a^2 + b^2}y + c = 0$ (b) $x^2 + y^2 + \frac{ab^2}{a^2 + b^2}x + \frac{a^2b}{a^2 + b^2}y + c = 0$
 (c) $(a^2 + b^2)(x^2 + y^2) + 2ab(bx + ay) + c = 0$ (d) None of these

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345. The equation of the circle drawn on the common chord of circles $(x-a)^2 + y^2 = a^2$ and $x^2 + (y-b)^2 = b^2$ as diameter is

[EAMCET 1989]

- (a) $(a^2 + b^2)(x^2 + y^2) = 2ab(bx + ay)$ (b) $(a^2 + b^2)(x^2 + y^2) = 2ab(ax + by)$
(c) $(a^2 - b^2)(x^2 + y^2) = 2ab(bx - ay)$ (d) $(a^2 - b^2)(x^2 + y^2) = 2ab(ax - by)$

Angle of Intersection of Two Circles and Orthogonal System of Circles

Basic Level

346. If a circle passes through the point (1, 2) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the equation of the locus of its centre is

[MNR 1992]

- (a) $x^2 + y^2 - 3x - 8y + 1 = 0$ (b) $x^2 + y^2 - 2x - 6y - 7 = 0$
(c) $2x + 4y - 9 = 0$ (d) $2x + 4y - 1 = 0$

347. The locus of centre of a circle passing through (p, q) and cuts orthogonally to circle $x^2 + y^2 = k^2$, is [IIT 1988]

- (a) $2px + 2qy - (p^2 + q^2 + k^2) = 0$ (b) $2px + 2qy - (p^2 - q^2 + k^2) = 0$
(c) $x^2 + y^2 - 3px - 4qy + (p^2 + q^2 - k^2) = 0$ (d) $x^2 + y^2 - 2px - 3qy + (p^2 - q^2 - k^2) = 0$

348. Two given circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + dx + ey + f = 0$ will intersect each other orthogonally, only when

- (a) $a + b + c = d + e + f$ (b) $ad + be = c + f$ (c) $ad + be = 2c + 2f$ (d) $2ad + 2be = c + f$

349. Two circles $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ cut each other orthogonally, then

[Rajasthan PET 1995]

- (a) $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ (b) $2g_1g_2 - 2f_1f_2 = c_1 + c_2$ (c) $2g_1g_2 + 2f_1f_2 = c_1 - c_2$ (d) $2g_1g_2 - 2f_1f_2 = c_1 - c_2$

350. If the circles of same radius a and centres at (2, 3) and (5, 6) cut orthogonally, then a = [EAMCET 1988]

- (a) 1 (b) 2 (c) 3 (d) 4

351. The circles $x^2 + y^2 + 4x + 6y + 3 = 0$ and $2(x^2 + y^2) + 6x + 4y + c = 0$ will cut orthogonally, if c equals [Kurukshetra CEE 1996]

- (a) 4 (b) 18 (c) 12 (d) 16

352. The equation of a circle that intersects the circle $x^2 + y^2 + 14x + 6y + 2 = 0$ orthogonally and whose centre is (0, 2) is [MP PET 1998]

- (a) $x^2 + y^2 - 4y - 6 = 0$ (b) $x^2 + y^2 + 4y - 14 = 0$ (c) $x^2 + y^2 + 4y + 14 = 0$ (d) $x^2 + y^2 - 4y - 14 = 0$

353. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is [IIT Screening 2000]

- (a) 2 or $-\frac{3}{2}$ (b) -2 or $\frac{3}{2}$ (c) 2 or $\frac{3}{2}$ (d) -2 or $\frac{3}{2}$

354. The locus of the centre of circle which cuts the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 6y + 4 = 0$ orthogonally is

[UPSEAT 2001]

- (a) $12x + 8y + 5 = 0$ (b) $8x + 12y + 5 = 0$ (c) $8x - 12y + 5 = 0$ (d) None of these

