CBSE Class 09

Mathematics

Sample Paper 5 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. The value of $\left\{5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^{3}\right\}^{\frac{1}{4}}$ is

   a. 1

   b. 27

   c. 8

   d. 5

2. The value of $249^2 - 248^2$ is

   a. 248

b. 1

c. 249

d. 497

3. The number of triangles that can be drawn having angles as 50^0, 60^0 and 70^0 are :

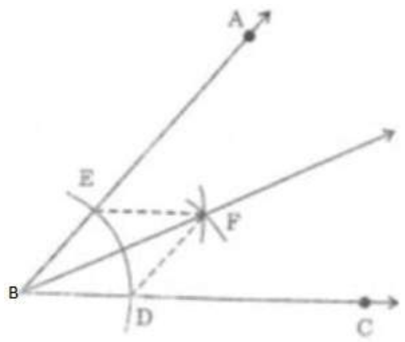
a. None of these

b. Two

c. Only one

d. Infinite

4. In the construction of the bisector of a given angle, as shown in the figure below $\triangle BEF \cong \triangle BDF$ by which congruence criterion?



a. SAS

b. SSS

c. RHS

d. AAS

5. The coefficient of x^3 in $2x + x^2 - 5x^3 + x^4$ is

a. -1

b. 2

c. 1

d. -5

6. A parallelogram has an area of 36 square an and base of the parallelogram is 9 cm. what is the corresponding altitude of parallelogram?

   a. 6 cm.

   b. 3 cm.

   c. 5 cm.

   d. 4 cm.

7. If x-1 if the factor of p(x)= $x^3 - 23x^2 + kx - 120$, then the value of 'k' is

   a. 120

   b. 124

   c. 142

   d. 140

8. The sides of a triangle are 35 cm, 54 cm and 61 cm, respectively and its area is $420\sqrt{5}\ cm^2$. The length of its longest altitude is

   a. $24\sqrt{5}\ cm$

   b. $10\sqrt{5}\ cm$

   c. $21\sqrt{5}$ cm

   d. 28 cm

9. The total surface area of a right circular cylinder of height 4 cm and radius 3 cm is

   a. $132cm^2$.

   b. 99 cm^2

   c. 66 cm^2

d. 198 cm^2

10. The probability of an event of a trial:

    a. is greater than 1

    b. is 0

    c. lies between 0 and 1 (both inclusive)

    d. is 1

11. Fill in the blanks:

    The two rational number which are their own multiplicative inverses are ________.

12. Fill in the blanks:

    The equation x = 7, in two variables can be written as ________.

    OR

    Fill in the blanks:

    If (1, -1) is a point on the line 2x - (2a + 5)y = 5, then the value of a is ________.

13. Fill in the blanks:

    The perpendicular distance of the point P(3, 4) from the Y-axis is ________.

14. Fill in the blanks:

    A point, whose distance from the centre of a circle is greater than its radius, lies in ________ of the circle.

15. Fill in the blanks:

    Volume of a cube is $648\sqrt{3}$ cm^3, then its side is _________ cm.

16. Express $0.\overline{1}$ in the form $\frac{p}{q}$.

17. Evaluate the following by using identities: 0.54 $\times$ 0.54 - 0.46 $\times$ 0.46

18. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.

OR

A hemi spherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

19. The angles of a quadrilateral are respectively 100^o, 98^o, 92^o. Find the fourth angle.

20. Express the following linear equation in the form ax + by + c = 0:
4 = 3x

21. Represent $\frac{8}{5}$ and $\frac{-8}{5}$ on the number line.

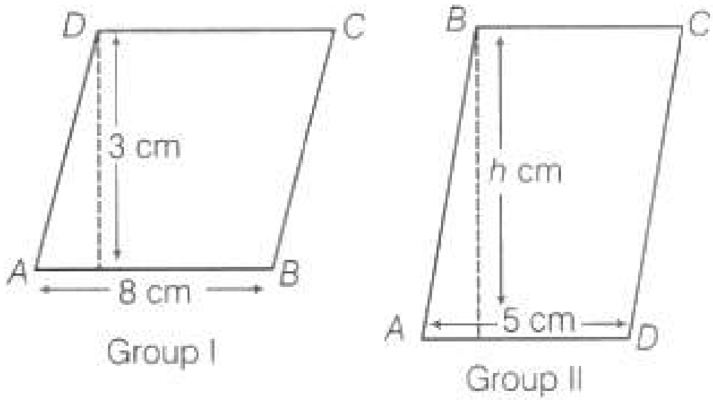
22. If the point (3, 4) lies on the graph of the equation 3y = ax + 7, find the value of a.

23. Factorise : $27x^3 + y^3 + z^3 - 9xyz$.

OR

Factorize: $a^6 - b^6$

24. In a class, teacher gave two identical cardboard pieces which are in the shape of a parallelogram to two groups. First group was asked to find area of parallelogram using AB as base. Then, another group was asked to find height h of the parallelogram with AD as base. How will they find value of h?



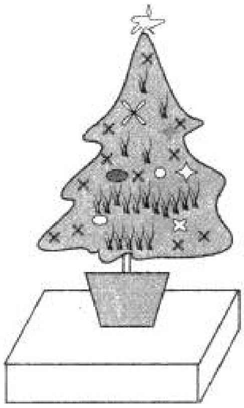
25. Convert the given frequency distribution into a continuous grouped frequency distribution:

Class - intervals	Frequency
150 - 153	7
154 - 157	7
158 - 161	15
162 - 165	10
166 - 169	5
170 - 173	6

OR

The traffic police recorded the speed (in km/hr) of 10 motorists as 47, 53, 49, 60, 39, 42, 55, 57, 52, 48. Later on an error in recording instrument was found. Find the correct average speed of the motorists if the instrument recorded 5 km/hr less in each case.

26. Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden block covered with coloured paper with picture of Santa Claus on it (see figure). She must know the exact quantity of paper to buy for this purpose. If the box has length, breadth and height as 80 cm, 40 cm and 20 cm respectively, then how many square sheets of paper of side 40 cm would she require?



