Sample Paper 16

Class- X Exam - 2022-23

Mathematics - Basic

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section - A consists of 20 questions of 1 mark each.)

1. The distance of the point (7, –8) from the origin is:

(a) $\sqrt{112}$ (b) $\sqrt{115}$

(c) $\sqrt{113}$ (d) 96 1

2. The ratio in which the line segment joining the points (–1, 7) and (4, –3) is divided by the point (1, 3) is:

(a) 2 : 3 (b) 3 : 2

(c) 1 : 4 (d) 2 : 5 1

3. For the following distribution,

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The sum of the lower limits of the median class and the modal class is:

(a) 20 (b) 35

(c) 30 (d) 25 1

4. An equilateral triangle ABC is inscribed in a circle with centre O. The measure of ∠BOC is:

(a) 120° (b) 130°

(c) 60° (d) 45° 1

5. The value of $4\tan^2 A - 4\sec^2 A$ is:

(a) –3 (b) 2

(c) –4 (d) 4 1

6. What is the perimeter of triangle with vertices (0, 0) (1, 0) and (0, 1)?

(a) $\left(1+\sqrt{2}\right)$ units (b) $\left(3+\sqrt{2}\right)$ units

(c) $\sqrt{2}$ units (d) $\left(2+\sqrt{2}\right)$ units 1

7. If $2\cos 3\theta = \sqrt{3}$ $(0° \le \theta \le 90°)$, then the value of θ is:

(a) 10° (b) 40°

(c) 20° (d) 60° 1

8. If the area of a sector of a circle of radius 2 cm is π sq m, then what is the central angle of the sector ?

(a) 90° (b) 45°

(c) 30° (d) 60° 1

9. If 18, a, b, – 3 are in A.P., then the value of $a + b$ is:

(a) 15 (b) 20

(c) 25 (d) 30 1

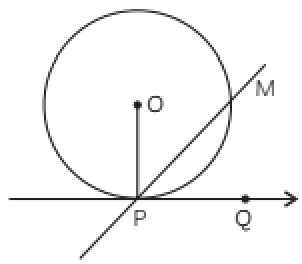
10. In $\triangle$ABC, D and E are prints on the sides AB and AC respectively, such that DE || BC.

If AD = 2.5 cm, BD = 3 cm and AE = 3.75 cm, then the value of AC is:

(a) 8 cm (b) 9.1 cm

(c) 8.25 cm (d) 9 cm 1

11. In the given figure, if $\angle$MPQ = 40°, then $\angle$OPM is:



(a) 30° (b) 20°

(c) 50° (d) 60° 1

12. A box contains 7 red balls and 6 blue balls. A ball is drawn at random from the box. The probability that this drawn is a red or blue ball is:

(a) 2 (b) 4

(c) 1 (d) 3 1

13. The empirical relationship between mean, median and mode is:

(a) Mode + Mean = Median

(b) 3 Median = Mode – Median

(c) Mode = Median + 2 Mean

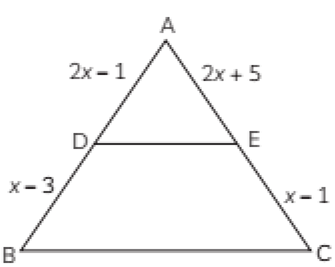
(d) 3 Median = Mode + 2 Mean 1

14. If the product of the zeros of the polynomial $ax^2 - 6x - 12$ is 4, then the value of 'a' is:

(a) 3 (b) 2

(c) 4 (d) –3 1

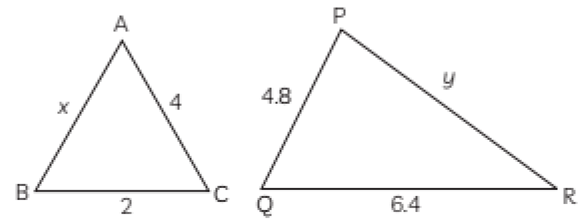
15. The value of x, in the adjoining figure, if DE || BC, is:



(a) 8 (b) 9

(c) 10 (d) 11 1

16. What is value of $x + y$, if $\triangle$ABC and $\triangle$PQR are similar ?



(a) 12.8 cm (b) 14.3 cm

(c) 12.5 cm (d) 14 cm 1

17. A quadratic polynomial whose zeros are – 7 and 5 is:

(a) $x^2 + 2x - 35$ (b) $x^2 - 2x + 35$

(c) $x^2 + 3x - 25$ (d) $x^2 - 3x - 35$ 1

18. The value for x and y :

$x + y = 2$ and $2x - y = 1$ is:

(a) 2 (b) 1

(c) 4 (d) 3 1

Direction for questions 19 and 20: In question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct option:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. Assertion (A) : In rhomus the diagonals are 20 cm and $10\sqrt{6}$ cm in length; then side length of rhombus is 15.8 cm.

Reason (R) : The sum of the sides of a rhomus is equal to the sum of the squares of its diagonals. 1

20. Assertion (A) : $\sec^2\theta = 1 + \tan^2\theta$ is a trigonometric identity.

Reason (R) : An equation involving trigonometric ratios of an angle is called trigonometric identity, which is true for all values of the angles involved. 1

SECTION - B

10 marks

(Section - B consists of 5 questions of 2 marks each.)

