Playing with Numbers

A **factor** of a number is an exact divisor of that number.
Example: 3 is an exact divisor of 15 (15 = 3 × 5). So, 3 is a factor of 15.
2 is not an exact factor of 17 (since any whole number multiplied with 2 does not give 17). So, 2 is not a factor of 17.

We can find the multiples of a given number by multiplying 1, 2, 3 …, to the number.
Example: To find the multiples of 12, we need to calculate as follows:
12 × 1 = 12, 12 × 2 = 24, 12 × 3 = 36, 12 × 4 = 48 …, and so on.
So, the numbers 12, 24, 36, 48, etc., are the multiples of 12.

- **Properties of natural numbers with respect to their factors:**
    - 1 is a factor of every number. For example, 5 = 1 × 5, 19 = 1 × 19
    - Every number is a factor of itself. For example, 20 = 20 × 1. This shows that 20 is a factor of 20.
    - Every factor of a number is less than or equal to the number. For example, factors of 12 are 1, 2, 3, 4, 6 and 12. Here, 1, 2, 3, 4, 6 and 12 are less than or equal to 12.
    - Every number has finite factors. For example, the factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30. Here, we may observe that there are only 8 factors of 30.

- **Properties of natural numbers with respect to their multiples:**
    - Every number is a multiple of itself. For example, 15 × 1 = 15, 7 × 1 = 7
    - Every multiple of a number is greater than or equal to that number.For example, the multiple of 6 is 6, 12, 18 … Here we may observe that the multiples of 6 are greater than or equal to 6.
    - Every number is a multiple of itself. For example, 18 is a multiple of itself, that is, 18. Therefore, every number is a factor as well as a multiple of itself.
    - The number of multiples of a given number is infinite. For example, the multiples of 11 are 11, 22, 33, 44 …. So, the number of multiples of 11 is infinite.

- **Perfect number** is a number for which sum of all its factors is equal to twice the number.

    For example, factors of 6 are 1, 2, 3 and 6 only.
    1 + 2 + 3 + 6 = 12 = 2 × 6

    So, 6 is a perfect number.

- The **common factors** of a given set of numbers are the numbers which are common to the factors of all those numbers.

  For example,
  Factors of 8 are 1, 2, 4 and 8.
  Factors of 10 are 1, 2, 5 and 10.
  Factors of 20 are 1, 2, 4, 5, 10 and 20.
  So, common factors of 8, 10 and 20 are 1 and 2.

- The **common multiples** of a given set of numbers are the numbers which are common to the multiples of all those numbers.

  For example,
  Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48………
  Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48 ……
  Multiples of 8 are 8, 16, 24, 32, 40, 48…………
  Common multiples of 4, 6 and 8 are 24, 48……….

- **HCF (highest common factor)** or **GCD (greatest common divisor)** is the greatest factor among all common factors of two or more numbers.
- **LCM (least common multiple)** is the least multiple among all common multiples of two or more numbers.

- Two numbers are called **co-prime**, if they have 1 as the only common factor.

  For example, the factors of 8 are 1, 2, 4 and 8. The factors of 15 are 1, 3, 5, and 15. The common factor of both 8 and 15 is 1. So, they are co-prime numbers.

- Expressing a number as the product of prime numbers is known as **prime factorisation**. To find the prime factorisation of a number, we need to divide it by prime numbers that are factors of the given number, till we get 1.

  For example, the prime factorisation of 840 can be done as follows:

2	840
2	420

2	210
3	105
5	35
7	7
	1

So, the prime factorisation of 840 is :

$840 = 2 \times 2 \times 2 \times 3 \times 5 \times 7$

A **factor** of a number is an exact divisor of that number.
Example: 3 is an exact divisor of 15 ($15 = 3 \times 5$). So, 3 is a factor of 15.
2 is not an exact factor of 17 (since any whole number multiplied with 2 does not give 17). So, 2 is not a factor of 17.

We can find the multiples of a given number by multiplying 1, 2, 3 …, to the number.
Example: To find the multiples of 12, we need to calculate as follows:
$12 \times 1 = 12$, $12 \times 2 = 24$, $12 \times 3 = 36$, $12 \times 4 = 48$ …, and so on.
So, the numbers 12, 24, 36, 48, etc., are the multiples of 12.

