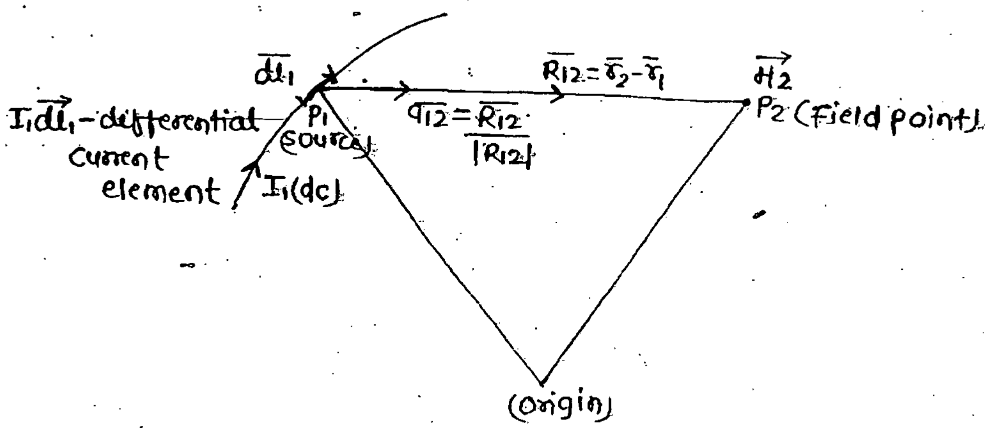
***Static megnatic field→** [Source → I(dc)]

*** Biot savart law→**



* Magnetic field intensity at point 2 (P_2) due to $I_1 d\overrightarrow{l_1}$ located at P_1 is

$$\overrightarrow{dH_2} = \frac{I_1 d\overrightarrow{l_1} \times \overrightarrow{a_{12}}}{4\pi |\overrightarrow{R_{12}}|^2} \left(\frac{\text{Amp}}{m}\right)$$

* Total magnetic field intensity at P_2 is

$$H_2 = \oint \frac{I_1 d\overrightarrow{l_1} \times \overrightarrow{a_{12}}}{4\pi |\overrightarrow{R_{12}}|^2}$$

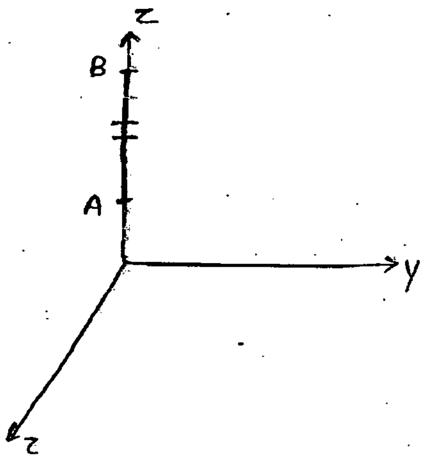
$\because \oint \rightarrow$ current flow in closed loop

$$\boxed{\overrightarrow{H_2} = \oint \frac{I_1 d\overrightarrow{l_1} \times \overrightarrow{R_{12}}}{4\pi |R_{12}|^3}}$$

↳ $\overrightarrow{R_{12}}$ is unknown ($\overrightarrow{R_{12}}$) = field point − Source point

Que.→ Find $\vec{H}$ in all the regions due to a finite long current line carrying I(DC) current lying on the z-axis from point A to point B as shown

(derivation not req.)



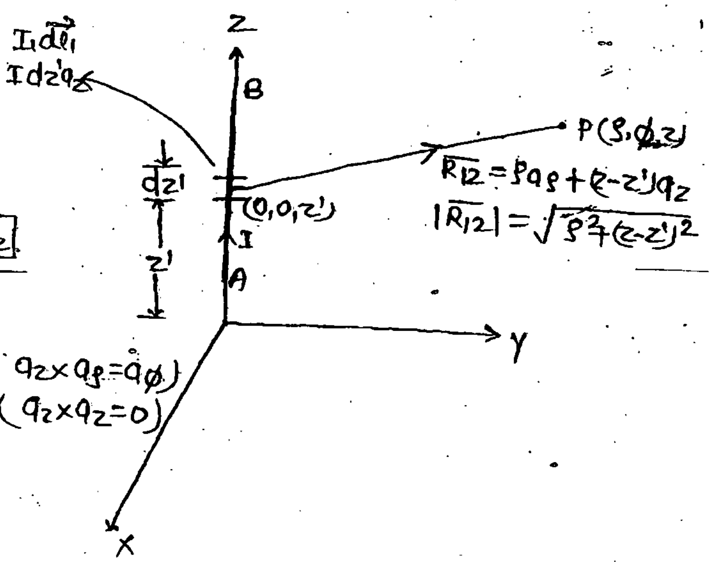
Sol$^n \rightarrow$

$$\vec{dH} = \frac{I_1 \vec{dl_1} \times \vec{R_{12}}}{4\pi |R_{12}|^3}$$

$$\vec{dH} = \frac{I dz' a_z \times [\rho a_\rho + (z-z') a_z]}{4\pi (\sqrt{\rho^2 + (z-z')^2})^3}$$

$$\vec{dH} = \frac{I dz' [\rho a_\phi + 0]}{4\pi (\sqrt{\rho^2 + (z-z')^2})^3}$$

$(\because a_z \times a_\rho = a_\phi)$
$(a_z \times a_z = 0)$



Total H field is

$$\vec{H} = \int_{z'=A}^{B} \frac{I \rho \, dz'}{4\pi [\sqrt{\rho^2 + (z-z')^2}]^3} a_\phi$$

Integration variable z' is in the denominator, so introduce angle terms.