21. Write any two irrational numbers whose product is a rational number. 2

22. If the zeros of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a and $a + b$, then find the values of a and b. 2

23. The product of A's age 5 years ago with his age 9 years later is 15. Find A's present age.

OR

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in the same two variables such that the geometrical representation of the pair of equations so formed is:

(A) parallel lines

(B) coincident lines 2

24. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20 × 16 m. Find the area of the field, that a cow can graze.

OR

Prove that : $(\tan\theta + 2)(2\tan\theta + 1) = 5\tan\theta + 2\sec^2\theta$ 2

25. Determine the mean of the following data :

x	2	4	3	7	9	5
f	5	6	8	12	10	7

2

SECTION - C

18 marks

(Section - C consists of 6 questions of 3 marks each.)

26. Prove that $\sqrt{3}$ is an irrational number.

OR

Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers. 3

27. Two friends Reena and Sunita applied for the post of Computer Engineer in two different companies and got selected. Reena has been offered a job with a starting monthly salary of ₹ 48000, with an annual increment of ₹ 1400 in her salary. Sunita has been offered a job with a starting monthly salary of ₹ 40000, with an annual increment of ₹ 1800 in her salary.

(A) Determine their monthly salaries for the 13th year.

(B) Find each of them's total salary of 13 years.

(C) Who will get more salary in 13 years? And how much more? 3

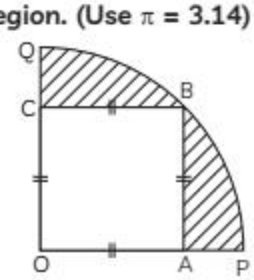
28. Find the roots of the equation : $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$ $(x \neq -4, 7)$

OR

Find a fraction which becomes $\frac{1}{2}$ when the denominator is increased by 4, and $\frac{1}{8}$ when the numerator is decreased by 5. 3

29. In the figure, a square OABC is inscribed in a quadrant OPBQ.

If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)

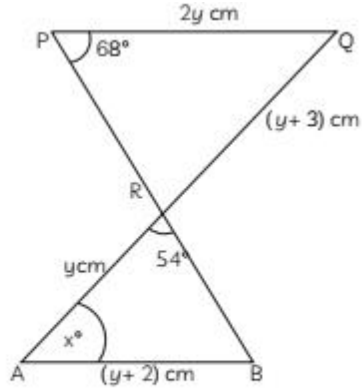


3

30. Given that ΔPQR is similar to ΔBAR. Find:

(A) the value of x;

(B) the value of y;



3

31. The annual rainfall record of a city for 66 days is given below in the table:

Rainfall (in cm)	0-10	10-20	20-30	30-40	40-50	50-60
Number of days	22	10	8	15	5	6

Calculate the median rainfall, using the formula. 3

SECTION - D

20 marks

(Section - D consists of 4 questions of 5 marks each.)

32. Draw the graphs of the equations:

$4x - y = 4$ and $4x + y = 12$

Hence, determine the vertices of the triangle formed by the lines representing these equations and the x – axis. Shade the triangular region so formed.

OR

Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$. 5

33. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Using the above result, prove the following:

If a circle touches all the four sides of a quadrilateral ABCD, prove that:

AB + CD = BC + DA. 5

34. Prove that :

$$\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}$$

OR

Prove that: $\dfrac{\tan A}{1 - \cot A} + \dfrac{\cot A}{1 - \tan A}$

$= 1 + \tan A + \cot A$ 5

35. Two stations are located at a distance of 'a' and 'b' from the foot of a leaning tower that leans in the direction of the north. If α and β be the elevations of the top of the tower from these stations, show that the inclination θ to the horizontal is given by $\cot\theta = \dfrac{b\cot\alpha - a\cot\beta}{b - a}$. 5

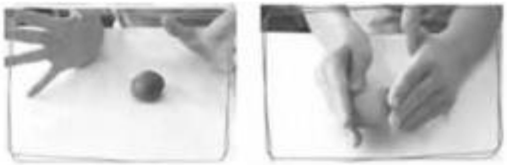
SECTION - E

12 marks

(Case Study Based Questions)

(Section E consists of 3 questions. All are compulsory.)

36. *To make the learning process more interesting, creative and innovative, Amayra's class teacher brings clay in the classroom, to teach the topic. Surface Areas and Volumes. With clay, she forms a cylinder of radius 6 cm and height 8 cm. Then she moulds the cylinder into a sphere and asks some questions to students.*



(A) Find the radius of the sphere so formed.