355. If the two circles $2x^2 + 2y^2 - 3x + 6y + k = 0$ and $x^2 + y^2 - 4x + 10y + 16 = 0$ cut orthogonally, then the value of k is

[Kerala (Engg.) 2002]

- (a) 41 (b) 14 (c) 4 (d) 0

356. The circles $x^2 + y^2 + x + y = 0$ and $x^2 + y^2 + x - y = 0$ intersect at an angle of

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$
357. The equation of the circle having its centre on the line $x + 2y - 3 = 0$ and passing through the point of intersection of the circles $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 - 4x - 2y + 4 = 0$ is [MNR 1992]
 (a) $x^2 + y^2 - 6x + 7 = 0$ (b) $x^2 + y^2 - 3x + 4 = 0$ (c) $x^2 + y^2 - 2x - 2y + 1 = 0$ (d) $x^2 + y^2 + 2x - 4y + 4 = 0$
358. The two circles $x^2 + y^2 - 2x - 2y - 7 = 0$ and $3(x^2 + y^2) - 8x + 29y = 0$ [Karnataka CET 1993]
 (a) Touch externally (b) Touch internally
 (c) Cut each other orthogonally (d) Do not cut each other
359. The locus of the centres of circles passing through the origin and intersecting the fixed circle $x^2 + y^2 - 5x + 3y - 1 = 0$ orthogonally is
 (a) A straight line of the slope $\frac{3}{5}$ (b) A circle
 (c) A pair of straight lines (d) None of these
360. The angle of intersection of circles $x^2 + y^2 + 8x - 2y - 9 = 0$ and $x^2 + y^2 - 2x + 8y - 7 = 0$ is [EAMCET 1987]
 (a) 45° (b) 90° (c) 60° (d) 30°
361. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is [AIEEE 2004]
 (a) $2ax - 2by - (a^2 + b^2 + 4) = 0$ (b) $2ax + 2by - (a^2 + b^2 + 4) = 0$
 (c) $2ax - 2by + (a^2 + b^2 + 4) = 0$ (d) $2ax + 2by + (a^2 + b^2 + 4) = 0$
362. The value of λ , for which the circle $x^2 + y^2 + 2\lambda x + 6y + 1 = 0$, intersects the circle $x^2 + y^2 + 4x + 2y = 0$ orthogonally is [MP PET 2004]
 (a) $-\frac{5}{2}$ (b) -1 (c) $-\frac{11}{8}$ (d) $-\frac{5}{4}$

Advance Level

363. The equation of a circle which cuts the three circles $x^2 + y^2 - 3x - 6y + 14 = 0$, $x^2 + y^2 - x - 4y + 8 = 0$ and $x^2 + y^2 + 2x - 6y + 9 = 0$ orthogonally is
 (a) $x^2 + y^2 - 2x - 4y + 1 = 0$ (b) $x^2 + y^2 + 2x + 4y + 1 = 0$
 (c) $x^2 + y^2 - 2x + 4y + 1 = 0$ (d) $x^2 + y^2 - 2x - 4y - 1 = 0$
364. The coordinates of the centre of the circle which intersects circles $x^2 + y^2 + 4x + 7 = 0$, $2x^2 + 2y^2 + 3x + 5y + 9 = 0$ and $x^2 + y^2 + y = 0$ orthogonally are
 (a) $(-2, 1)$ (b) $(-2, -1)$ (c) $(2, -1)$ (d) $(2, 1)$
365. The members of a family of circles are given by the equation $2(x^2 + y^2) + \lambda x - (1 + \lambda^2)y - 10 = 0$. The number of circles belonging to the family that are cut orthogonally by the fixed circle $x^2 + y^2 + 4x + 6y + 3 = 0$ is
 (a) 2 (b) 1 (c) 0 (d) None of these

Radical Axis and Radical Centre

Basic Level

366. The equation of radical axis of the circles $x^2 + y^2 + x - y + 2 = 0$ and $3x^2 + 3y^2 - 4x - 12 = 0$, is

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[Rajasthan PET 1984, 85, 86, 91, 2000]

