5. Matrices

Exercise 5A

1. Question

If $A = \begin{bmatrix} 5 & -2 & 6 & 1 \\ 7 & 0 & 8 & -3 \\ \sqrt{2} & \frac{3}{5} & 4 & 3 \end{bmatrix}$ then write

i. the number of rows in A,

ii. the number of columns in A,

iii. the order of the matrix A,

iv. the number of all entries in A,

v. the elements a_{23}, a_{31}, a_{14}, a_{33}, a_{22} of A.

Answer

(i) Number of rows = 3

(ii) Number of columns = 4

(iii) Order of matrix = Number of rows x Number of columns = (3 x 4)

(iv) Number of entries = (Number of rows) x (Number of columns)

= 3 x 4

= 12

(V) $a_{ij} = element\ of\ i^{th}\ row\ and\ j^{th}\ column$

$a_{23} = 8$

$a_{31} = \sqrt{2}$

$a_{14} = 1$

$a_{33} = 4$

$a_{22} = 0$

2. Question

Write the order of each of the following matrices:

i. $A = \begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & \frac{4}{9} \end{bmatrix}$

ii. $B = \begin{bmatrix} 6 & -5 \\ \frac{1}{2} & \frac{3}{4} \\ -2 & -1 \end{bmatrix}$

iii. $C = \begin{bmatrix} 7 - \sqrt{2} & 5 & 0 \end{bmatrix}$

iv. D = [8 -3]

v. $E = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

vi, F = [6]

Answer

i. $A = \begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & \frac{4}{9} \end{bmatrix}$

Order of matrix = Number of rows x Number of columns

= (2 x 4)

ii. $B = \begin{bmatrix} 6 & -5 \\ \frac{1}{2} & \frac{3}{4} \\ -2 & -1 \end{bmatrix}$

Order of matrix = Number of rows x Number of columns

= (4 x 2)

iii. $C = \begin{bmatrix} 7 - \sqrt{2} & 5 & 0 \end{bmatrix}$

Order of matrix = Number of rows x Number of columns

= (1 x 4)

iv. D = [8 -3]

Order of matrix = Number of rows x Number of columns

= (1 x 2)

v. $E = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

Order of matrix = Number of rows x Number of columns

= (3 x 1)

vi, F = [6]

Order of matrix = Number of rows x Number of columns

= (1 x 1)

3. Question

If a matrix has 18 elements, what are the possible orders it can have?

Answer

Number of entries = (Number of rows) x (Number of columns) = 18

If order is (a x b) then, Number of entries = a x b

So now a x b = 18 (in this case)

Possible cases are (1 x 18), (2 x 9), (3 x 6), (6 x 3), (9 x 2), (18 x 1)

Conclusion: If a matrix has 18 elements, then possible orders are (1 x 18), (2 x 9), (3 x 6), (6 x 3), (9 x 2), (18 x 1)

4. Question

Find all possible orders of matrices having 7 elements.

Answer

Number of entries = (Number of rows) x (Number of columns) = 7

If order is (a x b) then, Number of entries = a x b

So now a x b = 7 (in this case)

Possible cases are (1 x 7), (7 x 1)

Conclusion: If a matrix has 18 elements, then possible orders are (1 x 7), (7 x 1)

5. Question

Construct a 3 × 2 matrix whose elements are given by a_{ij} = (2i – j).

Answer

Given: a_{ij} = (2i – j)

Now, a_{11} = (2 × 1 – 1) = 2 – 1 = 1

a_{12} = 2 × 1 – 2 = 2 – 2 = 0

a_{21} = 2 × 2 – 1 = 4 – 1 = 3

a_{22} = 2 × 2 – 2 = 4 – 2 = 2

a_{31} = 2 × 3 – 1 = 6 – 1 = 5

a_{32} = 2 × 3 – 2 = 6 – 2 = 4

Therefore,

$$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$$

6. Question

Construct a 4 × 3 matrix whose elements are given by $a_{ij} = \frac{i}{j}$.

Answer

It is (4 x 3) matrix. So it has 4 rows and 3 columns

Given $a_{ij} = \frac{i}{j}$.

So, $a_{11} = 1,\ a_{12} = \frac{1}{2},\ a_{13} = \frac{1}{3},$

$a_{21} = 2,\ a_{22} = 1,\ a_{23} = \frac{2}{3},$

$a_{31} = 3,\ a_{32} = \frac{3}{2},\ a_{33} = 1,$

$a_{41} = 4,\ a_{42} = 2,\ a_{43} = \frac{4}{3}$

So, the matrix $= \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \\ 4 & 2 & \frac{4}{3} \end{bmatrix}$

Conclusion: Therefore, Matrix is $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \\ 4 & 2 & \frac{4}{3} \end{bmatrix}$

7. Question

Construct a 2 × 2 matrix whose elements are $a_{ij} = \frac{(i+2j)^2}{2}$.

Answer

It is a (2 x 2) matrix. So, it has 2 rows and 2 columns.

Given $a_{ij} = \frac{(i+2j)^2}{2}$

So, $a_{11} = \frac{9}{2}$, $a_{12} = \frac{25}{2}$,

$a_{21} = 8$, $a_{22} = 18$

So, the matrix $= \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

Conclusion: Therefore, Matrix is $= \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

8. Question

Construct a 2 × 3 matrix whose elements are $a_{ij} = \frac{(i-2j)^2}{2}$.

Answer

It is a (2 x 3) matrix. So, it has 2 rows and 3 columns.

Given $a_{ij} = \frac{(i-2j)^2}{2}$

So, $a_{11} = \frac{1}{2}$, $a_{12} = \frac{9}{2}$, $a_{13} = \frac{25}{3}$,

$a_{21} = 0$, $a_{22} = 2$, $a_{23} = 8$

So, the matrix $= \begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$

Conclusion: Therefore, Matrix is $\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$

9. Question

Construct a 3 × 4 matrix whose elements are given by $a_{ij} = \frac{1}{2}|-3i+j|$.

Answer

It is a (3 x 4) matrix. So, it has 3 rows and 4 columns.

Given $a_{ij} = \frac{|-3i+j|}{2}$

So, $a_{11} = 1$, $a_{12} = \frac{1}{2}$, $a_{13} = 0$, $a_{13} = \frac{1}{2}$,

$a_{21} = \frac{5}{2}$, $a_{22} = 2$, $a_{23} = \frac{3}{2}$, $a_{13} = 1$,

$a_{31} = 4$, $a_{32} = \frac{7}{2}$, $a_{33} = 3$, $a_{13} = \frac{5}{2}$

So, the matrix $= \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

Conclusion: Therefore, Matrix is $\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

Exercise 5B

1. Question

If $A = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix}$, verify that (A + B) = (B + A).

Answer

$A + B = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{bmatrix}$

$B + A = \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 5 & -1 & 3 \\ 3 & -3 & 4 \end{bmatrix} = B + A$

Therefore, A + B = B + A

This is true for any matrix

Conclusion: A + B = B + A

2. Question

If $A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}$, verify that (A + B) + C = A + (B+C).

Answer

$(A+B)+C = \left(\begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix}\right) + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}$

$= \left(\begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 4 & 2 \end{bmatrix}\right) + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$

$A+(B+C) = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \left(\begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}\right)$

$= \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \left(\begin{bmatrix} -1 & -1 \\ 7 & -2 \\ -1 & 9 \end{bmatrix}\right)$

$= \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$

Therefore, (A+B)+C = A+(B+C)

It is true for any matrix

Conclusion: (A+B)+C = A+(B+C)

3. Question

If $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix}$, find (2A – B).

Answer

$2A = 2\left(\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix}\right)$

$= \begin{bmatrix} 6 & 2 & 4 \\ 2 & 4 & -6 \end{bmatrix}$

$(2A-B) = \begin{bmatrix} 6 & 2 & 4 \\ 2 & 4 & -6 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 8 & 2 & 0 \\ -3 & 7 & -8 \end{bmatrix}$

Conclusion: $(2A-B) = \begin{bmatrix} 8 & 2 & 0 \\ -3 & 7 & -8 \end{bmatrix}$

4. Question

Let $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$ and $C=\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$. Find:

i. A + 2B

ii. B – 4c

iii. A – 2B + 3C

Answer

A + 2B $=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}+2(\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix})$

$=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}+\begin{bmatrix}2 & 6\\ -4 & 10\end{bmatrix}$

$=\begin{bmatrix}4 & 10\\ -1 & 12\end{bmatrix}$

Conclusion: (A+2B) = $\begin{bmatrix}4 & 10\\ -1 & 12\end{bmatrix}$

ii. B – 4c

B-4C $=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}-4(\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix})$

$=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}-\begin{bmatrix}-8 & 20\\ 12 & 16\end{bmatrix}$

$=\begin{bmatrix}9 & -17\\ -14 & -11\end{bmatrix}$

Conclusion: B-4C $=\begin{bmatrix}9 & -17\\ -14 & -11\end{bmatrix}$

iii. A – 2B + 3C

A-2B+3C $=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}-2\left(\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}\right)+3(\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix})$

$=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}-\begin{bmatrix}2 & 6\\ -4 & 10\end{bmatrix}+\begin{bmatrix}-6 & 15\\ 9 & 12\end{bmatrix}$

$=\begin{bmatrix}-6 & 13\\ 16 & 4\end{bmatrix}$

Conclusion: A_2B+3C $=\begin{bmatrix}-6 & 13\\ 16 & 4\end{bmatrix}$

5. Question

Let $A=\begin{bmatrix}0 & 1 & -2\\ 5 & -1 & -4\end{bmatrix}, B=\begin{bmatrix}1 & -3 & -1\\ 0 & -2 & 5\end{bmatrix}$ and $C=\begin{bmatrix}2 & -5 & 1\\ -4 & 0 & 6\end{bmatrix}$. Compute 5A – 3B + 4C.

Answer

5A-3B+4C $=5(\begin{bmatrix}0 & 1 & -2\\ 5 & -1 & -4\end{bmatrix})-3(\begin{bmatrix}1 & -3 & -1\\ 0 & -2 & 5\end{bmatrix})+4(\begin{bmatrix}2 & -5 & 1\\ -4 & 0 & 6\end{bmatrix})$

$=(\begin{bmatrix}0 & 5 & -10\\ 25 & -5 & -20\end{bmatrix})-(\begin{bmatrix}3 & -9 & -3\\ 0 & -6 & 15\end{bmatrix})+(\begin{bmatrix}8 & -20 & 4\\ -16 & 0 & 24\end{bmatrix})$

$=\begin{bmatrix}-3 & 14 & -7\\ 25 & 1 & -35\end{bmatrix}+\begin{bmatrix}8 & -20 & 4\\ -16 & 0 & 24\end{bmatrix}$

$=\begin{bmatrix}5 & -6 & -3\\ 9 & 1 & -11\end{bmatrix}$

Conclusion: 5A-3B+4C $=\begin{bmatrix}5 & -6 & -3\\ 9 & 1 & -11\end{bmatrix}$

6. Question

If $5A = \begin{bmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{bmatrix}$, find A.

Answer

$5A = \begin{bmatrix} 5 & 10 & -15 \\ 2 & 3 & 4 \\ 1 & 0 & -5 \end{bmatrix}$

$A = \begin{bmatrix} \frac{5}{5} & \frac{10}{5} & \frac{-15}{5} \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & \frac{0}{5} & \frac{-5}{5} \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & -3 \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & 0 & -1 \end{bmatrix}$

Conclusion: $A = \begin{bmatrix} 1 & 2 & -3 \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{5} & 0 & -1 \end{bmatrix}$

7. Question

Find matrices A and B, if $A + B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix}$ and $A - B = \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$.

Answer

Add (A+B) and (A-B)

We get (A+B)+(A-B) $= \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} + \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$

$2A = \begin{bmatrix} -4 & -4 & 10 \\ 16 & 6 & -6 \\ 6 & 10 & 12 \end{bmatrix}$

$A = \begin{bmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{bmatrix}$

Now Subtract (A-B) from (A+B)

(A+B)-(A-B) $= \begin{bmatrix} 1 & 0 & 2 \\ 5 & 4 & -6 \\ 7 & 3 & 8 \end{bmatrix} - \begin{bmatrix} -5 & -4 & 8 \\ 11 & 2 & 0 \\ -1 & 7 & 4 \end{bmatrix}$

$(2B) = \begin{bmatrix} 6 & 4 & -6 \\ -6 & 2 & -6 \\ 8 & -4 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 2 & -3 \\ -3 & 1 & -3 \\ 4 & -2 & 2 \end{bmatrix}$

Conclusion: $A = \begin{bmatrix} -2 & -2 & 5 \\ 8 & 3 & -3 \\ 3 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & -3 \\ -3 & 1 & -3 \\ 4 & -2 & 2 \end{bmatrix}$

8. Question

Find matrices A and B, if $2A - B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $2B + A = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$

Answer

Add 2(2A-B) and (2B+A)

2(2A-B)+(2B+A) $= 2\left(\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}\right) + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$

5A $= \left(\begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix}\right) + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$

5A $= \begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}$

A $= \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$

B $= 2\left(\begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}\right) - \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 6 & -4 & 2 \\ -4 & 2 & -2 \end{bmatrix} - \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$

B $= \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$

Conclusion: A $= \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$, B $= \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$

(GIVEN ANSWER IS WRONG for question 8)

9. Question

Find matrix X, if $\begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix} + X = \begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix}$.

Answer

Given $\begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix} + x = \begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix}$

$x = \begin{bmatrix} 6 & 2 & 3 \\ 4 & 8 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -9 \\ -1 & 4 & -7 \end{bmatrix}$

$= \begin{bmatrix} 3 & -3 & 12 \\ 5 & 4 & 13 \end{bmatrix}$

Conclusion : x $= \begin{bmatrix} 3 & -3 & 12 \\ 5 & 4 & 13 \end{bmatrix}$

10. Question

If $A = \begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}$, find a matrix C such that A + B - C = O.

Answer

Given A + B - C = 0

$\begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix} - C = 0$

$C = \begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 3 & 5 \\ -3 & 8 \\ 7 & -2 \end{bmatrix}$

Conclusion: $C = \begin{bmatrix} 3 & 5 \\ -3 & 8 \\ 7 & -2 \end{bmatrix}$

11. Question

Find the matrix X such that 2A – B + X = O,

where $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$.

Answer

Given 2A – B + X = 0

$2\left(\begin{bmatrix}3 & 1\\0 & 2\end{bmatrix}\right)-\begin{bmatrix}-2 & 1\\0 & 3\end{bmatrix}+X=0$

$X=\begin{bmatrix}-2 & 1\\0 & 3\end{bmatrix}-2(\begin{bmatrix}3 & 1\\0 & 2\end{bmatrix})$

$=\begin{bmatrix}-2 & 1\\0 & 3\end{bmatrix}-\begin{bmatrix}6 & 2\\0 & 4\end{bmatrix}$

$=\begin{bmatrix}-8 & -1\\0 & -1\end{bmatrix}$

Conclusion: $X=\begin{bmatrix}-8 & -1\\0 & -1\end{bmatrix}$

12. Question

If $A=\begin{bmatrix}1 & -3 & 2\\2 & 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1 & -1\\1 & 0 & -1\end{bmatrix}$, find a matrix C such that (A + B + C) is a zero matrix.