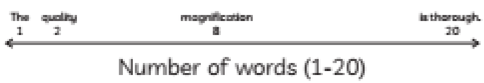
OR

Find the total surface area of the cylinder. 2

(B) What is the volume of the sphere so formed? 1

(C) Find the ratio of the volume of sphere to the volume of cylinder. 1

37. *A linguist is performing a statistical analysis of word frequency distributions as part of her quantitative stylistics to understand the measurable aspects of lexical structure. She picks a random newspaper sentence (structure of which is shown below) that has 20 words in it.*



The number of letters in each word is counted and the table below shows the frequency distribution:

Number of letters	2	3	4	5	6	7
Frequency	1	4	5	3	5	2

On the basis of the above information, answer the following questions:

(A) A word is chosen a random from the whole sentence. What is the probability that it has 4 letters? 1

(B) A word is chosen at random from those with an odd number of letters. What is the probability that it has 7 letters? 1

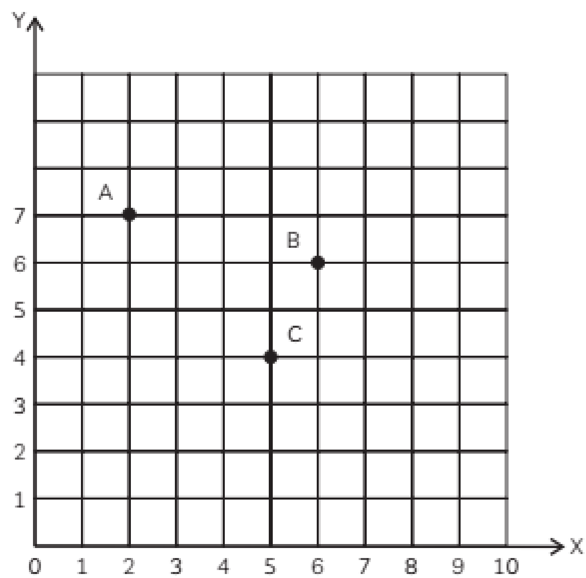
(C) First person chooses a word at random from the whole sentence, Another person chooses a word at random from the whole sentence. What is the probability that one person chooses a 2-letter word and the other chooses a 6-letter word?

OR

Find the mean number of letters in the whole sentence. 2

38. *Resident Welfare Association (RWA) of a M2K Society in Azadpur have put up three electric poles A, B and C in a society's common park near Tower A. Despite these three poles, some parts of the park are still in dark.*

So, RWA decides to have one more electric pole D in the park.



On the basis of the above information, answer the following questions:

(A) What is the distance of the pole B form the corner O of the park? 1

(B) Find the position of the fourth pole D so that four points A, B, C and D form a parallelogram.

OR

Find the distance between poles A and C. 2

(C) Find the distance between poles B and D. 1

SOLUTION

SECTION - A

1. *(c)* $\sqrt{113}$

Explanation: Distance of (7, – 8) from origin

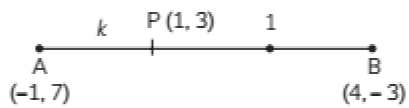
$$= \sqrt{(7-0)^2 + (-8-0)^2}$$

$$= \sqrt{49 + 64}$$

$$= \sqrt{113} \text{ units}$$

2. *(a)* 2 : 3

Explanation: Let P(1, 3) divide $\overline{AB}$ in the ratio k : 1



Then, $P(1, 3) = P\left(\frac{4k-1}{k+1}, \frac{-3k+7}{k+1}\right)$

$$\Rightarrow \quad \frac{4k-1}{k+1} = 1 \text{ and } \frac{-3k+7}{k+1} = 3$$

$$3k = 2 \qquad -6k = -4$$

$$\Rightarrow \quad k = \frac{2}{3}$$

Thus, the ratio is 2 : 3.

3. *(d)* 25

Explanation:

Class	Frequency	Cumulative Frequency
0–5	10	10
5–10	15	25
10–15	12	37
15-20	20	57
20-25	9	66

Here, the modal class is 15 – 20; and the median class is 10 – 15.

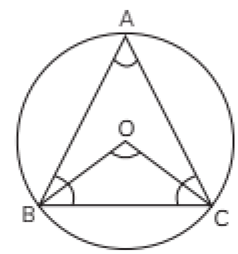
So, the sum of the two lower limits = 25

⚠ **Caution**

➥ *Remember cf is cumulative frequency of the class, preceeding the median class and f is frequency of the median class.*

4. *(a)* 120°

Explanation: As Δ ABC is equilateral



$\therefore \quad \angle A = 60°$

$\Rightarrow \quad \angle BOC = 2 \times \angle A = 120°$

[$\because$ Angle at the centre is twice the angle at circumference]

5. *(c)* –4

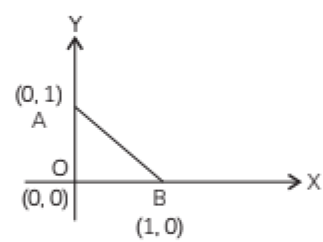
Explanation: $4 \tan^2 A - 4 \sec^2 A$

$$= 4 \tan^2 A - 4 (1 + \tan^2 A)$$

$$= -4.$$

6. *(d)* $2+\sqrt{2}$ *units*

Explanation: Perimeter of Δ AOB



$$= \overline{OA} + \overline{AB} + \overline{OB}$$

$$= 1 + \sqrt{2} + 1$$

$$= (2 + \sqrt{2}) \text{ units.}$$

7. *(a)* 10°

Explanation: Given, $2 \cos 3\theta = \sqrt{3}$

$\Rightarrow \quad \cos 3\theta = \frac{\sqrt{3}}{2} = \cos 30°$

$\Rightarrow \quad 3\theta = 30°$

$\Rightarrow \quad \theta = 10°$

8. *(a) 90°*

Explanation: Let the central angle be θ.

