Long Answer Questions-I (PYQ)

[4 Marks]

A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine:

(*i*) k (*ii*) $P(X < 3)$ (*iii*) $P(X > 6)$ (*iv*) $P(0 < X < 3)$

Ans.

$\because \quad \sum_{j=1}^{n} P_i = 1$

$\therefore \quad 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow \quad 10k^2 + 9k - 1 = 0$

$\Rightarrow \quad 10k^2 + 10k - k - 1 = 0$

$\Rightarrow \quad 10k(k+1) - 1(k+1) = 0$

$\Rightarrow \quad (k+1)(10k-1) = 0$

$\Rightarrow \quad k = -1 \text{ and } k = \frac{1}{10}$

But, k can never be negative as probability is never negative.

$\therefore \quad k = \frac{1}{10}$

Now,

i. $k = \frac{1}{10}$

ii. $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0 + k + 2k = 3k = \frac{3}{10}.$

iii. $P(X > 6) = P(X = 7) = 7k^2 + k = 7 \times \frac{1}{100} + \frac{1}{10} = \frac{17}{100}$

iv. $P(0 < X < 3) = P(X = 1) + P(X = 2) = k + 2k = 3k = \frac{3}{10}.$

Q.2. Three numbers are selected at random (without replacement) from first six positive integers. If *X* denotes the smallest of the three numbers obtained, find the probability distribution of *X*. Also, find the mean and variance of the distribution.

Ans.

First six positive integers are 1, 2, 3, 4, 5 and 6.

If three numbers are selected at random from above six numbers then the number of elements in sample space S is given by

i.e., $n(s) = {}^6C_3 = \frac{6!}{3!3!} = \frac{6 \times 5 \times 4}{3 \times 2} = 20$

Here X, smallest of the three numbers obtained, is random variable X may have value 1, 2, 3, and 4. Therefore, required probability distribution is given as

$P(X = 1)$ = Probability of event getting 1 as smallest number

$= \frac{{}^5C_2}{20} = \frac{5!}{2!3! \times 20} = \frac{5 \times 4}{2 \times 20} = \frac{10}{20} = \frac{1}{2} [{}^5C_2 \equiv$ selection of two numbers out of 2, 3, 4, 5, 6$]$

$P(X=2)$ = Probability of events getting 2 as smallest number.

$$= \frac{{}^4C_2}{20} = \frac{4!}{2!2!\times 20} = \frac{6}{20} = \frac{3}{10} \qquad [{}^4C_2 \equiv \text{selection of two numbers out of 3, 4, 5, 6}]$$

$P(X=3)$ = Probability of events getting 3 as smallest number

$$= \frac{{}^3C_2}{20} = \frac{3!}{2!1!\times 20} = \frac{3}{20} \qquad [{}^3C_2 \equiv \text{selection of two numbers out 4, 5, 6}]$$

$P(X=4)$ = Probability of events getting 4 as smallest number.

$$= \frac{{}^2C_2}{20} = \frac{1}{20} \qquad [{}^2C_2 \equiv \text{selection of two numbers out of 5, 6}]$$

Required probability distribution table is

***X* or x_i**	1	2	3	4
***P*(*X*) or p_i**	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$

Mean = $S(X) = Sp_ix_i$

$$= 1\times\frac{1}{2} + 2\times\frac{3}{10} + 3\times\frac{3}{20} + 4\times\frac{1}{20}$$

$$= \frac{1}{2} + \frac{6}{10} + \frac{9}{20} + \frac{4}{20} = \frac{10+12+9+4}{20} = \frac{35}{20} = \frac{7}{4}$$

Variance = $\sum x_1^2 p_i - (\sum X)^2$

$$= \left\{1^2\times\frac{1}{2} + 2^2\times\frac{3}{10} + 3^2\times\frac{3}{20} + 4^2\times\frac{1}{20}\right\} - \left(\frac{7}{4}\right)^2$$

$$= \frac{1}{2} + \frac{12}{10} + \frac{27}{20} + \frac{16}{20} - \frac{49}{16} = \frac{10+24+27+16}{20} - \frac{49}{16}$$

$$= \frac{77}{20} - \frac{49}{16} = \frac{308-245}{80} = \frac{63}{80}$$

Q.3. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

i. **the youngest is a girl?**
ii. **atleast one is a girl?**

Ans.

A family has 2 children,

then sample space = $S = \{BB, BG, GB, GG\}$ where B stands for boy and G for girl.

i. Let A and B be two event such that

A = Both are girls = $\{GG\}$

B = The youngest is a girl = $\{BG, GG\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \qquad [\because A \cap B = \{GG\}]$$

$$P\left(\frac{A}{B}\right) = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$

ii. Let C be event such that

C = at least one is a girl = $\{BG, GB, GG\}$

$$\text{Now } P(A/C) = \frac{P(A \cap C)}{P(C)} \qquad [\because A \cap C = \{GG\}]$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Q.4. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be at least 3 successes.

Ans.

An experiment succeeds thrice as often as it fails.

$\Rightarrow \quad p = (\text{getting success}) = \frac{3}{4}$ and $q = P(\text{getting failure}) = \frac{1}{4}$.

Here, number of trials $= n = 5$

By Binomial distribution, we have

$$P(x = r) = {}^nC_r p^r \cdot q^{n-r}$$

Now, $P(\text{getting at least 3 success}) = P(X = 3) + P(X = 4) + P(X = 5)$

$$= {}^5C_3\left(\frac{3}{4}\right)^3 \cdot \left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4 \cdot \left(\frac{1}{4}\right)^1 + {}^5C_5\left(\frac{3}{4}\right)^5 \cdot \left(\frac{1}{4}\right)^0$$

$$= \left(\frac{3}{4}\right)^3\left[{}^5C_3 \times \frac{1}{16} + {}^5C_4 \times \frac{3}{4} \times \frac{1}{4} + {}^5C_5\left(\frac{3}{4}\right)^2\right]$$

$$= \frac{27}{64}\left[\frac{10}{16} + \frac{15}{16} + \frac{9}{16}\right] = \frac{27}{64} \times \frac{34}{16} = \frac{459}{512}.$$

Q.5. Bag *I* contains 3 red and 4 black balls while another bag *II* contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag *II*.

Ans.

Let E_1 be the event of choosing the bag I, E_2 the event of choosing the bag II and A be the event of drawing a red ball.

Then $P(E_1) = P(E_2) = \frac{1}{2}$

Also $P(A/E_1) = P$ (drawing a red ball from bag I) $= \frac{3}{7}$

and $P(A/E_2) = P$ (drawing a red ball from bag II) $= \frac{5}{11}$

Now, the probability of drawing a ball from bag II, being given that it is red, is $P(E_2/A)$. By using Bayes' theorem, we have

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)} = \frac{\frac{1}{2} \times \frac{5}{11}}{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \times \frac{5}{11}} = \frac{35}{68}$$

Q.6. Three cards are drawn at random (without replacement) from a well shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence, find the mean of the distribution.