27. Prove that: $\frac{a^{-1}}{a^{-1}+b^{-1}} + \frac{a^{-1}}{a^{-1}-b^{-1}} = \frac{2b^2}{b^2-a^2}$

OR

Visualize 2.4646 on the number line using successive magnification.

28. Draw the graphs of the equations : $3x - 2y = 4$ and $x + y - 3 = 0$ in the same graph and find the co-ordinates of the point where two lines intersect.

29. Find at least 3 solutions for the following linear equation in two variables:

$2x + 3y = 4$

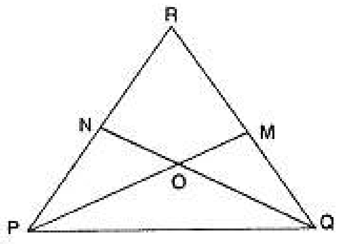
OR

Give the geometric representation of $y = 3$ as an equation in two variables.

30. Construct a triangle with perimeter 11.8 cm and base angles 60° and 45°.

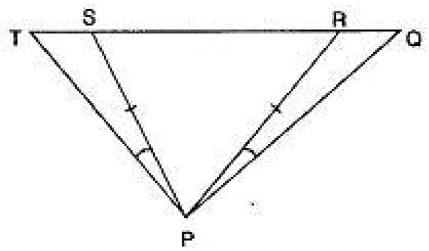
31. In a $\triangle$ABC, D, E, and F are respectively the mid-points of BC, CA, and AB. If the lengths of side AB, BC, and CA are 7 cm, 8 cm and 9 cm, respectively, find the perimeter of ΔDEF.

32. In figure, $\angle QPR = \angle PQR$ and M and N are respectively points on sides QR and PR or DPQR, such that QM = PN. Prove that OP = OQ, where O is the point of intersection of PM and QN.



OR

In figure, PS = PR, $\angle TPS = \angle QPR$. Prove that PT = PQ.



33. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 13 cm, 14 cm and 15 cm and the parallelogram stands on the base 14

cm, find the height of the parallelogram.

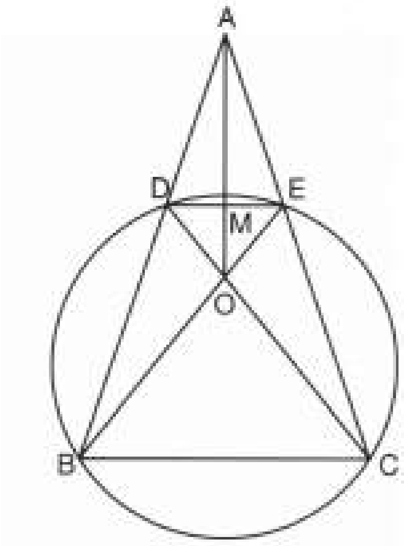
34. Two dice are thrown simultaneously 500 times. Each time the sum of two numbers appearing on their tops is noted and recorded as given in the following table:

SUM	**Frequency**
2	14
3	30
4	42
5	55
6	72
7	75
8	70
9	53
10	46
11	28
12	15

If the dice are thrown once more, what is the probability of getting a sum

i. more than 10?
ii. less than or equal to 5?
iii. between 8 and 12?

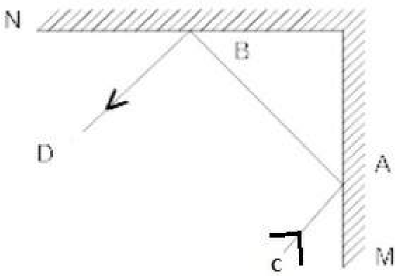
35. D and E are, respectively, the points on equal sides AB and AC of an isosceles triangle ABC such that B, C, E, and D are concyclic as shown in Fig. If O is the point of intersection of CD and BE, prove that AO is the bisector of line segment DE.



OR

D is the mid point of side BC of an isosceles triangle ABC with AB = AC. Prove that the circle drawn with either of the equal sides as a diameter passes through the point D.

36. In fig M and N are two plane mirrors perpendicular to each other; prove that the incident ray CA is parallel to reflected ray BD.



37. If a + b + c = 15 and $a^2 + b^2 + c^2 = 83$, find the value of $a^3 + b^3 + c^3 - 3abc$.

OR

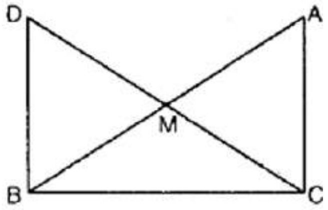
Let $p(x) = x^4 - 3x^2 + 2x + 5$. Find the remainder when p(x) is divided by (x - 1).

38. A hemispherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is Rs. 5 per 100 cm^2.

OR

The difference between outside and inside surface of a cylindrical metallic pipe 14 cm. long is 44 sq cm. if the pipe is made of 99 cu cm. of metal, find the outer and inner radius of the pipe.

39. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. (See figure)



Show that:

i. $\triangle AMC \cong \triangle BMD$
ii. $\angle DBC$ is a right angle.
iii. $\triangle DBC \cong \triangle ACB$
iv. CM = $\frac{1}{2}$ AB

40. Calculate the mean, median and mode for the following data.
23, 25, 28, 25, 16, 23, 17, 22, 25, 25

CBSE Class 09

Mathematics

Sample Paper 5 (2019-20)

Solution

Section A

1. (d) 5

   Explanation: $\left\{5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^{3}\right\}^{\frac{1}{4}}$

   $\Rightarrow\left\{5\left(2^{3\times\frac{1}{3}}+3^{3\times\frac{1}{3}}\right)^{3}\right\}^{\frac{1}{4}}$
   $\Rightarrow\left\{5(3+2)^{3}\right\}^{\frac{1}{4}}$
   $\Rightarrow\left\{5\times5^{3}\right\}^{\frac{1}{4}}$
   $\Rightarrow 5^{4\times\frac{1}{4}}$
   $\Rightarrow 5$

2. (d) 497

   Explanation: $(249)^{2}-(248)^{2}$
   = (249 + 248)(249 - 248) [Using identity $a^{2}-b^{2}=(a+b)(a-b)$]

   = 497 x 1

   = 497

3. (d) Infinite

   Explanation: As we know similar triangles can be drawn for any given triangle.