- (a) $2x^2 + 2y^2 - 5x + y - 14 = 0$ (b) $7x - 3y + 18 = 0$
 (c) $5x - y + 14 = 0$ (d) None of these
367. The radical centre of three circles described on the three sides of a triangle as diameter is [EAMCET 1994]
 (a) The orthocentre (b) The circumcentre (c) The incentre of the triangle (d)
368. The locus of centre of the circle which cuts the circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ orthogonally is [Karnataka CET 1991]
 (a) An ellipse (b) The radical axis of the given circles
 (c) A conic (d) Another circle
369. The coordinates of the radical centre of the three circles $x^2 + y^2 - 4x - 2y + 6 = 0$, $x^2 + y^2 - 4x - 2y + 6y = 0$, $x^2 + y^2 - 12x + 2y + 30 = 0$ are
 (a) (6, 30) (b) (0, 6) (c) (3, 0) (d) None of these
370. The equation of radical axis of the circles $2x^2 + 2y^2 - 7x = 0$ and $x^2 + y^2 - 4y - 7 = 0$ is [Rajasthan PET 1987, 89, 93, 96]
 (a) $7x + 8y + 14 = 0$ (b) $7x - 8y + 14 = 0$ (c) $7x - 8y - 14 = 0$ (d) None of these
371. The radical centre of the circles $x^2 + y^2 - 16x + 60 = 0$, $x^2 + y^2 - 12x + 27 = 0$, $x^2 + y^2 - 12y + 8 = 0$ is [Rajasthan PET 2000]
 (a) (13, 33/4) (b) (33/4, -13) (c) (33/4, 13) (d) None of these
372. The radical axis of two circles and the line joining their centres are [Karnataka CET 2001]
 (a) Parallel (b) Perpendicular
 (c) Neither parallel, nor perpendicular (d) Intersecting, but not fully perpendicular
373. Radical axis of the circles $3x^2 + 3y^2 - 7x + 8y + 11 = 0$ and $x^2 + y^2 - 3x - 4y + 5 = 0$ is [Rajasthan PET 2001]
 (a) $x + 10y + 2 = 0$ (b) $x + 10y - 2 = 0$ (c) $x + 10y + 8 = 0$ (d) $x + 10y - 8 = 0$
374. Two tangents are drawn from a point P on radical axis to the two circles touching at Q and R respectively then triangle formed by joining PQR is [UPSEAT 2002]
 (a) Isosceles (b) Equilateral (c) Right angled (d) None of these
375. Equation of radical axis of the circles $x^2 + y^2 - 3x - 4y + 5 = 0$ and $2x^2 + 2y^2 - 10x - 12y + 12 = 0$ is [Rajasthan PET 2003]
 (a) $2x + 2y - 1 = 0$ (b) $2x + 2y + 1 = 0$ (c) $x + y + 7 = 0$ (d) $x + y - 7 = 0$
376. If the circle $x^2 + y^2 + 6x - 2y + k = 0$ bisects the circumference of the circle $x^2 + y^2 + 2x - 6y - 15 = 0$, then $k =$ [EAMCET 2000]
 (a) 21 (b) -21 (c) 23 (d) -23
377. The locus of a point which moves such that the tangents from it to the two circles $x^2 + y^2 - 5x - 3 = 0$ and $3x^2 + 3y^2 + 2x + 4y - 6 = 0$ are equal, is given by
 (a) $2x^2 + 2y^2 + 7x + 4y - 3 = 0$ (b) $17x + 4y + 3 = 0$
 (c) $4x^2 + 4y^2 - 3x + 4y - 9 = 0$ (d) $13x - 4y + 15 = 0$
378. Two equal circles with their centres on x and y axes will possess the radical axis in the following form
 (a) $ax - by - \frac{a^2 + b^2}{4} = 0$ (b) $2gx - 2fy + g^2 - f^2 = 0$ (c) $g^2x + f^2y - g^4 - f^4 = 0$ (d) $2g^2x + 2f^2y - g^4 - f^4 = 0$
379. The equations of two circles are $x^2 + y^2 + 2\lambda x + 5 = 0$ and $x^2 + y^2 + 2\lambda y + 5 = 0$. P is any point on the line $x - y = 0$. If PA and PB are the lengths of the tangents from P to the two circles and $PA = 3$ then PB is equal to
 (a) 1.5 (b) 6 (c) 3 (d) None of these
380. The locus of a point from which the lengths of the tangents to the circles $x^2 + y^2 = 4$ and $2(x^2 + y^2) - 10x + 3y - 2 = 0$ are equal is