Answer

Given A+B+C is zero matrix i.e A+B+C = 0

$\begin{bmatrix}1 & -3 & 2\\2 & 0 & 2\end{bmatrix}+\begin{bmatrix}2 & -1 & -1\\1 & 0 & -1\end{bmatrix}+C=0$

$C=-\begin{bmatrix}1 & -3 & 2\\2 & 0 & 2\end{bmatrix}-\begin{bmatrix}2 & -1 & -1\\1 & 0 & -1\end{bmatrix}$

$=\begin{bmatrix}-3 & 4 & -1\\-3 & 0 & -1\end{bmatrix}$

Conclusion: $C=\begin{bmatrix}-3 & 4 & -1\\-3 & 0 & -1\end{bmatrix}$

13. Question

If A = diag [2, -5, 9], B = diag [-3, 7, 14] and C = diag [4, -6, 3], find:

(i) A + 2B

(ii)B + C – A

Answer

If Z = diag[a,b,c], then we can write it as

$Z=\begin{bmatrix}a & 0 & 0\\0 & b & 0\\0 & 0 & c\end{bmatrix}$

So, A+2B $=\begin{bmatrix}2 & 0 & 0\\0 & -5 & 0\\0 & 0 & 9\end{bmatrix}+2(\begin{bmatrix}-3 & 0 & 0\\0 & 7 & 0\\0 & 0 & 14\end{bmatrix})$

$=\begin{bmatrix}2 & 0 & 0\\0 & -5 & 0\\0 & 0 & 9\end{bmatrix}+\begin{bmatrix}-6 & 0 & 0\\0 & 14 & 0\\0 & 0 & 28\end{bmatrix}$

$=\begin{bmatrix}4 & 0 & 0\\0 & 9 & 0\\0 & 0 & 37\end{bmatrix}$

=diag[4,9,37]

Conclusion: A + 2B = diag[4,9,37]

(Given answer is wrong)

ii. B + C – A

If Z = diag[a,b,c], then we can write it as

$Z=\begin{bmatrix}a & 0 & 0\\0 & b & 0\\0 & 0 & c\end{bmatrix}$

B+C-A $= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

= diag[-1,6,8]

Conclusion: B+C-A = diag[-1,6,8]

iii. 2A + B – 5C

If Z = diag[a,b,c], then we can write it as

$$Z = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

2A+B-5C $= 2\left(\begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}\right) + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} - 5(\begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 3 \end{bmatrix})$

$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 18 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 14 \end{bmatrix} - \begin{bmatrix} 20 & 0 & 0 \\ 0 & -30 & 0 \\ 0 & 0 & 15 \end{bmatrix}$

$= \begin{bmatrix} -19 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 17 \end{bmatrix}$

= diag[-19,27,17]

Conclusion: 2A + B – 5C = diag[-19,27,17]

(Given answer is wrong)

14. Question

Find the value of x and y, when

i. $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

Answer

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$,

Then a=e, b=f, c=g, d=h

Given $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

So, x + y = 8 and x - y = 4

Adding these two gives 2x = 12

$\Rightarrow x = 6$

y =2

Conclusion : x = 6 and y =2

ii. $\begin{bmatrix} 2x+5 & 7 \\ 0 & 3y-7 \end{bmatrix} = \begin{bmatrix} x-3 & 7 \\ 0 & -5 \end{bmatrix}$

Given, $\begin{bmatrix} 2x+5 & 7 \\ 0 & 3y-7 \end{bmatrix} = \begin{bmatrix} x-3 & 7 \\ 0 & -5 \end{bmatrix}$

So, 2x+5 = x-3 and 3y-7 = -5

$\Rightarrow 3y = 2 \Rightarrow y = \frac{2}{3}$

$\Rightarrow 2x + 5 = x - 3 \Rightarrow x = -8$

Conclusion : x = -8 and y =$\frac{2}{3}$

iii. $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

2x+3 = 7 $\Rightarrow x = 2$

2y-4 = 14 $\Rightarrow y = 9$

Conclusion : x = 2 and y = 9

(Given answer is wrong)

15. Question

Find the value of (x + y) from the following equation :

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Answer

Given

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

So, 2+y = 5 and 2x+2 = 8

i.e y = 3 and x = 3

Therefore, x+y=6

Conclusion: Therefore x+y = 6

16. Question

If $\begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$ then write the value of (x + y).

Answer

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$,

Then a=e, b=f, c=g, d=h

Given, $\begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$,

So, x-y = 1, x+y =5, 2y = 4 and 2y+z = 9

Therefore, x+y = 5

Conclusion: x+y = 5

(Given answer is wrong)

Exercise 5C

1 A. Question

Compute AB and BA, which ever exists when

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$$

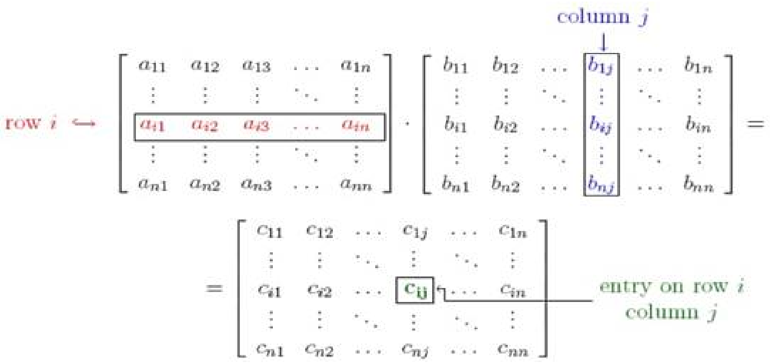
Answer

Given : $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$

Matrix A is of order 3 $\times$ 2, and Matrix B is of order 2 $\times$ 2

To find : matrix AB and BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = c = 2,d = 2 ,thus matrix AB is of order 3 $\times$ 2

Matrix AB = $\begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \times \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2(-2) + (-1)(0) & 2(3) + (-1)(4) \\ 3(-2) + 0(0) & 3(3) + 0(4) \\ -1(-2) + 4(0) & -1(3) + 4(4) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -4 + 0 & 6 - 4 \\ -6 + 0 & 9 + 0 \\ 2 + 0 & -3 + 16 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$

For matrix BA, a = 3,b = c = 2,d = 2 ,thus matrix BA exists, if and only if d=a

But 3 $\neq$ 2

Thus matrix BA does not exist

1 B. Question

Compute AB and BA, which ever exists when

$$A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$$

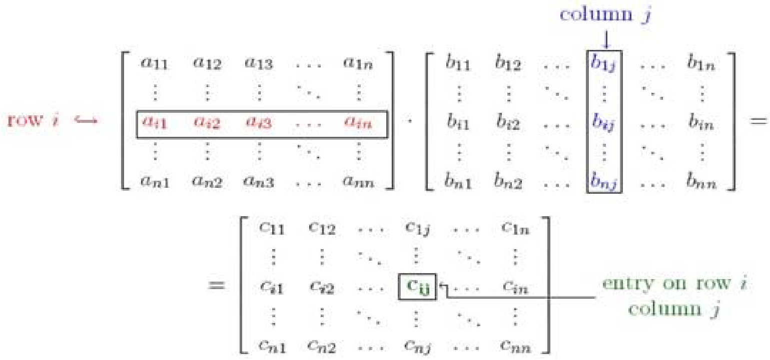
Answer

Given : $A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$

Matrix A is of order 3 × 2, and Matrice B is of order 3 × 3

To find : matrix AB and BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrice of order c × d ,then matrice AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrice of order c × d ,then matrice BA exists and is of order c × b ,if and only if d = a

For matrix AB, a = 3,b = 2,c = 3,d = 3 ,thus matrix AB does not exist, as d ≠ a

But 2 ≠ 3

Thus matrix AB does not exist

For matrix BA, a = 3,b = 2,c = 3,d = 3 ,thus matrix BA is of order 3 × 2

as d = a = 3

$$\text{Matrix BA} = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 3(-1) - 2(-2) + 1(-3) & 3(1) - 2(2) + 1(3) \\ 0(-1) + 1(-2) + 2(-3) & 0(1) + 1(2) + 2(3) \\ -3(-1) + 4(-2) - 5(-3) & -3(1) + 4(8) - 5(3) \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -3+4-3 & 3-4+3 \\ 0-2-6 & 0+2+6 \\ 3-8+15 & -3+8-15 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

$$\text{Matrix BA} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$$

1 C. Question

Compute AB and BA, which ever exists when

$A = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix}$

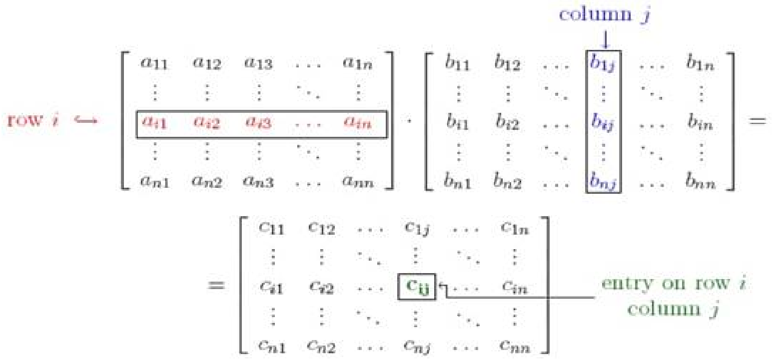
Answer

Given : $A = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix}$

Matrix A is of order 2 × 3 and Matrix B is of order 3 × 2

To find : matrices AB and BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix BA exists and is of order c × b ,if and only if d = a

For matrix AB, a = 2,b = 3,c = 3,d = 2 ,matrix AB exists and is of order 2 × 2,as

b = c = 3

Matrix AB = $\begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0(1)+1(-1)-5(0) & 0(3)+1(0)-5(5) \\ 2(1)+4(-1)+0(0) & 2(3)+4(0)+0(5) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 0-1-0 & 0+0-25 \\ 2-4+0 & 6+0+0 \end{bmatrix} = \begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$

For matrix BA, a = 2,b = 3,c = 3,d = 2 ,matrix BA exists and is of order 3 × 3,as

d = a = 2

Matrix BA = $\begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1(0)+3(2) & 1(1)+3(4) & 1(-5)+3(0) \\ -1(0)+0(2) & -1(1)+0(4) & -1(-5)+0(0) \\ 0(0)+5(2) & 0(1)+5(4) & 0(-5)+5(0) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 0+6 & 1+12 & -5+0 \\ 0+0 & -1+0 & 5+0 \\ 0+10 & 0+20 & 0+0 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{bmatrix}$

1 D. Question

Compute AB and BA, which ever exists when

A = [1 2 3 4] and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

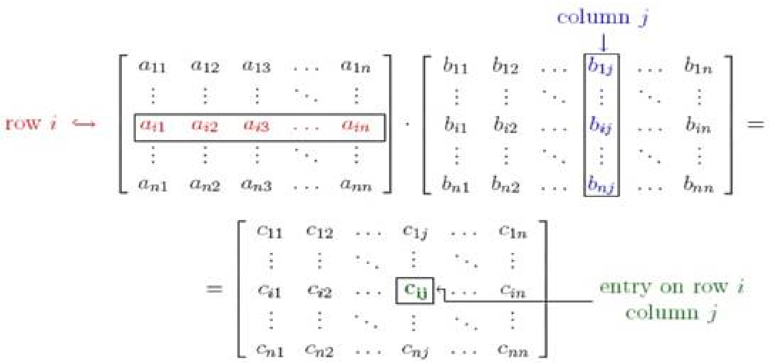
Answer

Given : A = [1 2 3 4] and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

Matrix A is of order 1 × 4 and Matrix B is of order 4 × 1

To find : matrices AB and BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix BA exists and is of order c × b ,if and only if d = a

For matrix AB, a = 1,b = 4,c = 4,d = 1 ,matrix AB exists and is of order 1 × 1,as

b = c = 4

Matrix AB = $[1 \quad 2\ 3 \quad 4] \times \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = [1(1) + 2(2) + 3(3) + 4(4)]$

Matrix AB = $[1 + 4 + 9 + 16] = [30]$

Matrix AB = $[30]$

Matrix AB = $[30]$

For matrix BA, a = 1,b = 4,c = 4,d = 1 ,matrix BA exists and is of order 4 × 4,as

d = a = 1

Matrix BA = $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \times [1 \quad 2\ 3 \quad 4] = \begin{bmatrix} 1(1) & 1(2) & 1(3) & 1(4) \\ 2(1) & 2(2) & 2(3) & 2(4) \\ 3(1) & 3(2) & 3(3) & 3(4) \\ 4(1) & 4(2) & 4(3) & 4(4) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$

1 E. Question

Compute AB and BA, which ever exists when

$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

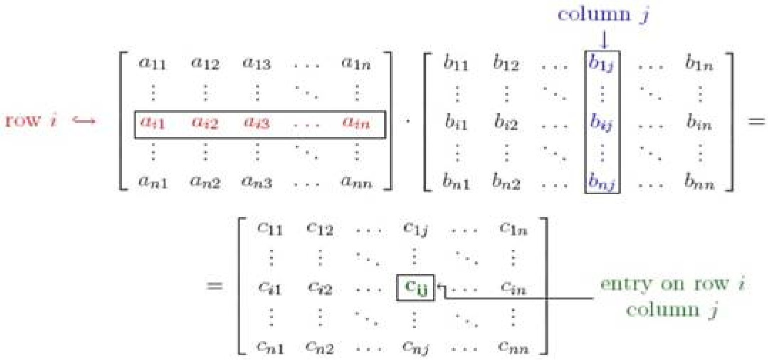
Answer

Given : $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

Matrix A is of order 3 × 2 and Matrix B is of order 2 × 3

To find : matrices AB and BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix BA exists and is of order c × b ,if and only if d = a

For matrix AB, a = 3,b = 2,c = 2,d = 3 ,matrix AB exists and is of order 3× 3,as

b = c = 2

Matrix AB = $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

For matrix BA, a = 3,b = 2,c = 2,d = 3 ,matrix BA exists and is of order 2× 2,as

d = a = 3

Matrix BA = $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1(2)+0(3)+1(-1) & 1(1)+0(2)+1(1) \\ -1(2)+2(3)+1(-1) & -1(1)+2(2)+1(1) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 2+0-1 & 1+0+1 \\ -2+6-1 & -1+4+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2 A. Question

Show that AB ≠ BA in each of the following cases :

$A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

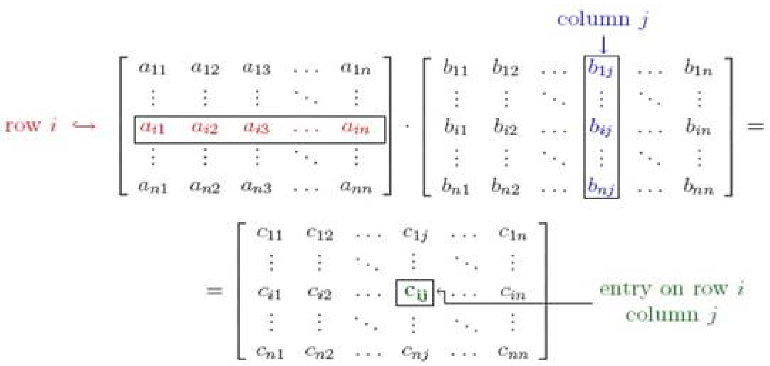
Answer

Given : $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

Matrix A is of order 2 $\times$ 2 and Matrix B is of order 2 $\times$ 2

To show : matrix AB $\neq$ BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 2,b = c = 2,d = 2 ,thus matrix AB is of order 2 $\times$ 2

Matrix AB = $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5(2)-1(3) & 5(1)-1(4) \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

For matrix BA, a = 2,b = c = 2,d = 2 ,thus matrix BA is of order 2 $\times$ 2

Matrix BA= $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$ and Matrix AB = $\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

Matrix AB $\neq$ BA

2 B. Question

Show that AB ≠ BA in each of the following cases :

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

Answer

Given : $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

Matrix A is of order 3 × 3, and Matrix B is of order 3 × 3

To show : matrix AB ≠ BA

The formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix BA exists and is of order c × b ,if and only if d = a

For matrix AB, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 × 3

Matrix AB =

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 1(-1) + 2(0) + 3(2) & 1(1) + 2(-1) + 3(3) & 1(0) + 2(1) + 3(4) \\ 0(-1) + 1(0) + 0(2) & 0(1) + 1(-1) + 0(3) & 0(0) + 1(1) + 0(4) \\ 1(-1) + 1(0) + 0(2) & 1(1) + 1(-1) + 0(3) & 1(0) + 1(1) + 0(4) \end{bmatrix}$$

Matrix AB = $\begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

For matrix BA, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 × 3

Matrix BA=

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1(1) + 1(0) + 0(1) & -1(2) + 1(1) + 0(1) & -1(3) + 1(0) + 0(0) \\ 0(1) - 1(0) + 1(1) & 0(2) - 1(1) + 1(1) & 0(3) - 1(0) + 1(0) \\ 2(1) + 3(0) + 4(1) & 2(2) + 3(1) + 4(1) & 2(3) + 3(0) + 4(0) \end{bmatrix}$$

Matrix BA = $\begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ 0-1+1 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$ and Matrix AB = $\begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

Matrix AB ≠ BA

3 A. Question

Show that AB = BA in each of the following cases:

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

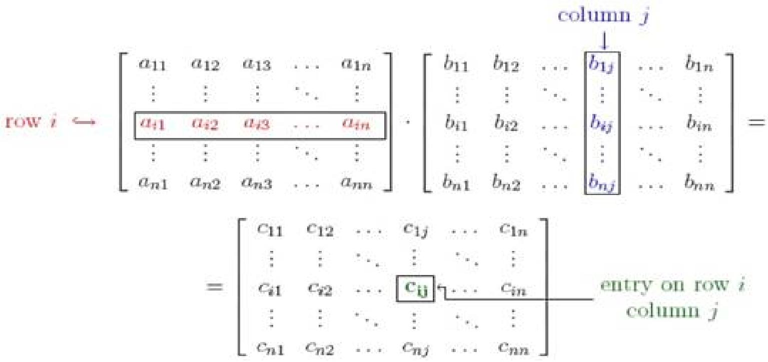
Answer

Given : $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$

Matrix A is of order 2 × 2 and Matrix B is of order 2 × 2

To show : matrix AB = BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix BA exists and is of order c × b ,if and only if d = a