Then, area of the sector

$$= \left[\frac{\theta}{360} \times \pi(2)^2\right] \text{sq cm}$$

Equating it to π sq cm, we have,

$$\frac{\theta}{360} \times \pi \times 4 = \pi$$

$\Rightarrow \quad \theta = 90°$

9. *(a) 15*

Explanation: Since 18, a, b, – 3 are is A.P., so their common difference will be same.

$\therefore\ a - 18 = b - a = -3 - b$

So, $\quad a - 18 = -3 - b$

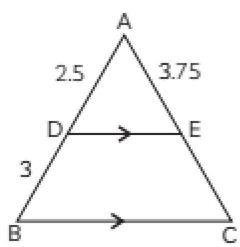
$\Rightarrow \quad a + b = 18 - 3$

$= 15$

10. *(c) 8.25 cm*

Explanation: Since, DE || BC

So, by B.P.T., we have



$$\frac{AD}{BD} = \frac{AE}{CE}$$

$\Rightarrow \quad \frac{2.5}{3} = \frac{3.75}{CE}$

$\Rightarrow \quad CE = \frac{3 \times 3.75}{2.5} = 4.5$

$\Rightarrow \quad AC = AE + CE = 3.75 + 4.5$

$= 8.25$ cm

11. *(c) 50°*

Explanation: In the figure,

$\angle OPQ = 90° \quad [\because \text{Tangent} \perp \text{Radius}]$

$\Rightarrow \quad \angle OPM = 90° - \angle MPQ$

$= 90° - 40°$

$= 50°$

12. *(c) 1*

Explanation: P (a red or blue ball)

$$= \frac{7+6}{13} = \frac{13}{13} = 1$$

13. *(d) 3 Median = Mode + 2 Mean*

Explanation: The emperical relation between the measures of central tendency which is given by

Mode = 3 Median – 2 Mean

14. *(d) –3*

Explanation: Given equation: $ax^2 - 6x - 12$

$$\text{Product of zeros} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$\Rightarrow \quad = \frac{(-12)}{a} = 4$

$a = -3$

15. *(a) 8*

Explanation: In ΔABC, DE || BC

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$

(By Thales theorem)

$\Rightarrow \quad \frac{2x-1}{x-3} = \frac{2x+5}{x-1}$

$\Rightarrow \quad (2x - 1)(x - 1) = (2x + 5)(x - 3)$

$\Rightarrow \quad 2x^2 - 2x - x + 1 = 2x^2 + 5x - 6x - 15$

$\Rightarrow \quad 2x = 16$

$\Rightarrow \quad x = 8$

Caution

- *Here DE || BC, so use the Thales theorem to find the value of x.*

16. *(b) 14.3 cm*

Explanation: As, ΔABC and ΔPQR are similar

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \quad \frac{x}{4.8} = \frac{2}{6.4}$

$\Rightarrow \quad x = 1.5$

Also, $\quad \dfrac{AC}{PR} = \dfrac{BC}{QR}$

$\Rightarrow \quad \dfrac{4}{y} = \dfrac{2}{6.4}$

$\Rightarrow \quad y = 12.8$

$\therefore \quad x + y = 1.5 + 12.8$

$= 14.3$ cm

17. *(a)* $x^2 + 2x - 35$

Explanation:

Sum of zeroes = – 7 + 5 = –2

Product of zeroes = – 7 × 5 = – 35

A quadratic polynomial with sum and product of zeroes is given as,

x^2 – (sum of zeroes) x + Product of zeroes.

$\Rightarrow x^2 - (-2)x - 35$

$\Rightarrow \quad x^2 + 2x - 35$

18. *(b)* 1

Explanation: On adding both the equations, we get

$$\begin{aligned} x + y &= 2 \\ 2x - y &= 1 \\ \hline 3x &= 3 \end{aligned}$$

$\Rightarrow \quad x = 1$

Then, $y = 2 - x = 2 - 1 = 1$

$\Rightarrow x = y = 1$ is the required solution.

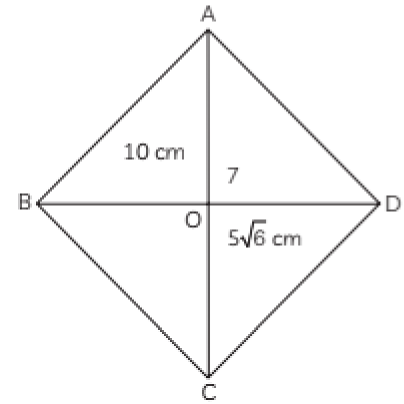
Caution

➥ *Derive the value of either x or y, but do which is more convenient and don't mess up the process.*

19. *(a) Both assertion (A) and reason (R) are correct and reason (R) is correct explanation of assertion (A).*

Explanation: The diagonals of a rhombus bisect each other at right angles.