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    - 1 is a factor of every number. For example, $5 = 1 \times 5$, $19 = 1 \times 19$
    - Every number is a factor of itself. For example, $20 = 20 \times 1$. This shows that 20 is a factor of 20.
    - Every factor of a number is less than or equal to the number. For example, factors of 12 are 1, 2, 3, 4, 6 and 12. Here, 1, 2, 3, 4, 6 and 12 are less than or equal to 12.
    - Every number has finite factors. For example, the factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30. Here, we may observe that there are only 8 factors of 30.

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- The number of multiples of a given number is infinite. For example, the multiples of 11 are 11, 22, 33, 44 …. So, the number of multiples of 11 is infinite.

- **Perfect number** is a number for which sum of all its factors is equal to twice the number.

  For example, factors of 6 are 1, 2, 3 and 6 only.
  $1 + 2 + 3 + 6 = 12 = 2 \times 6$

  So, 6 is a perfect number.

- **Prime numbers** are numbers having exactly two factors: 1 and the number itself. For example, the factors of 23 are 1 and 23 only. So, 23 is a prime number.
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- **Composite numbers** are numbers having more than two factors. For example, the factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42. Since there are 8 factors of 42, it is a composite number.
- 1 is neither prime nor composite as it has exactly one factor.
- The smallest prime number is 2.
- The smallest even prime number is 2 and the smallest odd prime number is 3.
- All even numbers except 2 are composite.
- The pairs of prime numbers whose difference is 2 are known as **twin primes**. For example, 11 and 13 are twin primes.

- The **common factors** of a given set of numbers are the numbers which are common to the factors of all those numbers.

  For example,
  Factors of 8 are 1, 2, 4 and 8.
  Factors of 10 are 1, 2, 5 and 10.
  Factors of 20 are 1, 2, 4, 5, 10 and 20.
  So, common factors of 8, 10 and 20 are 1 and 2.

- The **common multiples** of a given set of numbers are the numbers which are common to the multiples of all those numbers.

  For example,
  Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48………
  Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48 ……
  Multiples of 8 are 8, 16, 24, 32, 40, 48…………

Common multiples of 4, 6 and 8 are 24, 48……….

- **HCF (highest common factor)** or **GCD (greatest common divisor)** is the greatest factor among all common factors of two or more numbers.
- **LCM (least common multiple)** is the least multiple among all common multiples of two or more numbers.

- Two numbers are called **co-prime**, if they have 1 as the only common factor.

  For example, the factors of 8 are 1, 2, 4 and 8. The factors of 15 are 1, 3, 5, and 15. The common factor of both 8 and 15 is 1. So, they are co-prime numbers.

- A number is divisible by 10, if the digit in one’s place is zero.

  For example, the numbers 9520, 67120, 830, 1200, etc., are divisible by 10.

- A number is divisible by 5, if the digit in one’s place is either 0 or 5.

  For example, 3615, 92185, 370 are divisible by 5.

- A number is divisible by 2, if the digit in one’s place is either 0, 2, 4, 6, or 8.

  For example, the numbers 9218, 6054, 932 are divisible by 2.

- A number with two or more digits is divisible by 4, if the number formed by its last two digits (one’s and ten’s) is either 00 or divisible by 4.

  For example, the last two digits of 9584 is 84, which is divisible by 4. So, 9584 is divisible by 4.

- A number with three or more digits is divisible by 8, if the number formed by its last three digits (one’s, ten’s and hundred’s) is either 000 or divisible by 8.

  For example, the last three digits of 9368 is 368, which is divisible by 8. So, 9368 is divisible by 8.

- A number is divisible by 3, if the sum of its digits is divisible by 3.

  For example, 231456 is divisible by 3, since 2 + 3 + 1 + 4 + 5 + 6 = 21 is divisible by 3.

- A number is divisible by 9, if the sum of its digits is divisible by 9.

  For example, 253674 is divisible by 9, since 2 + 5 + 3 + 6 + 7 + 4 = 27 is divisible by 9.

- A number which is divisible by 9 is also divisible by 3.