- (a) A straight line inclined at $\pi/4$ with the line joining the centres of the circles
 (b) A circle
 (c) An ellipse
 (d) A straight line perpendicular to the line joining the centres of the circles
381. The equation of the radical axis of circles $x^2 + y^2 + 6x = 0$ and $x^2 + y^2 + 4x - 2y + 5 = 0$ is [Rajasthan PET 1986]
 (a) $2x + 2y = 5$ (b) $4x - 2y - 5 = 0$ (c) $2x + 2y + 5 = 0$ (d) None of these
382. The equation of the radical axis of circles $(x - a)^2 + (y - b)^2 = c^2$ and $(x - b)^2 + (y - a)^2 = c^2$ is
 (a) $x + y = 0$ (b) $x - y = 0$ (c) $x + y = c^2$ (d) None of these

Miscellaneous problems

Basic Level

383. If three circles are such that each intersects the remaining two, then their radical axes
 (a) Form a triangle (b) Are coincident (c) Concurrent (d) Parallel
384. If a circle S_1 bisects the circumference of another circle S_2 , then their radical axis
 (a) Passes through the centre of S_1 (b) Passes through the centre of S_2
 (c) Bisects the line joining their centres (d) None of these
385. If two circles intersect a third circle orthogonally, then their radical axis
 (a) Touches the third circle (b) Passes through the centre of the third circle
 (c) Does not intersect the third circle (d) None of these
386. The radical axis of two circles
 (a) Always intersects both the circles (b) Intersects only one circle
 (c) Bisects the line joining their centres (d) Bisects every common tangent to those circles
387. If the radical axis of circles $x^2 + y^2 - 6x - 8y + p = 0$ and $x^2 + y^2 - 8x - 6y + 14 = 0$ passes through the point $(1, -1)$, then p is equal to
 (a) -1 (b) 10 (c) -14 (d) 14
388. The radical centre of circles $x^2 + y^2 + 2ax + c = 0$, $x^2 + y^2 + 2by + c = 0$ and $x^2 + y^2 + 2ax + 2by + c = 0$ is
 (a) $(0, 0)$ (b) $(a, 0)$ (c) $(0, b)$ (d) (a, b)
389. The equation of the radical axis of circles $7x^2 + 7y^2 - 7x + 14y + 18 = 0$ and $4x^2 + 4y^2 - 7x + 8y + 20 = 0$ is [Roorkee 1989]
 (a) $21x - 68 = 0$ (b) $3y - 1 = 0$ (c) $3x^2 + 3y^2 + 6y - 6 = 0$ (d) None of these

Advance Level

390. If the radical axis of the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x + 2y + 1 = 0$, then
 (a) $g = \frac{3}{4}$ and $f \neq 2$ (b) $g \neq \frac{3}{4}$ and $f = 2$ (c) $g = \frac{3}{4}$ and $f = 2$ (d) None of these
391. If $(1, 2)$ is the radical centre of circle $x^2 + y^2 - 3x - 6y + d_1 = 0$, $x^2 + y^2 - x - 4y + d_2 = 0$ and $x^2 + y^2 + 2x - 6y + d_3 = 0$, then
 (a) $d_1 + d_3 = 5$ (b) $d_1 - d_3 = 5$ (c) $d_1 + d_3 = 10$ (d) $d_1 - d_3 = 10$
392. $x = 1$ is the equation of the radical axis of two circle which intersect orthogonally. If the equation of one of these circles is $x^2 + y^2 = 4$, then the equation of the other is [EAMCET 1983]
 (a) $x^2 + y^2 - 8x - 4 = 0$ (b) $x^2 + y^2 - 8x + 4 = 0$ (c) $x^2 + y^2 + 8x + 4 = 0$ (d) None of these