$$\vec{H} = \int_{z'=A}^{B} \frac{I \rho \, dz'}{4\pi |R_{12}|^3} a_\phi$$

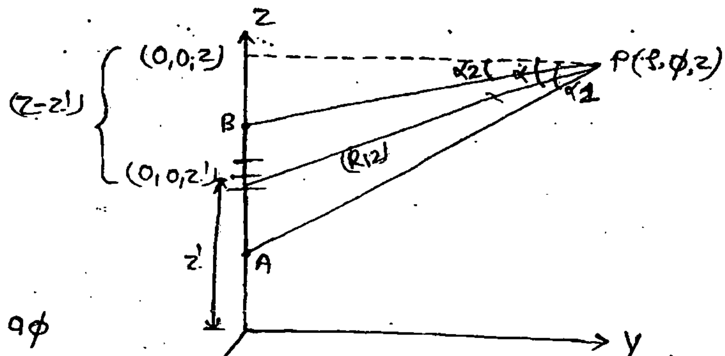
$$\vec{H} = \int_{z'=A}^{B} \frac{\frac{\rho}{|R_{12}|}}{|R_{12}|^2} dz' \, a_\phi$$

$$\vec{H} = \frac{I}{4\pi} \int_{\alpha=\alpha_1}^{\alpha_2} \frac{\cos\alpha}{\rho^2 \sec^2\alpha} (-\rho \sec^2\alpha \, d\alpha) a_\phi$$

$$\vec{H} = \frac{-I}{4\pi\rho} \int_{\alpha=\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha \cdot a_\phi$$

$$\vec{H} = \frac{-I}{4\pi\rho} (\sin\alpha)_{\alpha=\alpha_1}^{\alpha_2} a_\phi$$

$$\vec{H} = \frac{I}{4\pi\rho} [-(\sin\alpha_2 - \sin\alpha_1)] a_\phi$$



$\cos\alpha = \frac{\rho}{|R_{12}|}$; $|R_{12}| = \rho \sec\alpha$

$\tan\alpha = \frac{z-z'}{\rho}$, $(z-z') = \rho \tan\alpha$

diff wrt z', $0 - 1 = \rho \sec^2\alpha \cdot \frac{d\alpha}{dz'}$

$-dz' = \rho \sec^2\alpha \, d\alpha$

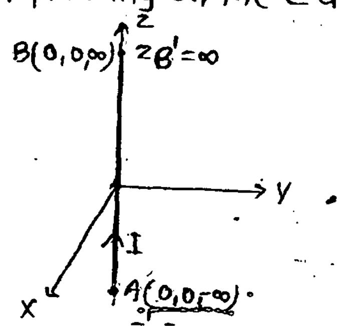
Case $\rightarrow$ Find $\vec{H}$ in all the regions due I(dc) current flowing on the z-axis

$$\tan\alpha = \frac{z-z'}{\rho}$$

$\alpha_1 \rightarrow$ angle of point A $(0,0,-\infty)$

$$\tan\alpha_1 = \frac{z - z'_A}{\rho} = \frac{z-(-\infty)}{\rho} = \infty = \tan\frac{\pi}{2}$$

$\alpha_1 = \pi/2$



$\underline{\alpha_2 \rightarrow}$ Angle of point $B(0,0,\infty)$

$$\tan\alpha_2 = \frac{z-z'_B}{\rho} = \frac{z-(\infty)}{\rho} = -\infty = \tan^{-1}\left(\frac{-\pi}{2}\right)$$

$$\alpha_2 = \frac{-\pi}{2}$$

$$\vec{H} = \frac{I}{4\pi\rho}\left[-(-1-1)a_\phi\right]$$

$$\vec{H} = \frac{I}{4\pi\rho}(2)\,a_\phi$$

$$\vec{H} = \frac{I}{2\pi\rho}a_\phi$$

<u>Note→</u> (1) $\vec{H}$ due to I(dc) flowing along z-axis is

$$\boxed{\vec{H} = \frac{I}{2\pi\rho}a_\phi} \quad H \propto \frac{1}{\rho}$$

* for current along z-axis $\vec{H}$ value do not depand on ϕ, z

(2) $\vec{H}$ in all the regions due to Idc current flowing from A to B on z-axis is

$$\vec{H} = \frac{I}{4\pi\rho}\left[-(\sin\alpha_2 - \sin\alpha_1)\right] a_H$$

$a_H = a_l \times a_\perp$ ($\perp$ perpandicular)

a_H = Unit vector in the dir^n of $\vec{H}$

a_l = Unit vector in the dir^n of current

$a_\perp$ = Unit vector in the dir^n of perpandicular line drawn from current line to field point.

* For current on the <u>z-axis</u>

$$\tan\alpha = \frac{z-z'}{\rho} = \frac{(z-z')}{\sqrt{x^2+y^2}}$$

For <u>x-axis:-</u>

$$\tan\alpha = \frac{x-x'}{\sqrt{y^2+x^2}}$$

(If current is on x-axis then $\tan\alpha = \dfrac{x-x'}{\sqrt{y^2+z^2}} = \dfrac{x-x'}{\rho}$)

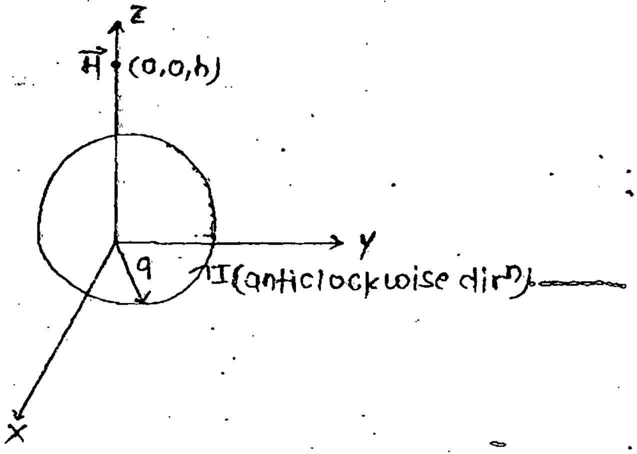
* $\alpha_1 \rightarrow$ angle of point A $(0,0,z_A')$ $P(\rho,\phi,z)$

$$\tan\alpha_1 = \frac{z - z_A'}{\rho}$$

* $\alpha_2 \rightarrow$ angle of point B $(0,0,z_B')$ $P(\rho,\phi,z)$.

$$\tan\alpha_2 = \frac{z - z_B'}{\rho}$$

Que. → Find $\vec{H}$ at $(0,0,h)$ due to I(dc) current flowing in a circular loop having a radius (with center coinciding with z-axis) lying on xy plane as shown below.