   These similar triangles will have the same angles as the original triangle (ie $\angle 50^{0}, \angle 60^{0}$ and $\angle 70^{0}$) and will be infinite in number.

4. (b) SSS

   Explanation:

   In $\triangle BEF$ and $\triangle BDF$,
   BE = BD (lines formed by same arc)

BF = BF (common)

EF = DF (sides from same arc)

hence, $\Rightarrow \triangle BEF \cong \triangle BDF$ (BY SSS)

5. (d) -5

   Explanation:

   The coefficient of x^3 in $2x + x^2 - 5x^3 + x^4$ is -5.

6. (d) 4 cm.

   Explanation: Area of parallelogram = Base x Corresponding altitude

   $\Rightarrow$ 36 = 9 x Corresponding altitude

   $\Rightarrow$ Corresponding Altitude = $\frac{36}{9} = 4$ cm

7. (c) 142

   Explanation: If x - 1is a factor of p(x), then

   p(1) = 0

   $(1)^3$ - 23 $(1)^2$ + k (1) - 120 = 0

   1 - 23 + k - 120 = 0

   1 - 143 + k = 0

   -142 + k = 0

   k = 142

8. (a) $24\sqrt{5}\ cm$

   Explanation: Since longest altitude us drawn opposite to the shortest side in a triangle.

   Area of triangle = $\frac{1}{2}$ x Base x Height

   $\Rightarrow 420\sqrt{5} = \frac{1}{2} \times 35$ x Height

   $\Rightarrow$ Height = $\frac{420\sqrt{5}\times 2}{35} = 24\sqrt{5}$ cm

9. (a) $132cm^2$.

Explanation:

Total surface area= $2\pi rh + 2\pi r^2$

$=2\pi r(4 + 3)$

$=2\pi r(7)$

$=2\times 22/7\times 7\times 3$

$= 132\ cm^2$

10. (c) lies between 0 and 1 (both inclusive)
    Explanation: Mathematically, the probability that an event will occur is expressed as a number between 0 and 1.

11. 1, -1

12. 1 x + 0 y = 7

OR

-1

13. 3

14. exterior

15. $6\sqrt{3}$

16. Let x = $0.\overline{1}$.

Then, we can write
x = 0.11111......(i)
$\Rightarrow$ 10 x = 1.11111....(ii)
On subtracting (i) from (ii), we get

10x - x = 1.11111 - 0.11111
9x = 1
$\Rightarrow$ x = $\frac{1}{9}$

$\Rightarrow 0.\overline{1} = \frac{1}{9}$

Thus, $0.\overline{1} = \frac{1}{9}$

17. We have,

$0.54 \times 0.54 - 0.46 \times 0.46$

$= (0.54)^2 - (0.46)^2 = (0.54 + 0.46)(0.54 - 0.46) = 1 \times 0.08 = 0.08$

18. Let the height of the right circular cylinder be 'h' m.

r = 0.7 m

Curved surface area = 4.4 m^2

$\Rightarrow 2\pi rh = 4.4$

$\Rightarrow 2 \times \frac{22}{7} \times 0.7 \times h = 4.4$

$\Rightarrow 4.4h = 4.4$

$\Rightarrow h = 1m$

$\therefore$ The height of the right circular cylinder is 1 m.

OR

The volume of water the bowl contain = $\frac{2}{3}\pi r^3$

Radius = $3.5cm$

Then volume = $\frac{2}{3} \times \frac{22}{7} \times (3.5)^3$

$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times \frac{35}{10}$

$= 89.8cm^3$

19. Let the measure of the fourth angle be x^o. We know that the sum of the angles of a quadrilateral is 360^o.

$\therefore 100 + 98 + 92 + x = 360$

$\Rightarrow 290 + x = 360 \Rightarrow x = 360 - 290 = 70$

Hence, the measure of fourth angle is 70^o

20. We have,

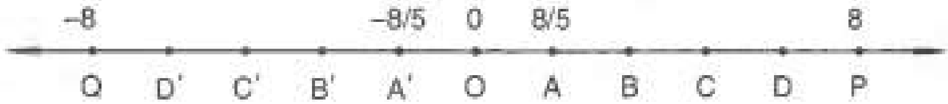
4 = 3x

$\Rightarrow -3x + 0y + 4 = 0$

On comparing this equation with ax + by + c = 0, we get

a = -3, b = 0 and c = 4

21. To represent $\frac{8}{5}$ and $\frac{-8}{5}$ on the number line, draw a number line and mark a point O on it to represent zero. Now, mark two points P and Q representing integers 8 and -8 respectively on the number line. Divide the segment OP into five equal parts. Let A, B, C, D be the points of division so that OA = AB = BC = CD = DP. By construction, OA is one-fifth of OP. So, A represents the rational number $\frac{8}{5}$.



Now, Q represents - 8 on the number line. Divide OQ into five equal parts OA', A'B', B'C', C'D' and D'Q. Since Q represents - 8, therefore, A represents the rational number $\frac{-8}{5}$.

22. If the point (3, 4) lies on the graph of the equation

3y = ax + 7, then

3(4) = a(3) + 7

⇒ 12 = 3a + 7

⇒ 3a = 12 – 7

⇒ 3a = 5

⇒ a = $\frac{5}{3}$

23. $27x^3 + y^3 + z^3 - 9xyz$.

$(3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)\{(3x)^2 + (y)^2 + (z)^2 - (3x)(y) - (y)(z) - (z)(3x)\}$

(Using Identity $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$)

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$.