  For example, in 252, the sum of its digits is 9, which is divisible by 3 and 9 both.

- A number is divisible by 6, if it is divisible by both 2 and 3.

  For example, 39612 is divisible by 2, since it is an even number. Sum of the digits of 39612 is 3 + 9 + 6 + 1 + 2 = 21, which is a multiple of 3. So, 39612 is divisible by 3. Now, 39612 is divisible by both 2 and 3. So, it is divisible by 6. 3513 is not divisible by 2, as it is an odd umber. But it is divisible by 3 (∵∵3 + 5 + 1+ 3 = 12, which is a multiple of 3). So, 3513 is not divisible by 6.

  Example:

  Write the smallest digit and the greatest digit in the blank space of the following number, so that the number is divisible by 6.

  931_

  Solution:

  Since the number is divisible by 6, it has to be divisible by 2. So, the unit's digit can be 0, 2, 4, 6 or 8. Also, the number has to be divisible by 3. For this, the sum of the digits should be a multiple of 3.
  Now, 9 + 3 + 1 = 13. If we add 2, 5, or 8 to 13, then we get a number which is a multiple of 3.
  If the digit in blank space is 2 or 8, then the obtained number will be divisible by 2 as well as 3. So, the required smallest number is 2 and the largest number is 8.

- A number is divisible by 11, if the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) is either 0 or multiple of 11.

  For example, for the number 82918, the sum of the digits at odd places = 8 + 9 + 8 = 25 and the sum of the digits at even places = 1 + 2 = 3. Now, 25 – 3 = 22, which is a multiple of 11. Hence, 82918 is divisible by 11.

- If a number is divisible by another number, then it is divisible by each factor of that number.

  For example, we know 40 is divisible by 20. So, 40 is divisible by each factor of 20 (i.e., by 1, 2, 4, 5, 10 and 20).

- If a number is divisible by two co-prime numbers, then it is divisible by their product as well.

  For example, 70 is divisible by two co-prime numbers 5 and 7. Also, 70 is divisible by $5 \times 7 = 35$.

- If two numbers are divisible by a number, then their sum is also divisible by that number.

  For example, 8 and 10 are divisible by 2. So, their sum, that is, $8 + 10 = 18$ is also divisible by 2.

- If two numbers are divisible by a number, then their difference is also divisible by the number.

  For example, 12 and 33 are divisible by 3. Now, their difference, that is, $33 – 12 = 21$ is also divisible by 3.

- Expressing a number as the product of prime numbers is known as **prime factorisation**. To find the prime factorisation of a number, we need to divide it by prime numbers that are factors of the given number, till we get 1.

  For example, the prime factorisation of 840 can be done as follows:

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So, the prime factorisation of 840 is :

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- The **highest common factor** (HCF) of two or more given numbers is the highest of their common factors.
- The **lowest common multiple** (LCM) of two or more given numbers is the least of their common multiples.
- We can find the HCF and LCM of given numbers by any of the following methods-

1. **Prime factorisation method**
2. **Common division method**

Example 1:

Find the HCF of numbers 90, 120 and 150.

Solution:

First of all we need to prime factorise the given numbers.

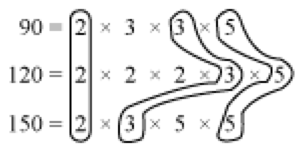
For example, the HCF of 90, 120, 150 can be found as:

2	90
3	45
3	15
5	5
	1

2	120
2	60
2	30
3	15
5	5

	1

2	150
3	75
5	25
5	5
	1



∴ HCF of 90, 120 and 150 = 2 × 3 × 5 = 30

Example 2:
Find the LCM of 90, 120 and 150.
Solution:
To find the LCM of 90, 120 and 150, we may proceed as follows:

2	90, 120, 150
2	45, 60, 75
2	45, 30, 75
3	45, 15, 75
3	15, 5, 25
5	5, 5, 25
5	1, 1,5
	1, 1, 1

∴ LCM of 60 and 120 = 2 × 2 × 2 × 3 × 3 × 5 × 5 = 1800

- Note: Product of LCM and HCF of two numbers = Product of the two numbers