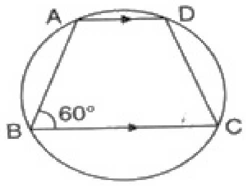
CBSE Test Paper 03

CH-10 Circles

1. If ABCD is a cyclic trapezium in which AD$||$ BC and $\angle B = 60^o$, then $\angle BCD$ is equal to



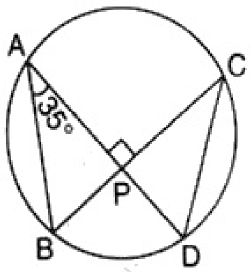
   a. 80^o

   b. 60^o

   c. 100^o

   d. 60^o

2. Chords AD and BC intersect each other at right angles at point P. $\angle DAB = 35^o$, then $\angle ADC$ is equal to



   a. 35^o

   b. 45^o

   c. 65^o

   d. 55^o

3. The constant distance of a point on a circle from the centre of the circle is called

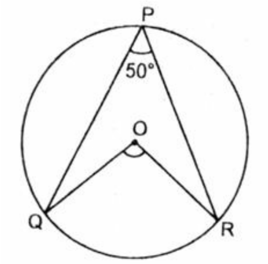
   a. diameter

b. circle

c. radius

d. centre

4. In the given figure, O is the centre of the circle. If $\angle QPR$ is 50^o, then $\angle QOR$ is :



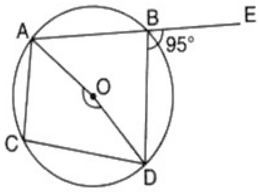
a. 100^o

b. 130^o

c. 40^o

d. 50^o

5. In the given figure, O is the centre of the circle ABE is a straight line. If $\angle DBE = 95^o$ then $\angle AOD$ is equal to



a. 170^o

b. 180^o

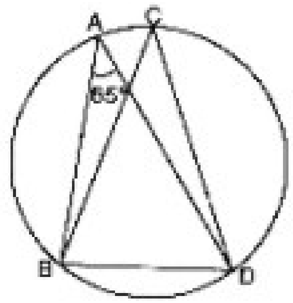
c. 190^o

d. 175^o

6. Fill in the blanks:

An arc is a ________ when its ends are the ends of a diameter.

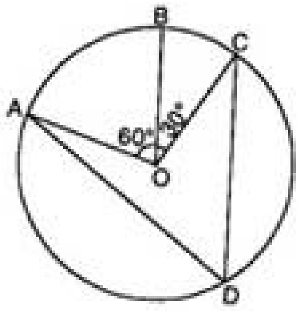
7. In the figure, $\angle BAD = 65^{\circ}$, then find $\angle BCD$



8. Draw different pairs of circles. How many points does each pair have in common? What is a maximum number of common points?

9. Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

10. In figure, A, B and C are three points on a circle with centre O such that $\angle BOC = 30^{\circ}$ and $\angle AOB = 60^{\circ}$. If D is a point on the circle other than the arc ABC, find $\angle ADC$



11. Prove that OM Bisect AB. If OM $\perp$ AB.

12. Two diameters of a circle intersect each other at right angles. Prove that the quadrilateral formed by joining their end points is a square.

13. In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

14. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

15. If two chords AB and CD of a circle AYDZBWCX intersect at right angles, prove that arc CXA + arc DZB = arc AYD + arc BWC = semicircle.

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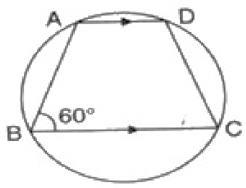
CH-10 Circles

Solution

1. (d) 60^o

   Explanation:

   Since ABCD is a cyclic quadrilateral

   $\angle$B + $\angle$D = 180^0

   60^0 + $\angle$D = 180^0

   $\angle$D = 120^0

   Now since AD is parallel to BC

   $\angle$C + $\angle$D = 180^0

   $\angle$C + 120^0 = 180^0

   $\angle$C = 60^0

2. (d) 55^o

   Explanation:

   From triangle APB, $\angle ABP = 180^0 - 90^0 - 35^0 = 55^0$
   Thus, $\angle ADC = 55^0$($\angle ABC = \angle ADC$)

3. (c) radius

   Explanation:

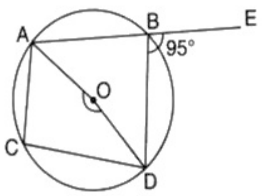
   Radius is the fixed distance of a fixed point from a point on the circle.

   Also more precisely, a circle is the locii or the path of a point that moves maintaining a fixed distance from a given point.

