EXERCISE: 13.4 (PROBABILITY)

Q.No 1 State which of the following are not probability distributions of a random variable. Give reason for your answer:

(i)

X	0	1	2
P(X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	−0.1	0.3

(iii)

X	−1	0	1
P(X)	0.6	0.1	0.2

(iv)

X	3	2	1	0	−1
P(X)	0.3	0.2	0.4	0.1	0.05

Sol. Note that for Prob. distribution of random variable

$0 \le P(X) \le 1$ and $\Sigma P(X) = 1$

(i) It is prob distribution of a random variable as

$0 \le P(X) \le 1$ and $\Sigma P(X) = 1$

(ii) It is not Prob. dist. of random variable as $P(3) = -0.1 < 1$ which is not possible.

(iii) It is not Prob. dist of random variable as

$\Sigma P(X) = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$

(iv) It is not a Prob. dist. of random variable as

$\Sigma P(X) = 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 \neq 1$

Q.No 2 An urn contains 5 red and 2 black balls. Two balls are randomly selected. Let X represent the no. of black balls. What are possible value of X. Is X a random variable.

Soln. X can take values 0, 1 and 2. Yes X is random variable.

QNo.3 Let X represents the difference between the no. of heads and no. of tailes obtained when a coin is tossed 6 times. What are possible values of X.

Sol. Let 'h' denotes the no. of heads and 't' denotes the no. of tails when a coin is tossed 6 times. Then $X = |h-t|$ (Difference)

Now consider the table.

h	0	1	2	3	4	5	6
t	6	5	4	3	2	1	0
X	6	4	2	0	2	4	6

Thus X can take values 0, 2, 4, 6.

QNo4 Find the prob. dist. of
(i) No. of heads in two tosses of coin.
(ii) No. of tails in simultaneous tosses of 3 coins.
(iii) No of heads in 4 tosses of coin.

Sol. (i) When a coin is tossed twice, then the sample space is $S = \{HH, HT, TH, TT\}$

Let X denote the number of heads in any outcome.

Then $X(HH) = 2$ $X(HT) = 1$ $X(TH) = 1$ $X(TT) = 0$

$\therefore P(X=0) = P(TT) = 1/4$

$P(X=1) = P(TH, HT) = 2/4 = 1/2$

$P(X=2) = P(HH) = 1/4$.

$\therefore$ Prob. dist of x is

X	0	1	2
$P(X)$	$1/4$	$1/2$	$1/4$

(ii) When a coin is tossed thrice, sample space is
S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}
Let X denote no. of tails in any outcome of S.
Then X can take values 0, 1, 2, 3.
$P(X=0) = P(\text{No tail}) = P(HHH) = 1/8$
$P(X=1) = P(\text{one tail two heads}) = P(\{HHT, HTH, THH\}) = 3/8$
$P(X=2) = P(\text{Two tails and one head}) = 3/8$
$P(X=3) = P(\text{3 tails}) = 1/8.$

∴ Prob. dist of X =

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

(iii) When coin is tossed 4 times, sample space is
S = {HHHH, HHHT, HHTH, HTHH, THHH, TTHH, HTHT, HTTH, THTH, THHT, HHTT, HTTT, THTT, TTHT, TTTH, TTTT}
Let X denote No. of head appears, then X can take values 0, 1, 2, 3 and 4 and.
$P(X=0) = P(\text{No head}) = 1/16$
$P(X=1) = P(\text{one head}) = 4/16$
$P(X=2) = P(\text{Two heads}) = 6/16$
$P(X=3) = P(\text{Three heads}) = 4/16$
$P(X=4) = P(\text{Four heads}) = 1/16$

∴ Probability dist of X =

X	0	1	2	3	4
P(X)	1/16	4/16	6/16	4/16	1/16

QNo5 Find Prob. dist. of Number of successes in two tosses of a die where a success is defined as
(i) A number greater that 4
(ii) Six appears on atleast one die

Soln. (i) When a die is tossed once,
the probability of throwing a number greater than

4 is $2/6 = 1/3$

$\therefore$ P(Success) = $\frac{1}{3}$

$\therefore$ P(Failure) = $1 - 1/3 = 2/3$

Here X takes values 0, 1, 2 as we have to throw die twice.

