Ohm's law and Continuity Equation:

$$\int (\nabla \cdot J)\, dv = \oint J \cdot ds = I = \frac{dQ}{dt} = \frac{d}{dt}\int \rho_v \, dv = \int \frac{\partial \rho_v}{dt}\, dv$$

i.e. $$I = \frac{dQ}{dt} = \frac{d}{dt}\int \rho_v \, dv = \int \frac{\partial \rho_v}{dt}\, dv$$

$$\Rightarrow \quad I = \oint J \cdot ds = \int (\nabla \cdot J)\, dv$$

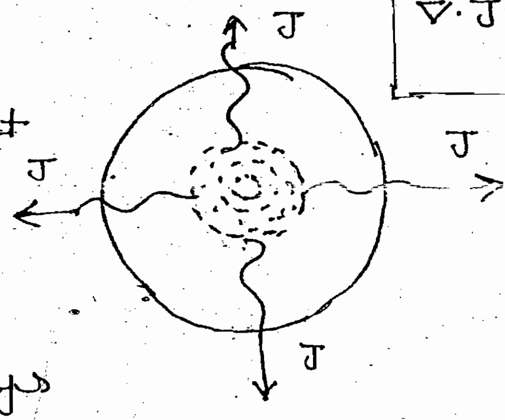
Divergence theorem

$$\boxed{\nabla \cdot J = \frac{\partial \rho_v}{\partial t}}$$

→ current has outflow and divergence at a rate depending on decrease of volume charges with time $\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$

→ Practically

Entering current = Leaving current



charge can neither be created nor be destroyed

$$\left.\begin{array}{l}\oint J \cdot ds = 0 \\ \nabla \cdot J = 0\end{array}\right] \longrightarrow \text{Always}$$

→ For a uni-directional current flow

$$\frac{\partial J}{\partial x} = \frac{\partial \rho_v}{\partial t}$$

$$\partial J = \partial \rho_v \frac{\partial x}{\partial t}$$

Integrating on both sides

$$J = \rho_v V_d$$

where $V_d = \frac{\partial x}{\partial t}$ = drift velocity

V_f = free velocity = $\sqrt{\frac{2qV}{m}}$

= vacuum tubes, CRO

= few $10^6 - 10^7$ m/s

$$\boxed{V_f \propto \sqrt{E}}$$

where $V_d = \frac{\partial x}{\partial t}$ = drift velocity

= charge in a material (bulk)

= few cm/sec

$V_d \propto E \Rightarrow \boxed{V_d = \mu E}$

μ → mobility of the particle

$J = \rho_v v_d$

$J = \rho_v \mu E$

$\Rightarrow \boxed{J = \sigma E}$ → Ohm's law in point form.

where σ = conductivity (mho/m) = $\rho_v \mu$

= ability to allow current

= available free carriers × mobility of carriers

Case-(1) :-

$\sigma = \infty$ → very good conductor

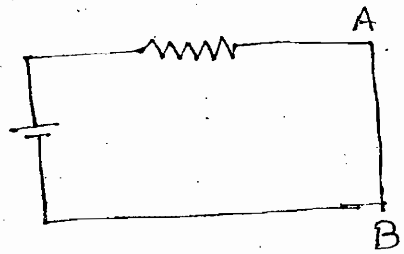
$\frac{J}{\sigma} = E = 0$

→ Electric field cannot exist in a very good conductor

→ $V = \int 0 \cdot dl$ = constant ;

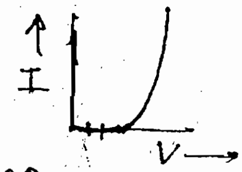
conductor is always a equi-potential region

→ Potential difference or voltage cannot exist in a very good conductor only current can flow but voltage difference good conductor cannot exist