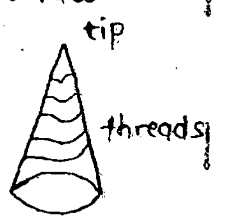


Soln →

$$* \ \vec{H}(0,0,h) = \frac{Ia^2}{2(a^2+h^2)^{3/2}} a_z \quad (I \text{ is in anticlockwise})$$

$$* \ \vec{H}(0,0,h) = \frac{Ia^2}{2(a^2+h^2)^{3/2}} (-a_z) \quad I \text{ is in clockwise dir}^n)$$

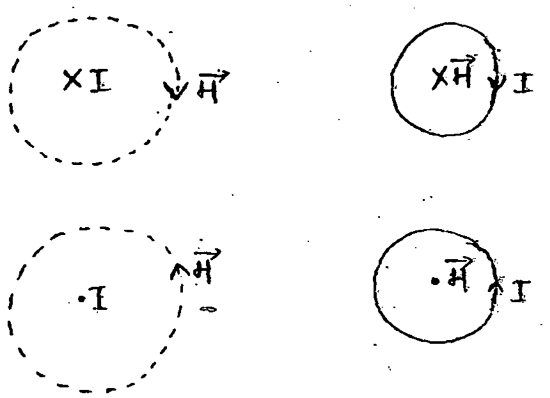
Note → Right hand screw law



* If tip is I then take threads as $\vec{H}$
* If tip is $\vec{H}$ then take threads as I

Notation → cross (x) → into the page
dot (·) → out of the page

Eg →



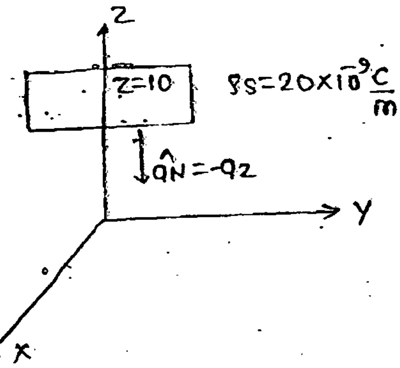
(16/55) line charge ~~$\rho_l = 30$ nC/m~~ at $y=3, z=5$ (y, z are constant)
line is parallel to x-axis

For ρ_l along x-axis $\vec{E}$ value do not depand ~~x-axis~~ value of field point.

$$\bar{E}\Big|_{(0,6,1)} = 64.7\,a_y - 86.3\,a_z$$

$$\bar{E}\Big|_{(5,6,1)} = \text{same}$$

(19/55)

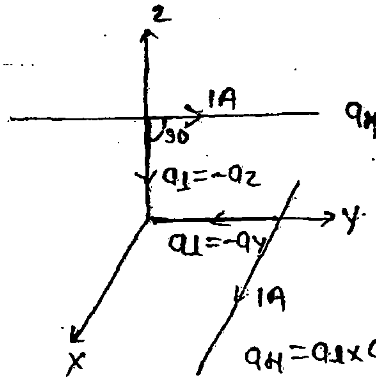


$\rho_s = 20\times10^{-9}\,\frac{C}{m^2}$

$$\bar{E} = \frac{\rho_s}{2\epsilon}\hat{a}_N = \frac{20\times10^{-9}}{2\times\frac{1}{36\pi}\times10^{-9}}(-a_z)$$

$$\bar{E} = -360\pi\, a_z$$

(17/55)



$$a_H = a_l \times a_\perp$$
$$= a_y \times (-a_z) = -a_x$$

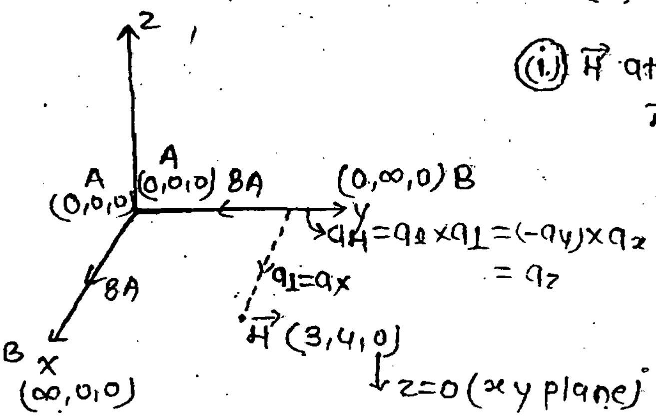
$$a_H = a_l \times a_\perp$$
$$= a_x \times (-a_y) = -a_z$$

(x, z) componants

$\frac{13}{55}$ * BA current is on +ve x-axis, +ve y-axis

* the dirn of current is from y to x . Field at H (3,4,0)

$\vec{H}$ is linear $\vec{H}_{(3,4,0)} = \vec{H}$ at (3,4,0) due to X line + $\vec{H}$ at (3,4,0) due to y line



(i.) $\vec{H}$ at (3,4,0) due to y line

$$\vec{H} = \frac{I}{4\pi\rho}\left[-(\sin\alpha_2 - \sin\alpha_1)\right] a_H$$

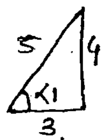
For I on y axis

$$\tan\alpha = \frac{y-y'}{\rho} = \frac{y-y'}{\sqrt{x^2+z^2}}$$

$\alpha_1 \Rightarrow$ angle of poin A (0,0,0)
(x', y', z')

ρ(3,4,0)
x, y, z

$$\tan\alpha_1 = \frac{4-0}{\sqrt{3^2+0}} = \frac{4}{3}$$



$\sin\alpha_1 = \frac{4}{5}$

$\alpha_2 \Rightarrow$ Angle of point B (0,∞,0)
(x', y', z')

P(3,4,0)

$$\tan\alpha_2 = \frac{4-\infty}{\sqrt{3^2+0^2}} = -\infty \;,\; \alpha_2 = -\frac{\pi}{2}$$

$$\vec{H} = \frac{I}{4\pi\rho}\left[-(\sin\alpha_2 - \sin\alpha_1)\right] a_H$$

$$= \frac{8}{4\pi(3)}\left[-\left(-1-\frac{4}{5}\right)a_z\right] = \frac{8}{4\pi(3)}\left[\frac{9}{5}a_z\right] = \frac{6}{5\pi}a_z$$

& similarly $\vec{H}$ at (3,4,0) due to x line $= \frac{4}{5\pi}a_z$

$$\vec{H} = \frac{6}{5\pi}a_z + \frac{4}{5\pi}a_z = \frac{10}{5\pi}a_z$$

$$= \frac{2}{\pi}a_z$$

standard question

(14/55) Ans(a)

(15/55) I in clockwise dir^n $\vec{H}(0,0,h) = \frac{Ia^2}{2(a^2+b^2)^{3/2}}(-a_z)$

given that $\vec{H}$ is required at the center $h=0$

$$\text{radius} = \frac{\text{diameter}}{2}$$

$$\vec{H}(0,0,0) = \frac{Ia^2}{2(a^2)^{3/2}}(-a_z) = \frac{I}{2a}(-a_z)$$

$$= \frac{I}{2\left(\frac{d}{2}\right)}(-a_z)$$

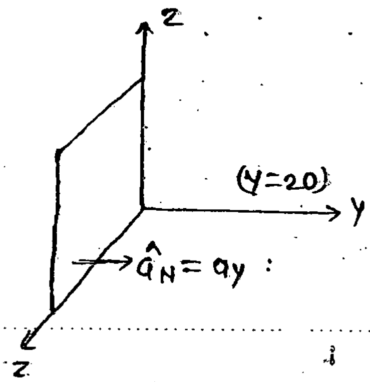
$$\vec{H} = \left(\frac{I}{d}\right)(-a_z)$$