Ans.

Let the number of red card in a sample of 3 cards drawn be random variable X. Obviously X may have values 0,1,2,3.

Now, $P(X=0)$ = Probability of getting no red card = $\frac{^{26}C_3}{^{52}C_3} = \frac{2600}{22100} = \frac{2}{17}$

$P(X=1)$ = Probability of getting one red card and two non-red cards

$$= \frac{^{26}C_1 \times {^{26}C_2}}{^{52}C_3} = \frac{8450}{22100} = \frac{13}{34}$$

$P(X=2)$ = Probability of getting two red cards and one non-red card

$$= \frac{^{26}C_2 \times {^{26}C_1}}{^{52}C_3} = \frac{8450}{22100} = \frac{13}{34}$$

$P(X=3)$ = Probability of getting 3 red cards = $\frac{^{26}C_3}{^{52}C_3} = \frac{2600}{22100} = \frac{2}{17}$

Hence, the required probability distribution in table as

X	0	1	2	3
$P(X)$	$\frac{2}{17}$	$\frac{13}{34}$	$\frac{13}{34}$	$\frac{2}{17}$

$\therefore$ Required mean = $E(X)$ = S$p_i x_i$

$$= 0 \times \frac{2}{17} + 1 \times \frac{13}{34} + 2 \times \frac{13}{34} + 3 \times \frac{2}{17}$$

$$= \frac{13}{34} + \frac{26}{34} + \frac{6}{17} = \frac{13+26+12}{34} = \frac{51}{34} = \frac{3}{2}$$

Q.7. Three numbers are selected at random (without replacement) from first six positive integers. Let *X* denote the largest of the three numbers obtained. Find the probability distribution of *X*. Also, find the mean and variance of the distribution.

Ans.

First six positive integers are 1, 2, 3, 4, 5, 6.

If three numbers are selected at random from above six numbers then the sample space S have 20 elements as

$$n(S) = {}^6C_3 = \frac{6!}{3!3!} = \frac{6\times5\times4}{3\times2} = 20$$

Here X, greatest of the three numbers obtained, is random variable. X may have value 3, 4, 5, 6. Therefore required probability distribution is given as

$P(X=3)$ = Probability of event getting 3 as greatest number.

$$= \frac{{}^2C_2}{20} = \frac{1}{20} \quad [{}^2C_2 \equiv \text{selection of two numbers out of 1, 2})$$

$P(X=4)$ = Probability of event getting 4 as greatest number.

$$= \frac{{}^3C_2}{20} = \frac{3!}{2!1!\times20} = \frac{3}{20} \quad [{}^3C_2 \equiv \text{selection of two numbers out of 1, 2, 3})$$

$P(X=5)$ = Probability of event getting 5 as greatest number.

$$= \frac{{}^4C_2}{20} = \frac{4!}{2!2!\times20} = \frac{6}{20} \quad [{}^4C_2 \equiv \text{selection of two numbers out of 1, 2, 3, 4})$$

$P(X=6)$ = Probability of event getting 6 as greatest number.

$$= \frac{{}^5C_2}{20} = \frac{5!}{2!3!20} = \frac{10}{20} \quad [{}^5C_2 \equiv \text{selection of two numbers out of 1, 2, 3, 4, 5})$$

Required probability distribution in tabular form as

***X* or *x*$_i$**	3	4	5	6
***P*(*X*) or *p*$_i$**	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

Mean = $\sum X = \sum p_i x_i = 3 \times \frac{1}{20} + 4 \times \frac{3}{20} + 5 \times \frac{6}{20} + 6 \times \frac{10}{20}$

$= \frac{3}{20} + \frac{12}{20} + \frac{30}{20} + \frac{60}{20} = \frac{105}{20} = \frac{21}{4} = 5.25$

Variance = $\sum p_i x_i^2 - (\sum X)^2$

$= \{3^2 \times \frac{1}{20} + 4^2 \times \frac{3}{20} + 5^2 \times \frac{6}{20} + 6^2 \times \frac{10}{20}\} - \left(\frac{21}{4}\right)^2$

$= \left(\frac{9}{20} + \frac{48}{20} + \frac{150}{20} + \frac{360}{20}\right) - \frac{441}{16} = \frac{567}{20} - \frac{441}{16}$

$= 28.35 - 27.56 = 0.79$

Q.8. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

Ans.

Total number of cards in the deck = 52

Number of red cards = 26

Number of cards drawn simultaneously = 2

∴ X = value of random variable = 0, 1, 2

X or x_i	$P(X)$	$x_i P(X)$	$x_i^2 P(X)$
0	$\frac{^{26}C_0 \times ^{26}C_2}{^{52}C_2} = \frac{25}{102}$	0	0
1	$\frac{^{26}C_1 \times ^{26}C_1}{^{52}C_2} = \frac{52}{102}$	$\frac{52}{102}$	$\frac{52}{102}$
2	$\frac{^{26}C_0 \times ^{26}C_2}{^{52}C_2} = \frac{25}{102}$	$\frac{50}{102}$	$\frac{100}{102}$
		$Sx_i P(X) = 1$	$\Sigma x_i^2 P(X) = \frac{152}{102}$

Mean = $\mu = \sum x_i P(X) = 1$

Variance = $\sigma^2 = \sum x_i^2 P(X) - \mu^2$ = $\frac{152}{102} - 1 = \frac{50}{102} = \frac{25}{51} = 0.49$

Q.9. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes and hence find its mean.

Ans.

Here, number of throws = 4

$P(\text{doublet})$ = $p = \frac{6}{36} = \frac{1}{6}$; $P(\text{not doublet})$ = $q = \frac{30}{36} = \frac{5}{6}$

Let X denote number of successes, then

$$P(X=0) = {}^4C_0 p^0 q^4 = 1 \times 1 \times \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$P(X=1) = {}^4C_1 \frac{1}{6} \times \left(\frac{5}{6}\right)^3 = 4 \times \frac{125}{1296} = \frac{500}{1296}$$

$$P(X=2) = {}^4C_2 \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2 = 6 \times \frac{25}{1296} = \frac{150}{1296}$$

$$P(X=3) = {}^4C_3 \left(\frac{1}{6}\right)^3 \times \frac{5}{6} = \frac{20}{1296}$$

$$P(X=4) = {}^4C_4 \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$$

Therefore the probability distribution of X is

X or x_i	0	1	2	3	4
$P(X)$ or p_i	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

$\therefore$ Mean (M) = S$x_i p_i$

$$= 0 \times \frac{625}{1296} + 1 \times \frac{500}{1296} + 2 \times \frac{150}{1296} + 3 \times \frac{20}{1296} + 4 \times \frac{1}{1296}$$

$$= \frac{500}{1296} + \frac{300}{1296} + \frac{60}{1296} + \frac{4}{1296} = \frac{864}{1296} = \frac{2}{3}$$

Q.10. *A* and *B* throw a pair of dice turn by turn. The first to throw 9 is awarded a prize. If *A* starts the game, show that the probability of *A* getting the prize is $\frac{9}{17}$.