Co-axial System of Circles and Limiting Points

Basic Level

393. Origin is a limiting point of a coaxial system of which $x^2 + y^2 - 6x - 8y + 1 = 0$ is a member. The other limiting point is [EAMCET 1994]

- (a) $(-2, -4)$ (b) $\left(\frac{3}{25}, \frac{4}{25}\right)$ (c) $\left(-\frac{3}{25}, -\frac{4}{25}\right)$ (d) $\left(\frac{4}{25}, \frac{3}{25}\right)$

394. If $(3, \lambda)$ and $(5, 6)$ are conjugate points with respect to circle $x^2 + y^2 = 3$, then λ equals [Rajasthan PET 1998]

- (a) 2 (b) -2 (c) 3 (d) 4

Advance Level

395. One of the limiting point of the coaxial system of circles containing $x^2 + y^2 - 6x - 6y + 4 = 0$, $x^2 + y^2 - 2x - 4y + 3 = 0$ is [EAMCET 1987]

- (a) $(-1, 1)$ (b) $(-1, 2)$ (c) $(-2, 1)$ (d) $(-2, 2)$

396. The co-axial system of circles given by $x^2 + y^2 + 2gx + c = 0$ for $c < 0$ represents. [Karnataka CET 2004]

- (a) Intersecting circles (b) Non intersecting circles
(c) Touching circles (d) Touching or non-intersecting circles

Miscellaneous problems

Basic Level

397. The limit of the perimeter of the regular n -gons inscribed in a circle of radius R as $n \rightarrow \infty$ is [MP PET 2003]

- (a) $2\pi R$ (b) πR (c) $4R$ (d) πR^2

398. A, B, C and D are the points of intersection with the coordinate axes of the lines $ax + by = ab$ and $bx + ay = ab$, then

- (a) A, B, C, D are concyclic (b) A, B, C, D form a parallelogram
(c) A, B, C, D form a rhombus (d) None of these

Advance Level

399. If the points $(2, 0)$, $(0, 1)$, $(4, 5)$ and $(0, c)$ are concyclic, then c is equal to [MNR 1982]

- (a) $-1, -\frac{3}{14}$ (b) $-1, -\frac{14}{3}$ (c) $\frac{14}{3}, 1$ (d) None of these

400. Line $Ax + By + C = 0$ cuts circle $x^2 + y^2 + ax + by + c = 0$ in P and Q and the line $A'x + B'y + C' = 0$ cuts the circle $x^2 + y^2 + a'x + b'y + c' = 0$ in R and S . If the four points P, Q, R and S are concyclic, then

$$D = \begin{vmatrix} a-a' & b-b' & C-C' \\ A & B & c \\ A' & B' & c' \end{vmatrix} =$$

[Roorkee 1986]

- (a) 1 (b) 0 (c) -1 (d) None of these

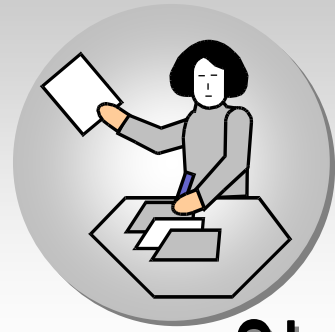
401. A circle is inscribed in an equilateral triangle of side a , the area of any square inscribed in the circle is [IIT 1994]

- (a) $\frac{a^2}{3}$ (b) $\frac{2a^2}{3}$ (c) $\frac{a^2}{6}$ (d) $\frac{a^2}{12}$