OR

We have,

$a^6 - b^6 = (a^3)^2 - (b^3)^2$

$\Rightarrow a^6 - b^6 = (a^3 - b^3)(a^3 + b^3)$ [using $a^2 - b^2 = (a-b)(a+b)$]

$\Rightarrow a^6 - b^6 = (a - b)(a^2 + ab + b^2)(a + b)(a^2 - ab + b^2)$

$\Rightarrow a^6 - b^6 = (a - b)(a + b)(a^2 + ab + b^2)(a^2 - ab + b^2)$

24. Area of both the parallelograms are equal as they are identical.

Thus,

Area of IIgm of Group I = Area of IIgm of Group II

$8 \times 3 = 5 \times h$

$h = \frac{8 \times 3}{5}$

$= \frac{24}{5}$ = 4.8 cm

25.

Class interval	Continuous class intervals	Frequency
150 - 153	149.5 - 153.5	7
154 - 157	153.3 - 157.5	7
158 - 161	157.5 - 161.5	15
162 - 165	161.5 - 165.5	10
166 - 169	165.5 - 169.5	5
170 - 173	169.5 - 173.3	6
Total		30

OR

The speed of 10 motorists (in km/hr) = 47, 53, 49, 60, 39, 42, 55, 57, 52, 48

Later on it was discovered that the instruments recorded 5 km/hr less in each case.

$\therefore$ Correct values are = 52, 58, 54, 65, 44, 47, 60, 62, 57, 53

$\therefore$ Correct mean = $\frac{52+58+54+65+44+47+60+62+57+53}{10}$

= $\frac{552}{10}$ = 55.2 km/hr

26. Since mary wants to paste the paper on the outer surface of the box; the quantity of paper required would be equal to the surface area of the box which is of the shape of a cuboid.

The dimensions of the box are:

Length, l = 80 cm, Breadth, b = 40 cm, Height, h = 20 cm.

The surface area of the box = 2(lb + bh + hl)

= $2[(80 \times 40) + (40 \times 20) + (20 \times 80)]$

= 2[3200 + 800 + 1600]

= 2 $\times$ 5600 cm^2 = 11200 cm^2

The area of each sheet of the paper = 40 $\times$ 40 = 1600 cm^2

Therefore, number of sheets required = $\frac{surface\ area\ of\ box}{area\ of\ one\ sheet\ of\ paper}$ = $\frac{11200}{1600}$ =7

27. Let's start by simplifying the L.H.S,

$\frac{a^{-1}}{a^{-1}+b^{-1}} + \frac{a^{-1}}{a^{-1}-b^{-1}}$

= $\frac{1/a}{\frac{1}{a}+\frac{1}{b}} + \frac{1/a}{\frac{1}{a}-\frac{1}{b}}$

= $\frac{1}{a} \cdot \frac{ab}{b+a} + \frac{1}{a} \cdot \frac{ab}{b-a}$

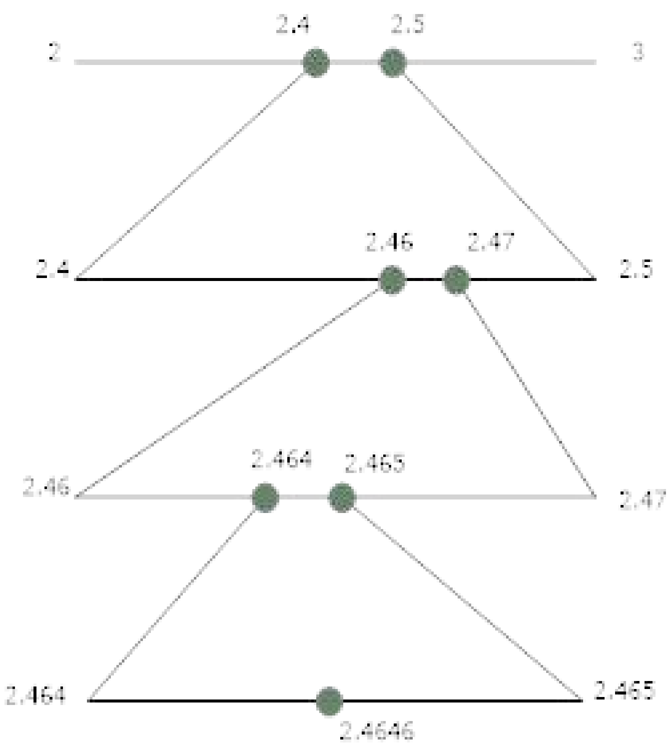
= $\frac{b}{b+a} + \frac{b}{b-a}$

= $\frac{b(b-a)+b(b+a)}{(b+a)(b-a)}$

= $\frac{b^2-ba+b^2+ab}{b^2-a^2}$ = $\frac{2b^2}{b^2-a^2}$

Hence Proved.

OR



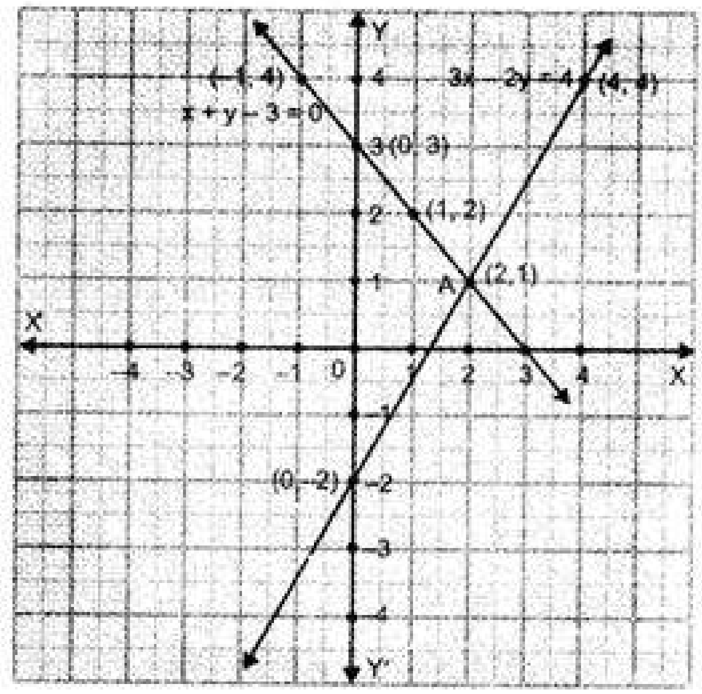
28. Graph of equation 3x - 2y = 4,

We have, 3x - 2y = 4, 3x - 4 = 2y

$\Rightarrow y = \frac{3}{2}x - 2$
Let x = 0 : $y = \frac{3}{2}(0) - 2 = 0 - 2 = -2$
Let x = 2 : $y = \frac{3}{2}(2) - 2 = 3 - 2 = 1$
Let x = 4 : $y = \frac{3}{2}(4) - 2 = 6 - 2 = 4$
Thus, we have the following table :