Ans.

Let E be the event that sum of number on two dice is 9.

$$E = \{(3, 6), (4, 5), (5, 4), (6, 3)\}$$

$$P(E) = \frac{4}{36} = \frac{1}{9} \quad \Rightarrow \quad P(E') = \frac{8}{9}$$

$$P\,(A \text{ getting the prize}) = P\,(A) = \frac{1}{9} + \frac{8}{9} \times \frac{8}{9} \times \frac{1}{9} + \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{1}{9} + \ldots\ldots..$$

$$= \frac{1}{9}\left(1 + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^4 + \left(\frac{8}{9}\right)^6 + \ldots..\right)$$

$$= \frac{1}{9}\frac{1}{\left[1-\left(\frac{8}{9}\right)^2\right]} = \frac{1}{9}.\frac{9^2}{(9^2-8^2)} = \frac{9}{17}$$

Q.11. The probability that *A* hits a target is $\frac{1}{3}$ and the probability that *B* hits it is $\frac{2}{5}$ If each one of *A* and *B* shoots at the target, what is the probability that

i. **the target is hit?**
ii. **exactly one of them hits the target?**

Ans.

Let $P(A)$ = Probability that A hits the target = $\frac{1}{3}$

$P(B)$ = Probability that B hits the target = 2/5

i. $P(\text{target is hit}) = P(\text{at least one of } A, B \text{ hits})$

$= 1 - P \text{ (none hits)}$

$= 1 - \frac{2}{3} \times \frac{3}{5} = \frac{9}{15} = \frac{3}{5}$

ii. $P(\text{exactly one of them hits}) = P\,(A \text{ and } \bar{B} \text{ or } \bar{A} \text{ and } B) = P(A \cap \bar{B} \cup \bar{A} \cap B)$

$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$

$= \frac{1}{3} \times \frac{3}{5} + \frac{2}{3} \times \frac{2}{5} = \frac{7}{15}$

Q.12. A family has 2 children. Find the probability that both are boys, if it is known that

i. **at least one of the children is a boy**
ii. **the elder child is a boy.**

Ans.

A family has 2 children, then

Sample space $= S = \{BB, BG, GB, GG\}$, where $B =$ Boy, $G =$ Girl

i. Let us define the following events:

A : at least one of the children is boy : $\{BB, BG, GB\}$

B : both are boys: $\{BB\}$

$\therefore \quad A \cap B = \{BB\}$

$$\Rightarrow \quad P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{3/4} = \frac{1}{3}$$

ii. Let A : elder child boy : $\{BB, BG\}$

B : both are boys: $\{BB\}$

$\therefore \quad A \cap B : \{BB\}$

$$\Rightarrow \quad P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{2/4} = \frac{1}{2}$$

Q.13. An experiment succeeds twice often as it fails. Find the probability that in the next six trials there will be at least 4 successes.

Ans.

An experiment succeeds twice as often as it fails.

$\therefore \quad p = P(\text{success}) = \frac{2}{3}$ and $q = P(\text{failure}) = \frac{1}{3}$

Number of trials $= n = 6$

By the help of Binomial distribution,

$$P(r) = {}^6C_r\left(\frac{2}{3}\right)^r\left(\frac{1}{3}\right)^{6-r}$$

$P(\text{at least four success}) = P(4) + P(5) + P(6)$

$$= {}^6C_4\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4 + {}^6C_5\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)^5 + {}^6C_6\left(\frac{2}{3}\right)^6$$

$$= \left(\frac{2}{3}\right)^4\left[\frac{1}{9}\ ({}^6C_4) + \frac{2}{9}\ ({}^6C_5) + \frac{4}{6}\ ({}^6C_6)\right]$$

$$= \left(\frac{2}{3}\right)^4\left[\frac{15}{9} + \frac{2}{9} \times 6 + \frac{4}{9}\right] = \frac{16}{81} \times \frac{31}{9} = \frac{496}{726}$$

Q.14. Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Ans.

The repeated throws of a die are Bernoulli trials.

Let X denotes the number of sixes in 6 throws of die.

Obviously, X has the binomial distribution with $n = 6$

and $p = \frac{1}{6}, q = 1 - \frac{1}{6} = \frac{5}{6}$

where, p is probability of getting a six and q is probability of not getting a six

Now, Probability of getting at most 2 sixes in 6 throws $= P(X = 0) + P(X = 1) + P(X = 2)$

$$= {}^6C_0 . p^0 . q^6 + {}^6C_1 p^1 q^5 + {}^6C_2 p^2 q^4$$

$$= \left(\frac{5}{6}\right)^6 + \frac{6!}{1!5!} . \frac{1}{6} . \left(\frac{5}{6}\right)^5 + \frac{6!}{2!4!} . \left(\frac{1}{6}\right)^2 . \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^6 + 6 . \frac{1}{6}\left(\frac{5}{6}\right)^5 + \frac{6 \times 5}{2} \times \left(\frac{1}{6}\right)^2 . \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^4\left[\frac{25}{36} + \frac{5}{6} + \frac{5}{12}\right]$$

$$= \left(\frac{5}{6}\right)^4 \times \frac{25+30+15}{36} = \left(\frac{5}{6}\right)^4 \times \frac{70}{36} = \frac{21875}{23328}$$

Q.15. A bag *A* contains 4 black and 6 red balls and bag *B* contains 7 black and 3 red balls. *A* die is thrown. If 1 or 2 appears on it, then bag *A* is chosen, otherwise bag *B*. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

Ans.

Let E, F and A be three events such that

E = selection of bag A. and F = selection of bag B.

A = getting one red and one black ball out of two.

Here, $P(E) = P(\text{getting 1 or 2 in a throw of die}) = \frac{2}{6} = \frac{1}{3}$

$\therefore \quad P(F) = 1 - \frac{1}{3} = \frac{2}{3}$

Also, $P(A/E) = P$ (getting one red and one black if bag A is selected) $= \frac{{}^6C_1 \times {}^4C_1}{{}^{10}C_2} = \frac{24}{45}$

and $P(A/F) = P$ (getting one red and one black if bag B is selected) $= \frac{{}^3C_1 \times {}^7C_1}{{}^{10}C_2} = \frac{21}{45}$

Now, by theorem of total probability,

$$P(A) = P(E).\,P(A/E) + P(F).\,P(A/F)$$

$$\Rightarrow \quad P(A) = \frac{1}{3} \times \frac{24}{45} + \frac{2}{3} \times \frac{21}{45} = \frac{8+14}{45} = \frac{22}{45}$$

Q.16. For 6 trials of an experiment, let *X* be a binomial variate which satisfies the relation

9*P* (*X* = 4) = *P* (*X* = 2). Find the probability of success.