$\therefore P(X=0) = P(\underset{\text{twice}}{\text{Failure}}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$P(X=1) - P(SF, FS) = \frac{1}{3}\times\frac{2}{3} + \frac{2}{3}\times\frac{1}{3} = \frac{4}{9}$

$P(X=2) = P(SS) = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$

So. Prob. dist. of X is

X	0	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) In this case either will get 'No Six' or Six will appear on atleast one dice. Hence, either there is no success or ~~at~~ there is one success.

ie X take values 0 and 1.

Now P(getting six) = $\frac{1}{6}$

P(Not getting six) = $1 - 1/6 = 5/6$

$\therefore P(X=0) = P(\text{No six}) = P(\text{Not six}) \times P(\text{Not six})$

$= \frac{5}{6}\times\frac{5}{6} = \frac{25}{36}$

$P(X=0) = P(\text{six and Not six or Not six and six or six and six})$

$= \frac{1}{6}\times\frac{5}{6} + \frac{5}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} = 11/36$

Or $P(X=0) = P(\text{atleast one six}) = 1 - P(\text{No six})$

$= 1 - \frac{5}{6}\times\frac{5}{6} = \frac{11}{36}$

$\therefore$ P(X) =

X	0	1
P(X)	$\frac{25}{36}$	$\frac{11}{36}$

Q No 6. From a lot of 30 bulbs, which include 6 defective bulbs an sample of 4 bulbs is drawn at random with replacement. find probability distn. of No. of defective bulbs.

Sol. Total bulbs = 30, Defective bulbs = 6

$\therefore$ P(Drawing defective bulb) $= \frac{6}{30} = \frac{1}{5}$

$\therefore$ P(Drawing Good bulb) $= 1 - \frac{1}{5} = \frac{4}{5}$

Let x denote No. of defective bulbs, then x can take values 0, 1, 2, 3, 4

$\therefore P(X=0) = P(\text{No defective bulb}) = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{256}{625}$

$P(x=1) = P(\text{one defective bulb in 4 drawings})$

$= {}^4C_1 \times \left(\frac{1}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}\right) = \frac{256}{625}$

$P(X=2) = P(\text{Two defective bulbs in 4 drawings})$

$= {}^4C_2 \times \left(\frac{1}{5} \times \frac{1}{5} \times \frac{4}{5} \times \frac{4}{5}\right) = \frac{4 \times 3}{2 \times 1} \times \frac{16}{625} = \frac{96}{625}$

$P(X=3) = P(\text{3 defective bulb in 4 drawings})$

$= {}^4C_3 \times \left(\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{4}{5}\right) = \frac{4 \times 3 \times 2}{3 \times 2 \times 1} \times \frac{4}{625} = \frac{16}{625}$

$P(x=4) = P(\text{4 defective bulbs in 4 drawings})$

$= \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{625}$

$\therefore$ Prob. dist. is

X	0	1	2	3	4
P(x)	256/625	256/625	96/625	16/625	1/625

Q.No 7. A coin is biased so that head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

Sol. ATQ $P(H) = 3(P(T))$; H for head, T for tail

Now $P(H) + P(T) = 1$

$3P(T) + P(T) = 1$

$\Rightarrow P(T) = \frac{1}{4}$

$\therefore P(H) = \frac{3}{4}$.

When coin is tossed twice let X denotes number of tails. The X takes values 0, 1, 2

$P(X=0) = (\text{No tail and No tail}) = P(H)\,P(H) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

$P(X=1) = P(HT \text{ or } TH) = \frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} = \frac{6}{16}$

$P(X=2) = P(TT) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

$\therefore$ Reqd. Prob. dist. is

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

Q.No 8 A random variable X has following Prob. distribution.