$\therefore$ In ΔAOD, using pythagoras theorem,



$AD^2 = OA^2 + OD^2$

$= 10^2 + \left(5\sqrt{6}\right)^2$

$= 100 + 150$

$= 250$

$= 15.8$ cm

20. *(a) Both assertion (A) and reason (R) are correct and reason (R) is correct explanation of assertion (A).*

Explanation: Here, $\sec^2 \theta = 1 + \tan^2 \theta$

Put, $\quad \theta = 45°$

$\sec^2 45° = \left(\sqrt{2}\right)^2 = 2$

$1 + \tan^2 \theta = 1 + (\tan 45°)$

$= 1 + 1 = 2$

Thus, $\sec \theta = 1 + \tan^2 \theta$

Caution

➥ *Apply deduction of trigonometric identities, wherever necessary.*

SECTION - B

21. Consider two irrationals as, $5 - 2\sqrt{2}$ and $5 + 2\sqrt{2}$

Here,

$(5 - 2\sqrt{2})(5 + 2\sqrt{2}) = 5^2 - (2\sqrt{2})^2$

$= 25 - 8 = 17$

(a rational number)

22. As $(a - b)$, a and $(a + b)$ are zeroes of $x^3 - 3x^2 + x + 1$, we have:

$a - b + a + a + b = 3$

$\left[\because \text{Sum of zeroes} = -\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}\right]$

$\Rightarrow \quad 3a = 3$, or $a = 1 \quad$ ...(i)

Also,

$a(a - b) + a(a + b) + (a - b)(a + b) = 1$

$\left[\because \text{Sum of product of zeroes} = -\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}\right]$

$\Rightarrow \quad 3a^2 - b^2 = 1 \quad$ .....(ii)

and $\quad (a - b)\, a\, (a + b) = -1$

$\left[\because \text{Product of Zeroes} = -\dfrac{\text{Constant term}}{\text{Coefficient of } x^3}\right]$

$\Rightarrow \quad a(a^2 - b^2) = -1 \quad ...(iii)$

From (i) and (ii), we have $b = \pm\sqrt{2}$

Thus, $\quad a = 1, b = \pm\sqrt{2}$

23. Let A's present age (in years) be x. Then,

$$(x - 5)(x + 9) = 15$$
$$\Rightarrow \quad x^2 + 4x - 45 = 15$$
$$\Rightarrow \quad x^2 + 4x - 60 = 0$$
$$\Rightarrow \quad x^2 + 10x - 6x - 60 = 0$$
$$\Rightarrow x(x + 10) - 6(x + 10) = 0$$
$$\Rightarrow \quad (x + 10)(x - 6) = 0$$
$$\Rightarrow \quad x - 6 = 0 \quad (\because x + 10 \neq 0)$$
$$\Rightarrow \quad x = 6$$

Thus, A's present age is 6 years.

Caution

➥ *Read the word problem twice, before formulating it into an equation. Be clear about what is asked and how it go through.*

OR

(A) For parallel lines, we must have equation

$$ax + by + c = 0$$

which must satisfy $\dfrac{2}{a} = \dfrac{3}{b} \neq \dfrac{-8}{c}$

So, we can write the required equation as

$$2x + 3y - 2 = 0$$

(B) For coincident lines, we must have equation

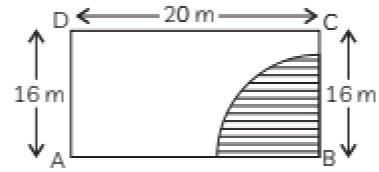
$$ax + by + c = 0$$

which must satisfy $\dfrac{2}{a} = \dfrac{3}{b} = \dfrac{-8}{c}$

So, we can write the required equation as

$$4x + 6y - 16 = 0$$

24. Shaded area can be grazed by the cow tied at the corner B.



$\therefore$ Required area $= \dfrac{1}{4}[\pi(14)^2]$ sq cm

$= 154$ sq cm.

OR

$(\tan\theta + 2)(2\tan\theta + 1)$

$= 2\tan^2\theta + 4\tan\theta + \tan\theta + 2$

$= 2\tan^2\theta + 5\tan\theta + 2$

$= 2(\tan^2\theta + 1) + 5\tan\theta$

$= 2\sec^2\theta + 5\tan\theta$

25.

x	f	fx
2	5	10
4	6	24
3	8	24
7	12	84
9	10	90
5	7	35

Here, $\Sigma f = 48$ and $\Sigma fx = 267$

We know,

$$\text{mean} = \frac{\Sigma fx}{\Sigma f} = \frac{267}{48} = 5.5625$$

SECTION - C

26. Let us suppose that $\sqrt{3}$ is a rational number.

Then $\sqrt{3}$ can be written in the form $\dfrac{p}{q}$ where p, q are co-prime i.e., they do not have common factor other than 1.

Now, $\quad \sqrt{3} = \dfrac{p}{q}$

$\Rightarrow \quad 3 = \dfrac{p^2}{q^2}$ [squaring both sides]

$\Rightarrow \quad p^2 = 3q^2$

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ 3 divides p

$\Rightarrow$ 3 is factor of p. ...(i)

$\therefore$ Let $\quad p = 3m.$

$\Rightarrow \quad p^2 = 3q^2$

$\Rightarrow \quad (3m)^2 = 3q^2$

$\Rightarrow \quad 3m^2 = q^2$

$\Rightarrow$ 3 divides q^2

$\Rightarrow$ 3 divides q

It means, 3 is a factor of both p and q. But p and q cannot have any common factor other than 1.