4. (a) 100^o

Explanation:

Angle made by a chord at the centre is twice the angle made by it on any point on the circumference. Therefore,
$\angle QOR = 2\angle QPR = 50^0 * 2 = 100^0$

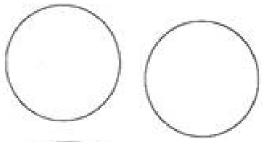
5. (a) 170^o
Explanation:


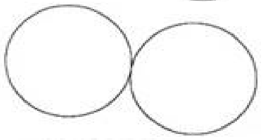
$\angle ABD = 180^0 - 95^0 = 85^0$(Linear Pair)
Since, $\angle AOD = 2\angle ABD = 2 \times 85^0 = 170^0$

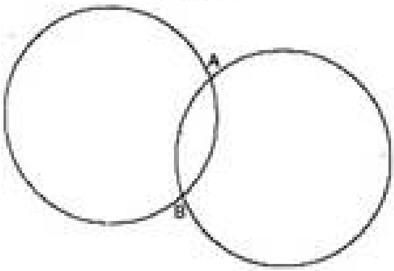
6. semi-circle

7. $\angle BCD = \angle BAD = 65^\circ$ $\angle s$ of the same segment are equal.

8. Each pair has at the most two common points.

i.  No point common

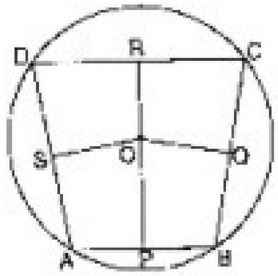
ii.  One point common

iii. 

Two points common

The maximum number of common points is two.
Thus, the above cases represent possible ways of drawing.

9. Given: ABCD is a cyclic quadrilateral of the circle with centre O.
To prove: Perpendicular bisectors of the sides AB, BC, CD and DA are concurrent.



Proof: We know that perpendicular bisector of a chord of a circle passes through the centre of the circle.

The centre O of the circle will lie on each perpendicular bisector.

Hence, the perpendicular bisectors of the sides are concurrent and their point of concurrence is the centre of the circle.

10. $\angle ADC = \frac{1}{2}\angle AOC$

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$= \frac{1}{2}(\angle AOB + \angle BOC)$

$= \frac{1}{2}(60^\circ + 30^\circ)$

$= \frac{1}{2}(90^\circ) = 45^\circ$

11. AB is a chord of the circle with centre O.

OM $\perp$ AB

OA=OB (radii of same circle)

OM=OM (common)

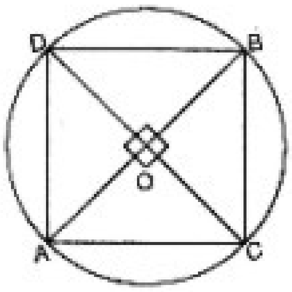
$\angle OMA = \angle OMB$ [each 90°]

ΔOAM $\cong$ Δ OBM (by SAS)

AB = BM

Hence OM bisects AB

12. Given: Two diameters AB and CD of a circle intersect each other at right angles.



To prove: The quadrilateral ACBD formed by joining their end points is a square.

Proof: A diameter essentially passes through the centre of the circle.

$\therefore$ Diameters AB and CD intersect each other at O, the centre of the circle.

$\angle A = \angle B = \angle C = \angle D = 90^\circ (\text{ each })$ [$\because$ Angle in a semi-circle is 90^o]

Quadrilateral ACBD is a rectangle .... (1)

In $\triangle OAC$ and ΔOAD

$\angle AOC = \angle AOD$ |Each = 90^o

OA = OA [common]

OC = OD [Radii of the same circle]

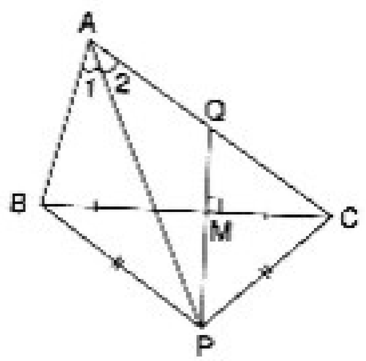
$\therefore \Delta OAC \cong \Delta OAD$ [SAS]

$\therefore AC = AD$ .... (2) [c.p.c.t]

In view of (1) and (2)

Quadrilateral ABCD is a square

13. Given: Bisector AP of angle A of $\triangle ABC$ and the perpendicular bisector PQ of its opposite side BC intersect at P.



To prove: P lies on the circumcircle of the triangle ABC.

Construction: Draw the circle through three non-collinear points A, B and P.