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Determine (i) k (ii) $P(X<3)$ (iii) $P(X>7)$ (iv) $P(0<x<3)$

Sol. (i)

Now $\sum P(X) = 1$

$\Rightarrow 0+k+2k+3k+k^2+2k^2+7k^2+k = 1$

$\Rightarrow 10k^2 - 9k - 1 = 0$

$\Rightarrow k = \frac{9 \pm \sqrt{81+40}}{20} = \frac{9 \pm 11}{20} = \frac{1}{10}, -1$

Since $P(1) = K$ and $K \geqslant 0$

$\therefore$ $\therefore K \neq -1$

$\therefore$ $K = \frac{1}{10}$

(ii) $P(X<3) = P(0, 1 \text{ or } 2) = P(0) + P(1) + P(2)$

Now $k = \frac{1}{10}$

$\therefore P(X<3) = 0 + \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$

(iii) $P(X>6) = P(7) = 7K^2 + K$

$= 7\left(\frac{1}{100}\right) + k = \frac{7}{100} + \frac{1}{10} = \frac{17}{100}$

(iv) $P(0<X<3) = P(1 \text{ or } 2) = P(1) + P(2)$

$= K + 2K = 3K = 3 \times \frac{1}{10} = \frac{3}{10}$

Q.No. 9 The random variable X has probability distribution P(X) of the following form, where k is some number

$$P(X) = \begin{cases} k & ; x = 0 \\ 2k & ; x = 1 \\ 3k & ; x = 2 \\ 0 & ; \text{otherwise} \end{cases}$$

(a) Determine the value of k

b) Find $P(X<2)$, $P(X \leq 2)$, $P(X \geq 2)$

Sol.

(a) Since $\sum P(X) = 1$

$\therefore$ $K + 2K + 3K + 0 = 1$

$\Rightarrow$ $6K = 1 \Rightarrow K = \frac{1}{6}$

(b) $P(X<2) = P(0) + P(1)$

$= K + 2K = 3K = 3 \times \frac{1}{6} = \frac{1}{2}$

$P(X \leq 2) = P(0) + P(1) + P(2)$

$= K + 2K + 5K = 6K = 1 \times \frac{1}{6} = 1$

$P(X \geq 2) = P(2) = 3K = 3 \times \frac{1}{6} = \frac{1}{2}$

Q.No. 10 Find Mean Number of heads in three tosses of fair coin.

Sol Let X denote the number of heads obtained in three tosses of a coin.

Then X can take value 0, 1, 2, 3

Now $P(\text{getting head}) = \frac{1}{2} = p(\text{say})$

and $P(\text{not getting head}) = \frac{1}{2} = q(\text{say})$

$\therefore P(X=0) = q \times q \times q = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$P(X=1) = pqq + qpq + qqp = 3\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{3}{8}$

$P(X=2) = 3(ppq) = 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$

$P(X=3) = p \times p \times p = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$\therefore$

X or x_i	0	1	2	3
$P(X)$ or p_i	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\therefore$ Mean $\mu = \sum_{i=1}^{n} x_i p_i = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$

$= \frac{3+6+3}{8} = \frac{12}{8} = \frac{3}{2}$

Q.No. 11 Two dice are thrown simultaneously. If X denote the number of sixes find expectation of X

Sol. Let X denote no. of sixes in throwing two dice simul.

Here X can take values 0, 1, 2.

Now $P(\text{getting six}) = 1/6$

$P(\text{Not getting six}) = 1 - \frac{1}{6} = \frac{5}{6}$

$\therefore P(X=0) = P(\text{Not getting six on both dice})$

$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$P(X=0) = P(\text{six on first die and not on other or six on 2nd. die and not on first})$

$= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{10}{36}$

$P(X=2) = P(\text{six on both dice})$

$= \frac{1}{6} \times \frac{1}{6} = 1/36$

Hence prob. dist. is

X	0	1	2
P(x)	25/36	10/36	1/36

$\therefore$ Expectation of X = Mean of X = $\mu = \sum x P(x)$

$= 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36} = \frac{12}{36} = \frac{1}{3}$