It means, our assumption is wrong.

Hence, $\sqrt{3}$ is an irrational number.

OR

Number are of two types – prime and composite

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13$

$= 13 \times (7 \times 11 + 1)$

$= 13 \times (77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors.

Therefore, it s a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorized further

Therefore, the given expression has 5 and 1009 as its factors.

Hence, it is a composite number.

27. Reena's yearly amount (in ₹) of monthly salary.

48000, 49400, 50800, ...

It is A.P. with $a = 48000$ and $d = 1400$.

Sunita's yearly amount (in ₹) of monthly salary.

40000, 41800, 43600, ...

It is A.P. with $a' = 40000$ and $d' = 1800$.

(A) So, Reena's 13th year monthly salary

$= a + 12d$

$= 48000 + 12 \times 1400$

$=$ ₹ 64800

Sunita's 13th year monthly salary

$= a + 12d$

$= 40000 + 12 \times 1800$

$=$ ₹ 61600

(B) Reena's total salary of 13 years

$= \frac{13}{2}[a_1 + a_{13}] \times 12$

$= 78\,[48000 + 64800]$

$=$ ₹ 8,7,98,400

Similarly, Sunita's total salary of 13 years

$= \frac{13}{2}[40000 + 61600] \times 12$

$=$ ₹ 79,24,800

(C) Reena will get more than Sunita by ₹ (8798400 – 7924800) *i.e.,* ₹ 8,73,600.

28. The given equation is:

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\Rightarrow \frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow (x+4)(x-7) = -30$$

$$\Rightarrow x^2 + 4x - 7x - 28 = -30$$

$$\text{or} \quad x^2 - 3x + 2 = 0$$

$$\text{or} \quad (x-2)(x-1) = 0$$

$$\Rightarrow x - 2 = 0, x - 1 = 0$$

$$\Rightarrow x = 2, 1$$

So, the two roots are $x = 2$ and $x = 1$.

OR

Let the required fraction be $\frac{p}{q}$. Then,

$$\frac{p}{q+4} = \frac{1}{2}$$

$$\Rightarrow 2p - q - 4 = 0 \quad \text{...(i)}$$

$$\frac{p-5}{q} = \frac{1}{8}$$

$$8p - q - 40 = 0 \quad \text{...(ii)}$$

Subtracting the first equation from the second equation, we get

$$6p - 36 = 0$$

$$\Rightarrow p = 6.$$

Substituting this value $p = 6$ in either equations, we get

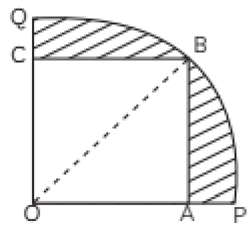
$$q = 8$$

Thus, the required fraction is $\frac{6}{8}$.

29. As, OA = 20 cm

∴ Diagonal,

$$OB = \sqrt{20^2 + 20^2}$$
$$= \sqrt{400 + 400}$$
$$= \sqrt{800}$$
$$= 20\sqrt{2} \text{ cm}$$



So, radius of the quadrant

$$= 20\sqrt{2} \text{ cm}$$

Area of the quadrant

$$= \frac{\pi}{4}(20\sqrt{2})^2 \text{ sq cm}$$
$$= 200\pi \text{ sq cm}$$

Area of the square

$$= (20)^2 \text{ sq cm, } i.e. \text{ 400 sq cm}$$

Area of the shaded region

$$= (200\pi - 400) \text{ sq cm}$$
$$= (628 - 400) \text{ sq cm}$$
$$= 228 \text{ sq cm}$$

30. (A) Here,

$\angle PRQ = 54°$ (vertically opposite angles)

Now, In ΔPQR,

$$\angle PQR = 180° - (68° + 54°) = 58°$$

$\because \Delta PQR \sim \Delta BAR$

$\therefore \angle Q = \angle A$

$\Rightarrow x = 58°$

(B) Again, $\Delta PQR \sim \Delta BAR$,

$$\therefore \frac{PQ}{BA} = \frac{QR}{AR}$$

$$\Rightarrow \frac{2y}{y+2} = \frac{y+3}{y}$$
$$\Rightarrow 2y^2 = y^2 + 5y + 6$$
$$\Rightarrow y^2 - 5y - 6 = 0$$
$$\Rightarrow (y-6)(y+1) = 0$$
$$\Rightarrow y = 6 \; (\because y \neq -1)$$

31. The cumulative frequency table for the given data is:

Ranifall (in cm)	Frequency	Cumulative frequency
0–10	22	22
10–20	10	32
20–30	8	40
30–40	15	55
40–50	5	60
50–60	6	66

Here, $N = 66$

So, $\frac{N}{2} = 33$

Cumulative frequency just greater than 33 is 40, which belongs to class 20 – 30.

So, the median class is 20 – 30.