Ans.

Let the probability of success be p.

$\therefore$ the probability of failure = $1 - p$.

Here X is a binomial variate with parameters $n = 6$, p and $(1 - p)$.

Now, according to question

$$9P(X = 4) = P(X = 2)$$

$$\Rightarrow \quad 9.\,{}^6C_4.p^4.(1-p)^2 = {}^6C_2 p^2 (1-p)^4$$

$$\Rightarrow \quad \frac{9\ {}^6C_4}{{}^6C_2} = \frac{p^2(1-p)^4}{p^4(1-p)^2}$$

$$\Rightarrow \quad \frac{9.6!}{4!2!} \times \frac{2!4!}{6!} = \frac{(1-p)^2}{p^2}$$

$$\Rightarrow \quad 9 = \frac{(1-p)^2}{p^2}$$

$$\Rightarrow \quad 9p^2 = 1 - 2p + p^2$$

$$\Rightarrow \quad 8p^2 + 2p - 1 = 0$$

$$\Rightarrow \quad 8p^2 + 4p - 2p - 1 = 0$$

$$\Rightarrow \quad 4p(2p + 1) - 1(2p + 1) = 0$$

$$\Rightarrow \quad (4p - 1)(2p + 1) = 0$$

$$\Rightarrow \quad 4p - 1 = 0 \text{ or } 2p + 1 = 0$$

$$\Rightarrow \quad p = \frac{1}{4} \text{ or } p = -\frac{1}{2} \text{ (Not acceptable due to being negative)}$$

$$\Rightarrow \quad p = \frac{1}{4} \text{ is the required probability of success.}$$

Q.17. Three persons *A, B* and *C* apply for a job of manager in a private company. Chances of their selection (*A*, *B* and *C*) are in the ratio 1 : 2 : 4. The probabilities that *A*, *B* and *C* can introduce changes to improve profits of the company are 0.8,

0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of *C*.

Ans.

Let E_1, E_2, E_3 and A be events such that

E_1 = Person selected is A

E_2 = Person selected is B

E_3 = Person selected is C

A = Changes to improve profit does not take place.

Now $P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$

$$p\left(\frac{A}{E_1}\right) = 1 - \frac{8}{10} = \frac{2}{10}$$

$$P\left(\frac{A}{E_2}\right) = 1 - \frac{5}{10} = \frac{5}{10}$$

$$P\left(\frac{A}{E_3}\right) = 1 - \frac{3}{10} = \frac{7}{10}$$

We require $P\left(\frac{E_3}{A}\right)$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3).P\left(\frac{A}{E_3}\right)}{p(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{4}{7} \times \frac{7}{10}}{\frac{1}{7} \times \frac{2}{10} + \frac{2}{7} \times \frac{5}{10} + \frac{4}{7} \times \frac{7}{10}}$$

$$= \frac{28}{70} \times \frac{70}{2 + 10 + 28}$$

$$= \frac{28}{40} = \frac{7}{10}$$

Q.18. In a game, a man wins ₹5 for getting a number greater than 4 and loses ₹1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.

Ans.

Let X be random variable, which is possible value of winning or loosing of rupee occur with probability of getting a number greater than 4 in 1st, 2nd, 3rd or in any throw respectively.

Obviously X may have value ₹5, ₹4, ₹3 and – ₹3 respectively.

Now, $P(X = 5) = P$ (getting number greater than 4 in first throw)

$$= \frac{2}{6} = \frac{1}{3}$$

$P(X = 4) = P$ (getting number greater than 4 in 2nd throw)

$$= \frac{4}{6} \times \frac{2}{6} = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

$P(X = 3) = P$ (getting number greater than 4 in 3rd throw)

$$= \frac{4}{6} \times \frac{4}{6} \times \frac{2}{6} = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$$

$P(X = -3) = P$ (getting number greater than 4 in no throw)

$$= \frac{4}{6} \times \frac{4}{6} \times \frac{4}{6} = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

Therefore, probability distribution is as

***X* or x_i**	5	4	3	–3
***P*(*X*) or p_i**	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{4}{27}$	$\frac{8}{27}$

∴ Expected value of the amount he wins/loses = $E(x)$

$$\sum x_i P(x_i) = 5 \times \frac{1}{3} + 4 \times \frac{2}{9} + 3 \times \frac{4}{27} + (-3) \times \frac{8}{27}$$

$$= \frac{5}{3} + \frac{8}{9} + \frac{12}{27} - \frac{24}{27} = \frac{45+24+12-24}{27} = \frac{57}{27} = ₹\,\frac{19}{9} = ₹\,2\frac{1}{9}$$

Q.19. A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

Ans.

There may be three situations as events.

E_1 = Bag contains 2 white balls.

E_2 = Bag contains 3 white balls.

E_3 = Bag contains all 4 white balls.

A = Getting two white balls.

We have required $P\left(\frac{E_3}{A}\right) = ?$

Now, $P(E_1) = \frac{1}{3}; \quad P(E_2) = \frac{1}{3}; \qquad P(E_3) = \frac{1}{3}$

$$P\left(\frac{A}{E_1}\right) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}$$

$$P\left(\frac{A}{E_2}\right) = \frac{{}^3C_2}{{}^4C_2} = \frac{3}{6} = \frac{1}{2}$$

$$P\left(\frac{A}{E_3}\right) = \frac{{}^4C_2}{{}^4C_2} = 1$$

Now, $P\left(\frac{E_3}{A}\right) = \dfrac{P(E_3).P\left(\frac{A}{E_3}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1} = \frac{\frac{1}{3}}{\frac{1}{18}+\frac{1}{6}+\frac{1}{3}}$$

$$= \frac{\frac{1}{3}}{\frac{10}{18}} = \frac{1}{3}\times\frac{18}{10} = \frac{3}{5}$$

Q.20. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, then that at least one girl must be there in the committee.

Ans.

Let A and B be two events such that

A = Selection of committee having exactly 2 boys.

B = Selection of committee having at least one girl.