x	0	2	4
y	-2	1	4

Now, plot the points (0, -2), (2, 1) and (4, 4) on a graph paper and join them by a line.



Graph of the equation x + y - 3 = 0

x + y - 3 = 0

$\Rightarrow y = -x + 3$

Let x = 0 : y = -0 + 3 = 3

Let x = 1 : y = -1 + 3 = 2

Let x = -1 : y = - (-1) + 3 = 1 + 3 = 4

Thus, we have the following table :

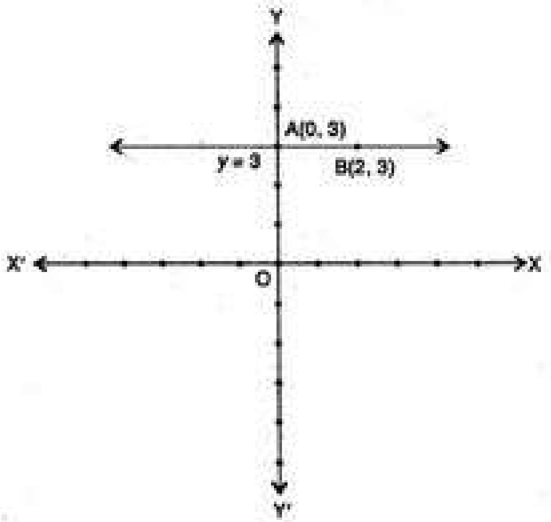
x	0	1	-1
y	3	2	4

By plotting the points (0, 3), (1, 2) and (-1, 4) on the graph paper and joining them by a line, we obtain the graph of x + y - 3 = 0
The lines represented by the equations 3x - 2y = 4 and x + y - 3 = 0 intersect at point A whose co-ordinates are (2, 1).

29. 2x + 3y = 4
⇒ 3y = 4 – 2x
⇒ $y = \frac{4-2x}{3}$
Put x = 0, then $y = \frac{4-2(0)}{3} = \frac{4}{3}$
put x = 1, then $y = \frac{4-2(1)}{3} = \frac{2}{3}$
Put x = 2, then $y = \frac{4-2(2)}{3} = 0$
Put x = 3, then $y = \frac{4-2(3)}{3} = \frac{-2}{3}$
$\therefore (0, \frac{4}{3}), (1, \frac{2}{3}), (2, 0)\ and (3, \frac{-2}{3})$, are the solutions of the equation 2x + 3y = 4.

OR

The given equation is
y = 3
⇒ 0.x + 1.y = 3



It is a linear equation in two variables x and y. This is represented by a line. All the values of x are permissible because 0.x is always 0. However, y must satisfy the relation y = 3. Hence, two solutions of the given equation are x = 0, y = 3 and x = 2, y = 3.

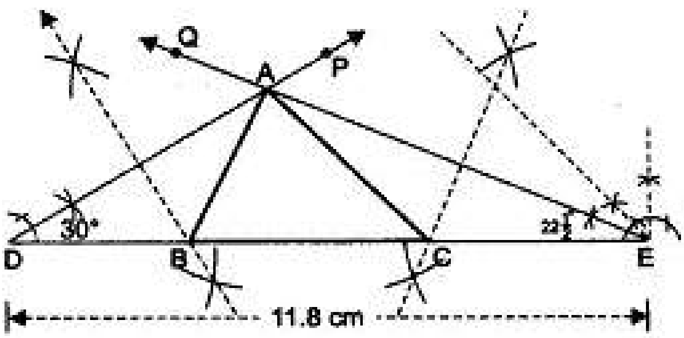
Thus the graph AB is a line parallel to the x-axis at a distance of 3 units above it.

30. Given: In $\triangle$ABC, AB + BC + CA = 11.8 cm. $\angle$B = 60^{o} and $\angle$C = 45^{o}.

Required: To construct the $\triangle$ABC.

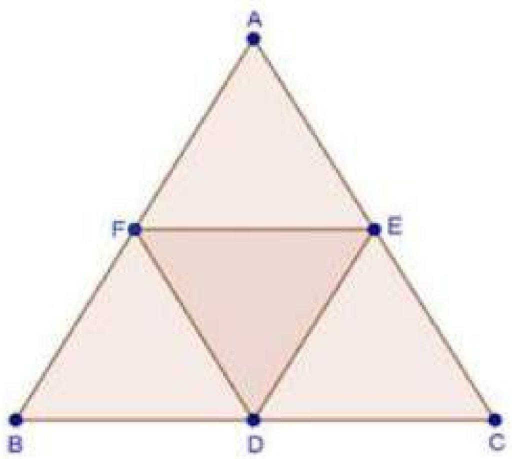
Steps of construction :

i. Draw DE = 11.8 cm.
ii. At D, construct $\angle EDP = \frac{1}{2}$ of $60^{o} = 30^{o}$ and at E, construct $\angle DEQ = \frac{1}{2}$ of $45^{o} = 22\frac{1}{2}^{o}$
iii. Let DP and EQ meet at A.
iv. Draw perpendicular bisector of AD to meet DE at B.
v. Draw perpendicular bisector of AE to meet DE at C.
vi. Join AB and AC.