Proof : $\angle BAP = \angle CAP$

$\Rightarrow \overline{\text{BP}} \cong \overline{\text{CP}}$

$\Rightarrow$ chord BP = chord CP

In $\triangle BMP$ and $\triangle CMP$

BM = CM

BP = CP

MP = MP

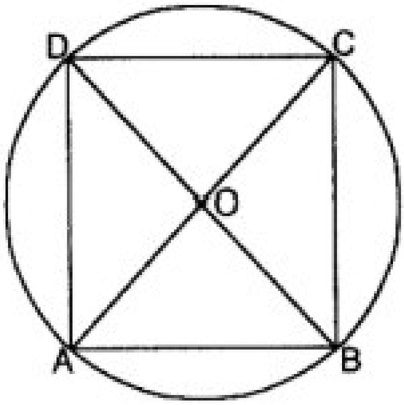
$\therefore \triangle BMP \cong \triangle CMP$ |SSS

$\therefore \angle BMP = \angle CMP$ |c.p.c.t

But $\angle BMP + \angle CMP = 180^\circ$ |Linear Pair Axiom

$\therefore \angle BMP = \angle CMP = 90^\circ$

$\Rightarrow$ PM is the right bisector of BC.

14. 

$\triangle$OAB and $\triangle$ OCD

OA = OC [Radii of a circle]

OB = OD |Radii of a circle

$\angle$AOB = $\angle$COD [Vertically opposite angles]

$\therefore$ $\triangle$OAB $\cong$ $\triangle$ OCD |SAS rule

$\therefore$ AB = CD [c.p.c.t]

$\Rightarrow$ Ar CAB = Ar CCD---- (1)

Similarly, we can show that

$\Rightarrow$ Arc AD = Arc CB ---- (2)

Adding (1) and (2), we get

Arc AB + Arc AD = Arc CD + Arc CB

$\Rightarrow$ Ar cBAD = ArcBCD

$\Rightarrow$BD divides the circle into two equal parts (each a semicircle)

$\therefore$ $\angle$A = 90^0, $\angle$C [Angle of a semi-circle is 90^0]

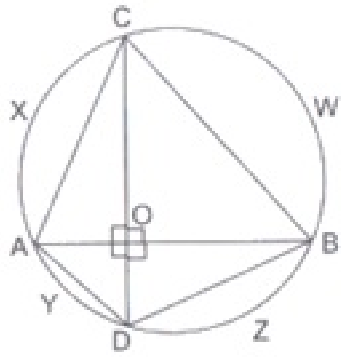
Similarly, we can show that

$\angle$B = 90^0, $\angle$D = 90^0

$\therefore$ $\angle$A = $\angle$B = $\angle$C = $\angle$D = 90^0

$\therefore$ABCD is a rectangle.

15. Here it is given that the two chords AB and CD of circle AYDZBWCX intersect at right angles.



We need to prove that: arc CXA + arc DZB = arc AYD + arc BWC = semicircle.

Construction: Join AC, AD, BD and BC as shown in the figure.

Proof: O is any point inside the circle.

Now, consider the chord CA. The angle subtended by the chord AC at the circumference is $\angle CBA$ and similarly, the angle subtended by the chord BD at the circumference is $\angle BCD$

Now, consider the right triangle BOC, we have:

$\angle COB + \angle CBA + \angle BCD = 180^\circ$ (by angle sum property)

$\Rightarrow 90^\circ + \angle CBA + \angle BCD = 180^\circ$

$\Rightarrow \angle CBA + \angle BCD = 180^\circ - 90^\circ$

$\Rightarrow \angle CBA + \angle BCD = 90^\circ$

That is the sum of angle subtended by the arc CXA and the angles subtended by the arc BZD = 90^o.

a$rc\widehat{CXA} + \text{arc}\,\widehat{BZD} = 90^\circ$ ....(1)

Now, consider the chord BC.

The angles subtended by the chord BC at the centre is $\angle BAC$

Similarly, the angle subtended by the Chord AD at the centre is $\angle ACD$

Now, consider the right triangle AOC, we have:

$\angle COA + \angle BAC + \angle ACD = 180^\circ$ (by angle sum property)

$\Rightarrow 90^\circ + \angle BAC + \angle ACD = 180^\circ$

$\Rightarrow \angle BAC + \angle ACD = 180^\circ - 90^\circ$

$\Rightarrow \angle BAC + \angle ACD = 90^\circ$

That is the sum of angle subtended by the are CWB and the angle subtended by the are AYD = 90°.

$\mathrm{a}rc\widehat{CWB} + \mathrm{arc}\,\widehat{AYD} = 90^\circ$ ....(2)

From equations (1) and (2), we obtain

$\mathrm{a}rc\widehat{CXA} + \mathrm{arc}\,\widehat{BZD} = \mathrm{a}rc\widehat{CWB} + \mathrm{arc}\,\widehat{AYD} = 90^\circ$

We know that the arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

Thus, we have

$\mathrm{a}rc\widehat{CXA} + \mathrm{arc}\,\widehat{BZD} = \mathrm{a}rc\widehat{CWB} + \mathrm{arc}\,\widehat{AYD} =$ semicircle

Hence proved.