The required probability is $P\left(\frac{A}{B}\right)$

Now, $P\left(\frac{A}{B}\right) = \frac{P\ (A \cap\ B)}{P(B)}$

$$P(B) = \frac{{}^4C_1 \times {}^7C_3 + {}^4C_2 \times {}^7C_2 + {}^4C_3 \times {}^7C_1 + {}^4C_4}{{}^{11}C_4}$$

$$= \frac{\frac{4!}{1!3!} \times \frac{7!}{3! \times 4!} + \frac{4!}{2!2!} \times \frac{7!}{2!5!} + \frac{4!}{3!1!} \times \frac{7!}{1\ !\ 6!} + \frac{4!}{4\ !\ 0!}}{\frac{11\ !}{4!7!}}$$

$$= \frac{4 \times \frac{7 \times 6 \times 5}{3 \times 2} + \frac{4 \times 3}{2 \times 1} \times \frac{7 \times 6}{2 \times 1} + 4 \times 7 + 1}{\frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1}}$$

$$= \frac{140+126+28+1}{330} = \frac{295}{330} = \frac{59}{66}$$

$$P(A \cap \mathrm{B}) = \frac{{}^4C_2 \times {}^7C_2}{{}^{11}C_2} = \frac{\frac{4!}{2!2!} \times \frac{7!}{2!5!}}{\frac{11!}{2!7!}} = \frac{\frac{4 \times 3}{2 \times 1} \times \frac{7 \times 6}{2 \times 1}}{330}$$

$$= \frac{126}{330} = \frac{21}{55}$$

$$\therefore \quad P\left(\frac{A}{B}\right) = \frac{\frac{21}{55}}{\frac{59}{66}} = \frac{21}{55} \times \frac{66}{59} = \frac{126}{295}$$

Q.21. A bag *X* contains 4 white balls and 2 black balls, while another bag *Y* contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag *Y*.

Ans.

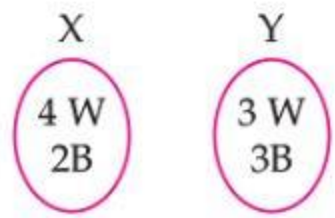
Let E_1, E_2 and A be three events.

E_1 = selection of bag X

E_2 = selection of bag Y

A = getting one black and one white ball.

Now, $P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}$



$$P\left(\frac{A}{E_1}\right) = \frac{{}^4C_1 \times {}^2C_1}{{}^6C_2} = \frac{4\times2}{\frac{6!}{2!4!}} = \frac{4\times2\times2}{6\times5} = \frac{8}{15}$$

$$P\left(\frac{A}{E_2}\right) = \frac{{}^3C_1 \times {}^3C_1}{{}^6C_2} = \frac{3\times3}{\frac{6!}{2!4!}} = \frac{3\times3\times2}{6\times5} = \frac{9}{15}$$

We require $P\left(\frac{E_2}{A}\right)$

$$\therefore \quad P\left(\frac{E_2}{A}\right) = \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{9}{15}}{\frac{1}{2}\times\frac{8}{15}+\frac{1}{2}\times\frac{9}{15}} = \frac{\frac{9}{30}}{\frac{8}{30}+\frac{9}{30}} = \frac{9}{17}$$

Q.22. Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Ans.

Let E_1 , E_2 and A be event such that

E_1 = Selecting male person

E_2 = Selecting women (female person)

A = Selecting grey haired person.

Then $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$

$P\left(\frac{A}{E_1}\right) = \frac{5}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{0.25}{100}$

Here, required probability is $P\left(\frac{E_1}{A}\right)$.

$$\therefore \quad P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right)}$$

$$\therefore \quad P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{2} \times \frac{5}{100}}{\frac{1}{2} \times \frac{5}{100} + \frac{1}{2} \times \frac{0.25}{100}} = \frac{5}{5+0.25} = \frac{500}{525} = \frac{20}{21}$$

Q.23. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

Ans.

Let E_1 , E_2 , E_3 be events such that

$E_1 \equiv$ Selection of Box I ;

$E_2 \equiv$ Selection of Box II ;

$E_3 \equiv$ Selection of Box III

Let A be event such that

$A \equiv$ the coin drawn is of gold

Now, $P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{3},$

$$P\left(\frac{A}{E_1}\right) = P \text{ (a gold coin from box } I) = \frac{2}{2} = 1$$

$$P\left(\frac{A}{E_2}\right) = P \text{ (a gold coin from box II)} = 0$$

$$P\left(\frac{A}{E_3}\right) = P \text{ (a gold coin from box III)} = \frac{1}{2}$$

The probability that the other coin in the box is also of gold $= P\left(\frac{E_1}{A}\right)$

$$\therefore \quad P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times 0+\frac{1}{3}\times\frac{1}{2}} = \frac{2}{3}$$

Q.24. There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let *X* denote the sum of the numbers on the two drawn cards. Find the mean and variance of *X*.

Ans.

If two cards, from four cards having numbers 1, 2, 3, 4 each are drawn at random then sample space S is given by

$$S = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (4, 1), (4, 2), (4, 3), (3, 1), (3, 2), (3, 4)\}$$

Let X, sum of the numbers, be random variable. X may have value 3, 4, 5, 6, 7.

Now $P(X = 3)$ = Probability of event getting $(1, 2), (2, 1) = \frac{2}{12} = \frac{1}{6}$

$P(X = 4)$ = Probability of event getting $(1, 3), (3, 1) = \frac{2}{12} = \frac{1}{6}$

$P(X = 5)$ = Probability of event getting $(1, 4), (4, 1), (2, 3), (3, 2) = \frac{4}{12} = \frac{1}{3}$

$P(X = 6)$ = Probability of event getting $(4, 2)\ (2, 4) = \frac{2}{12} = \frac{1}{6}$

$P(X = 7)$ = Probability of event getting $(4, 3)\ (3, 4) = \frac{2}{12} = \frac{1}{6}$

Thus, probability distribution is represented in tabular form as

X	3	4	5	6	7
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$
$X.P(X)$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{3}$	$\frac{6}{6}$	$\frac{7}{6}$
$X^2 P(X)$	$\frac{9}{6}$	$\frac{16}{6}$	$\frac{25}{3}$	$\frac{36}{6}$	$\frac{49}{6}$

$\therefore \quad \text{Mean} = \sum X.P(X) = \frac{3}{6} + \frac{4}{6} + \frac{5}{3} + \frac{6}{6} + \frac{7}{6}$

$= \frac{3+4+10+6+7}{6} = \frac{30}{6} = 5$

$\text{Variance} = \sum X^2 P(X) - (\sum X. P(X))^2$

$= \left(\frac{9}{6} + \frac{16}{6} + \frac{25}{3} + \frac{36}{6} + \frac{49}{6}\right) - (5)^2$

$= \frac{9+16+50+36+49}{6} - 25$

$= \frac{160}{6} - 25 = \frac{160 - 150}{6} = \frac{10}{6} = \frac{5}{3}.$

Q.25. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the probability of the lost card being of clubs.

Ans.