For matrix AB, a = 2,b = c = 2,d = 2 ,thus matrix AB is of order 2 × 2

Matrix AB =

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} \cos\emptyset & -\sin\emptyset \\ \sin\emptyset & \cos\emptyset \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$$

Matrix AB = $\begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$

Matrix AB = $\begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$

For matrix BA, a = 2,b = c = 2,d = 2 ,thus matrix BA is of order 2 × 2

Matrix BA=

$$\begin{bmatrix} \cos\emptyset & -\sin\emptyset \\ \sin\emptyset & \cos\emptyset \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\emptyset\cos\theta - \sin\emptyset\sin\theta & -\cos\emptyset\sin\theta - \sin\emptyset\cos\theta \\ \sin\emptyset\cos\theta + \cos\emptyset\sin\theta & -\sin\emptyset\sin\theta + \cos\emptyset\cos\theta \end{bmatrix}$$

Matrix BA = $\begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$

Matrix BA = Matrix AB = $\begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$

Thus Matrix AB = BA

3 B. Question

Show that AB = BA in each of the following cases:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

Answer

Given : $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$

Matrix A is of order 3 $\times$ 3 and Matrix B is of order 3 $\times$ 3

To show : matrix AB $\neq$ BA

Formula used : $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3$\times$ 3

Matrix AB $= \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} =$

$$\begin{bmatrix} 1(10) + 2(-11) + 1(-9) & 1(-4) + 2(5) + 1(-5) & 1(-1) + 2(0) + 1(1) \\ 3(10) + 4(-11) + 2(-9) & 3(-4) + 4(5) + 2(-5) & 3(-1) + 4(0) + 2(1) \\ 1(10) + 3(-11) + 2(-9) & 1(-4) + 3(5) + 2(-5) & 1(-1) + 3(0) + 2(1) \end{bmatrix}$$

Matrix AB $= \begin{bmatrix} 10 - 22 - 9 & -4 + 10 - 5 & -1 + 0 + 1 \\ 30 - 44 - 18 & -12 + 20 - 10 & -3 + 0 + 2 \\ 10 - 33 - 18 & -4 + 15 - 10 & -1 + 0 + 2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -32 & -2 & -1 \\ -41 & 1 & 1 \end{bmatrix}$

Matrix AB $= \begin{bmatrix} -3 & 1 & 0 \\ -32 & -2 & -1 \\ -41 & 1 & 1 \end{bmatrix}$

For matrix BA, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3$\times$ 3

Matrix BA=

$$\begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 10(1) - 4(3) - 1(1) & 10(2) - 4(4) - 1(3) & 10(1) - 4(2) - 1(2) \\ -11(1) + 5(3) + 0(1) & -11(2) + 5(4) + 0(3) & -11(1) + 5(2) + 0(2) \\ 9(1) - 5(3) + 1(1) & 9(2) - 5(4) + 1(3) & 9(1) - 5(2) + 1(2) \end{bmatrix}$$

Matrix BA $= \begin{bmatrix} 10 - 12 - 1 & 20 - 16 - 3 & 10 - 8 - 2 \\ -11 + 15 + 0 & -22 + 20 + 0 & -11 + 10 + 0 \\ 9 - 15 + 1 & 18 - 20 + 3 & 9 - 10 + 2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix}$

Matrix AB $\neq$ BA

3 C. Question

Show that AB = BA in each of the following cases:

$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$

Answer

Given : $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$

Matrix A is of order 3 $\times$ 3 and Matrix B is of order 3 $\times$ 3

To show : matrix AB = BA

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix AB = $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} =$

$$\begin{bmatrix} 1(-2)+3(-1)-1(-6) & 1(3)+3(2)-1(9) & 1(-1)+3(-1)-1(-4) \\ 2(-2)+2(-1)-1(-6) & 2(3)+2(2)-1(9) & 2(-1)+2(-1)-1(-4) \\ 3(-2)+0(-1)-1(-6) & 3(3)+0(2)-1(9) & 3(-1)+0(-1)-1(-4) \end{bmatrix}$$

Matrix AB = $\begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4-2+6 & 6+4-9 & -2-2+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

For matrix BA, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix BA = $\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} -2(1)+3(2)-1(3) & -2(3)+3(2)-1(0) & -2(-1)+3(-1)-1(-1) \\ -1(1)+2(2)-1(3) & -1(3)+2(2)-1(0) & -1(-1)+2(-1)-1(-1) \\ -6(1)+9(2)-4(3) & -6(3)+9(2)-4(0) & -6(-1)+9(-1)-4(-1) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} -2+6-3 & -6+6+0 & 2-3+1 \\ -1+2-3 & -3+4+0 & 1-2+1 \\ -6+18-12 & -18+18+0 & 6-9+4 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix AB = Matrix BA = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Question

If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, shown that AB = A and BA = B.

Answer

Given : $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$,

Matrix A is of order 3 $\times$ 3 and Matrix B is of order 3 $\times$ 3

To show : matrix AB = A, BA = B

Formula used : $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$$= \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix AB = $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} =$

$$\begin{bmatrix} 2(2)-3(-1)-5(1) & 2(-2)-3(3)-5(-2) & 2(-4)-3(4)-5(-3) \\ -1(2)+4(-1)+5(1) & -1(-2)+4(3)+5(-2) & -1(-4)+4(4)+5(-3) \\ 1(2)-3(-1)-4(1) & 1(-2)-3(3)-4(-2) & 1(-4)-3(4)-4(-3) \end{bmatrix}$$

Matrix AB = $\begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & +2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+8 & -4-12+12 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ = Matrix A

Matrix AB = Matrix A

For matrix BA, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix BA = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-3)-2(4)-4(-3) & 2(-5)-2(5)-4(-4) \\ -1(2)+3(-1)+4(1) & -1(-3)+3(4)+4(-3) & -1(-5)+3(5)+4(-4) \\ 1(2)-2(-1)-3(1) & 1(-3)-2(4)-3(-3) & 1(-5)-2(5)-3(-4) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & +3+12-12 & +5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ = Matrix B

Matrix BA = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ = Matrix B

MATRIX AB = A and MATRIX BA = B

5. Question

If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, show that AB is a zero matrix.

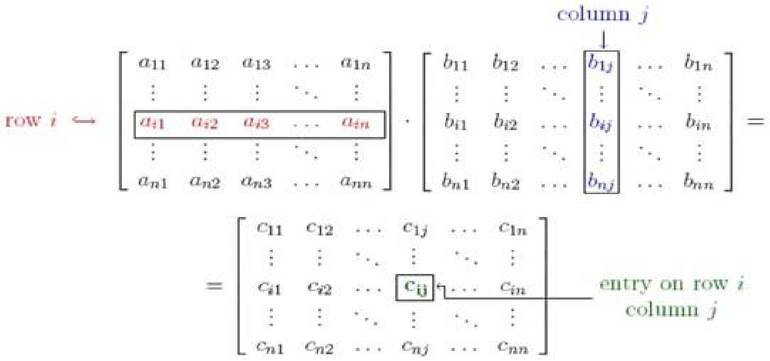
Answer

Given : $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

Matrix A is of order 3 $\times$ 3 , matrix B is of order 3 $\times$ 3 and matrix C is of order 3 $\times$ 3

To show : AB is a zero matrix

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 \times a^2 + c \times ab - b \times ac & 0 \times ab + c \times b^2 - b \times bc & 0 \times ac + c \times bc - b \times c^2 \\ -c \times a^2 + 0 \times ab + a \times ac & -c \times ab + 0 \times b^2 + a \times bc & -c \times ac + 0 \times bc + a \times c^2 \\ b \times a^2 - a \times ab + \times ac & b \times ab - a \times b^2 + \times bc & b \times ac - a \times bc + \times c^2 \end{bmatrix}$$

$$= \begin{bmatrix} abc - abc & b^2c - b^2c & bc^2 - bc^2 \\ -a^2c + a^2c & -abc + abc & -ac^2 + ac^2 \\ a^2b - a^2b & ab^2 - ab^2 & abc - abc \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

= 0 matrix

Hence, Proved

16 A. Question

For the following matrices, verify that A(BC) = (AB)C :

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

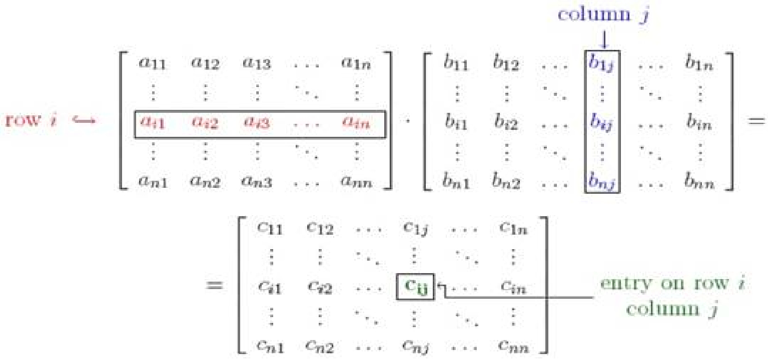
Answer

Given : $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$

Matrix A is of order 2 $\times$ 3 , matrix B is of order 3 $\times$ 3 and matrix C is of order 3 $\times$ 1

To show : matrix A(BC) = (AB)C

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix BC, a = 3,b = c = 3,d = 1 ,thus matrix BC is of order 3$\times$ 1

Matrix BC = $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2(1)+3(4)+0(5) \\ 1(1)+0(4)+4(5) \\ 1(1)-1(4)+2(5) \end{bmatrix} = \begin{bmatrix} 2+12+0 \\ 1+0+20 \\ 1-4+10 \end{bmatrix}$

Matrix BC = $\begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix}$

For matrix A(BC),a = 2 ,b = c = 3 ,d = 1 ,thus matrix A(BC) is of order 2 x 1

Matrix A(BC) = $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix} = \begin{bmatrix} 1(14)+2(21)+5(7) \\ 0(14)+1(21)+3(7) \end{bmatrix} = \begin{bmatrix} 14+42+35 \\ 0+21+21 \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 91 \\ 42 \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 91 \\ 42 \end{bmatrix}$

For matrix AB, a = 2,b = c = 3,d = 3 ,thus matrix BC is of order 2$\times$ 3

Matrix AB =$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 1(2)+2(1)+5(1) & 1(3)+2(0)+5(-1) & 1(0)+2(4)+5(2) \\ 0(2)+1(1)+3(1) & 0(3)+1(0)+3(-1) & 0(0)+1(4)+3(2) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 2+2+5 & 3+0-5 & 0+8+10 \\ 0+1+3 & 0+0-3 & 0+4+6 \end{bmatrix} = \begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix}$

For matrix (AB)C, a = 2,b = c = 3,d = 1 ,thus matrix (AB)C is of order 2$\times$ 1

Matrix (AB)C = $\begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 9(1)-2(4)+18(5) \\ 4(1)-3(4)+10(5) \end{bmatrix}$

Matrix (AB)C = $\begin{bmatrix} 9-8+90 \\ 4-12+50 \end{bmatrix} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$

Matrix (AB)C = $\begin{bmatrix} 91 \\ 42 \end{bmatrix}$

Matrix A(BC) = (AB)C = $\begin{bmatrix} 91 \\ 42 \end{bmatrix}$

6 B. Question

For the following matrices, verify that A(BC) = (AB)C :

$A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and C = [1 -2]

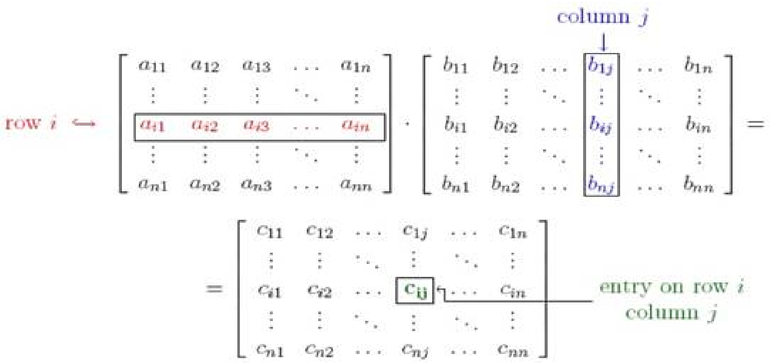
Answer

Given : $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and C = [1 -2]

Matrix A is of order 2 × 3 , matrix B is of order 3 × 1 and matrix C is of order 1 × 2

To show : matrix A(BC) = (AB)C

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix BA exists and is of order c × b ,if and only if d = a

For matrix BC, a = 3,b = c = 1,d = 2 ,thus matrix BC is of order 3× 2

Matrix BC = $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \end{bmatrix} = \begin{bmatrix} 1(1) & 1(-2) \\ 1(1) & 1(-2) \\ 2(1) & 2(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$

Matrix BC = $\begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$

For matrix A(BC),a = 2 ,b = c = 3 ,d = 2 ,thus matrix A(BC) is of order 2 x 2

Matrix A(BC) = $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 2(1)+3(1)-1(2) & 2(-2)+3(-2)-1(-4) \\ 3(1)+0(1)+2(2) & 3(-2)+0(-2)+2(-4) \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 2+3-2 & -4-6+4 \\ 3+0+4 & -6+0-8 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

For matrix AB, a = 2,b = c = 3,d = 1 ,thus matrix BC is of order 2× 1

Matrix AB = $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2(1)+3(1)-1(2) \\ 3(1)+0(1)+2(2) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 2+3-2 \\ 3+0+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

For matrix (AB)C, a = 2,b = c = 1,d = 2 ,thus matrix (AB)C is of order 2 $\times$ 2

Matrix (AB)C = $\begin{bmatrix}3\\7\end{bmatrix} \times \begin{bmatrix}1 & -2\end{bmatrix} = \begin{bmatrix}3(1) & 3(-2)\\7(1) & 7(-2)\end{bmatrix} = \begin{bmatrix}3 & -6\\7 & -14\end{bmatrix}$

Matrix (AB)C = $\begin{bmatrix}3 & -6\\7 & -14\end{bmatrix}$

Matrix A(BC) = (AB)C = $\begin{bmatrix}3 & -6\\7 & -14\end{bmatrix}$

7 A. Question

Verify that A(B + C) = (AB + AC), when

$A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}, B = \begin{bmatrix}2 & 0\\1 & -3\end{bmatrix}$ and $C = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$.

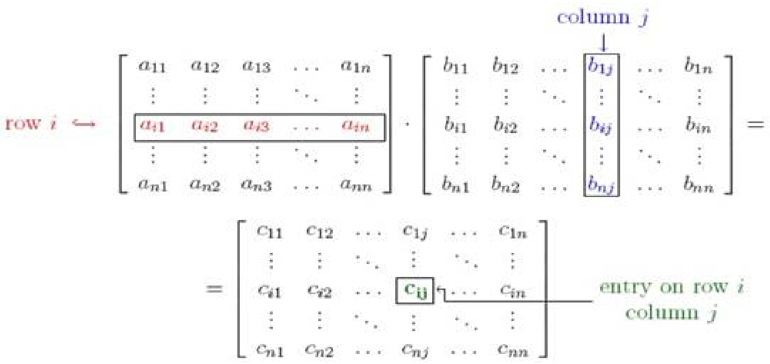
Answer

Given : $A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}, B = \begin{bmatrix}2 & 0\\1 & -3\end{bmatrix}$ and $C = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$.