For this class,

$l = 20, cf = 32, f = 8, \frac{N}{2} = 33$ and $h = 10$

So,

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$
$$= 20 + \frac{33 - 32}{8} \times 10$$
$$= 20 + \frac{1}{8} \times 10$$
$$= 21.25$$

Thus the median rain fall (in cm) is 21.25

SECTION - D

32. Table of values of

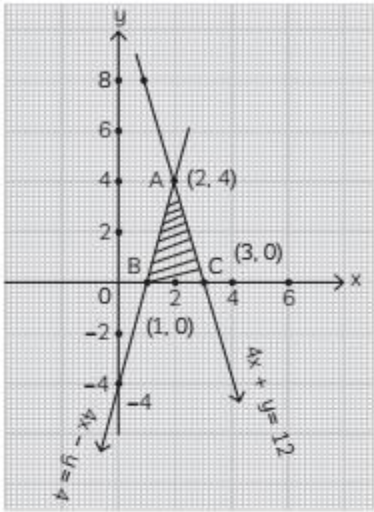
$4x - y = 4$

x	1	0	2
y	0	–4	4

Table of values of

$4x + y = 12$

x	1	2	3
y	8	4	0



From the graph, we find the vertices A, B, C of Δ ABC as A(2, 4), B(1, 0), C(3, 0).

Also, triangular region ABC is shaded.

OR

$$2x + 3y = 11 \quad ...(i)$$

$$2x - 4y = -24 \quad ...(ii)$$

Using equation (ii), we can say that

$$2x = -24 + 4y$$

$$\Rightarrow \quad x = -12 + 2y$$

Putting this in equation (i), we get

$$2(-12 + 2y) + 3y = 11$$

$$\Rightarrow \quad -24 + 4y + 3y = 11$$

$$\Rightarrow \quad 7y = 35$$

$$\Rightarrow \quad y = 5$$

Putting value of y in equation (i), we get

$$2x + 3(5) = 11$$

$$\Rightarrow \quad 2x + 15 = 11$$

$$\Rightarrow \quad 2x = 11 - 15 = -4$$

$$\Rightarrow \quad x = -2$$

Therefore, $x = -2$ and $y = 5$

Putting values of x and y in $y = mx + 3$, we get

$$5 = m(-2) + 3$$

$$\Rightarrow \quad 5 = -2m + 3$$

$$\Rightarrow \quad -2m = 2$$

$$\Rightarrow \quad m = -1$$

33. **Ist-Part:**

We are given a circle with centre O, a point A lying outside the circle and two tangents AX and AY on the circle from the point A.

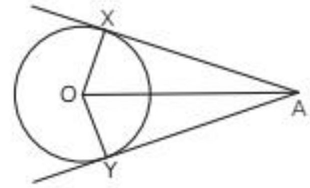
We need to prove that,

$$AX = AY$$

Join OX, OY and AO.

We know, tangent is perpendicular to radius, at the point of contact.

$$\angle AXO = \angle AYO = 90°$$



Now, in right triangles AXO and AYO, we have

AO = AO (common)

XO = YO (radii of the same circle)

$\angle AXO \cong \angle AYO$ (each 90°)

Therefore, by R.H.S. congruence criterion,

$$\Delta\, AXO \cong \Delta\, AYO$$

$$\Rightarrow \quad AX = BY$$

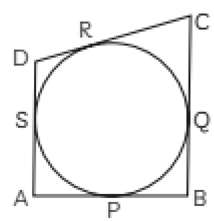
IInd-Part:

Here, a circle touches all the four sides of quadrilateral ABCD.

From the figure, using the above result, we have,

AS = AP, BP = BQ, CQ = CR

and DS = DR



Now, AB + CD = (AP + BP) + (CR + DR)

= (AP + DR) + (BP + CR)

= (AS + DS) + (BQ + CQ)

= AD + BC

34. $$\text{L.H.S.} = \frac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1}$$

$$= \frac{\cot\theta + \text{cosec}\,\theta - (\text{cosec}^2\theta - \cot^2\theta)}{\cot\theta - \text{cosec}\,\theta + 1}$$

$[\because \text{cosec}^2\theta - \cot^2\theta = 1]$

$$= \frac{(\cot\theta + \text{cosec}\,\theta) - (\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta)}{\cot\theta - \text{cosec}\,\theta + 1}$$

$$= \frac{(\cot\theta + \text{cosec}\,\theta)(1 - \text{cosec}\,\theta + \cot\theta)}{\cot\theta - \text{cosec}\,\theta + 1}$$

$$= \cot\theta + \text{cosec}\,\theta$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}$$

$$= \frac{\cos\theta + 1}{\sin\theta} \text{ or } \frac{1 + \cos\theta}{\sin\theta}$$

= R.H.S.