Let A_1 , E_1 and E_2 be the events defined as follows:

A : cards drawn are both clubs

E_1 : lost card is club

E_2 : lost card is not a club

Then, $P(E_1) = \frac{13}{52} = \frac{1}{4}, \quad P(E_2) = \frac{39}{52} = \frac{3}{4}$

$P(A / E_1)$ = Probability of drawing both club cards when lost card is club $= \frac{12}{51} \times \frac{11}{50}$

$P(A / E_2)$ = Probability of drawing both club cards when lost card is club $= \frac{13}{51} \times \frac{12}{50}$

To find : $P(E_1 / A)$

By Baye's Theorem,

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{4} \times \frac{12}{51} \times \frac{11}{50}}{\frac{1}{4} \times \frac{12}{51} \times \frac{11}{50} + \frac{3}{4} \times \frac{13}{51} \times \frac{12}{50}}$$

$$= \frac{12 \times 11}{12 \times 11 + 3 \times 13 \times 12}$$

$$= \frac{11}{11+39} = \frac{11}{50}$$

Q.26.

p and $P(X = 2) = P(X = 3)$ such that $\sum pi\, xi^2 = 2\sum p_i x_i$, find the value of p.

Ans.

Given X is a random variable with values 0, 1, 2, 3. Given probability distributions are as

$X(x_i)$	0	1	2	3
$P(x)(p_i)$	p	p	a	a
$x_i p_i$	0	p	$2a$	$3a$
$x_i^2 p_i$	0	p	$4a$	$9a$

$\therefore \quad \sum x_i p_i = 0 + p + 2a + 3a = p + 5a$

$\sum x_i^2 p_i p_i = 0 + p + 4a + 9a = p + 13a$

According to question

$\sum p_i x_i^2 = 2\sum p_i x_i$

$p + 13a = 2p + 10a \quad \Rightarrow \quad p = 3a$

Also $p + p + a + a = 1$

$2\,p + 2a = 1$

$2a = 1 - 2p \quad \Rightarrow \quad a = \frac{1-2p}{2}$

$\therefore \quad p = 3 \times \frac{(1-2p)}{2} \quad \Rightarrow \quad 2\,p = 3 - 6p$

$\Rightarrow \quad 8p = 3 \quad \Rightarrow \quad p = \frac{3}{8}$

Q.27. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the mean and variance of the number of successes.

Ans.

Here number of throws = 4

$P(\text{doublet}) = p = \frac{6}{36} = \frac{1}{6}$

$P(\text{not doublet}) = q = \frac{30}{36} = \frac{5}{6}$

Let X denotes number of successes, then

$$P(X=0) = {}^4C_0 P^0 q^4 = 1 \times 1 \times \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$P(X=1) = {}^4C_1 \frac{1}{6} \times \left(\frac{5}{6}\right)^3 = 4 \times \frac{125}{1296} = \frac{500}{1296}$$

$$P(X=2) = {}^4C_2\left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2 = 6 \times \frac{25}{1296} = \frac{150}{1296}$$

$$P(X=3) = {}^4C_3\left(\frac{1}{6}\right)^3 \times \frac{5}{6} = \frac{20}{1296}$$

$$P(X=4) = {}^4C_4\left(\frac{1}{6}\right)^4 \times \frac{5}{6} = \frac{1}{1296}$$

Being a binomial distribution with

$$n = 4, p = \frac{1}{6} \text{ and } q = \frac{5}{6}$$

$$\mu = \text{mean} = \text{np} = 4 \times \frac{1}{6} = \frac{2}{3}$$

$$\sigma^2 = \text{variance} = \text{npq} = 4 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{9}.$$

Q.28. Three machines E_1, E_2, E_3 in a certain factory produce 50%, 25% and 25% respectively, of the total daily output of electric tubes. It is known that 4% of the tube produced on each of machines E_1 and E_2 are defective and that 5% of those produced on E_3, are defective. If one tube is picked up at random from a day's production, calculate the probability that it is defective.

Ans.

Let A be the event that the picked up tube is defective.

Let A_1, A_2, A_3 be events such that

A_1 = event of producing tube by machine E_1

A_2 = event of producing tube by machine E_2

A_3 = event of producing tube by machine E_3

$$P(A_1) = \frac{50}{100} = \frac{1}{2}, \qquad P(A_2) = \frac{25}{100} = \frac{1}{4}, \quad P(A_3) = \frac{25}{100} = \frac{1}{4}$$

Also, $P\left(\frac{A}{A_1}\right) = \frac{4}{100} = \frac{1}{25}$

$$P\left(\frac{A}{A_2}\right) = \frac{4}{100} = \frac{1}{25} \text{ and } P\left(\frac{A}{A_3}\right) = \frac{5}{100} = \frac{1}{20}$$

Now, $P(A)$ is required.

From concept of total probability,

$$P(A) = P(A_1).P\left(\frac{A}{A_1}\right) + P(A_2).P\left(\frac{A}{A_2}\right) + P(A_3).P\left(\frac{A}{A_3}\right)$$

$$= \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20} = \frac{1}{50} + \frac{1}{100} + \frac{1}{80}$$

$$= \frac{8+\ 4+5}{400} = \frac{17}{400} = 0.0425$$

Q.29. In a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Ans.

Let p = probability of correct answer = $\frac{1}{3}$

q = probability of incorrect answer = $\frac{2}{3}$

Here, total number of questions = 5

$P(4 \text{ or more correct}) = P(4 \text{ correct}) + P(5 \text{ correct})$

$$=^5 C_4 p^4 q^1 +^5 C_5 p^5 q^0 \quad \left[\text{Using } P(r \text{ success}) = {}^nC_r p^r q^{n-r}\right]$$

$$= 5 \times \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) + 1 \times \left(\frac{1}{3}\right)^5 = 5 \times \frac{1}{81} \times \frac{2}{3} + \frac{1}{243} = \frac{11}{243}$$

Q.30. *A* speaks truth in 60% of the cases, while *B* in 90% of the cases. In what per cent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of *B* will carry more weight as he speaks truth in more number of cases than *A*?

Ans.

Let E be the event that A speaks truth and F be the event that B speaks truth. Then, E and F are independent events such that

$$P(E) = \frac{60}{100} = \frac{3}{5} \text{ and } P(F) = \frac{90}{100} = \frac{9}{10}$$

A and B will contradict each other in narrating the same fact in the following mutually exclusive ways:

(I) A speaks truth and B tells a lie *i.e.,* $E \cap \bar{F}$

(II) A tells a lie and B speaks truth *i.e.,* $\bar{E} \cap F$

$\therefore$ P (A and B contradict each other)

$$= P\ (I \text{ or II}) = P\ (I \cup \text{II}) = P\ [(E \cap \bar{F}) \cup (\bar{E} \cap F)]$$

$$= P(E \cap \bar{F}) + P\ (\bar{E} \cap F) \qquad [\because E \cap \bar{F} \text{ and } \bar{E} \cap F \text{ are mutually exclusive}]$$

$$= P\ (E)\ P\ (\bar{F}) + P\ (\bar{E})\ P\ (F) \qquad [\ E \text{ and } F \text{ are independent}]$$

$$= \frac{3}{5} \times \left(1 - \frac{9}{10}\right) + \left(1 - \frac{3}{5}\right) \times \frac{9}{10} = \frac{3}{5} \times \frac{1}{10} + \frac{2}{5} \times \frac{9}{10} = \frac{21}{50}$$

Yes, the statement of B will carry more weight as the probability of B to speak truth is more than that of A.