Matrix A is of order 2 $\times$ 2 , matrix B is of order 2 $\times$ 2 and matrix C is of order 2 $\times$ 2

To verify : A(B + C) = (AB + AC)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

B + C = $\begin{bmatrix}2 & 0\\1 & -3\end{bmatrix} + \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix} = \begin{bmatrix}2+1 & 0-1\\1+0 & -3+1\end{bmatrix} = \begin{bmatrix}3 & -1\\1 & -2\end{bmatrix}$

B + C = $\begin{bmatrix}3 & -1\\1 & -2\end{bmatrix}$

Matrix A(B + C) is of order 2 x 2

A(B + C) = $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} \times \begin{bmatrix}3 & -1\\1 & -2\end{bmatrix} = \begin{bmatrix}1(3)+2(1) & 1(-1)+2(-2)\\3(3)+4(1) & 3(-1)+4(-2)\end{bmatrix}$

A(B + C) = $\begin{bmatrix}3+2 & -1-4\\9+4 & -3-8\end{bmatrix} = \begin{bmatrix}5 & -5\\13 & -11\end{bmatrix}$

A(B + C) = $\begin{bmatrix}5 & -5\\13 & -11\end{bmatrix}$

For matrix AB, a = b = c = d = 2 ,matrix AB is of order 2 x 2

Matrix AB = $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} \times \begin{bmatrix}2 & 0\\1 & -3\end{bmatrix} = \begin{bmatrix}1(2)+2(1) & 1(0)+2(-3)\\3(2)+4(1) & 3(0)+4(-3)\end{bmatrix}$

Matrix AB = $\begin{bmatrix}2+2 & 0-6\\6+4 & 0-12\end{bmatrix} = \begin{bmatrix}4 & -6\\10 & -12\end{bmatrix}$

Matrix AB = $\begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix}$

For matrix AC, a = b = c = d = 2 ,matrix AC is of order 2 x 2

Matrix AC = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(1)+2(0) & 1(-1)+2(1) \\ 3(1)+4(0) & 3(-1)+4(1) \end{bmatrix}$

Matrix AC = $\begin{bmatrix} 1+0 & -1+2 \\ 3+0 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

Matrix AC = $\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

Matrix AB + AC = $\begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 4+1 & -6+1 \\ 10+3 & -12+1 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$

Matrix AB + AC = A(B + C) = $\begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$

A(B + C) = (AB + AC)

7 B. Question

Verify that A(B + C) = (AB + AC), when

$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}.$

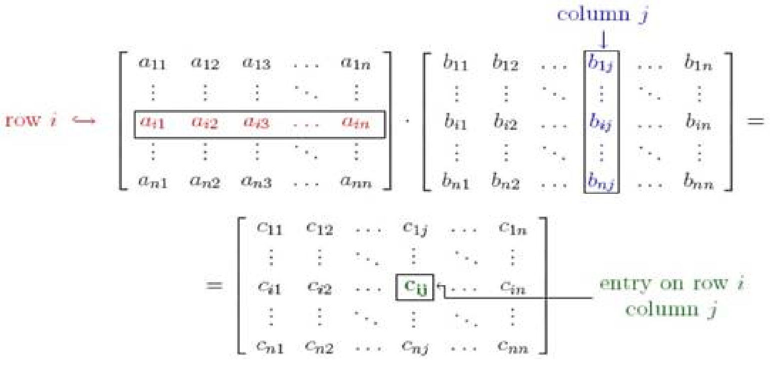
Answer

Given : $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}.$

Matrix A is of order 3 × 2 , matrix B is of order 2 × 2 and matrix C is of order 2 × 2

To verify : A(B + C) = (AB + AC)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix BA exists and is of order c × b ,if and only if d = a

B + C = $\begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5-1 & -3+2 \\ 2+3 & 1+4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix}$

B + C = $\begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix}$

For Matrix A(B + C), a = 3,b = c = d = 2,thus matrix A(B + C) is of order 3 x 2

A(B + C) = $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 2(4)+3(5) & 2(-1)+3(5) \\ -1(4)+4(5) & -1(-1)+4(5) \\ 0(4)+1(5) & 0(-1)+1(5) \end{bmatrix}$

A(B + C) = $\begin{bmatrix} 8+15 & -2+15 \\ -4+20 & 1+20 \\ 0+5 & 0+5 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$

A(B + C) = $\begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$

For matrix AB, a = 3, b = c = d = 2 ,matrix AB is of order 3 x 2

Matrix AB = $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(5)+3(2) & 2(-3)+3(1) \\ -1(5)+4(2) & -1(-3)+4(1) \\ 0(5)+1(2) & 0(-3)+1(1) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 10+6 & -6+3 \\ -5+8 & 3+4 \\ 0+2 & 0+1 \end{bmatrix} = \begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix}$

For matrix AC, a = 3, b = c = d = 2 ,matrix AC is of order 3 x 2

Matrix AC = $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2(-1)+3(3) & 2(2)+3(4) \\ -1(-1)+4(3) & -1(2)+4(4) \\ 0(-1)+1(3) & 0(2)+1(4) \end{bmatrix}$

Matrix AC = $\begin{bmatrix} -2+9 & 4+12 \\ 1+12 & -2+16 \\ 0+3 & 0+4 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix}$

Matrix AC = $\begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix}$

Matrix AB + AC = $\begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 16+7 & 16-3 \\ 3+13 & 7+21 \\ 2+3 & 1+4 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$

Matrix AB + AC = A(B + C) = $\begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$

A(B + C) = (AB + AC)

8. Question

If $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$; verify that A(B – C) = (AB – AC).

Answer

Given : $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$;

Matrix A is of order 3 × 3; matrix B is of order 3 × 3 and matrix C is of order 3 × 3

To verify : A(B – C) = (AB – AC).

The formula used : $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{bmatrix}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix BA exists and is of order c × b ,if and only if d = a

B - C = $\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0-1 & 5-5 & -4-2 \\ -2+1 & 1-1 & 3-0 \\ 1-0 & 0+1 & 2-1 \end{bmatrix}$

B - C = $\begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

For Matrix A(B - C), a = 3,b = c = d = 3,thus matrix A(B + C) is of order 3 x 3

A(B - C) = $\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

A(B - C) = $\begin{bmatrix} 1(-1)+0(-1)-2(1) & 1(0)+0(0)-2(1) & 1(-6)+0(3)-2(1) \\ 3(-1)-1(-1)+0(1) & 3(0)-1(0)+0(1) & 3(-6)-1(3)+0(1) \\ -2(-1)+1(-1)+1(1) & -2(0)+1(0)+1(1) & -2(-6)+1(3)+1(1) \end{bmatrix}$

A(B - C) = $\begin{bmatrix} -1+0-2 & 0+0-2 & -6+0-2 \\ -3+1+0 & 0+0+0 & -18-3+0 \\ 2-1+1 & 0+0+1 & 12+3+1 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$

A(B - C) = $\begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$

For matrix AB, a = 3, b = c = d = 3 ,matrix AB is of order 3 x 3

Matrix AB = $\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 1(0)+0(-2)-2(1) & 1(5)+0(1)-2(0) & 1(-4)+0(3)-2(2) \\ 3(0)-1(-2)+0(1) & 3(5)-1(1)+0(0) & 3(-4)-1(3)+0(2) \\ -2(0)+1(-2)+1(1) & -2(5)+1(1)+1(0) & -2(-4)+1(3)+1(2) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 0+0-2 & 5+0+0 & -4+0-4 \\ 0+2+0 & 15-1+0 & -12-3+0 \\ 0-2+1 & -10+1+0 & 8+3+2 \end{bmatrix} = \begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix}$

For matrix AC, a = 3, b = c = d = 3 ,matrix AC is of order 3 x 3

Matrix AC = $\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

Matrix AC = $\begin{bmatrix} 1(1)+0(-1)-2(0) & 1(5)+0(1)-2(-1) & 1(2)+0(0)-2(1) \\ 3(1)-1(-1)+0(0) & 3(5)-1(1)+0(-1) & 3(2)-1(0)+0(1) \\ -2(1)+1(-1)+1(0) & -2(5)+1(1)+1(-1) & -2(2)+1(0)+1(1) \end{bmatrix}$

Matrix AC = $\begin{bmatrix} 1+0+0 & 5+0+2 & 2+0-2 \\ 3+1+0 & 15+1+0 & 6+0+0 \\ -2-1+0 & -10+1-1 & -4+0+1 \end{bmatrix} = \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix}$

Matrix AC = $\begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix}$

Matrix AB - AC = $\begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix} = \begin{bmatrix} -2-1 & 5-7 & -8-0 \\ 2-4 & 14-16 & -15-6 \\ -1+3 & -9+10 & 13+3 \end{bmatrix}$

Matrix AB - AC = $\begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$

A(B – C) = (AB – AC) = $\begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$

9. Question

If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $A^2 = O$.

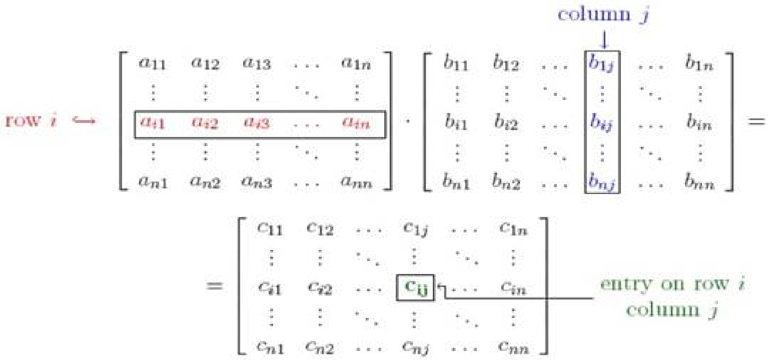
Answer

Given : $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$,

Matrix A is of order 2 × 2

To show : $A^2 = O$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

$$A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \times \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} = \begin{bmatrix} ab(ab) + b^2(-a^2) & ab(b^2) + b^2(-ab) \\ -a^2(ab) - ab(-a^2) & -a^2(b^2) - ab(-ab) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A^2 = O$

10. Question

If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that $A^2 = A$.

Answer

Given : $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$,

Matrix A is of order 3 × 3

To show : $A^2 = A$

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

$$A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-2)-2(3)-4(-2) & 2(-4)-2(4)-4(-3) \\ -1(2)+3(-1)+4(1) & -1(-2)+3(3)+4(-2) & -1(-4)+3(4)+4(-3) \\ 1(2)-2(-1)-3(1) & 1(-2)-2(3)-3(-2) & 1(-4)-2(4)-3(-3) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

11. Question

If $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$, show that $A^2 = I$.

Answer

Given : $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$,

Matrix A is of order 3 × 3

To show : $A^2 = I$

Formula used : $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$$= \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

$$A^2 = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \times \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4(4)-1(3)-4(3) & 4(-1)-1(0)-4(-1) & 4(-4)-1(-4)-4(-3) \\ 3(4)+0(3)-4(3) & 3(-1)+0(0)-4(-1) & 3(-4)+0(-4)-4(-3) \\ 3(4)-1(3)-3(3) & 3(-1)-1(0)-3(-1) & 3(-4)-1(-4)-3(-3) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 16-3-12 & -4+0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

12. Question

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $(3A^2 - 2B + I)$.

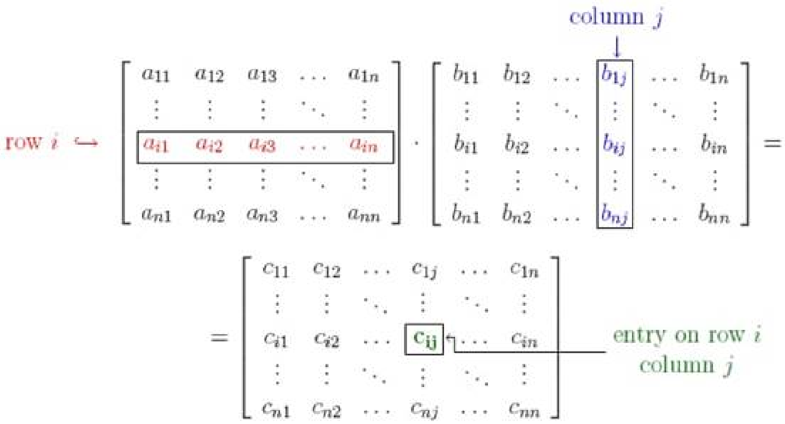
Answer

Given : $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$,

Matrix A is of order 2 × 2, Matrix B is of order 2 × 2

To find : $3A^2 - 2B + I$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2(2)-1(3) & 2(-1)-1(2) \\ 3(2)+2(3) & 3(-1)+2(2) \end{bmatrix} = \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$

$3A^2 = 3 \times \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix}$

$3A^2 = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix}$

$2B = 2 \times \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$2B = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-0+1 & -12-8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

13. Question

If $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$ then find $(-A^2 + 6A)$.

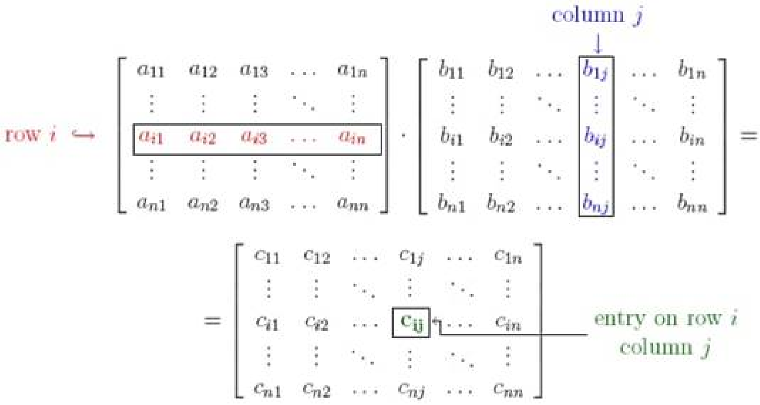
Answer

Given : $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$

Matrix A is of order 2 $\times$ 2.

To find : $-A^2 + 6A$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

$A^2 = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2(2)-2(-3) & 2(-2)-2(4) \\ -3(2)+4(-3) & -3(-2)+4(4) \end{bmatrix} = \begin{bmatrix} 4+6 & -4-8 \\ -6-12 & 6+16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix}$

$-A^2 = -\begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix} = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix}$

$6A = 6 \times \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$

$6A = \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$

$-A^2 + 6A = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix} + \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix} = \begin{bmatrix} -10+12 & 12-12 \\ 18-18 & -22+24 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$-A^2 + 6A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

14. Question

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that (A2 – 5A + 7I) = O.

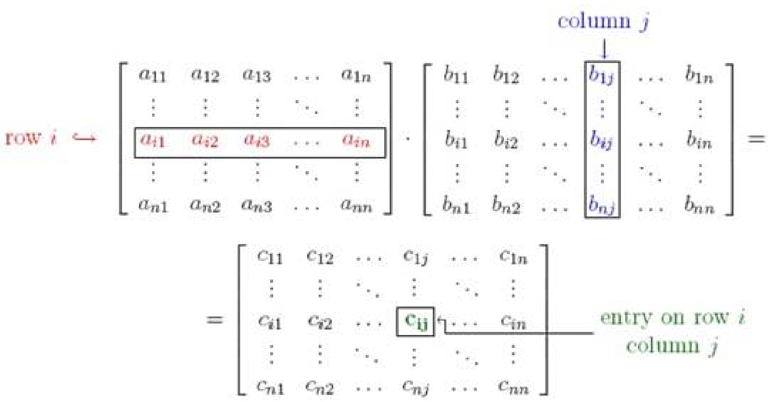
Answer

Given : $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$,

Matrix A is of order 2 × 2.

To show : $A^2 - 5A + 7I = 0$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b =

c

A^2 = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

A^2 = $\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

5A = 5 × $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

5A = $\begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7I = 7 × $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

7I = $\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

A^2 - 5A + 7I = $\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

A^2 - 5A + 7I = 0

15. Question

Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation A^3 – 4A^2 + A = O.

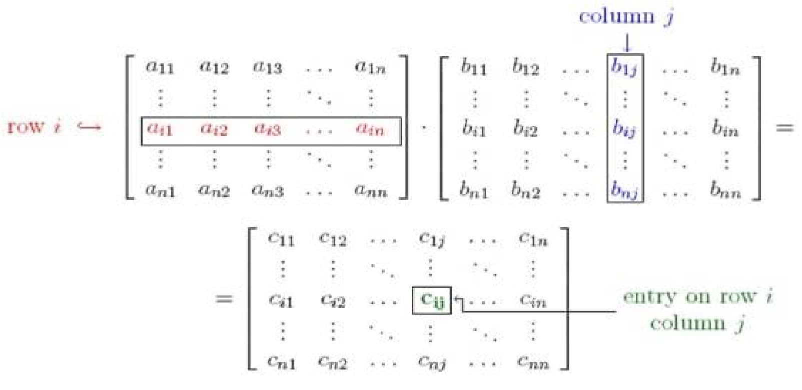
Answer

Given : $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

Matrix A is of order 2 × 2.

To show : A^3 - 4A^2 + A = 0

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots + a_{in}b_{nj}$

A^2 and A^3 are matrices of order 2 x 2.