- 402.** Any circle through the points of intersection of the lines $x + \sqrt{3}y = 1$ and $\sqrt{3}x - y = 2$ if intersects these lines at points P and Q , then the angle subtended by the arc PQ at its centre is [MP PET 1998]
 (a) 180° (b) 90° (c) 120° (d) Depends on centre of radius
- 403.** The area of the triangle formed by joining the origin to the points of intersection of the line $x\sqrt{5} + 2y = 3\sqrt{5}$ and circle $x^2 + y^2 = 10$ is [Roorkee 1998]
 (a) 3 (b) 4 (c) 5 (d) 6
- 404.** Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then the locus of the centroid of the $\triangle PAB$ as P moves on the circle is [IIT Screening 2001]
 (a) A parabola (b) A circle (c) An ellipse (d) A pair of straight lines
- 405.** A square is inscribed in the circle $x^2 + y^2 - 2x + 4y - 93 = 0$ with its sides parallel to the coordinate axes. The coordinates of its vertices are
 (a) $(-6, -9), (-6, 5), (8, -9), (8, 5)$ (b) $(-6, 9), (-6, -5), (8, -9), (8, 5)$
 (c) $(-6, -9), (-6, 5), (8, 9), (8, 5)$ (d) $(-6, -9), (-6, 5), (8, -9), (8, -5)$
- 406.** If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ cut the coordinate axes in concyclic points, then
 (a) $a_1a_2 = b_1b_2$ (b) $a_1b_1 = a_2b_2$ (c) $a_1b_2 = a_2b_1$ (d) None of these
- 407.** Let P be a point on the circle $x^2 + y^2 = 9$, Q a point on the line $7x + y + 3 = 0$, and the perpendicular bisector of PQ be the line $x - y + 1 = 0$. Then the coordinates of P are
 (a) $(3, 0)$ (b) $(0, 3)$ (c) $\left(\frac{72}{25}, -\frac{21}{25}\right)$ (d) $\left(-\frac{72}{25}, \frac{21}{25}\right)$
- 408.** A line meets the coordinate axes in A and B . A circle is circumscribed about the triangle OAB . The distances from the end points A, B of the side AB to the tangent at O are equal to m and n respectively. Then the diameter of the circle is
 (a) $m(m+n)$ (b) $n(m+n)$ (c) $m-n$ (d) None of these
- 409.** If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is touched by $y = x$ at P such that $OP = 6\sqrt{2}$, then the value of c is
 (a) 36 (b) 144 (c) 72 (d) None of these
- 410.** One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y = x + 7$. If A and B are the points $(-3, 4)$ and $(5, 4)$ respectively, then the area of the rectangle is
 (a) 16 sq. units (b) 24 sq. units (c) 32 sq. units (d) None of these
- 411.** The maximum number of points with rational coordinates on a circle whose centre is $(\sqrt{3}, 0)$ is
 (a) One (b) Two (c) Four (d) Infinite
- 412.** The locus of co-ordinates of the centre of the circumcircle of the regular hexagon whose two consecutive vertices have the coordinates $(-1, 0)$ and $(1, 0)$ and which lies wholly above the x -axis, are
 (a) $x^2 + y^2 - 2\sqrt{3}y - 1 = 0$ (b) $x^2 + y^2 - \sqrt{3}y - 1 = 0$ (c) $x^2 + y^2 - 2\sqrt{3}x - 1 = 0$ (d) None of these
- 413.** For each $k \in N$, let C_k denote the circle whose equation is $x^2 + y^2 = k^2$. On the circle C_k , a particle moves k units in the anticlockwise direction. After completing its motion on C_k , the particle moves to C_{k+1} in the radial direction. The motion of the particle continues in this manner. The particle starts at $(1, 0)$. If the particle crosses the positive direction of the x -axis for the first time on the circle C_n , then n is
 (a) 7 (b) 6 (c) 2 (d) None of these
- 414.** A ray of light incident at the point $(-2, -1)$ gets reflected from the tangent at $(0, -1)$ to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line along which the incident ray moved is
 (a) $4x - 3y + 11 = 0$ (b) $4x + 3y + 11 = 0$ (c) $3x + 4y + 11 = 0$ (d) None of these
- 415.** The point P moves in the plane of a regular hexagon such that the sum of the squares of its distances from the vertices of the hexagon is $6a^2$. If the radius of the circumcircle of the hexagon is $r (< a)$ then the locus of P is
 (a) A pair of straight lines (b) An ellipse
 (c) A circle of radius $\sqrt{a^2 - r^2}$ (d) An ellipse of major axis a and minor axis r
- 416.** The equation of a circle is $x^2 + y^2 = 4$. A regular hexagon is inscribed in the circle whose one vertex is $(2, 0)$. Then a consecutive vertex has the coordinates
 (a) $(\sqrt{3}, 1)$ (b) $(1, -\sqrt{3})$ (c) $(\sqrt{3}, -1)$ (d) $(1, \sqrt{3})$