Q.31. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is actually a six.

Ans.

Let E_1, E_2 and E be three events such that

E_1 = six occurs

E_2 = six does not occur

E = man reports that six occurs in the throwing of the dice.

Now, $P(E_1) = \frac{1}{6}$, $P(E_2) = \frac{5}{6}$

$$P\left(\frac{E}{E_1}\right) = \frac{4}{5}, P\left(\frac{E}{E_2}\right) = 1 - \frac{4}{5} = \frac{1}{5}$$

We have to find $P\left(\frac{E_1}{E}\right)$

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1).P\left(\frac{E}{E_1}\right)}{P(E_1).P\left(\frac{E}{E_1}\right) + P(E_2).P\left(\frac{E}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}} = \frac{4}{30} \times \frac{30}{4+5} = \frac{4}{9}$$

Q.32. In shop *A*, 30 tin pure ghee and 40 tin adulterated ghee are kept for sale while in shop *B*, 50 tin pure ghee and 60 tin adulterated ghee are there. One tin of ghee is purchased from one of the shops randomly and it is found to be adulterated. Find the probability that it is purchased from shop *B*.

Ans.

Let the event be defined as

E_1 = Selection of shop A.

E_2 = Selection of shop B.

A = Purchasing of a tin having adulterated ghee.

$P(E_1) = \frac{1}{2}$ $\qquad$ $P(E_2) = \frac{1}{2}$

$P\left(\frac{A}{E_1}\right) = \frac{40}{70} = \frac{4}{7},$ $\qquad$ $P\left(\frac{A}{E_2}\right) = \frac{60}{110} = \frac{6}{11}$

$P\left(\frac{E_2}{A}\right)$ = required

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{6}{11}}{\frac{1}{2}\cdot\frac{4}{7}+\frac{1}{2}\cdot\frac{6}{11}} = \frac{\frac{3}{11}}{\frac{2}{7}+\frac{3}{11}} = \frac{21}{43}$$

Q.33. The probabilities of two students *A* and *B* coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, '*A* coming in time' and '*B* coming in time' are independent, find the probability of only one of them coming to the school in time.

Ans.

Let E_1 and E_2 be two events such that

E_1 = A coming to the school in time.

E_2 = B coming to the school in time.

Here, $P(E_1) = \frac{3}{7}$ and $P(E_2) = \frac{5}{7}$

$$P(\bar{E}_1) = \frac{4}{7}, P(\bar{E}_2) = \frac{2}{7}$$

P (only one of them coming to the school in time)

$$= P(E_1) \times P(\bar{E}_2) + P(\bar{E}_1) \times P(E_2)$$

$$= \frac{3}{7} \times \frac{2}{7} + \frac{5}{7} \times \frac{4}{7}$$

$$= \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$$

Q.34. In a hockey match, both teams *A* and *B* scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not.

Ans.

Let E_1, E_2 be two events such that

E_1 = The captain of team 'A' gets a six.

E_2 = The captain of team 'B' gets a six.

Here, $P(E_1) = \frac{1}{6}$, $P(E_2) = \frac{1}{6}$

$$P(E_1)' = 1 - \frac{1}{6} = \frac{5}{6}, P(E_2)' = 1 - \frac{1}{6} = \frac{5}{6}$$

Now, P (winning the match by team A) $= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots\ldots$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 . \frac{1}{6} + \left(\frac{5}{6}\right)^4 . \frac{1}{6} + \ldots\ldots = \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

P (winning the match by team B) $= 1 - \frac{6}{11} = \frac{5}{11}$

[**Note:** If a be the first term and r the common ratio then sum of infinite terms $S_\infty = \frac{a}{1-r}$]

Long Answer Questions-I (OIQ)

[4 Marks]

Q.1. A problem in mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the (*i*) problem is solved? (*ii*) exactly one of them will solve it?

Ans.

(i) Let A, B, C be the respective events of solving the problem. Then $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$. Clearly A, B, C are independent events and the problem is solved if at least one student solves it.

$\therefore$ Required probability = $P(A \cup B \cup C) = 1 - P(\overline{A})P(\overline{B})P(\overline{C})$

$= 1 - \left[1 - \frac{1}{2}\right]\left[1 - \frac{1}{3}\right]\left[1 - \frac{1}{4}\right] = 1 - \frac{1}{2}.\frac{2}{3}.\frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$

(ii) Required probability = $P(A \cap \overline{B} \cap \overline{C}) + P(\overline{A} \cap B \cap \overline{C}) + P(\overline{A} \cap \overline{B} \cap C)$

$= P(A).\ P(\overline{B}).\ P(\overline{C}) + P(\overline{A}).\ P(B).\ P(\overline{C}) + P(\overline{A}).\ P(\overline{B}).\ P(C)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} = \frac{6+3+2}{24} = \frac{11}{24}$

Q.2. If two dice are rolled 12 times, obtain the mean and variance of the distribution of success, if getting a total greater than 4 is considered a success.

Ans.

Let X denote the number of success in 12 trials. Then X follows binomial distribution with parameters n = 12 and p = probability of getting a total greater than 4 in a single throw of a pair of dice

$= 1 - \frac{6}{36} = \frac{5}{6}$

$\therefore\ q = 1 - p = 1 - \frac{5}{6} = \frac{1}{6}$

Now, Mean = $np = \frac{5}{6} \times 12 = 10$

and Variance = $npq = 12 \times \frac{5}{6} \times \frac{1}{6} = \frac{5}{3}$

Q.3. Six coins are tossed simultaneously. Find the probability of getting

i. **3 heads**
ii. **no heads**
iii. **at least one head.**

Ans.

Let p be the probability of getting a head in the toss of a coin.