(18/55) Ans.(c)

Note→ $\vec{H}$ in all the regions due to surface (or) sheet current density $\bar{K}\left(\frac{amp}{m}\right)$ is $\boxed{\vec{H} = \frac{1}{2}\vec{K}\times\hat{a}_N}$ Unknown $\hat{a}_N$

$$\left(\frac{amp}{m}\right) = \left(\frac{amp}{m}\right)$$

(22/56) Given $\vec{K} = 30a_z \frac{mA}{m}$ on $y=0$ plane. Find $\vec{H}(1,20,-2)$



$$\vec{H} = \frac{1}{2}\cdot\vec{K}\times\hat{a}_N$$

$$= \frac{1}{2}(30a_z\times10^{-3})\times a_y$$

$$= 15\times10^{-3}(-a_x)$$

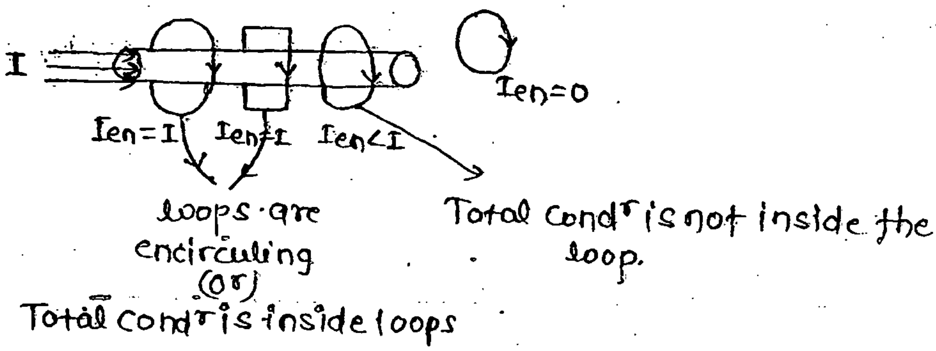
$$= -15a_x\left(\frac{mA}{m}\right)$$

$$= -15\hat{i}\,\frac{mA}{M}$$

Ampere's circuital law → This law says that the closed line integral of $\vec{H}$ is equal to the current enclosed by that closed line.

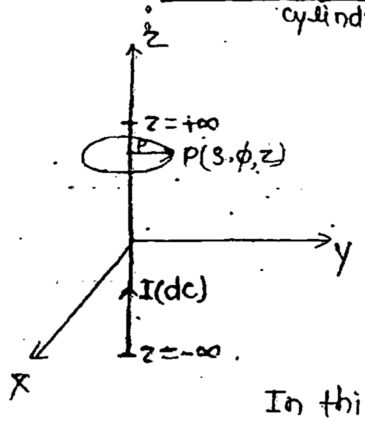
mathematically $\boxed{\oint \vec{H} \cdot \vec{dl} = I_{enclosed}}$, unknown $I_{enclosed}$

example →



Que. → Find $\vec{H}$ in all the regions due to I(dc) flowing along z-axis.
cylindrical (ρ, ϕ, z)

Solⁿ →



$$\oint \vec{H} \cdot \vec{dl} = I_{enclosed} \quad \text{——— (i)}$$

$\cancel{H_\phi (2\pi\rho) = I}$ $(2\pi\rho = \text{circumference})$

In cylindrical system

$$\oint \vec{H} \cdot \vec{dl} = H_\phi \, 2\pi\rho \quad \text{——— (ii)}$$

In this case $I_{enclosed} = I$ ——— (3)

from ② ③ in eqⁿ (i)

$$H_\phi (2\pi\rho) = I$$

$$H_\phi = \frac{I}{2\pi\rho} \quad \text{——— (4.)}$$

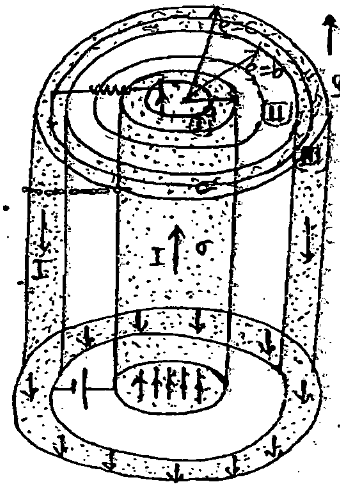
$$\vec{H} = H_\phi \, a_\phi$$

$$\vec{H} = \frac{I}{2\pi\rho} a_\phi$$

(IES conventional)

Que → Find $\vec{H}$ in all the regions due to I(dc) in inner cond^r, −I(dc) in outer cond^r as shown :-

sol^n →



(i) $0<\rho<a$

$a<\rho<b$

$b<\rho<c$

$c>\rho$

*(1.) $\vec{H}$ in the region $(0<\rho<a)$

$$\because \oint \vec{H}\cdot \vec{dl} = I_{enclosed} \quad \text{———} (i)$$

In cylindrical $\oint H \cdot dl = H_\phi (2\pi\rho)$ ——— (ii)

$$\pi a^2 (m^2) \xrightarrow{\text{current}} I$$

$$1 \xrightarrow{\text{current}} \frac{I}{\pi r^2}\left(\frac{A}{m^2}\right)$$

$$\pi\rho^2 \xrightarrow{\text{Current}} \frac{I}{\pi a^2}\times(\pi\rho^2) = I\cdot\frac{\rho^2}{a^2} = I_{enclosed} \quad \text{———} (iii)$$

From (2.),(3.) in (1.)