QNo12 Two numbers are selected at random (without replacement) from first six +ve numbers integers. Let x denotes the lagest of two numbers obtained. find $E(X)$

Sol: Since two no. are selected without replacement.
$\therefore$ Sample space has $6 \times 5 = 30$ equally likely sample points.
and $S = \{(x,y);\ x,y \in \{1,2,3,4,5,6\},\ x \neq y\}$

Since X = largest of (x,y)
$\therefore X$ will take values $2,3,4,5,6$

Now $P(X=2) = P(\text{larger of two selected integers is } 2)$

$= P\{(1,2)(2,1)\} = \frac{2}{30} = \frac{1}{15}$

$P(X=3) = P\{(1,3)(2,3)(3,2)(3,1)\} = 4/30 = \frac{2}{15}$

$P(X=4) = P\{(1,4)(2,4)(3,4),(4,3)(4,2)(4,1)\} = \frac{6}{30} = \frac{3}{15}$

$P(X=5) = P\{(1,5)(2,5)(3,5)(4,5)(5,4)(5,3)(5,2)(5,1)\} = \frac{8}{30} = \frac{4}{15}$

$P(X=6) = P\{(1,6)(2,6)(3,6)(4,6)(5,6)(6,5)(6,4)(6,3)(6,2)(6,1)\} = \frac{10}{30} = \frac{5}{15}$

$\therefore$ Prob. dist. is.

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

$\therefore$ Expectation $E(X) = \mu = \sum x P(x)$

$$= 2\times\frac{1}{15} + 3\times\frac{2}{15} + 4\times\frac{3}{15} + 5\times\frac{4}{15} + 6\times\frac{5}{15}$$

$$= \frac{2+6+12+20+30}{15} = \frac{70}{15} = \frac{14}{3}$$

Q.No.13 Let X denote the sum of Numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X

Sol. When a pair of fair dice is rolled once the sample space is $S = \{(x,y);\ x,y \in \{1,2,3,4,5,6\}\}$ which contains 36 sample pts.

Let here $X = x+y$

$\therefore$ X can take values 2 to 12

$\therefore P(X=2) = P((1,1)) = 1/36$

$P(X=3) = P\{(1,2)(2,1)\} = 2/36$

$P(X=4) = P\{(1,3)(2,2)(3,1)\} = 3/36$

$P(X=5) = P\{(1,4)(2,3)(3,2)(4,1)\} = 4/36$

$P(X=6) = P\{(1,5)(2,4)(3,3),(4,2)(5,1)\} = 5/36$

$P(X=7) = P\{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\} = 6/36$

$P(X=8) = P\{(2,6)(3,5)(4,4)(5,3)(6,2)\} = 5/36$

$P(X=9) = P\{(3,6)(4,5)(5,4)(6,3)\} = 4/36$

$P(X=10) = P\{(4,6)(5,5)(6,4)\} = 3/36$

$P(X=11) = P\{(5,6)(6,5)\} = 2/36$

$P(X=12) = P\{(6,6)\} = 1/36$

$\therefore$ Prob. dist =

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$\therefore$ Mean $\mu = E(X) = \sum x P(x)$

$$= 2\times\frac{1}{36} + 3\times\frac{2}{36} + 4\times\frac{3}{36} + 5\times\frac{4}{36} + 6\times\frac{5}{36} + 7\times\frac{6}{36} + 8\times\frac{5}{36} + 9\times\frac{4}{36} + 10\times\frac{3}{36} + 11\times\frac{2}{36} + 12\times\frac{1}{36}$$

$$= \frac{2+6+12+20+30+42+40+36+30+22+12}{36} = \frac{252}{36} = 7.$$

$$\text{Variance} = \sum x^2 P(x) - \mu^2$$

$$= \frac{2^2\times1+3^2\times2+4^2\times3+5^2\times4+6^2\times4+7^2\times6+8^2\times5+9^2\times4+10^2\times3+11^2\times2+12^2\times1}{36} - (7)^2$$

$$= \frac{1974}{36} - 49 = \frac{1974-1764}{36} = \frac{210}{36} = \frac{35}{6}$$

Standard deviation $SD = \sqrt{\text{Variance}} = \sqrt{\frac{35}{6}} = \frac{1}{6}\sqrt{210}$

Q No. 14 A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 yrs. One student is selected in such a manner that each has the same chance of being chosen and age X of selected student is noted. What is prob. dist. of X. Find mean, variance and S.D of X.