Then, $p = \frac{1}{2} \quad \Rightarrow \quad q = 1 - p = \frac{1}{2}$

Let X = Number of successes in the experiment, then X can take the values, 0, 1, 2, 3, 4, 5, 6

Here $n = 6$. Now by Binomial distribution, we have

$$P(X = r) = {}^{n}C_r \, p^r \, . \, q^{n-r}$$

(i) $P(X = 3) = {}^{6}C_3 \left(\frac{1}{2}\right)^3 . \left(\frac{1}{2}\right)^{6-3} = \frac{20}{2^6} = \frac{20}{64} = \frac{5}{16}$

(ii) $P(X = 0) = {}^{6}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0} = \frac{1}{64}$

(iii) P (at least one head) = $1 - P$ (no head) = $1 - P(X = 0) = 1 - \frac{1}{64} = \frac{63}{64}$.

Q.4. The probability that a student entering a university will graduate is 0.4. Find out the probability that out of 3 students of the university:

i. **none will graduate**
ii. **only one will graduate**
iii. **all will graduate.**

Ans.

Let p denote the probability of a student of university will graduate.

$p = 0.4$ and $q = 1 - p = 1 - 0.4 = 0.6$

Let X denote the number of graduates entering a university. Then, X is a binomial variate with parameters, $n = 3$, $p = 0.4$, $q = 0.6$

Now, probability of getting r graduates = $P(X = r) = {}^{3}C_r (0.4)^r . (0.6)^{3-r}$, $r = 0, 1, 2, 3$

i. Probability of none will graduate = $P(X = 0) = {}^{3}C_0 (0.4)^0 (0.6)^{3-0} = 1 \times 1 \times (0.6)^3 = 0.216$

ii. Probability of only one will graduate =

$P(X = 1) = {}^{3}C_1 (0.4)^1 (0.6)^{3-1} = 3 \times 0.4 \times 0.36 = 0.432$

iii. Probability of all will graduate = $P(X = 3) = {}^{3}C_3 (0.4)^3 = 1 \times 0.4 \times 0.4 \times 0.4 = 0.064$

Q.5. A student is given a test with 8 items of true-false type. If he gets 6 or more items correct, he is declared pass. Given that he guesses the answer to each item, compute the probability that he will pass in the test.

Ans.

Let E_1, E_2 and A be the following events:

E_1 : Student makes a guess and gets true answer.

E_2 : Student makes a guess and gets false answer.

A : The event that the student will pass the test.

$\therefore P(E_1) = P(E_2) = \frac{1}{2}$;

$P(A/E_1) = \frac{3}{8}$ (because if he gets 6, 7, or 8 items correct then he will pass *i.e.,* 3 cases must be required to pass)

$P(A/E_2) = \frac{5}{8}$

$$\therefore \quad P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{3}{8}}{\frac{1}{2}\times\frac{3}{8}+\frac{1}{2}\times\frac{5}{8}} = \frac{3/8}{3/8+5/8} = \frac{3}{8}$$

Q.6. If each element of a second order determinant is either 0 or 1, what is the probability that the value of determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value assumed with probability $\frac{1}{2}$.

Ans.

There are four entries in determinant of 2×2 order.

Each entry may be filled up in two ways with 0 or 1.

Therefore, number of determinant that can be formed $2^4 = 16$

The value of determinant is positive in the following cases

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \qquad i.e., \quad 3 \text{ determinants.}$$

Thus, the probability that the determinants is positive = $\frac{3}{16}$.

Q.7. Let d_1, d_2, d_3 be three mutually exclusive diseases. Let S = {S_1, S_2, ...S_6} be the set of observable symptoms of these diseases. For example, S_1 is the shortness of breath, S_2 is the loss of weight, S_3 is fatigue, etc. Suppose a random sample of 10000 patients contains 3200 patients with disease d_1, 3500 with disease d_2 and 3300 with disease d_3. Also, 3100 patients with disease d_1 3300 with disease d_2 and 3000 with disease d_3 show the symptoms *S*. Knowing that the patient has symptoms *S*, the doctor wishes to determine the patient's illness. On the basis of this information, what should the doctor conclude?

Ans.

Let D_1 denote the event that the patient has disease d_1 The events D_2 and D_3 are defined similarly.

Then, $P(D_1) = \frac{3200}{1000} = 0.32$, $\quad P(D_2) = \frac{3500}{10000} = 0.35 \quad$ and $\quad P(D_3) = \frac{3300}{10000} = 0.33$

Let S be the event that the patient shows the symptom S

Then, $P(S/D_1) = \frac{P(S \cap D)}{P(D_1)} = \frac{3100}{3200} = \mathbf{0.97}$ (approx.)

$P(S/D_2) = \frac{3300}{3500} = 0.94$ (approx.) and $P(S/D_3) = \frac{3000}{3300} = 0.97$ (approx.)

Using Bayes' theorem, we get

$P(D_1/S)$ = The probability that the patient has disease d_1 knowing that he/she has symptoms S_1. S_2, ..., S_6

$$P(D_1/S) = \frac{P(D_1)P(S/D_1)}{P(D_1)P(S/D_1)+P(D_2)P(S/D_2)+P(D_3)(S/D_3)}$$

$$= \frac{0.32\times0.97}{0.32\times0.97+0.35\times0.94+0.33\times0.91}$$

$$= \frac{0.3104}{0.3104+0.329+0.3003} = \frac{0.3104}{0.9397} = 0.33 \text{ approx.}$$

Similarly, $P(D_2/S) = \frac{0.329}{0.9397} = 0.35$ approx.

$P(D_3/S) = \frac{0.3003}{0.9397} = 0.32$ approx.

Thus, knowing that the patient has symptoms S_1, S_2, ..., S_6, the probability that he has disease d_1 is 0.33, the probability that he has disease d_2 is 0.35, the probability that he has disease d_3 is 0.32. Therefore, the doctor should conclude that the patient is most likely to have disease d_2.

Q.8. Let *X* denote the number of hours you study during a randomly selected school day. The probability that *X* can take the values *x*, has the following form, where *k* is some unknown constant.

$$P(X=x) = \begin{cases} 0.1 & \text{if } x=0 \\ \text{kx}, & \text{if } x = 1 \text{ or } 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

a. **Find the value of *k*.**
b. **What is the probability that you study (*i*) At least two hours? (*ii*) Exactly two hours? (*iii*) At most two hours?**

Ans.

The probability distribution of *X* is:

X	0	1	2	3	4
***P*(*X*)**	0.1	*k*	2*k*	2*k*	*k*

a. We know that $\sum_{i=1}^{n} p_i = 1$

Therefore $0.1 + k + 2k + 2k + k = 1$

i.e., $k = 0.15$

b. P (you study at least two hours) $= P(X \geq 2)$

$= P(X = 2) + P(X = 3) + P(X = 4)$

$= 2k + 2k + k = 5k = 5 \times 0.15 = 0.75$

P (you study exactly two hours) $= P(X = 2) = 2 \times 0.15 = 0.3$

P (you study at most two hours) $= P(X \leq 2)$

$= P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.1 + k + 2k = 0.1 + 3k = 0.1 + 3 \times 0.15 = 0.55$