A^2 = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2(2)+3(1) & 2(3)+3(2) \\ 1(2)+2(1) & 1(3)+2(2) \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$

A^2 = $\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

A^3 = $\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7(2)+12(1) & 7(3)+12(2) \\ 4(2)+7(1) & 4(3)+7(2) \end{bmatrix} = \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$

A^3 = $\begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$

4A^2 = 4 × $\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix}$

$4A^2 = \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix}$

$A^3 - 4A^2 + A = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^3 - 4A^2 + A = 0$

16. Question

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

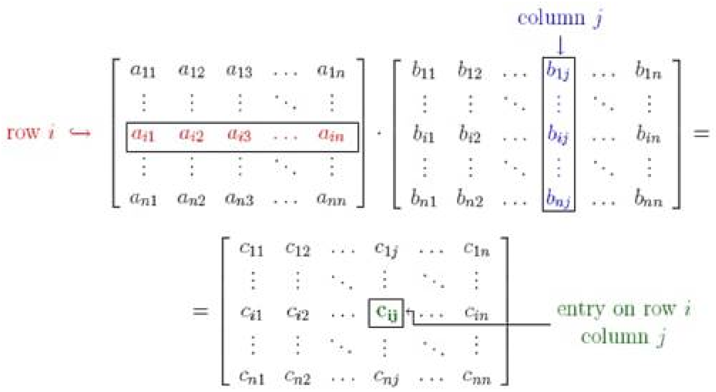
Answer

Given : $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, $A^2 = kA - 2I$.

Matrix A is of order 2 × 2.

To find : k

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3(3)-2(4) & 3(-2)-2(-2) \\ 4(3)-2(4) & 4(-2)-2(-2) \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

$kA = k \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$

$kA - 2I = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - 2 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

It is the given that $A^2 = kA - 2I$

$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

Equating like terms,

$3k - 2 = 1$

$3k = 1 + 2 = 3$

$3k = 3$

$k = \frac{3}{3} = 1$

$k = 1$

17. Question

If $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, find f(A), where $f(x) = x^2 - 2x + 3$.

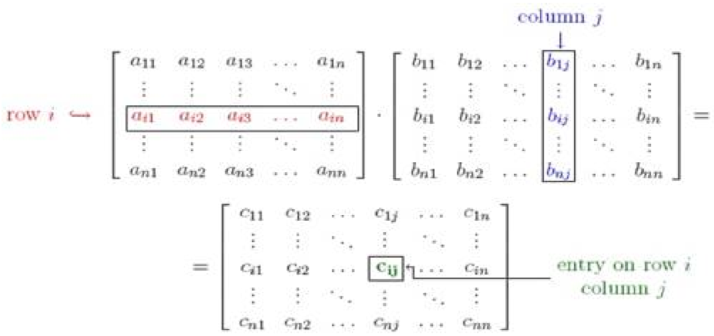
Answer

Given : $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, and f(x) = x^2 – 2x + 3.

Matrix A is of order 2 × 2.

To find : f(A)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

f(x) = x^2 – 2x + 3

f(A) = A^2 – 2A + 3I

$A^2 = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -1(-1) + 2(3) & -1(2) + 2(1) \\ 3(-1) + 1(3) & 3(2) + 1(1) \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 + 6 & -2 + 2 \\ -3 + 3 & 6 + 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$2A = 2 \times \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix}$

$2A = \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix}$

$3I = 3 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

f(A) = $A^2 - 2A + 3I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7 + 2 + 3 & -4 + 0 \\ 0 - 6 + 0 & 7 - 2 + 3 \end{bmatrix}$

f(A) = $A^2 - 2A + 3I = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$

f(A) = $A^2 - 2A + 3I = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$

18. Question

If $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and f(x) = $2x^3$ + 4x + 5, find f(A).

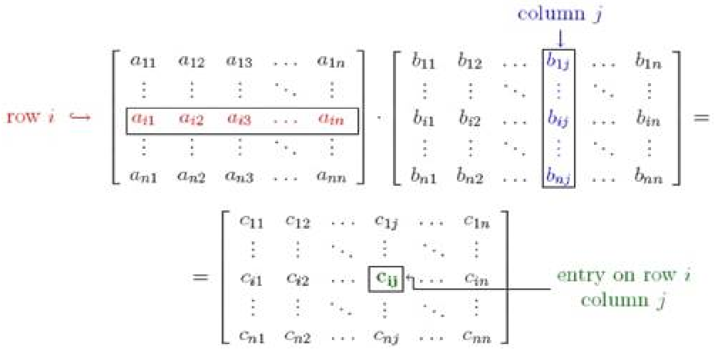
Answer

Given : $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and f(x) = $2x^3$ + 4x + 5

Matrix A is of order 2 × 2.

To find : f(A)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^3 is a matrix of order 2 x 2.

$f(x) = 2x^3 + 4x + 5$

$f(A) = 2A^3 + 4A + 5I$

$A^2 = \begin{bmatrix}1 & 2\\4 & -3\end{bmatrix} \times \begin{bmatrix}1 & 2\\4 & -3\end{bmatrix} = \begin{bmatrix}1(1)+2(4) & 1(2)+2(-3)\\4(1)-3(4) & 4(2)-3(-3)\end{bmatrix} = \begin{bmatrix}1+8 & 2-6\\4-12 & 8+9\end{bmatrix}$

$A^2 = \begin{bmatrix}9 & -4\\-8 & 17\end{bmatrix}$

$A^3 = \begin{bmatrix}9 & -4\\-8 & 17\end{bmatrix} \times \begin{bmatrix}1 & 2\\4 & -3\end{bmatrix} = \begin{bmatrix}9(1)-4(4) & 9(2)-4(-3)\\-8(1)+17(4) & -8(2)+17(-3)\end{bmatrix}$

$A^3 = \begin{bmatrix}9-16 & 18+12\\-8+68 & -16-51\end{bmatrix} = \begin{bmatrix}-7 & 30\\60 & -67\end{bmatrix}$

$A^3 = \begin{bmatrix}-7 & 30\\60 & -67\end{bmatrix}$

$2A^3 = 2 \times \begin{bmatrix}-7 & 30\\60 & -67\end{bmatrix} = \begin{bmatrix}-14 & 60\\120 & -134\end{bmatrix}$

$2A^3 = \begin{bmatrix}-14 & 60\\120 & -134\end{bmatrix}$

$4A = 4 \times \begin{bmatrix}1 & 2\\4 & -3\end{bmatrix} = \begin{bmatrix}4 & 8\\16 & -12\end{bmatrix}$

$4A = \begin{bmatrix}4 & 8\\16 & -12\end{bmatrix}$

$5I = 5 \times \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$

$5I = \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$

$2A^3 + 4A + 5I = \begin{bmatrix}-14 & 60\\120 & -134\end{bmatrix} + \begin{bmatrix}4 & 8\\16 & -12\end{bmatrix} + \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix} = \begin{bmatrix}-14+4+5 & 60+8+0\\120+16+0 & -134-12+5\end{bmatrix}$

$f(A) = 2A^3 + 4A + 5I = \begin{bmatrix}-5 & 68\\136 & -141\end{bmatrix}$

$f(A) = 2A^3 + 4A + 5I = \begin{bmatrix}-5 & 68\\136 & -141\end{bmatrix}$

19. Question

Find the values of x and y, when

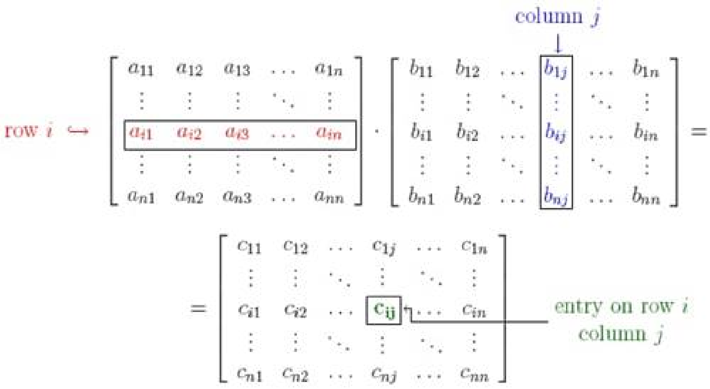
$$\begin{bmatrix}2 & -3\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}$$

Answer

Given : $\begin{bmatrix}2 & -3\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}$

To find : x and y

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

The resulting matrix obtained on multiplying $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is of order 2 × 1

$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Equating similar terms,

2x – 3y = 1 equation 1

x + y = 3 equation 2

equation 1 + 3(equation 2) and solving the above equations,

$$\begin{array}{l} 2x - 3y = 1 \\ + \\ 3x + 3y = 9 \\ \hline 5x = 10 \end{array}$$

$x = \frac{10}{5} = 2$

x = 2 , substituting x = 2 in equation 2,

2 + y = 3

y = 3 – 2 = 1

x = 2 and y = 1

20. Question

Solve for x and y, when

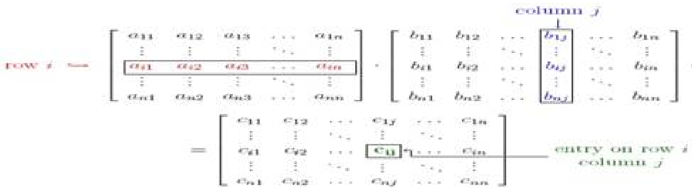
$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}.$

Answer

Given : $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}.$

To find : x and y

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

The resulting matrix obtained on multiplying $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is of order 2 × 1

$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x - 4y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$

$\begin{bmatrix} 3x - 4y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$

Equating similar terms,

3x – 4y = 3 equation 1

x + 2y = 11 equation 2

equation 1 + 2(equation 2) and solving the above equations,

3x – 4y = 3
+
2x + 4y = 22

5x = 3 + 22 = 25

5x = 25

$x = \frac{25}{5} = 5$

x = 5 , substituting x = 2 in equation 2,

5 + 2y = 11

2y =11 – 5 = 6

2y = 6

$y = \frac{6}{2} = 3$

x = 5 and y = 3

21. Question

If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$.

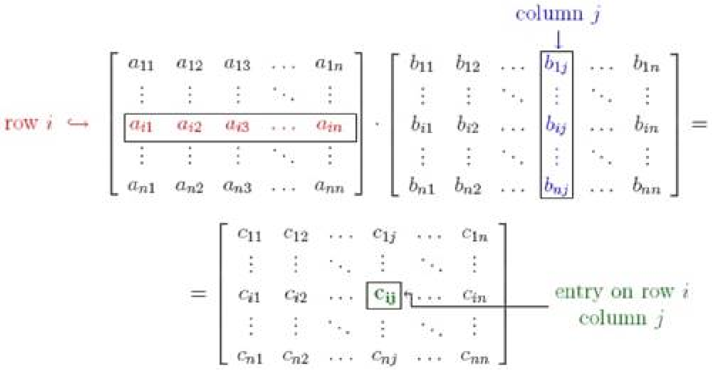
Answer

Given : $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, $A^2 + xI = yA$.

A is a matrix of order 2 x 2

To find : x and y

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

A^2 is a matrix of order 2 x 2

$A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3(3)+1(7) & 3(1)+1(5) \\ 7(3)+5(7) & 7(1)+5(5) \end{bmatrix} = \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$

$A^2 = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$

$xI = x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$

$xI = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$

$A^2 + xI = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 16+x & 8+0 \\ 56+0 & 32+x \end{bmatrix} = \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix}$

$A^2 + xI = \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix}$

$yA = y \times \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

$yA = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

It is given that $A^2 + xI = yA$,

$\begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

Equating similar terms in the given matrices,

16 + x = 3y and 8 = y,

hence y = 8

Substituting y = 8 in equation 16 + x = 3y

16 + x = 3 × 8 = 24

16 + x = 24

x = 24 – 16 = 8

x = 8

x = 8, y = 8

22. Question

If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the value of a and b such that $A^2 + aA + bI = O$.

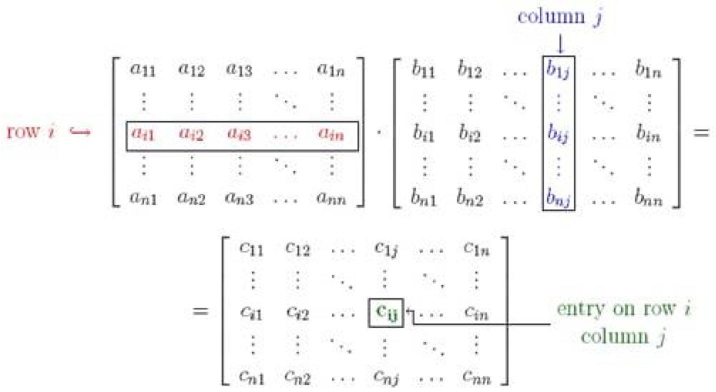
Answer

Given : $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, $A^2 + aA + bI = O$

A is a matrix of order 2 x 2

To find : a and b

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

A^2 is a matrix of order 2 x 2

$A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3(3)+2(1) & 3(2)+2(1) \\ 1(3)+1(1) & 1(2)+1(1) \end{bmatrix} = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

$aA = a \times \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3a & 2a \\ 1a & 1a \end{bmatrix}$

$bI = b \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$

$bI = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$

$A^2 + aA + bI = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ 1a & 1a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix}$

$A^2 + aA + bI = \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix}$

It is given that $A^2 + aA + bI = 0$

$\begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Equating similar terms in the matrices,we get

4 + a = 0 and 3 + a + b = 0

a = 0 – 4 = -4

a = -4

substituting a = -4 in 3 + a + b = 0

3 – 4 + b = 0

-1 + b = 0

b = 0 + 1 = 1

b = 1

a = -4 and b = 1

23. Question

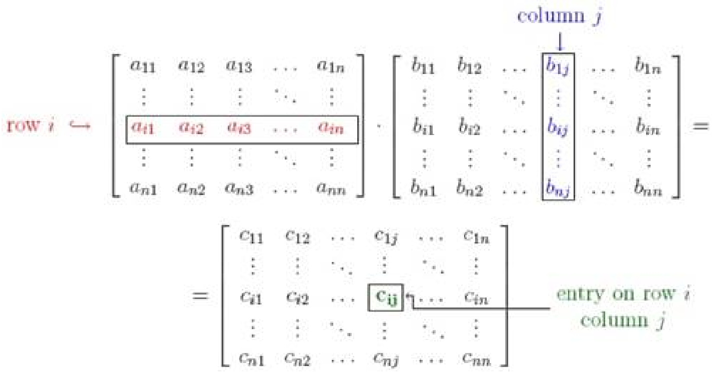
Find the matrix A such that $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}. A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$.

Answer

Given : $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}. A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$.

To find : matrix A

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots + a_{in}b_{nj}$

IF XA = B, then $A = X^{-1}B$

$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} . A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

To find $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1}$

Determinant of given matrix = $\begin{vmatrix} 5 & -7 \\ -2 & 3 \end{vmatrix}$ = 5(3) – (-7)(-2) = 15 – 14 = 1

Adjoint of matrix $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} = \frac{1}{1} \times \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 3(-16) + 7(7) & 3(-6) + 7(2) \\ 2(-16) + 5(7) & 2(-6) + 5(2) \end{bmatrix} = \begin{bmatrix} -48 + 49 & -18 + 14 \\ -32 + 35 & -12 + 10 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$

24. Question

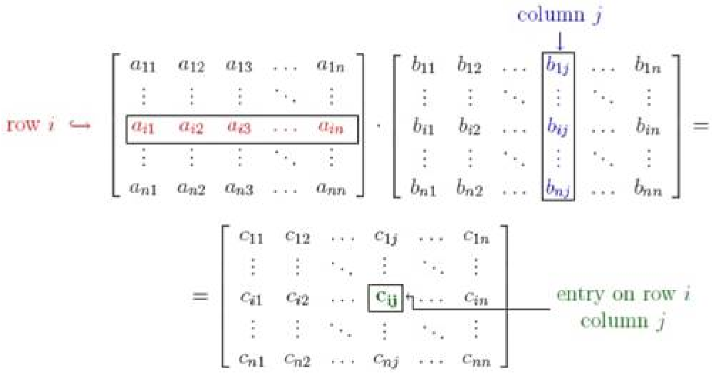
Find the matrix A such that A. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$.

Answer

Given : A. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$.

To find : matrix A

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

IF AX = B, then $A = BX^{-1}$

A. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1}$

To find $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1}$

Determinant of given matrix = $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$ = 5(2) – (4)(3) = 10 – 12 = -2

Adjoint of matrix $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{1}{-2} \times \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \frac{1}{-2} . \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{1}{-2} . \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \frac{1}{-2} . \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$A = \frac{1}{-2} \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \frac{1}{-2} . \begin{bmatrix} 0(5) - 4(-4) & 0(-3) - 4(2) \\ 10(5) + 3(-4) & 10(-3) + 3(2) \end{bmatrix}$

$A = \frac{1}{-2} . \begin{bmatrix} 0 + 16 & 0 - 8 \\ 50 - 12 & -30 + 6 \end{bmatrix} = \frac{1}{-2} . \begin{bmatrix} 16 & -8 \\ 38 & -24 \end{bmatrix} = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$

$A = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$

$A = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$

25. Question

If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = (A^2 + B^2)$ then find the values of a and b.