$$H_\phi(2\pi\rho) = I\cdot\frac{\rho^2}{a^2}$$

$$H_\phi = \frac{I}{2\pi}\cdot\frac{\rho}{a^2} \quad \text{———} (4.)$$

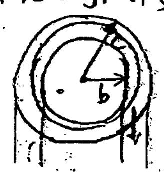
*(2.) $\vec{H}$ in the region $(a<\rho<b)$

$$\oint \vec{H}\cdot\vec{dl} = I_{enclosed} \quad \text{———} (5.)$$

$$H_\phi(2\pi\rho) = I$$

$$H_\phi = \frac{I}{2\pi\rho} \quad \text{———} (6.)$$

*(3.) $\vec{H}$ in the region $(b<\rho<c)$



$$\xrightarrow{\text{area}} \pi c^2 - \pi b^2 \xrightarrow{\text{current}} -I$$

$$1 \xrightarrow{\text{current}} \frac{-I}{\pi c^2 - \pi b^2}$$

$$\pi\rho^2 - \pi b^2 \xrightarrow{\text{current}} \frac{-I}{(\pi c^2 - \pi b^2)}\times(\pi\rho^2 - \pi b^2) = \frac{-I}{I}\left(\frac{\rho^2 - b^2}{c^2 - b^2}\right)$$

$$I_{enclosed} = I_{inner} + I_{outer} \text{(Inside green line)}$$

$$= I - I\left(\frac{\rho^2 - b^2}{c^2 - b^2}\right)$$

$$I_{enclosed} = I\left(\frac{c^2-\rho^2}{c^2-b^2}\right)$$

$$\oint H \cdot dl = I_{enclosed}$$

$$H_\phi (2\pi\rho) = I\left(\frac{c^2-\rho^2}{c^2-b^2}\right)$$

$$H_\phi = \frac{I}{2\pi\rho} \times \frac{(c^2-\rho^2)}{(c^2-b^2)}$$

1★(4.) $\vec{H}$ in the region $\rho > c$

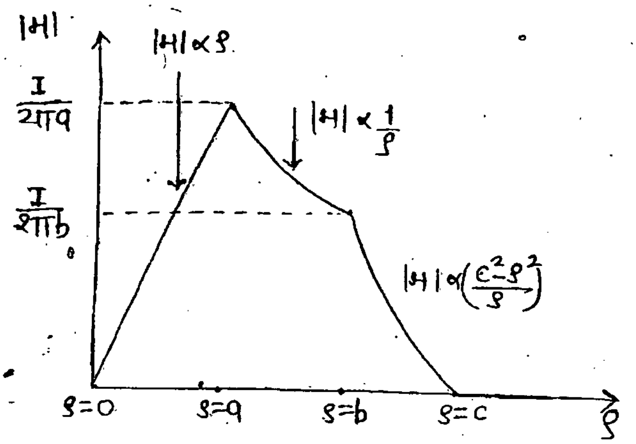
$$\oint H \cdot dl = I_{enclosed}$$

$$H_\phi (2\pi\rho) = I + (-I)$$

$$H_\phi (2\pi\rho) = 0$$

Total value of the magnetic field

$$\vec{H} = \begin{cases} \frac{I}{2\pi} \cdot \frac{\rho}{a^2} a_\phi & ; 0 < \rho < a \\ \frac{I}{2\pi\rho} \cdot a_\phi & ; a < \rho < b \\ \frac{I}{2\pi\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right) a_\phi & ; b < \rho < c \\ 0 & ; \rho > c \end{cases}$$



R → Bad cond^r
C → Good dielectr
L → magnetic material

65

Note → (1.) magnetic flux density $\left(\bar{B} = \frac{\phi}{area}\right)$ is given by

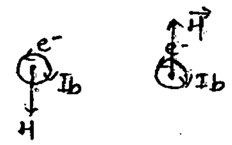
$$\boxed{\vec{B} = \mu\vec{H}} \left(\frac{wb}{m^2}\right) \text{ (or) Tesla} \quad \text{———(i)}$$

$\mu = \mu_0 \cdot \mu_r$

μ = Permeability of the medium $\left(\frac{H}{m}\right)$

$\mu_0 = 4\pi \times 10^{-7} \left(\frac{H}{m}\right)$ = permeability of free space.

$\mu_r = \frac{\mu}{\mu_0}$ = relative permeability (unitless)



I_b = Bound current {current bounded by the nucleas}

(2.) If $\vec{B}$ is given then magnetic flux (ϕ) is given by :-

$$\boxed{\phi = \iint \vec{B} \cdot \vec{ds}} \quad \text{———(2.)}$$

$wb = \frac{wb}{m^2} \cdot m^2$

(3.) :

$$\boxed{L = \frac{N\phi}{I} \text{ (Henery)}} \quad \text{———(3.)}$$

ϕ = magnetic flux linking each turn (wb)

N = No. of turns in the coil

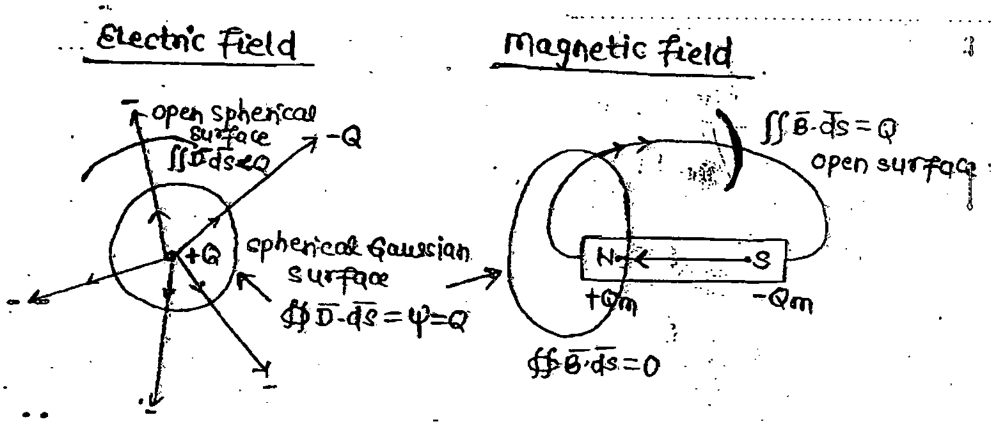
$N\phi$ = Total magetic flux.

I = Current in the coil

L = Inductance

for single turn (or) single loop N = 1

Note →



Maxwells equation →

(1.) Ampere circuital law

$$\oint \vec{H}\cdot d\vec{l} = I \quad \text{——— (i)}$$

By defination $I = \iint \vec{J}\cdot d\vec{s}$ ——— (ii)

By stokes theorem $\oint \vec{H}\cdot d\vec{l} = \iint (\vec{\nabla}\times\vec{H})\cdot d\vec{s}$ ——— (3.)

From (2), (3) in (1.)

$$\iint (\vec{\nabla}\times\vec{H})\, ds = \iint \vec{J}\cdot d\vec{s}$$

Compare $\boxed{\vec{\nabla}\times\vec{H} = \vec{J}}$

(2.) Gauss law of magnetic field

$$\oiint \vec{B}\cdot d\vec{s} = 0 \quad \text{——— (1)}$$

Using divergance theorem

$$\oiint \vec{B}\cdot d\vec{s} = \iiint (\vec{\nabla}\cdot\vec{B})\, dv \quad \text{——— (2.)}$$

From (2) in (1.) $\iiint (\vec{\nabla}\cdot\vec{B})\, dv = 0$

$$\iiint (\vec{\nabla}\cdot B)\, dv = \iiint 0\cdot dv$$

Compare $\vec{\nabla}\cdot\vec{B} = 0$

Note → maxwells eqn for static fields.