OR

$$\text{L.H.S.} = \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

$$= \frac{\frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}} + \frac{\frac{\cos A}{\sin A}}{1 - \frac{\sin A}{\cos A}}$$

$$= \frac{\sin A \cdot \frac{\sin A}{\cos A}}{\sin A - \cos A} + \frac{\cos A \cdot \frac{\cos A}{\sin A}}{\cos A - \sin A}$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A(\sin A - \cos A)}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\sin A \cos A(\sin A - \cos A)}$$

$\because \quad a^3 - b^3 = (a - b)(a^2 - b^2 + ab)$

$$= \frac{1 + \sin A \cos A}{\sin A \cos A}$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$\text{R.H.S.} = 1 + \tan A + \cot A = 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

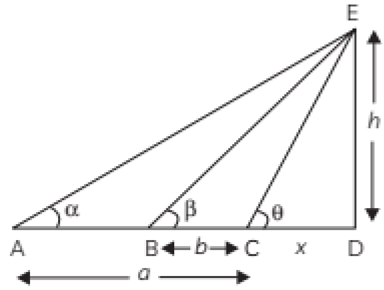
$$= \frac{\sin A \cos A + \sin^2 A + \cos^2 A}{\sin A + \cos A}$$

$$= \frac{\sin A \cos A + 1}{\sin A \cos A}$$

Thus, L.H.S. = R.H.S.

35. Let, the height of the tower DE = h

Distance between first station to foot of tower AD = $a + x$



Distance between second station to foot of tower BD = $b + x$

Distance between C and D = x

Given, α and β are angles of elevation of two station to the top of the tower.

i.e., $\angle DAE = \alpha$, $\angle DBE = \beta$, $\alpha DCE = \theta$

In ΔCDE,

$$\cot\theta = \frac{x}{h} \quad \text{...(i)}$$

In ΔBDE,

$$\cot b = \frac{b+x}{h}$$

$\Rightarrow \quad b + x = h\cot\beta$

multiply 'a' on both sides

$$ab + ax = ha\cot\beta \quad \text{...(ii)}$$

In ΔADE,

$$\cot\alpha = \frac{a+x}{h}$$

$\Rightarrow \quad a + x = h\cot\alpha$

Multiply 'b' on both sides

$$ba + bx = bh\cot\alpha \quad \text{...(iii)}$$

Subtract (iii) from (ii)

$$(b - a)x = h(b\cot\beta - a\cot\beta)$$

$$\frac{x}{h} = \frac{b\cot\alpha - a\cot\beta}{b - a}$$

$$\cot\theta = \frac{b\cot\alpha - a\cot\beta}{b - a}$$

[from (i)]

Hence, proved

SECTION - E

36. (A) Since, volume of sphere = volume of cylinder

$\Rightarrow \frac{4}{3}\pi R^3 = \pi r^2 h$, where R, r are the radii of sphere and cylinder respectively.

$$\Rightarrow \quad R^3 = \frac{6 \times 6 \times 8 \times 3}{4} = (6)^3$$

$\Rightarrow \quad$ R = 6 cm

$\therefore$ Radius of sphere = 6 cm

OR

Total surface area of the cylinder = $2\pi r(r + h)$

$= 2 \times \frac{22}{7} \times 6(6 + 8)$

$= 2 \times 2 \times \frac{22}{7} \times 6 \times 14 = 528 \text{ cm}^2$

(B) Volume of sphere = $\frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6 = 905.14 \text{ cm}^3$

(C) Volume of sphere = Volume of cylinder

$\therefore$ Required ratio = 1 : 1

37. (A) No. of four letter words = 5

Total number of words = 20

$\therefore$ P(4 letter words) = $\frac{5}{20} = \frac{1}{4}$

(B) No. of words with 7 letters =2

Total no. of words with odd letters = 9

$\therefore$ P(word has 7 letter) = $\frac{2}{9}$

(C) If first person chooses a 2-letter word then second person chooses a 6-letter word or vice-versa.

$\therefore$ Required Probability

$= \left(\frac{1}{20} \times \frac{5}{20}\right) + \left(\frac{5}{20} \times \frac{1}{20}\right)$

$= \frac{1}{80} + \frac{1}{80}$

$= \frac{2}{80} = \frac{1}{40}$

OR

$$\text{Mean} = \frac{2\times1+3\times4+4\times5+5\times3+6\times5+7\times2}{20}$$

$$= \frac{2+12+20+15+30+14}{20}$$

$$= \frac{93}{20} = 4.65$$

38. (A) Coordinates of B are (6, 6)

Distance from origin

$= \sqrt{(6-0)^2+(6-0)^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$ units

(B) If ABCD forms a parallelogram, then the diagonals bisects each other.

Mid-point of AC

$= \left(\frac{2+5}{2}, \frac{7+4}{2}\right) = (3.5, 5.5)$

Now, mid-point of diagonal, BD will be same. Let, the coordinates of D be (x, y)

Then,

$\frac{6+x}{2} = 3.5$ and $\frac{6+y}{2} = 5.5$

$x = 1$ and $y = 5$

OR

Coordinates of A are (2, 7)

Coordinates of C are (5, 4)

Distance of AC

$= \sqrt{(5-2)^2+(4-7)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$ units

(C) Coordinates of B(6, 6)

Coordinates of D(1, 5)

Distance between BD

$= \sqrt{(6-1)^2+(6-5)^2}$

$= \sqrt{5^2+1^2}$

$= \sqrt{25+1}$

$= \sqrt{26}$ units