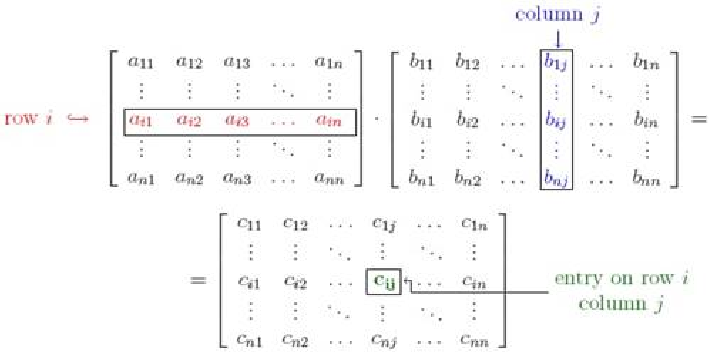
Answer

Given : $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix}$

$(A + B)^2 = (A^2 + B^2)$

To find : a and b

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

$A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 1+a & -1-1 \\ 2+b & -1-1 \end{bmatrix} = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix}$

$A + B = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix} \times \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix} = \begin{bmatrix} (1+a)(1+a)-2(2+b) & (1+a)(-2)-2(-2) \\ (2+b)(1+a)-2(2+b) & (2+b)(-2)-2(-2) \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} 1+a^2+2a-4-2b & -2-2a+4 \\ 2+2a+b+ab-4-2b & -4-2b+4 \end{bmatrix} = \begin{bmatrix} a^2+2a-2b-3 & 2-2a \\ 2a-b+ab-2 & -2b \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} a^2+2a-2b-3 & 2-2a \\ 2a-b+ab-2 & -2b \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1(1)-1(2) & 1(-1)-1(-1) \\ 2(1)-1(2) & 2(-1)-1(-1) \end{bmatrix} = \begin{bmatrix} 1-2 & -1+1 \\ 2-2 & -2+2 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

$B^2 = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} \times \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a(a)-1(b) & a(-1)-1(-1) \\ b(a)-1(b) & b(-1)-1(-1) \end{bmatrix} = \begin{bmatrix} a^2-b & -a+1 \\ ab-b & -b+1 \end{bmatrix}$

$B^2 = \begin{bmatrix} a^2-b & -a+1 \\ ab-b & -b+1 \end{bmatrix}$

$(A^2 + B^2) = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a^2-b & -a+1 \\ ab-b & -b+1 \end{bmatrix} = \begin{bmatrix} -1+a^2-b & -a+1 \\ ab-b & -b+1 \end{bmatrix}$

$(A^2 + B^2) = \begin{bmatrix} -1+a^2-b & -a+1 \\ ab-b & -b+1 \end{bmatrix}$

It is given that $(A + B)^2 = (A^2 + B^2)$

$\begin{bmatrix} a^2+2a-2b-3 & 2-2a \\ 2a-b+ab-2 & -2b \end{bmatrix} = \begin{bmatrix} -1+a^2-b & -a+1 \\ ab-b & -b+1 \end{bmatrix}$

Equating similar terms in the given matrices we get,

2 – 2a = -a + 1 and -2b = -b + 1

2 – 1 = -a + 2a and -2b + b = 1

1 = a and -b = 1

a = 1 and b = -1

26. Question

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that F(x) . F(y) = F(x + y).

Answer

Given : $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$,

To show : F(x) . F(y) = F(x + y).

Formula used : $\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\times\begin{bmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{bmatrix}$

$$=\begin{bmatrix}a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33}\\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33}\\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33}\end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

F(x) = $\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

F(y) = $\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x + y) = $\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x) . F(y) = $\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}\cos x(\cos y)-\sin x(\sin y)+0(0) & \cos x(-\sin y)-\sin x(\cos y)+0(0) & \cos x(0)-\sin x(0)+0(1)\\ \sin x(\cos y)+\cos x(\sin y)+0(0) & \sin x(-\sin y)+\cos x(\cos y)+0(0) & \sin x(0)+\cos x(0)+0(1)\\ 0(\cos y)+0(\sin y)+1(0) & 0(-\sin y)+0(\cos y)+1(0) & 0(0)+0(0)+1(1)\end{bmatrix}$$

F(x) . F(y) = $\begin{bmatrix}\cos x\cos y-\sin x\sin y & -\cos x\sin y-\sin x\cos y & 0\\ \sin x\cos y+\cos x\sin y & -\sin x\sin y+\cos x\cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

We know that,

cosx(cosy) – sinx (siny) = cos(x+y) and -cosx(siny) - sinx(cosy) = -sin(x+y)

F(x) . F(y) = $\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x + y) = F(x) . F(y) = $\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x + y) = F(x) . F(y)

27. Question

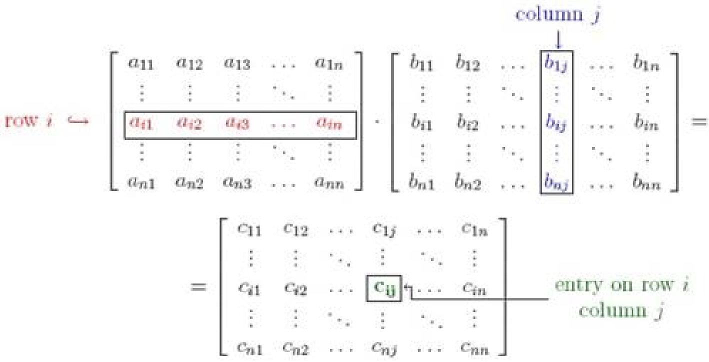
If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, show that $A^2=\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{bmatrix}$

Answer

Given : $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$,

To show : $A^2=\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{bmatrix}$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \times \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha(\cos\alpha) + \sin\alpha(-\sin\alpha) & \cos\alpha(\sin\alpha) + \sin\alpha(\cos\alpha) \\ -\sin\alpha(\cos\alpha) + \cos\alpha(-\sin\alpha) & -\sin\alpha(\cos\alpha) + \cos\alpha(\cos\alpha) \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & -2\sin\alpha\cos\alpha \\ -2\sin\alpha\cos\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$

We know that cos2α = $\cos^2\alpha - \sin^2\alpha$ and sin2α = $2\sin\alpha\cos\alpha$

$A^2 = \begin{bmatrix} \cos2\alpha & -\sin2\alpha \\ -\sin2\alpha & \cos2\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos2\alpha & -\sin2\alpha \\ -\sin2\alpha & \cos2\alpha \end{bmatrix}$

28. Question

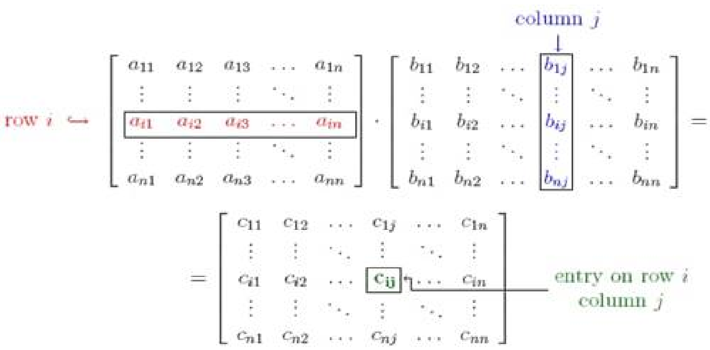
If $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = O$, find x.

Answer

Given : $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = O,$

To find : x

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$$[1 \quad x \quad 1]\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\-2\\3\end{bmatrix} = 0$$

$$[1 \quad x \quad 1] \times \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix} = [1(1)+x(4)+1(3) \quad 1(2)+x(5)+1(2) \quad 1(3)+x(6)+1(5)]$$

$$[1 \quad x \quad 1] \times \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix} = [1+4x+3 \quad 2+5x+2 \quad 6x+8]$$

$$[1 \quad x \quad 1] \times \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix} = [4x+4 \quad 5x+4 \quad 6x+8]$$

$$[1 \quad x \quad 1]\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\-2\\3\end{bmatrix} = [4x+4 \quad 5x+4 \quad 6x+8] \times \begin{bmatrix}1\\-2\\3\end{bmatrix}$$

$$[4x+4 \quad 5x+4 \quad 6x+8] \times \begin{bmatrix}1\\-2\\3\end{bmatrix} = [(4x+4)(1)+(5x+4)(-2)+(6x+8)(3)]$$

$$[4x+4 \quad 5x+4 \quad 6x+8] \times \begin{bmatrix}1\\-2\\3\end{bmatrix} = [4x+4-10x-8+18x+24] = [12x+20]$$

$$[1 \quad x \quad 1]\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\-2\\3\end{bmatrix} = [12x+20] = 0$$

12x + 20 = 0

12x = -20

$x = \frac{-20}{12} = \frac{-5}{3}$

$x = \frac{-5}{3}$

29. Question

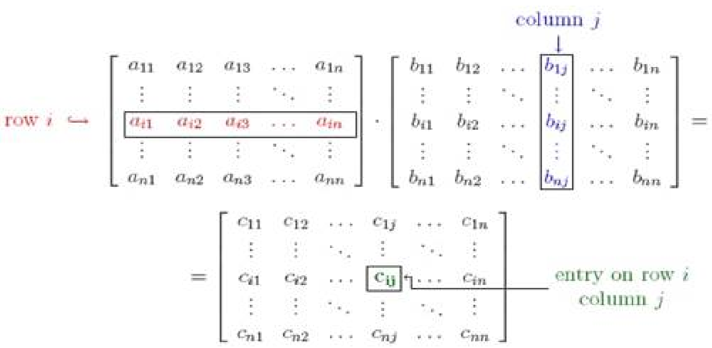
If $\begin{bmatrix}x & 4 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix} = O$, find x.

Answer

Given : $\begin{bmatrix}x & 4 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix} = O,$

To find : x

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$[x \quad 4 \quad 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix} = 0$

$[x \quad 4 \quad 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix} = [x(2)+4(1)+1(0) \quad x(1)+4(0)+1(2) \quad x(2)+4(2)+1(-4)]$

$[x \quad 4 \quad 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix} = [2x+4 \quad x+2 \quad 2x+4]$

$[2x+4 \quad x+2 \quad 2x+4]\begin{bmatrix}x\\4\\-1\end{bmatrix} = [(2x+4)(x)+4(x+2)+(2x+4)(-1)]$

$[x \quad 4 \quad 1]\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix} = [2x^2+4x+4x+8-2x-4] = [2x^2+6x+4] = 0$

$2x^2 + 6x + 4 = 0$

$x^2 + 3x + 2 = 0$

$(x + 1)(x + 2) = 0$

$x + 1 = 0$ or $x + 2 = 0$

$x = -1$ or $x = -2$

$x = -1$ or $x = -2$

30. Question

Find the values of a and b for which

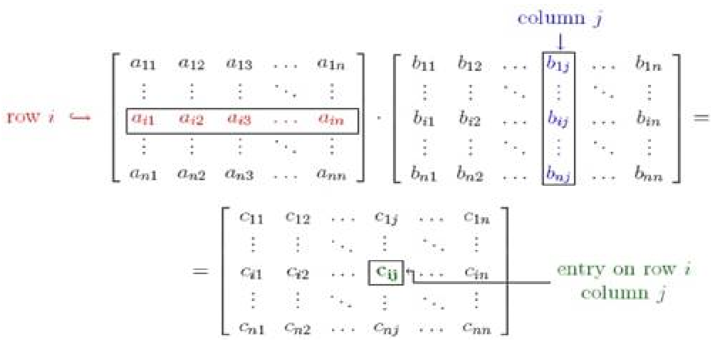
$\begin{bmatrix}a & b\\-a & 2b\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix}=\begin{bmatrix}5\\4\end{bmatrix}.$

Answer

Given : $\begin{bmatrix}a & b\\-a & 2b\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix}=\begin{bmatrix}5\\4\end{bmatrix}.$

To find : a and b

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots\dots\dots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$\begin{bmatrix}a & b\\-a & 2b\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$

$\begin{bmatrix}a & b\\-a & 2b\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}a(2)+b(-1)\\-a(2)+2b(-1)\end{bmatrix} = \begin{bmatrix}2a-b\\-2a-2b\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$

$\begin{bmatrix}2a-b\\-2a-2b\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$

Equating similar terms,

2a – b = 5

-2a – 2b = 4

Adding the above two equations,we get

-3b = 9

b = $\frac{9}{-3}$ = -3

b = -3

substituting b = -3 in 2a – b = 5,we get

2a + 3 = 5

2a = 5 – 3 = 2

a = 1

a = 1 and b = -3

31. Question

If $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, find f(A), where f(x) = x2 – 5x + 7.

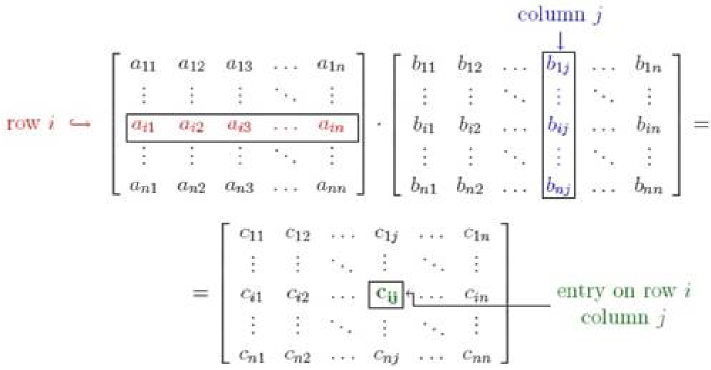
Answer

Given : $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, and f(x) = x^2 – 5x + 7.

Matrix A is of order 2 × 2.

To find : f(A)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

f(x) = x^2 – 5x + 7

f(A) = A^2 – 5A + 7I

$A^2 = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 3(3)+4(-4) & 3(4)+4(-3) \\ -4(3)-3(-4) & -4(4)-3(-3) \end{bmatrix} = \begin{bmatrix} 9-16 & 12-12 \\ -12+12 & -16+9 \end{bmatrix}$

$A^2 = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$

$5A = 5 \times \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}$

$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$f(A) = A^2 - 5A + 7I = \begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix} - \begin{bmatrix}15 & 20\\ -20 & -15\end{bmatrix} + \begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix} = \begin{bmatrix}-7-15+7 & 0-20+0\\ 0+20+0 & -7+15+7\end{bmatrix}$

$f(A) = A^2 - 5A + 7I = \begin{bmatrix}-15 & -20\\ 20 & 15\end{bmatrix}$

$f(A) = A^2 - 5A + 7I = \begin{bmatrix}-15 & -20\\ 20 & 15\end{bmatrix}$

32. Question

If $A = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, prove that $A^n = \begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$ for all $n \in N$.

Answer

Given : $A = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$,

Matrix A is of order 2×2.

To prove : $A^n = \begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$

Proof :

$A = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

Let us assume that the result holds for A^{n-1}

$A^{n-1} = \begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix}$

We need to prove that the result holds for A^n by mathematical induction .

$A^n = A^{n-1} \times A = \begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix} \times \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1(1)+(n-1)(0) & 1(1)+(n-1)(1)\\ 0(1)+1(0) & 0(1)+1(1)\end{bmatrix}$

$A^n = \begin{bmatrix}1+0 & 1+n-1\\ 0+0 & 0+1\end{bmatrix} = \begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$

$A^n = \begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$

33. Question

Given an example of two matrices A and B such that

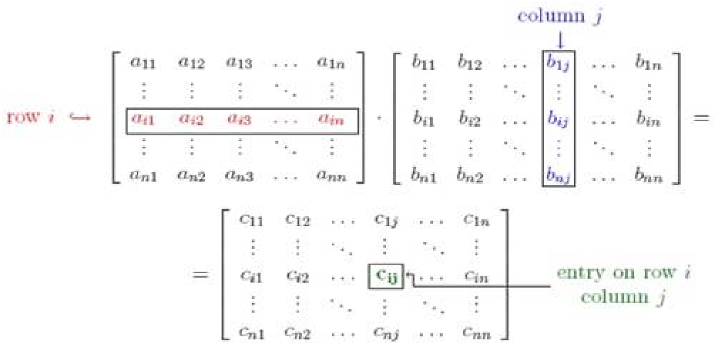
$A \neq O$, $B \neq O$, $AB = O$ and $BA \neq O$.