(1.) $\vec{\nabla}\cdot\vec{D} = \rho_v$

(2.) $\vec{\nabla}\times\vec{E} = 0$ (Static elec. field is irrotational)

(3.) $\vec{\nabla}\times\vec{H} = J$ (Static mag. field is rotational)

(4.) $\vec{\nabla}\cdot\vec{B} = 0$ (Static mag. field is divergance lines)

(derivation not Req.)

Que. → Find inductance between $\rho = a$ & $\rho = b$ of a coaxial ~~cble~~ for 'h' height.

Ans. →

$$L = \frac{\mu h}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\vec{H} = \frac{I}{2\pi\rho}\, a_\phi, \quad a<\rho<b$$

$$\vec{B} = \mu H = \frac{\mu I}{2\pi\rho}\, a_\phi$$

$$\phi = \iint \vec{B}\cdot\vec{ds} = -\iint\left(\frac{\mu I}{2\pi\rho}a_\phi\right)(d\rho\, dz\, a_\phi)$$

$$= \frac{\mu I}{2\pi}\int_{\rho=a}^{b}\frac{d\rho}{\rho}\cdot\int_{z=0}^{h} dz$$

$$= \frac{\mu I}{2\pi}(\ln\rho)_a^b\,(z)_0^h$$

$$\phi = \frac{\mu I}{2\pi}\ln\left(\frac{b}{a}\right)h$$

$$\boxed{L = \frac{\phi}{I} = \frac{\mu h}{2\pi}\ln\left(\frac{b}{a}\right) \text{ Henery}}$$

✱ Inductance per meter length of a coaxial cable is –

$$\boxed{\frac{L}{h} = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)}\left(\frac{H}{m}\right)$$

(2.) Inductance of N turn coil (Solenoidal)

$$\vec{H} = \frac{NI}{d}a_z\left(\frac{amp}{m}\right)$$

$$\vec{B} = \mu\vec{H}$$

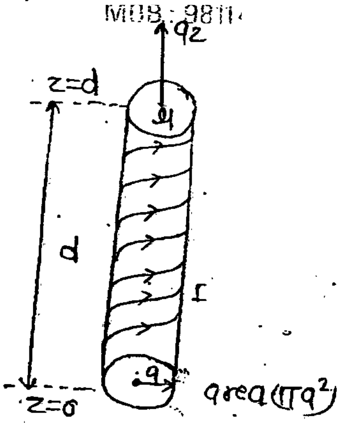
$$\vec{B} = \frac{\mu NI}{d}a_z$$

$$\phi = \iint \vec{B}\cdot\vec{ds} = \iint\left(\frac{\mu NI}{d}\right)a_z\,[\rho\, d\rho\, d\phi\, a_z]$$

$$= \frac{\mu NI}{d}\int_{\rho=0}^{a}\rho\, d\rho\int_{\phi=0}^{2\pi}d\phi$$

$$= \frac{\mu NI}{d}\left(\frac{\rho^2}{2}\right)_0^a(\phi)_0^{2\pi}$$

$$\phi = \frac{\mu NI}{d}(\pi a^2)$$



$$L = \frac{N\phi}{I}$$

$$L = N\cdot\frac{\mu NI}{d}(\pi a^2)$$

$$\boxed{L = \frac{\mu N^2}{d}(\pi a^2)}$$

$$L = \frac{\mu N^2}{d}\left[\text{Cross sectional area}\right]$$

Note →

*(1.) NT ⇒ $\phi = LI$; $W_{ind^r} = \frac{1}{2}LI^2 = \frac{1}{2}\phi I = \frac{1}{2}\frac{\phi^2}{L}$ (Joule)

EMT ⇒ $B = \mu H$; $W_{mag.field} = \frac{1}{2}\mu|H|^2$

$$\begin{pmatrix}\text{Energy stored in} \\ \text{magnetic field}\end{pmatrix} = \iiint \frac{1}{2}\mu|H|^2 dv = \iiint \frac{1}{2}\bar{B}\cdot\bar{H}\, dv = \iiint \frac{|B|^2}{2\mu} dv$$

(Joule)

*(2.) magnetic energy density $= \frac{1}{2}\mu|\bar{H}|^2$

$$= \frac{1}{2}\vec{B}\cdot\vec{H}$$

$$= \frac{1}{2}\frac{|\bar{B}|^2}{\mu} \left(\frac{\text{Joule}}{m^3}\right)$$

*(3.) $JE = \sigma\vec{E}\cdot E = \sigma|E|^2 \left(\frac{\text{watts}}{m^3}\right)$

$\left(\frac{A}{m^2}\right)\left(\frac{V}{m}\right) = \left(\frac{W}{m^3}\right)$

$R = \frac{l}{\sigma A}$

= Power dessipation density of a material

(1.) Scalar Electric Potential →

$$\vec{E} = -\vec{\nabla} V$$

$\left(\frac{\text{Volt}}{m}\right)$ scalar electric potential

(2.) Scalar Magnetic Potential →

$$\boxed{\vec{H} = -\vec{\nabla} V_m \quad \left(\frac{\text{Amp}}{m}\right) \quad \left(\frac{1}{m}\right) \rightarrow \text{Amp}}$$

$\boxed{\text{provided } \bar{J} = 0}$ where V_m = Scalar magnetic potential

From maxwell eq^n: $\nabla \times \vec{H} = \bar{J}$

$$-\nabla \times (\nabla V_m) = \bar{J}$$

curl of grad $= 0$

$$0 = \bar{J}$$

The above concept is applicable for $\underline{\bar{J} = 0}$

(3.) Vector magnetic Potential →

we know that $\vec{\nabla}\cdot\vec{B} = 0$ ——(1) {EMT}

& also ⇒ $\nabla\cdot(\nabla\times A) = 0$ ——(2) {maths}

From eqn (1) & (2)

$$\nabla\cdot\vec{B} = \cancel{\nabla}\cdot(\nabla\times\vec{A})$$

$$\boxed{\begin{array}{c}\vec{B} = \vec{\nabla}\times\vec{A} \\ \left(\frac{wb}{m^2}\right) \quad \left(\frac{1}{m}\right)\underline{\left(\frac{wb}{m}\right)} \\ ?\end{array}} \quad \left\{ \vec{H} = \frac{1}{\mu}(\vec{\nabla}\times\vec{A}) \right\}$$

where; $\bar{A}$ = Vector magnetic Potential $\left(\frac{wb}{m}\right)$

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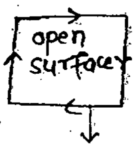

$$\oint\vec{A}\cdot\vec{dl} = \phi$$

$$\left(\frac{wb}{m}\right)\cdot m = wb$$

$$\oint\vec{A}\cdot\vec{dl} = \iint(\nabla\times\bar{A}\cdot\bar{ds})$$

$$wb = wb$$

Ans. (a)