ABC is the required triangle.

31.



Given: In an $\triangle$ABC, D, E, and F are, respectively, the mid-points of BC, CA, and AB. Where AB = 7cm, BC = 8cm, and CA = 9cm

To find: The perimeter of ΔDEF

In $\triangle$ABC

$\therefore$ F & E are the mid-points of AB and AC

$\therefore$ EF = $\frac{1}{2}$ BC [Mid-point theorem]

Similarly

DF = $\frac{1}{2}$ AC, DE = $\frac{1}{2}$ AB

Perimeter of $\triangle$DEF = DE + EF + DF

= $\frac{1}{2}$ AB + $\frac{1}{2}$ BC + $\frac{1}{2}$ AC

= $\frac{1}{2}$ (AB+BC+AC)

= $\frac{1}{2} \times (7 + 8 + 9)$

= $\frac{1}{2}$ (24) = 12cm

$\therefore$ Perimeter of $\triangle$DEF = 12cm.

32. In DQMP and DPNQ,

QM = PN . . . .[Given]

$\angle$PQR = $\angle$QPR . . .[Given]

$\Rightarrow$ $\angle$PQM = $\angle$QPN

PQ = PQ . . . . [Common]

$\therefore$ DQMP$\cong$DPNQ . . . .[By SAS property]

$\therefore$ $\angle$QMP = $\angle$PNQ . . . [c.p.c.t.]

In DONP and D OMQ

$\angle$PON = $\angle$QOM . . . . [Vertically opposite angles]

$\angle$QMP = $\angle$PNQ . . . [As proved above]

$\Rightarrow$ $\angle$QMO = $\angle$PNO

QM = PN . . . . [Given]

$\therefore$ DONP$\cong$DOMQ

$\therefore$ OP = OQ . . . . [c.p.c.t.]

OR

In $\triangle$PSR,

PS = PR …… [Given]

$\angle$PRS = $\angle$PSR ……. [Angles opposite to equal sides]

180^{o} – $\angle$PRS = 180^{o} - $\angle$PSR

$\angle$PRQ = $\angle$PST .... (1)

In $\triangle$PST and $\triangle$PRQ

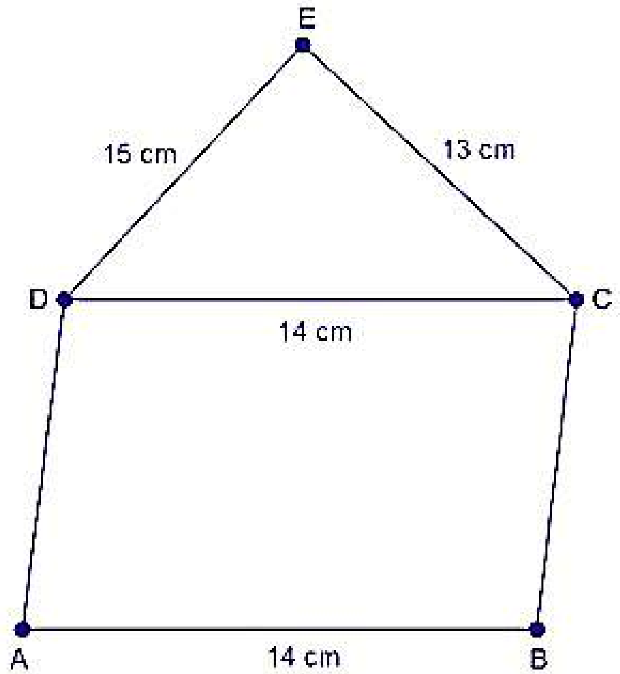
PS = PR ...... [Given]

$\angle$TPS = $\angle$QPR . . . [Given]

$\angle$PST = $\angle$PRQ . . .[From (1)]

$\therefore \triangle PST \cong \triangle PRQ$ .... [By SAS property]

$\therefore$ PT = PQ . . .[c.p.c.t.]

33. 

Let h be the height of parallelogram ABCD.

Now 2s = (DC + CE + ED)

$\Rightarrow$ s = $\frac{1}{2}$(DC + CE + ED)

= $\frac{1}{2}$(15 + 13 + 14) = $\frac{1}{2}$ $\times$ 42 = 21 cm

Area of $\triangle$DCE = $\sqrt{s(s-DC)(s-CE)(s-ED)}$ [Heron's Formula]

= $\sqrt{21(21-14)(21-13)(21-15)}$

= $\sqrt{21 \times 7 \times 8 \times 6}$

= 84 cm^2

Its given that area of $\triangle$DCE = area of ABCD

$\Rightarrow$ area of parallelogram ABCD = 84 cm^2

$\Rightarrow$ 14 $\times$ h = 84 [area of parallelogram = base $\times$ height]

$\Rightarrow$ h = 6 cm

34. i. P (getting a sum more than 10)

= P (getting a sum of 11) + P (getting a sum of 12)

= $\frac{28}{500} + \frac{15}{500} = \frac{28+15}{500} = \frac{43}{500} = 0.086 = 0.09$

ii. P (getting a sum less than or equal to 5)

= P (getting a sum of 5) + P (getting a sum of 4) + P (getting a sum of 3) + P (getting a sum of 2)

= $\frac{55}{500} + \frac{42}{500} + \frac{30}{500} + \frac{14}{500} = \frac{141}{500} = 0.282$

iii. P (getting a sum between 8 and 12)

= P (getting a sum of 9) + P (getting a sum of 10) + P (getting a sum of 11)

= $\frac{53}{500} + \frac{46}{500} + \frac{28}{500} = \frac{127}{500} = 0.254$

35. In $\triangle$ABC, we have

AB = AC

$\Rightarrow \angle$B = $\angle$C ...(i)

Since B, C, E, and D are concyclic. Therefore, BCED is a cyclic quadrilateral.