146 Circle and System of Circles

- 417.** A point $P(\sqrt{3}, 1)$ moves on the circle $x^2 + y^2 = 4$ and after covering a quarter of the circle leaves it tangentially. The equation of a line along which the point moves after leaving the circle is
(a) $y = \sqrt{3}x + 4$ (b) $\sqrt{3}y = x + 4$ (c) $\sqrt{3}y = x - 4$ (d) $y = \sqrt{3}x - 4$
- 418.** If the curves $ax^2 + 4xy + 2y^2 + x + y + 5 = 0$ and $ax^2 + 6xy + 5y^2 + 2x + 3y + 8 = 0$ intersect at four concyclic points then the value of a is
(a) 4 (b) - 4 (c) 6 (d) - 6



Answer Sheet

Circle and System of Circles

Assignment (Basic and Advance level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	b	a	b	a	b	b	a	b	b	c	c	a	a	a	c	a	d	a	a
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
d	c	a	b	c	a	d	c	b	d	c	d	a	a,b, c	a	b	a	d	d	b
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
d	a	d	c	b	c	d	b	c	b	a	a	c	a	a	c	b	a	c	b
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
a	c	a	c	b	c	c	a	a	a	a	b	c	d	b	d	a	b	b	b
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
b	a	c	b	b	c	a	b	c	b	a	d	b	a	a	b	b	d	c	c
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
a	d	a	b	b	b	a	a	a	c	d	c	b	b	a	b,c	b	c	a	c
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
a,c	b	d	b	b	a	b	a	a	d	d	a	b	d	c	a	d	b	d	c
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
a	b	a	b	c	b	b	b	b	c	b	a	c	c	d	a	b	b	a	a
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
b	c	b	a,d	b	b	b	a	c	b	c	c	c	b,c	c	d	c	a	a	c
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
d	b	b	b	b	b	b	a	c	c	b	a	a	a	c	c	a	a,c	d	b
201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
d	d	c	c	a	a	c	a,c	b	a	c	b	c	c	a	c	a	c	c	c
221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
b	b	b	c	b	b,c	a	b	c	d	c	d	b	c	a	c,d	c	c	c	a

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241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260
d	d	a	d	a	c	c	a	d	b	a	c	c	c	a	a,c	b	b	a	c
261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280
b	a	b	d	a	a	a	c	b	a	b	a	a,c	a	a	a	d	a	a	a
281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300
a,d	a	d	a	a,b,c d	b	b	b	a	d	c	b	b	a	b	a	b	c	b	c
301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320
a	b	c	b	a	a	a	d	b	c	b	a	b	b	b	d	c	c	d	b
321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340
b	c	a	c	b	b	b	a	a	d	b	c	b	d	a	c	a	d	d	b
341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360
b	a	c	a	a	c	a	c	a	c	b	d	a	c	c	d	a	c	d	b
361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380
b	d	a	b	a	b	c	b	d	c	d	b	b	a	a	d	b	b	c	d
381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400
a	b	c	b	b	d	b	a	a	c	b	b	b	b	a	a	a	a	c	b
401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418		
c	a	c	b	a	a	a,d	d	c	c	b	a	a	b	c	b	b	b		