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$$\frac{1}{2}\bar{J}\cdot\bar{A} = \frac{\text{magnetic energy}}{m^3} \qquad W_L = \frac{1}{2}LI^2 = \frac{1}{2}\phi I$$

$$\left(\frac{A}{m^2}\right)\left(\frac{wb}{m}\right) \qquad (wb)(Amp) = \text{magnetic energy}$$

= magnetic energy density

Ans (b)

* Boundry Conditions →

* Boundry is a surface (whose thickness → 0) which connects medium1, medium 2.
* Boundry condition give the relation between fields in one medium, fields in other medium.
* If fields in one medium are known, then using boundry condn we can find fields in other medium.

$z<0$	$z=0$	$z>0$
Electromegnatic wave is travelling in z dirn		
medium(1)		medium(2)
$\mu_1\ \epsilon_1\ \sigma_1$		$\mu_2\ \epsilon_2\ \sigma_2$
$\vec{E_1} = \vec{E}_{tangential1} + \vec{E}_{Normal1}$		$\vec{E_2} = \vec{E_{T2}} + \vec{E_{N2}}$
$\vec{D_1} = \vec{D_{T1}} + \vec{D_{N1}}$		$\vec{D_2} = \vec{D_{T2}} + \vec{D_{N2}}$.
$\vec{H_1} = \vec{H_{T1}} + \vec{H_{N1}}$		$\vec{H_2} = \vec{H_{T2}} + \vec{H_{N2}}$
$\vec{B_1} = \vec{B_{T1}} + \vec{B_{N1}}$		$\vec{B_2} = \vec{B_{T2}} + \vec{B_{N2}}$

tangential intensities $(\bar{E}, \bar{H})$

Normal densities $(\bar{D}, \bar{B})$

Q, ρ_l, ρ_s, ρ_v I $\vec{K}$ $\bar{J}$

C/m^2 A/m

	E_{t1}		E_{t2}	$E_{t1} = E_{t2}$
C/m^2	D_{N1}	ρ_s	$D_{N2} + \rho_s (C/m^2)$	$D_{N1} - D_{N2} = \rho_s$ (scalar) (C/m^2)
A/m	H_{t1}	$\bar{K}$	$H_{t2} + \bar{K}\left(\frac{A}{m}\right)$	$H_{t1} - H_{t2} = \cancel{\bar{K}}\ a_{N12} \times \bar{K}$ (vector) (A/m)
	B_{N1}		B_{N2}	$B_{N1} = B_{N2}$

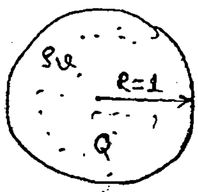
$\frac{26}{56}$ $\vec{H} = \frac{I}{2\pi} \cdot \frac{\rho}{a^2} a_\phi$, $\quad 0<\rho<a$

given $\rho = r$, $a = R$ $(r<R)$

$$\vec{H} = \frac{I}{2\pi} \cdot \frac{r}{R^2} a_\phi$$

$$|\vec{H}| = \frac{I}{2\pi} \cdot \frac{r}{R^2}$$

$\frac{28}{56}$



within a unit sphere ($R=1$ meter $\rightarrow$ volume chare configuration)

$$Q = \iiint \rho_v \, dv$$

$V = \frac{-6r^5}{\epsilon_0}$ For (If in question not given then take Homogeneous)

From poisson's eqn

$$\nabla^2 V = \frac{-\rho_v}{\epsilon_0}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = \frac{-\rho_v}{\epsilon_0}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left\{\frac{-6r^5}{\epsilon_0}\right\}\right) = \frac{-\rho_v}{\epsilon_0}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \times \frac{-5r^4 \times 6}{\epsilon_0}\right] = \frac{-\rho_v}{\epsilon_0}$$

$$\frac{1}{r^2}(-30)6\frac{r^5}{\epsilon_0} = \frac{-\rho_v}{\epsilon_0}$$

$$\frac{r^3}{\cancel{\epsilon_0}} \times 180 = \frac{\rho_v}{\cancel{\epsilon_0}}$$

$$\rho_v = 180 r^3$$

$$Q = \iiint \rho_v \, dv$$

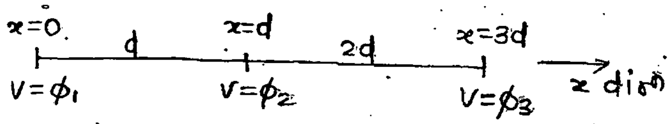
$$= \iiint 180 r^3 (r^2 \sin\theta \, dr\, d\theta\, d\phi)$$

$$= 180 \int_{r=0}^{1} r^5 dr . \int_{\theta=0}^{\pi} \sin\theta\, d\theta . \int_{\phi=0}^{2\pi} d\phi$$

$$Q = 120C$$

(29/56) One direction $\rightarrow$ Let x dir^n

given that Laplace eq^n is satiesfied.



(sol^n is in rectangular sys)

$$\nabla^2 V = 0 \quad \text{——— (i)}$$

$$\frac{\partial^2 V}{\partial x^2} = 0$$

$$\int \frac{\partial^2 V}{\partial x^2} dx = \int 0 \, dx$$

$$\frac{\partial V}{\partial x} = 0 + A$$

$$\partial V = A\,dx$$

$$V = Ax + B \quad \text{——— (ii)}$$

given $v = \phi_1$, at $x = 0$ ——— (3)

From (3) in (2) $\phi_1 = A(0) + B$, $B = \phi_1$ ——— (4)

$$V = Ax + \phi_1$$

$V = \phi_2$ at $x = d$

$\phi_2 = A(d) + \phi_1 \Rightarrow A = \frac{\phi_2 - \phi_1}{d}$

$V = \left(\frac{\phi_2 - \phi_1}{d}\right) x + \phi_1$

$V = \phi_3$ at $x = 3d$

$\phi_3 = \frac{\phi_2 - \phi_1}{d}(3d) + \phi_1$

$\phi_3 = 3\phi_2 - 3\phi_1 + \phi_1$

$\phi_3 + 2\phi_1 = 3\phi_2$

$\phi_2 = \frac{\phi_3 + 2\phi_1}{3}$

(36/57) $\epsilon_r = 5$, $\bar{D} = 2C/m^2$

$\bar{P} = \left(1 - \frac{1}{\epsilon_r}\right)\bar{D}$

$\bar{P} = 1.6 \frac{C}{m^2}$

(37/57)