$\Rightarrow \angle$ADE = $\angle$C and $\angle$AED = $\angle$B

$\Rightarrow \angle$ADE = $\angle$AED [$\because \angle$B = $\angle$C (From (i)] ...(ii)

$\Rightarrow$ AD = AE and DE || BC ...(iii)

But, AB = AC

$\therefore$ AD = AE

$\Rightarrow$ AB - AD = AC - AE

$\Rightarrow$ BD = CE.

In $\triangle_S$ BOD and COE, we have

BD = CE

$\angle$OBD = $\angle$OCE [Angles in the same segment]

and, $\angle$BDO = $\angle$CEO [Angles in the same segment]

So, by ASA congruence criterion, we obtain

$\triangle$BOD $\cong$ $\triangle$COE

$\Rightarrow$ OD = OE [c.p.c.t.]

In $\triangle$s ADO and AEO, we have

AD = AE

$\angle$ADO = $\angle$AEO [$\because \angle$ADE = $\angle$AED and $\angle$ODE = $\angle$OED $\therefore \angle$ADE + $\angle$ODE = $\angle$AED + $\angle$OED $\Rightarrow \angle$ADO = $\angle$AEO] and, OD = OE

$\therefore \triangle ADO \cong \triangle AEO$

$\Rightarrow \angle DAO = \angle EAO$

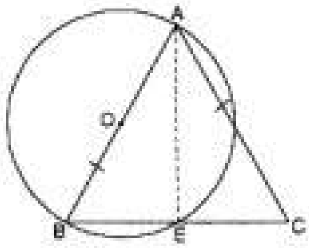
$\Rightarrow$ OA is the bisector of $\angle DAE$

$\Rightarrow$ AM is the bisector of $\angle DAE$ in the isosceles triangle ADE

$\Rightarrow$ AO is the perpendicular bisector of segment DE.

OR

Given: D is the mid-point of side BC of an isosceles triangle ABC with AB = AC.



To prove The circle drawn with either of the equal sides as a diameter passes through the point D.

Proof : In $\triangle AEB$ and $\triangle AEC$

AB = AC |Given

$\angle BEA = 90^o$ |Angle in semi-circle

$\angle BEA + \angle CEA$ [Linear Pair]

$\Rightarrow 90^o + \angle CEA = 180^o$

$\Rightarrow \angle CEA = 90^o$

$\therefore \angle BEA = \angle CEA = 90^o$

$\therefore \angle BEA = \angle CEA = 90^o$

AE = AE |common

$\therefore \triangle AEB \cong \triangle AEC$ [R.H.S]

$\therefore$ BE = CE [c.p.c.t]

$\Rightarrow$ E is the mid-point of BC

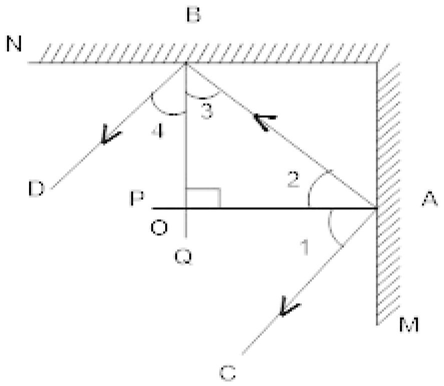
But the mid-point of BC is given to be D.

D and E coincide

Hence, the circle drawn with AB as diameter passes through the point D.

Similarly, we can prove that the circle drawn with AC as diameter passes through the point D.

36. Draw AP $\perp$ M and BQ $\perp$ N



$\therefore$ BQ $\perp$ N and AP $\perp$ M and M $\perp$ N

$\therefore\ \angle BOA = 90^0$

$\Rightarrow\ BQ \perp AP$

In Δ BOA $\angle 2 + \angle 3 + \angle BOA = 180^0$ [By angle sum property]

$\Rightarrow\ \angle 2 + \angle 3 + 90^0 = 180^0$

$\therefore\ \angle 2 + \angle 3 = 90^0$

Also

$\Rightarrow\ \angle 1 + \angle 4 = \angle 2 + \angle 3 = 90^0$ [Because, $\angle 1 = \angle 2$ and $\angle 4 = \angle 3$]

$\Rightarrow\ \angle 1 + \angle 4 = \angle 2 + \angle 3 = 90^0$

$\therefore\ (\angle 1 + \angle 4) + (\angle 2 + \angle 3) = 90^0 + 90^0 = 180^0$

$\Rightarrow\ (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180^0$

or $\angle CAB + \angle DBA = 180^0$

$\therefore\ CA \parallel BD$ [By sum of interior angles of same side of transversal]

37. Given: a + b + c = 15 and $a^2 + b^2 + c^2 = 83$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)\{(a^2 + b^2 + c^2) - (ab + bc + ca)\}$ ...(i)

It follows from the above identity that we require the values of a + b + c, $a^2 + b^2 + c^2$ and ab + bc + ca to get the value of $a^3 + b^3 + c^3 - 3abc$.

The values of a + b + c and $a^2 + b^2 + c^2$ are known to us. So, we require the value of ab + bc + ca.

Now,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$15^2 = 83 + 2(ab + bc + ca)$

$\Rightarrow 225 = 83 + 2(ab + bc + ca)$

$\Rightarrow 142 = 2(ab + bc + ca)$

$\Rightarrow ab + bc + ca = \frac{142}{2} = 71$

Substituting the value of ab + bc + ca in (i), we get,

$a^3 + b^3 + c^3 - 3abc = 15 \times (83 - 71) = 15 \times 12 = 180$

OR

$p(x) = x^4 - 3x^2 + 2x + 5$

We have,

$$
\begin{array}{r|l}
x-1 & x^4 + 0x^3 - 3x^2 + 2x + 5 \;\big(\, x^3 + x^2 - 2x \\
 & -(x^4 - x^3) \\
\hline
 & \quad x^3 - 3x^2 + 2x + 5 \\
 & \quad -(x^3 - x^2) \\
\hline
 & \qquad -2x^2 + 2x + 5 \\
 & \qquad -(-2x^2 + 2x) \\
\hline
 & \qquad\qquad 5
\end{array}
$$

Clearly, the remainder is 5.