Sol. We construct the table

X (Age)	No. of students	P(X)	X P(X)	X^2 P(X)
14	2	2/15	28/15	392/15
15	1	1/15	15/15	225/15
16	2	2/15	32/15	512/15
17	3	3/15	51/15	867/15
18	1	1/15	18/15	324/15
19	2	2/15	38/15	722/15
20	3	3/15	60/15	1200/15
21	1	1/15	21/15	441/15
	15		$\sum xp(x) = \frac{263}{15}$	$\sum x^2 p(x) = \frac{4683}{15}$

$\therefore$ Mean $\mu = \sum x P(x) = \frac{263}{15}$

$$\text{Variance} = \sum x^2 P(x) - (\mu)^2 = \frac{4683}{15} - \left(\frac{263}{15}\right)^2 = 312.2 - 307.4 = 4.8$$

$$S.D = \sqrt{\text{Variance}} = \sqrt{4.8} = 2.19 \text{ (approx)}$$

Q.No15 In a meeting 70% of members favours and 30% oppose a certain proposal. A member is selected at random and we take $X=0$ if he opposed and $X=1$ if he favours. find $E(X)$ and $Var(X)$.

Sol: Here X taken the values 0 and 1

and $P(X=0) = \frac{30}{100}$

$P(X=1) = \frac{70}{100}$

∴ Prob. dist is

X	0	1
P(X)	$\frac{30}{100}$	$\frac{70}{100}$

∴ Mean $X = \mu = \sum xP(x) = 0 \times \frac{30}{100} + 1 \times \frac{70}{100} = \frac{70}{100} = 0.7$.

$$\text{Variance} = \sum x^2 P(x) - \left(\sum x P(x)\right)^2$$

$$= \left(0^2 \times \frac{30}{100} + 1^2 \times \frac{70}{100}\right) - (0.7)^2$$

$$= \frac{70}{100} - \frac{49}{100} = \frac{21}{100} = 0.21$$

Q.No16. The mean of the numbers obtained on throwing one die having written 1 on three faces, 2 on two faces and 5 on 1 face is (A) 1 (B) 2 (C) 5 (D) 8/3.

Sol. Let X denote the no. of die, then X take the values 1, 2 and 5.

$P(X=1) = 3/6 = 1/2$

$P(X=2) = 2/6 = 1/3$

$P(X=5) = 1/6$

· Prob. dist is

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$\therefore$ Mean $\mu = \sum x P(x) = \frac{1}{2} + \frac{2}{3} + \frac{5}{6} = \frac{3+4+5}{6} = \frac{12}{6} = 2$

$\therefore$ Correct option is (B)

Q No. 17 Suppose that two Cards are drawn at random from a deck of Cards. Suppose X be the no. of Aces obtained. Find E(X)

(A) $\frac{37}{221}$ (B) $\frac{5}{13}$ (C) $\frac{1}{13}$ (D) $\frac{2}{13}$

Sol.. Here X takes values 0, 1, 2.

$P(X=0) = \frac{48}{52} \times \frac{47}{51} = \frac{188}{221}$

$P(X=1) = 2 \times \frac{4}{52} \times \frac{48}{51} = \frac{32}{221}$

$P(X=2) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

$\therefore$ Prob. dist. is

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$\therefore E(X) = \sum X P(X) = 0 + \frac{32}{221} + \frac{2}{221} = \frac{34}{221} = \frac{2}{13}$

$\therefore$ (D) is correct option.

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