$\bar{P}_i = (\epsilon_r - 1)\epsilon_0 E_i$

$= (3-1)\epsilon_0 \cdot \frac{6qx}{3} \quad \left(\because E_i \propto \frac{E_0}{\epsilon_r}\right)$

$\bar{P}_i = 4\epsilon_0 q x$

$E_0 = 6qx$; $\epsilon = 3\epsilon_0$; $P_i = (\epsilon_r - 1)\epsilon_0 E_i$; $x=0$; $x=5$

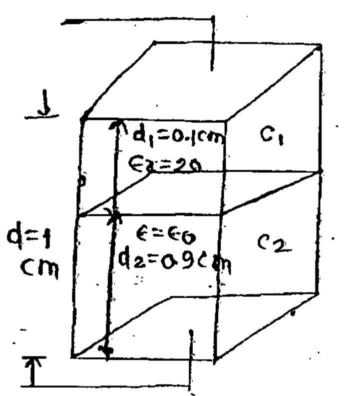
(38/57) $\because w = \frac{1}{2}QV, \quad V = \frac{Q}{4\pi\epsilon r}$

$w \propto V \propto \frac{1}{r}$

$\frac{w_2}{w_1} = \frac{r_1}{r_2}$

$\frac{w_2}{w_1} = \frac{0.5}{1}, \quad 2w_2 = w_1$

(40/57)



$C_1 = \frac{\epsilon_1 A}{d_1}, \quad C_2 = \frac{\epsilon_2 A}{d_2}$

$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{1}{\frac{d_1}{\epsilon_1 A} + \frac{d_2}{\epsilon_2 A}}$

$D = \frac{\psi}{A} = \frac{Q}{A} = \frac{CV}{A}$

$E_1 = \frac{D}{\epsilon_1} \qquad E_2 = \frac{D}{\epsilon_2}$

$$\cancel{D} = \frac{\cancel{C \times 400}}{\frac{1}{1\left(\cancel{\frac{d_1}{\epsilon_1}}\right)}} \qquad \cancel{D} = \cdot 1$$

$$\frac{V_1}{d_1} = \frac{D}{\epsilon_1} \qquad \frac{V_2}{d_2} = \frac{D}{\epsilon_2}$$

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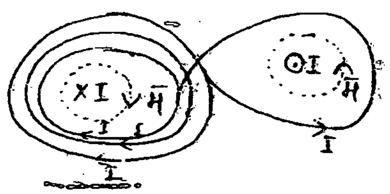
$$\oint H \cdot dl = I_{enclosed}$$



$3I + I = 4I$

***Remaining part of the boundry cond^n:-**

*There are 4 boundry conditions:-

(1.) The tangential componants of electric field intensities are contineous at the boundry surface.

mathematicaly $E_{tan1} = E_{tan2}$ (or) $\vec{E}_{tan1} = \vec{E}_{tan2}$
(magnitude) (mag.) (vec) (vec)

(2.) The normal componants of electric flux densities are discontineous (not equal) by an amount equal to the surface charge density on the boundry surface

$$\boxed{D_{N1} - D_{N2} = \rho_s} \qquad \text{if } \rho_s = 0 \text{ then } D_{N1} = D_{N2} \left\{ \vec{D}_{N1} = \vec{D}_{N2} \right\}$$

(3.) The tangential componants of magnetic field intensities are discontineous at the boundry by an amount equal to surface current density ($\vec{K}$) on the boundry surface

$$\boxed{\vec{H}_{t1} - \vec{H}_{t2} = \vec{a}_{N12} \times \vec{K}} \quad \text{if } \vec{K} = 0 \text{ then } \vec{H}_{t1} = \vec{H}_{t2}$$

(4.) The normal componants of magnetic flux densities are contineous (equal) at the boundry surface

$$\boxed{B_{N1} = B_{N2}} \qquad \vec{B}_{N1} = \vec{B}_{N2}$$

$\vec{a}_{N12}$ = normal unit vector from medium1 to medium2

Example→

$z<0$ $z=0$ $z>0$ $\longrightarrow a_z$ (by defination z value increasing)

medium 2 | medium 1

$\overline{a_{N12}} \longleftarrow$

(from $\overrightarrow{a_{N12}}$ defination)

Compare $\overrightarrow{a_{N12}}$, a_z both are in opposite direction $\overrightarrow{a_{N12}} = -a_z$

Note→ for $z=0$ {(or) z = constant} boundry surface z compared of the vector is normal (90°) · other two are tengential (parallel to the surface)

Types of boundries→

(1) Dielectric & dielectric boundry→

medium(1)	medium(2)
$\sigma_1 \simeq 0$; $\mu_1 = \mu_0 \mu_{r1}$	$\sigma_2 \simeq 0$, $\mu_2 = \mu_0 \mu_{r2}$
$\epsilon_1 = \epsilon_0 \epsilon_{r1}$	$\epsilon_2 = \epsilon_0 \epsilon_{r2}$

In this case boundry conditions are same as above 4 condⁿ

(2) Dielectric & condʳ boundry→

medium(1)	medium(2)
$\sigma_1 \simeq 0$, μ_1, ϵ_1	$\sigma_2 \simeq \to \infty$, μ_2, ϵ_2

$$\overline{E_2} = \frac{\overline{J_2}}{\sigma_2 (\to \infty)} = 0$$

$$E_2 = 0$$

$$\overrightarrow{E_{t2}} + \overrightarrow{E_{N2}} = 0. \quad E_{t2} = 0 \text{ —— (i)}$$

$$E_{N2} = 0 \text{ —— (ii)}$$

(i) In general boundry condⁿ $E_{t1} = E_{t2}$ —— (iii)

from (i) in (iii) $\boxed{E_{t1} = 0 \text{ (or) } \overline{E_{t1}} = 0}$

$\boxed{\text{On the cond}^r \text{ boundry (or) surface } E_{tangential} \text{ is } 0.}$

(ii) $D_{N1} - D_{N2} = \rho_s$

$D_{N1} - \epsilon_2 E_{N2} = \rho_s$ —— (iv)

From (2) in (4)

$$D_{N_1} - 0 = \rho_s$$

$$\boxed{\begin{aligned} D_{N_1} &= \rho_s \\ \epsilon_1 E_{N_1} &= \rho_s \end{aligned}}$$

Normal componant of electric flux density is equal to ρ_s on the cond^r surface