Let us now compute p(1) i.e. value of p(x) when x is replaced by 1.

We have,

$p(1) = 1 - 3 + 2 + 5 = 5$

Thus, we find that the remainder when p(x) is divided by x - 1 is equal to p(1) i.e., the value of p(x) at x = 1.

38. Since only the rounded surface of the dome is to be painted, we would need to find the curved surface area of the hemisphere to know the extent of painting that needs to be done. Now, circumference of the dome = 17.6 m. Therefore, $17.6 = 2\pi r$

$2 \times \frac{22}{7} r = 17.6$ m

So, the radius of the dome = $17.6 \times \frac{7}{2 \times 22}$ m = 2.8 m

The curved surface area of the dome = $2\pi r^2$

= $2 \times \frac{22}{7} \times 2.8 \times 2.8$ m^2

= 49.28 m^2

Now, the cost of painting 100 cm^2 is Rs. 5.

So, the cost of painting 1 m^2 = Rs. 500

Therefore, the cost of painting the whole dome

= Rs. 500 $\times$ 49.28

= Rs. 24640

OR

Let r_1 cm and r_2 cm can be the inner and outer radii respectively of the pipe

Area of the outside surface = $2\pi r_2 h\ sq\ unit$

Area of the inside surface = $2\pi r_1 h\ sq\ unit$

$\therefore$ By the given condition

$2\pi r_2 h - 2\pi r_1 h$ = 44

or $2\pi h\left(r_2 - r_1\right) = 44$

$\therefore\ 2 \times \frac{22}{7} \times 14 \times \left(r_2 - r_1\right)$

$= 44\left(\because h = 14cm\right)$

Or, $88\left(r_2 - r_1\right) = 44$

$\therefore\ \left(r_2 - r_1\right) = \frac{1}{2}\ (i)$

Again volume of the metal used in the pipe $= \pi\left(r_2{}^2 - r_1{}^2\right) hcu\ units$

$\therefore \frac{22}{7}\left(r_2{}^2 - r_1{}^2\right) \times 14 = 99$ (given)

or $44\left(r_2{}^2 - r_1{}^2\right) = \frac{99}{44} = \frac{9}{4}\ (ii)$

Dividing (ii) by (i) we get

$\frac{\left(r_2{}^2 - r_1{}^2\right)}{r_2 - r_1} = \frac{9}{4} \div \frac{1}{2}$

or, $r\frac{\left(r_2 - r_1\right)\left(r_2 + r_1\right)}{\left(r_2 - r_1\right)} = \frac{9}{4} \times \frac{2}{1}$

$\therefore \left(r_2 + r_1\right) = \frac{9}{2}$

Also, $\left(r_2 - r_1\right) = \frac{1}{2}$ [From (i)]

$2r_2 = 5$

Adding

$\therefore\ r_2 = \frac{5}{2}$

And, $\frac{5}{2} + r_1 = \frac{9}{2}$

$\therefore\ r_1 = \frac{9}{2} - \frac{5}{2}$

Or, $r_1 = 2$ Thus outer radius = 2.5 cm

And inner radius = 2 cm

39. i. In $\triangle$AMC and $\triangle$and BMD,

    AM = BM [M is the mid-point of AB]

    $\angle$AMC =$\angle$ BMD [Vertically opposite angles]

    CM = DM [Given]

    $\therefore$ $\triangle$AMC $\cong$ $\triangle$BMD [By SAS congruency]

    $\therefore$ $\angle$ ACM =$\angle$ BDM ...(i)

    ii. For two lines AC and DB and transversal DC, we have,

    $\angle$ACD = $\angle$BDC [Alternate angles]

    $\therefore$ AC $\parallel$ DB

    Now for parallel lines AC and DB and for transversal BC.

    $\angle$ DBC = $\angle$ ACB [Alternate angles] ...(ii)

    ButΔ ABC is a right-angled triangle, right-angled at C.

    $\therefore$ $\angle$ACB = 90° (iii)

    Therefore $\angle$ DBC = 90° [Using eq. (ii) and (iii)]

    $\Rightarrow$$\angle$ DBC is a right angle.

    iii. Now in $\triangle$DBC and $\triangle$ ABC,

    DB = AC [Proved in part (i)]

    $\angle$ DBC = $\angle$ACB = 90° [Proved in part (ii)]

    BC = BC [Common]

    $\therefore$ $\triangle$DBC $\cong$ $\triangle$ACB [By SAS congruency]

    iv. Since $\triangle$DBC $\cong$ $\triangle$ACB [Proved above]

    $\therefore$ DC = AB

    $\Rightarrow$ AM + CM = AB

    $\Rightarrow$ CM + CM = AB [$\because$ DM = CM]

    $\Rightarrow$ 2CM = AB

    $\Rightarrow$ CM = $\frac{1}{2}$ AB

40. Firstly, to find out the measures of central tendency, that is mean, median and mode, we will write the given data in ascending order. 16, 17, 22, 23, 23, 25, 25, 25, 25, 28

    We can represent above data in the following table.

Item (x_i)	**Frequency (f_i)**	**$x_i f_i$**
16	1	16
17	1	17
22	1	22
23	2	46
25	4	100
28	1	28
Total	$N = \sum f_i = 10$	$\sum f_i x_i = 229$

Now, $\sum f_i x_i = 229$ and $\sum f_i = 10$

$\therefore$ Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{229}{10} = 22.9$

Here, n = 10 [which is an even number]

$\therefore$ Median = $\frac{1}{2}$ [Value of $(\frac{n}{2})^{th}$ term + Value of $(\frac{n}{2} + 1)^{th}$ term]

= $\frac{1}{2}$ [Value of 5^{th} term + Value of 6^{th} term]

= $\frac{1}{2}$ (23 + 25) = 24

Mode = value of variable which occurs most frequently = 25.

Thus, the value of mean for the given data is 22.9,

The value of median for the given data is 24

And, the value of mode for the given data is 25.