Answer

Given : $A \neq 0, B \neq 0, AB = 0, BA \neq 0$

To Find : matrix A and B

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots\dots\dots + a_{in}b_{nj}$

Let $A = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ and $B = \begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$

$A \neq 0, B \neq 0$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(0)+0(1) & 1(0)+0(0) \\ 0(0)+0(1) & 0(0)+0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0(1)+0(0) & 0(0)+0(0) \\ 1(1)+0(0) & 1(0)+0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

34. Question

Give an example of three matrices A, B, C such that

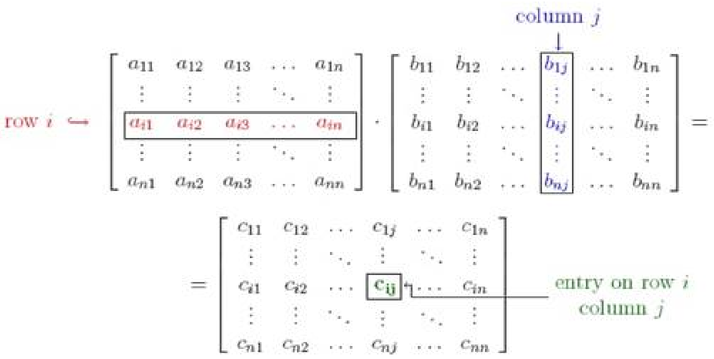
$AB = AC$ but $B \neq C$.

Answer

Given : $AB = AC$ and $B \neq C$.

To Find : matrix A and B

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots + a_{in}b_{nj}$

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$B \neq C$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(0)+0(1) & 1(0)+0(0) \\ 0(0)+0(1) & 0(0)+0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(0)+0(0) & 1(0)+0(1) \\ 0(0)+0(0) & 0(0)+0(1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$AC = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$AB = AC = 0$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

35. Question

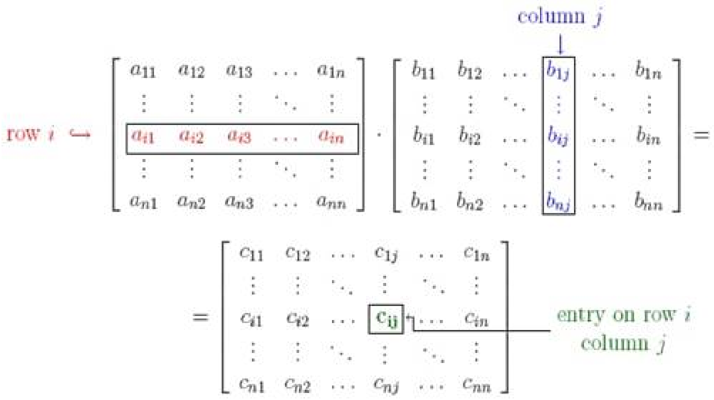
If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $(3A^2 - 2B + I)$.

Answer

Given : $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, Matrices A and B are of order 2 × 2.

To find : $(3A^2 - 2B + I)$.

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1(1)+0(-1) & 1(0)+0(7) \\ -1(1)+7(-1) & -1(0)+7(7) \end{bmatrix}$

$A^2 = \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$

$3A^2 = 3 \times \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix}$

$3A^2 = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix}$

$2B = 2 \times \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$2B = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-0+1 & 0-8+0 \\ -24+2+0 & 147-14+1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 4 & -8 \\ -22 & 134 \end{bmatrix}$

36. Question

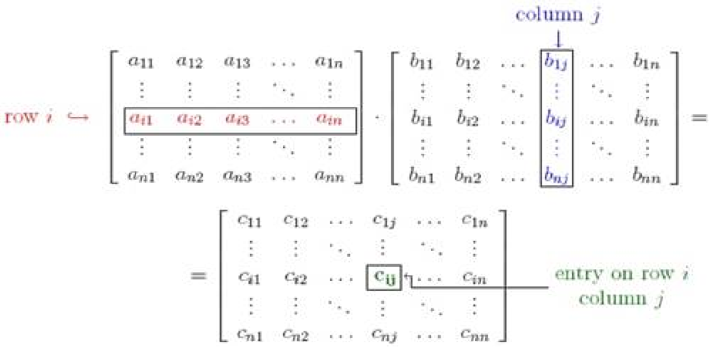
If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, find the value of x.

Answer

Given : $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$,

To find : x

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots\dots + a_{in}b_{nj}$

$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 2(1)+3(-2) & 2(-3)+3(4) \\ 5(1)+7(-2) & 5(-3)+7(4) \end{bmatrix} = \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

Equating similar terms in the two matrices, we get

x = 13

x = 13

Exercise 5D

1. Question

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix}$, verify that (A')' = A.

Answer

Transpose of a matrix is obtained by interchanging the rows and the columns of matrix A. It is denoted by A'.

e.g. $A_{12} = A_{21}$

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix}$$

Hence transpose of matrix A is,

$$A' = \begin{bmatrix} 2 & 0 \\ -3 & 7 \\ 5 & -4 \end{bmatrix}$$

$(A')' = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix}$ (A')' = AHence, Proved.

2. Question

If $A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$, verify that (2A)' = 2A'.

Answer

Given $A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$

To Prove: (2A)’ = 2A’

Proof: Let us consider, B = 2A

Now, $B = 2\begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 10 \\ -4 & 0 \\ 8 & -12 \end{bmatrix}$

LHS$\Rightarrow$ $B' = \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix}$

Again to find RHS, we will find the transpose of matrix A

$A' = \begin{bmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{bmatrix}$

RHS = 2A’

$\Rightarrow 2\begin{bmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix}$

LHS = RHS

Hence proved.

3. Question

If $A = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$, verify that (A + B)’ = (A’ + B’).

Answer

Given $A = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$

To Prove: (A + B)’ = A’ + B’

Proof: Let us consider C = A + B

$C = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix} + \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -1 & -3 & -3 \\ -2 & 1 & 2 \end{bmatrix}$

Now LHS = C’

$\Rightarrow \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$

To find RHS, we will find transpose of matrix A and B

$$A' = \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix} \text{ And } B' = \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix}$$

RHS = A' + B'

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix} + \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$$

LHS = RHS

Hence proved.

4. Question

If $P = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix}$ and $P = \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$, verify that (P + Q)' = (P' + Q').

Answer

Given $P = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix}$ and $Q = \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$

To Prove: (P + Q)' = P' + Q'

Proof: Let us consider R = P + Q,

$$R = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & -1 \\ -2 & -1 \\ 2 & 11 \end{bmatrix}$$

LHS = R $\Rightarrow$(P + Q)'

$$LHS = \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

To find RHS, we will first find the transpose of matrix P and Q

$$P' = \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix} \text{ And } Q' = \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$$

RHS = P' + Q'

$$\Rightarrow \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

LHS = RHS

Hence proved.

5. Question

If $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, show that (A + A’) is symmetric.

Answer

Given $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$

To Prove: A + A’ is symmetric.(Note:A matrix P is symmetric if P’ = P)

Proof: We will find A’,

$$A' = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$$

Now let us take P = A + A’

$$P = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$

Now $P' = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$

$$\Rightarrow P' = P$$

Hence A + A’ is a symmetric matrix.

6. Question

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that (A + A’) is skew-symmetric.

Answer

Given $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

To prove: A-A’ is a skew-symmetric matrix.(Note: A matrix P is skew-symmetric if P’ = -P)

Proof: First we will find the transpose of matrix A

$$A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

Let us take P = A-A’

$$P = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$P = {}'\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

$\Rightarrow$P' = P

Hence A-A' is a skew symmetric matrix.

7. Question

Show that the matrix $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is skew-symmetric.

HINT: Show that A' = -A.

Answer

Given $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

To Prove: A is a skew symmetric matrix.

Proof: As for a matrix to be skew symmetric A' = -A

We will find A'.

$$A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$\Rightarrow$A' = -A

So A is A skew symmetric matrix.

8. Question

Express the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

Answer

Given $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, As for a symmetric matrix A' = A hence

A + A' = 2A

A $= \frac{1}{2}(A + A') \Rightarrow P$ (Symmetric Matrix)

Similarly for a skew symmetric matrix since A' = -A hence

A-A' = 2A

A $= \frac{1}{2}(A - A') \Rightarrow Q$ (Skew Symmetric Matrix)

So a matrix can be represented as a sum of a symmetric matrix P and skew symmetric matrix Q.

First, we will find the transpose of matrix A,

$$A' = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

Now using the above formulas,

$P = \frac{1}{2}(A + A')$

$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\right)$

$\Rightarrow \frac{1}{2}\begin{bmatrix} 4 & 2 \\ 2 & 8 \end{bmatrix}$

$P = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$

$Q = \frac{1}{2}\left(\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\right)$

$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Hence A = P + Q

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ [Matrix A as the sum of P and Q]

$\Rightarrow \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$

9. Question

Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

Answer

Given $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$,to express as sthe um of symmetric matrix P and skew symmetric matrix Q.

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,we will find transpose of matrix A

$A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

Now using the above formulas

$P = \frac{1}{2}(A + A')$

$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right)$

$\Rightarrow \frac{1}{2}\begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

Hence A = P + Q

$$\Rightarrow \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$ [Matrix A as the sum of P and Q]

$$\Rightarrow \begin{bmatrix} 3 & \frac{-8}{2} \\ \frac{2}{2} & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

10. Question

Express the matrix $A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Answer

Given $A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q.

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,

First, we find A’

$$A' = \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}$$

Now using the above mentioned formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} -2 & 7 & 8 \\ 7 & 6 & 4 \\ 8 & 4 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 9 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & 3 & -6 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{3}{2} & -3 \\ \frac{-3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$

Now A = P + Q

$$\Rightarrow \begin{bmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 9 \end{bmatrix} + \begin{bmatrix} 0 & \frac{3}{2} & -3 \\ \frac{-3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$ [Matrix A as sum of P and Q]

$$\Rightarrow \begin{bmatrix} -1 & \frac{10}{2} & 1 \\ \frac{4}{2} & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$$

11. Question

Express the matrix A as the sum of a symmetric and a skew-symmetric matrix, where $A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$.

Answer

Given $A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,

First we will find A',

$$A' = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}$$

Now using above mentioned formulas,

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 6 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & -3 & -1 \\ 3 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix}$$

Now A = P + Q

$$\Rightarrow \begin{bmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix}$$ [Matrix A as sum of P and Q]

$$\Rightarrow \begin{bmatrix} 3 & \frac{-2}{2} & 0 \\ \frac{4}{2} & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$$

12. Question

Express the matrix $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as sum of two matrices such that one is symmetric and the other is skew-symmetric.

Answer

Given $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q.

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,

First we will find A'

$$A' = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}$$

Now using above mentioned formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

Now A = P + Q

$$\Rightarrow \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$$

13 A. Question

For each of the following pairs of matrices A and B, verify that (AB)’ = (B’ A’) :

$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$

Answer

Let us take C = AB

$$C = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}$$

$$LHS \Rightarrow C' = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ And $B' = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$

RHS = B’A’

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+6 & 2+8 \\ 3+16 & 8+20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

LHS = RHS

Hence proved.

13 B. Question

For each of the following pairs of matrices A and B, verify that (AB)’ = (B’ A’) :

$A = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

Answer

Let us take C = AB

$$C = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+(-2) & -9+1 \\ 2+(-4) & -6+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -8 \\ -2 & -4 \end{bmatrix}$$

$$LHS \Rightarrow C' = \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$B' = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}$ And $A' = \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$

RHS = B’A’

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+(-2) & 2+(-4) \\ -9+1 & -6+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$$

LHS = RHS

Hence proved.

13 C. Question

For each of the following pairs of matrices A and B, verify that (AB)’ = (B’ A’) :

$A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and B = [-2 -1 -4]

Answer

Let us take C = AB

$C = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}$

LHS = C’

$\Rightarrow \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$

To find RHS we will find transpose of matrix A and B,

$A' = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$ And $B' = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix}$

RHS = B’A’

$\Rightarrow \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$

LHS = RHS

Hence proved.

13 D. Question

For each of the following pairs of matrices A and B, verify that (AB)’ = (B’ A’) :

$A = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$

Answer

Let us take C = AB

$$C = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3+4+3 & 4+2+0 \\ 12+(-10)+(-6) & -16+(-5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 6 \\ -4 & -21 \end{bmatrix}$$

LHS = C’

$$\Rightarrow \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$A' = \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix}$ And $B' = \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix}$

RHS = B’A’

$$\Rightarrow \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3+4+3 & 12+(-10)+(-6) \\ 4+2 & -16+(-5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

LHS = RHS

Hence proved.

14. Question

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that A’A = I.

Answer

Given $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, We will find A’

$$A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

LHS = A’A

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha.\sin\alpha + (-\sin\alpha.\cos\alpha) \\ \sin\alpha.\cos\alpha + (-\cos\alpha.\sin\alpha) & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ [Using $\cos^2\alpha + \sin^2\alpha = 1$ and commutative law a.b = b.a i.e. $\sin\alpha.\cos\alpha = \cos\alpha.\sin\alpha)$]

RHS = I $\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

LHS = RHS

Hence proved.

15. Question

If matrix A = [1 2 3], write AA'.

Answer

Given $A =$ [1 2 3]

We will find A' to calculate AA',

$$A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Now

$$AA' = [123]\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow$[1 + 4 + 9]

$\Rightarrow$[14]

Exercise 5E

1. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}[A|I] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right]$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right] \xrightarrow{R_2-3R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

2. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A|I\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array}\right]$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -5 & -2 & 1 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right]$$

Here, the matrix A is converted into the Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

3. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A|I\right] = \left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -3 & 1 & 0 & 1 \end{array}\right]$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -3 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 + R_1} \left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -1 & 6 & 1 & 1 \end{array}\right] \xrightarrow{R_1 + R_2} \left[\begin{array}{cc|cc} 1 & 11 & 2 & 1 \\ -1 & 6 & 1 & 1 \end{array}\right] \xrightarrow{R_2 + R_1} \left[\begin{array}{cc|cc} 1 & 11 & 2 & 1 \\ 0 & 17 & 3 & 2 \end{array}\right]$$

$$\xrightarrow{\frac{1}{17}R_2} \left[\begin{array}{cc|cc} 1 & 11 & 2 & 1 \\ 0 & 1 & \frac{3}{17} & \frac{2}{17} \end{array}\right] \xrightarrow{R_1 - 11R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{17} & -\frac{5}{17} \\ 0 & 1 & \frac{3}{17} & \frac{2}{17} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} \frac{1}{17} & -\frac{5}{17} \\ \frac{3}{17} & \frac{2}{17} \end{bmatrix} = \frac{1}{17}\begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

4. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 0 & 11 & -2 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 11 & -2 & 1 \end{array}\right] \xrightarrow{\frac{1}{11}R_2} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{11} & \frac{1}{11} \end{array}\right]$$

$$\xrightarrow{R_1+\frac{3}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{5}{22} & \frac{3}{22} \\ 0 & 1 & -\frac{2}{11} & \frac{1}{11} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} \frac{5}{22} & \frac{3}{22} \\ -\frac{2}{11} & \frac{1}{11} \end{bmatrix}$$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

5. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{cc|cc} 4 & 0 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc}4 & 0 & 1 & 0\\ 2 & 5 & 0 & 1\end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{cc|cc}0 & -10 & 1 & -2\\ 2 & 5 & 0 & 1\end{array}\right] \xrightarrow{R_1\leftrightarrow R_2} \left[\begin{array}{cc|cc}2 & 5 & 0 & 1\\ 0 & -10 & 1 & -2\end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc}1 & \frac{5}{2} & 0 & \frac{1}{2}\\ 0 & -10 & 1 & -2\end{array}\right]$$

$$\xrightarrow{-\frac{1}{10}R_2} \left[\begin{array}{cc|cc}1 & \frac{5}{2} & 0 & \frac{1}{2}\\ 0 & 1 & -\frac{1}{10} & \frac{1}{5}\end{array}\right] \xrightarrow{R_1-\frac{5}{2}R_2} \left[\begin{array}{cc|cc}1 & 0 & \frac{1}{4} & 0\\ 0 & 1 & -\frac{1}{10} & \frac{1}{5}\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix}\frac{1}{4} & 0\\ -\frac{1}{10} & \frac{1}{5}\end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

6. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix}6 & 7\\ 8 & 9\end{bmatrix}$$

Answer

Let, $A = \begin{bmatrix}6 & 7\\ 8 & 9\end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A|I\right] = \left[\begin{array}{cc|cc}6 & 7 & 1 & 0\\ 8 & 9 & 0 & 1\end{array}\right]$, where $I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc}6 & 7 & 1 & 0\\ 8 & 9 & 0 & 1\end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{cc|cc}6 & 7 & 1 & 0\\ 2 & 2 & -1 & 1\end{array}\right] \xrightarrow{R_1\leftrightarrow R_2} \left[\begin{array}{cc|cc}2 & 2 & -1 & 1\\ 6 & 7 & 1 & 0\end{array}\right] \xrightarrow{R_2-3R_1} \left[\begin{array}{cc|cc}2 & 2 & -1 & 1\\ 0 & 1 & 4 & -3\end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc}1 & 1 & -\frac{1}{2} & \frac{1}{2}\\ 0 & 1 & 4 & -3\end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{cc|cc}1 & 0 & -\frac{9}{2} & \frac{7}{2}\\ 0 & 1 & 4 & -3\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix}-\frac{9}{2} & \frac{7}{2}\\ 4 & -3\end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

7. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A \middle| I\right] = \left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - 3R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 + 4R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 4 & -3 & 1 \end{array}\right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

A^{-1} = $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

8. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & -4 & 4 & 2 & 0 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 1 & -4 & 4 & 2 & 0 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 0 & 5 & -5 & -3 & 0 & 2 \end{array}\right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|ccc} 1 & -4 & 4 & 2 & 0 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 0 & 0 & -5 & -2 & -1 & 2 \end{array}\right]$$

$$\xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 1 & -1 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 0 & 0 & -5 & -2 & -1 & 2 \end{array}\right] \xrightarrow{\frac{1}{5}R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 1 & -1 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & -5 & -2 & -1 & 2 \end{array}\right] \xrightarrow{-\frac{1}{5}R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 1 & -1 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array}\right]$$

$$\xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & \frac{6}{5} & \frac{4}{5} & -1 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array}\right] \xrightarrow{R_1 - 4R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{5} & 0 & \frac{3}{5} \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 2 & 0 & -3 \\ 1 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

9. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 1 & 5 & 9 & 0 & 1 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array}\right]$$, where $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 1 & 5 & 9 & 0 & 1 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 3 & 0 & 2 & 1 & 0 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 3 & 0 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - 3R_1} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 0 & -15 & -25 & 1 & -3 & 0 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_2 + 4R_3} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 - 4R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 0 & 55 & 26 & 12 & -15 \end{array}\right] \xrightarrow{\frac{1}{55}R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right]$$

$$\xrightarrow{R_2 + 13R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & 0 & -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{157}{55} & \frac{64}{55} & -\frac{80}{55} \\ 0 & 1 & 0 & -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right]$$

$$\xrightarrow{R_1 - 6R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{55} & -\frac{8}{55} & \frac{10}{55} \\ 0 & 1 & 0 & -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} \frac{1}{55} & -\frac{8}{55} & \frac{10}{55} \\ -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{bmatrix} = -\frac{1}{55}\begin{bmatrix} -1 & 8 & -10 \\ 47 & 9 & -25 \\ -26 & -12 & 15 \end{bmatrix}$$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

10. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & -3 & -4 & 0 & 0 & 1 \end{array}\right]$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 3 & -3 & -4 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 1 & -6 & -6 & 0 & -1 & 1 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & 1 & 5 & -1 & 1 & 0 \\ 1 & -6 & -6 & 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & 1 & 5 & -1 & 1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -1 & 8 & -2 & 1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & -7 & -11 & 1 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_3+7R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & 0 & -67 & 15 & -9 & 1 \end{array}\right] \xrightarrow{-\frac{1}{67}R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -8 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right]$$

$$\xrightarrow{R_2+8R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{39}{67} & -\frac{10}{67} & \frac{16}{67} \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right]$$

$$\xrightarrow{R_1+3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

11. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\middle|I\right] = \left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-R_1} \left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_3+2R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & -5 & 6 & 1 \end{array}\right] \xrightarrow{\frac{1}{4}R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -2 & 3 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right]$$

$$\xrightarrow{R_2-R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 2 & 0 & -\frac{3}{4} & \frac{6}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right]$$

$$\xrightarrow{R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & \frac{5}{8} & -\frac{2}{8} & -\frac{1}{8} \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right] \xrightarrow{R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{5}{8} & \frac{10}{8} & \frac{1}{8} \\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

A^{-1} = $\begin{bmatrix} -\frac{5}{8} & \frac{10}{8} & \frac{1}{8} \\ -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8} \\ -\frac{5}{4} & \frac{6}{4} & \frac{1}{4} \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} 5 & -10 & -1 \\ 3 & -6 & 1 \\ 10 & -12 & -2 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

12. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\mathrm{Aug}\left[A \middle| I\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_2+3R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 9 & -7 & 3 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 5 & -4 & 2 & 0 & -1 \end{array}\right] \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array}\right]$$

$$\xrightarrow{R_2-4R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 5 & 9 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array}\right] \xrightarrow{R_2\leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & -2 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right] \xrightarrow{R_1+2R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -5 & 10 & 18 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right] \xrightarrow{R_1-3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -3 \\ 0 & 1 & 0 & -2 & 4 & 7 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

A^{-1} = $\begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

13. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 7 & 0 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 7 & 0 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3+2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 9 & -2 & 2 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{R_1-3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -1 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

14. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Answer

Let, A = $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}[A|I] = \left[\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 9 & 2 & -2 & 3 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-2R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-4R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array}\right] \xrightarrow{R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 9 & -13 & 9 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array}\right]$$

$$\xrightarrow{R_1+3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -4 & 3 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

15. Question

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Answer

$$\text{Let, } A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|ccc} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-3R_2} \left[\begin{array}{ccc|ccc} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 3 & 5 & 1 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -2 & -3 & 1 & -2 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \xrightarrow{R_3+3R_1} \left[\begin{array}{ccc|ccc} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{array}\right] \xrightarrow{R_1-2R_3} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{array}\right]$$

$$\xrightarrow{R_3-R_1} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 5 & -4 & 3 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \\ 0 & 0 & 1 & 5 & -4 & 3 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Exercise 5F

1. Question

Construct a 3 × 2 matrix whose elements are given by

$$a_{ij} = \frac{1}{2}(i - 2j)^2$$

Answer

Here, i is the subscript for a row, and j is the subscript for column

And the given matrix is 3×2, so 1≤ i ≤ 3 and 1≤j ≤2

Hence for i=1, j=1, $a_{11} = \frac{1}{2}(1 - (2 \times 1))^2 = \frac{1}{2}$

For i=1, j=2, $a_{12} = \frac{1}{2}(1 - (2 \times 2))^2 = \frac{9}{2}$

For i=2, j=1 $a_{21} = \frac{1}{2}(2 - (2 \times 1))^2 = 0$

For i=2, j=2 $a_{22} = \frac{1}{2}(2 - (2 \times 2))^2 = 2$

For i=3, j=1 $a_{31} = \frac{1}{2}(3 - (2 \times 1))^2 = \frac{1}{2}$

For i=3, j=2 $a_{32} = \frac{1}{2}(3 - (2 \times 2))^2 = \frac{1}{2}$

Hence the required matrix is :- $\begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

2. Question

Construct a 2 × 3 matrix whose elements are given by

$$a_{ij} = \frac{1}{2}|-3i + j|.$$

Answer

The elements of the matrix are given by, $a_{ij} = \frac{1}{2}| - 3j + j|$

Matrix is 2$\times$ 3 hence, $1 \le i \le 2, 1 \le j \le 3$

Here, i is the subscript for a row, and j is the subscript for column

For i=1, j=1, $a_{11} = \frac{1}{2}|-3(1) + 1| = 1$

For i=1, j=2, $a_{12} = \frac{1}{2}|-3(1) + 2| = \frac{1}{2}$

For i=1, j=3, $a_{13} = \frac{1}{2}|-3(1) + 3| = 0$

For i=2, j=1, $a_{21} = \frac{1}{2}|-3(2) + 1| = \frac{5}{2}$

For i=2, j=2, $a_{22} = \frac{1}{2}|-3(2) + 2| = 2$

For i=2, j=3, $a_{23} = \frac{1}{2}|-3(2) + 3| = \frac{3}{2}$

Hence the required matrix is :-

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ \frac{5}{2} & 2 & \frac{3}{2} \end{bmatrix}$$

3. Question

If $\begin{bmatrix} x + 2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the values of x and y.

Answer

On comparing L.H.S. and R. H.S we get,

$\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$

On comparing each term we get,

$x + 2y = -4$ ....(i)

$-y = 3$ ...(ii)

$3x = 6$ .....(iii)

From (i), (ii) and (iii), we get,

$y = -3 \; and \; x = 2$

4. Question

Find the values of x and y, if

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$

Answer

Given,

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Using the property of matrix multiplication such that h is scalar, $h\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ah & bh \\ ch & dh \end{bmatrix}$

Using the matrix property of matrix addition, when two matrices are of the same order then, each element gets added to the corresponding element,

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

$\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix}$

$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Comparing each element we get,

2+y=5, ⇒ y=3

2x+2=8, ⇒x=3

5. Question

If $x.\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y.\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the values of x and y.

Answer

Given, $x. \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y.\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix}$

And we have,

$\begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Solving the linear equations, we get,

$x = 3, y = -4$

6. Question

If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find the values of x, y, z, ω.

Answer

Given,

$\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$

On comparing each element of the two matrices we get,

x=3,

3x-y=2

y=7

2x+z=4,

z=-2,

3y-w=7,

w=14

7. Question

If $\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & w \end{bmatrix}$, find the values of x, y, z, ω.

Answer

Given,

$\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & w \end{bmatrix}$

First applying matrix addition then, comparing each element of the matrix with the corresponding element we get,

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

$\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}$

$\begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}$

We now have, $x + 4 = 3x$, ...(i)

x=2

$2w + 3 = 3w$, ...(ii)

$w = 3$

6+x+y=3y, substituting x from (i) we get,

y = 4,

And -1+z+w=3z, substituting w from (ii), we get,

z=1

8. Question

If A = diag (3 -2, 5) and B = diag (1 3 -4), find (A + B).

Answer

We are given two diagonal matrices A and B,

On adding the two diagonal matrices of order (3×3) we get an diagonal matrix of order (3×3)

Each of the elements get added to the corresponding element hence, we get after adding,

$$\begin{bmatrix}3 & 0 & 0\\0 & -2 & 0\\0 & 0 & 5\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\0 & 3 & 0\\0 & 0 & -4\end{bmatrix}=\begin{bmatrix}4 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Hence, we get A+B = diag(4 1 1)

9. Question

Show that

$$\cos\theta\cdot\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\cdot$$

$$\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}=I$$

Answer

We have to show that

$$\cos\theta.\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}+sin\theta.\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Multiplying the scalars with we get,

$$\begin{bmatrix}\cos\theta\times\cos\theta & \cos\theta\times\sin\theta\\\cos\theta\times(-\sin\theta) & \cos\theta\times\cos\theta\end{bmatrix}+\begin{bmatrix}\sin\theta\times\sin\theta & \sin\theta\times(-\cos\theta)\\\sin\theta\times\cos\theta & \sin\theta\times\sin\theta\end{bmatrix}$$

$$\begin{bmatrix}cos^2\theta+sin^2\theta & 0\\0 & cos^2\theta+sin^2\theta\end{bmatrix}$$

And we know that $cos^2\theta+sin^2\theta=1$

$$\begin{bmatrix}cos^2\theta+sin^2\theta & 0\\0 & cos^2\theta+sin^2\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Hence, proved.

10. Question

If $A=\begin{bmatrix}1 & -5\\-3 & 2\\4 & -2\end{bmatrix}$ and $B=\begin{bmatrix}3 & 1\\2 & -1\\-2 & 3\end{bmatrix}$, find the matrix C such that A + B + C is a zero matrix

Answer

Given, A+B+C = zero matrix

We know that zero matrix is a matrix whose all elements are zero, so we have,

$$A=\begin{bmatrix}1 & -5\\-3 & 2\\4 & -2\end{bmatrix},B=\begin{bmatrix}3 & 1\\2 & -1\\-2 & 3\end{bmatrix}$$

WE have A+B+C=0,

So C = -A+B,

$$-C=\begin{bmatrix}1 & -5\\-3 & 2\\4 & -2\end{bmatrix}+\begin{bmatrix}3 & 1\\2 & -1\\-2 & 3\end{bmatrix}$$

$$C=\begin{bmatrix}-4 & 4\\1 & -1\\-2 & -1\end{bmatrix}$$

11. Question

If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{bmatrix}$ then find the least value of α for which A + A’ = I.

Answer

Given, $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{bmatrix}$

Here, A' i.e. A transpose is $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

We are given that A+A'=I

So, $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

After doing addition of matrices, we get,

$\begin{bmatrix} \cos\alpha + \cos\alpha & \sin\alpha - \sin\alpha \\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing the elements we get,

$2\cos\alpha = 1$

This implies, $\cos\alpha = \frac{1}{2}$

For α belongs 0 to π, $\alpha = \frac{\pi}{3}$

12. Question

Find the value of x and y for which

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Answer

Given,

$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Applying matrix multiplication we get,

$\begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

On comparing the elements we get, 2x-3y = 1,

x+y = 3,

On solving the equations we get, x=2, y=1

13. Question

Find the value of x and y for which

$$\begin{bmatrix} x & y \\ 3y & x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

Answer

Given,

$\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

Applying matrix multiplication we have, $\begin{bmatrix} x + 2y \\ 3y + 2x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

On comparing the elements with each other we get,

The linear equations, x+2y=3, 3y+2x=5

On solving these equations we get x = 1, y = 1

14. Question

If $A = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$, show that (A + A') is symmetric

Answer

Given, $A = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} and\ A' = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$

Then, (A +A') will be, $\begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$

The matrix $\begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$ is a symmetrical matrix.

15. Question

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, and show that (A – A') is skew-symmetric

Answer

Given,

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, and

$A' = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

(A - A') = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

The matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is skew-symmetric.

16. Question

If $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$, find a matrix X such that A + 2B + X = O.

Answer

Given, $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$

We need to a matrix X such that, A +2B + X = 0

We have, X = -(A + 2B),

$X = - \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} + 2\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$

$X = -\begin{bmatrix} 2+(-2) & -3+(2\times 2) \\ 4+0 & 5+(2\times 3) \end{bmatrix}$

$X = \begin{bmatrix} 0 & -1 \\ -4 & -11 \end{bmatrix}$

17. Question

If $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$, find a matrix X such that

3 A – 2B + X = O.

Answer

Given, $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} and\ B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

We have 3A – 2B + X = 0

So X = -(3A – 2B)

Thus,

$X = -\ 3\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} - 2\begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

$X = -\begin{bmatrix} 3\times4+2\times2 & 3\times2-2\times1 \\ 3\times1-2\times3 & 3\times3-2\times2 \end{bmatrix}$

$X = \begin{bmatrix} -16 & -4 \\ 3 & -5 \end{bmatrix}$

18. Question

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that A' A = I.

Answer

Given, $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Then , $AA' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Applying matrix multiplication we get,

$AA' = \begin{bmatrix} \cos\alpha\times\cos\alpha+\sin\alpha\times\sin\alpha & \cos\alpha\times(-\sin\alpha)+\sin\alpha\times\cos\alpha \\ (-\sin\alpha)\times\cos\alpha+\cos\alpha\times\sin\alpha & (-\sin\alpha)\times(-\sin\alpha)+\cos\alpha\times\cos\alpha \end{bmatrix}$

$AA' = \begin{bmatrix} cos^2\alpha+sin^2\alpha & 0 \\ 0 & cos^2\alpha+sin^2\alpha \end{bmatrix}$

Hence, $AA' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

As we know that $cos^2\alpha + sin^2\alpha = 1$

19. Question

If A and B are symmetric matrices of the same order, show that (AB – BA) is a skew symmetric matrix.

Answer

We are given that A and B are symmetric matrices of the same order then, we need to show that (AB – BA) is a skew symmetric matrix.

Let us consider P is a matrix of the same order as A and B

And let P = (AB – BA),

we have A = A' and B = B'

then, P' = (AB – BA)'

P' = ((AB)' – (BA)') ....using reversal law we have (CD)'=D'C'

P' = (B'A' – A'B')

P' = (BA – AB)

P' = -P

Hence, P is a skew symmetric matrix.

20. Question

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and f(x) = x^2 – 4x + 1, find f(A).

Answer

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

f(x) = x^2 – 4x + 1,

f(A) = A^2 – 4A + I,

f(A) = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

f(A) = $\begin{bmatrix} 4+3-8+1 & 6+6-12+0 \\ 2+2-4+0 & 3+4-8+1 \end{bmatrix}$

f(A) = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

21. Question

If the matrix A is both symmetric and skew-symmetric, show that A is a zero matrix.

Answer

Given that matrix A is both symmetric and skew symmetric, then,

We have A = A’ ……(i)

And A = -A’ ……(ii)

From (i) and (ii) we get,

A’ = -A’,

2A’ = 0

A’ = 0

Then, A = 0

Hence proved.