$V_{AB} = 0$ but current flows

$V \uparrow$ $V_{AB} = 0$ $I_{AB} \uparrow$



eg:-(1) diode current after cuting voltage

(II) Accumulation — does not exist — only flow exists

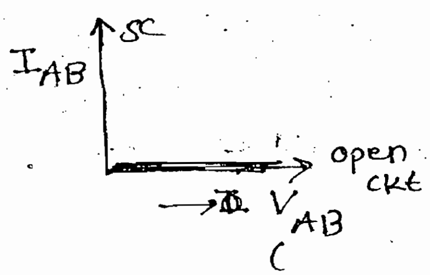
case-(II) :-

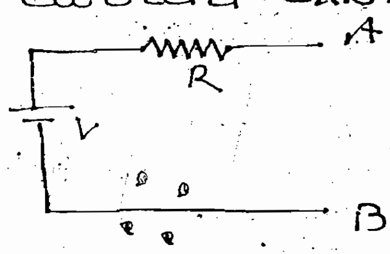
$\sigma = 0$ very good dielectric

$J = 0$

→ current cannot exist in a very good dielect

→ Voltage or potential difference or Electric field can exist but flow current exist





→ Accumulation can exist but flow can't exist.

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

$$J = \sigma E$$

$$\nabla \cdot (\sigma E) = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \sigma (\nabla \cdot E) = -\frac{\partial \rho_v}{\partial t}$$

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• The ρ_v is time solution is

$$\boxed{\rho_v(t) = \rho_{v_0} \cdot e^{-\frac{t\sigma}{\epsilon}}}$$

Note:-

Every charge density exponentially spread on the medium at a rate depending on $\frac{\epsilon}{\sigma}$ that called the relaxation time

$$\frac{\epsilon}{\sigma} = \text{Relaxation time} = \frac{\text{Farad}}{m \times \frac{\text{mho}}{m}}$$

$$= \text{ohms} \times \text{farad}$$

$$= \text{second}$$

Boundary Conditions at conductor surfaces:-

(1) $E_{t_1} = E_{t_2}$ (General)

As $\sigma = \infty$ along the conductor

$\frac{J}{\sigma} = E = 0$ along the surface

$\Rightarrow \boxed{E_{tang.} = 0}$

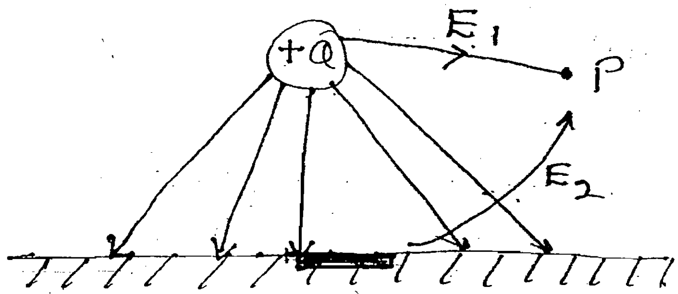
(11) $D_{n_2} - D_{n_1} = \rho_s$ (General)

Electric field can be normal to a conductor which depends on ρ_s on the surface

$$\boxed{D_{normal} = \rho_s}$$

Conductors, Induction, Method of Images:-

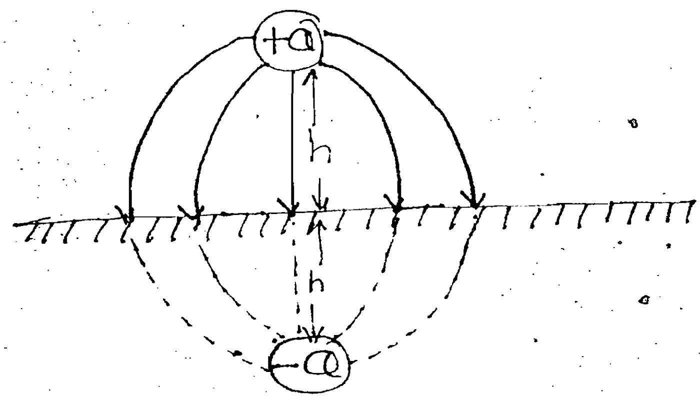
→ When a charge is placed near a conductor surface the field is as shown below



→ The field has tangential component which displaces the free carrier. Hence they are periodically accumulated all ~~the other~~ along the conductor surface. This is called as induced charge

→ The resultant field at any point is the vector sum of field due to actual charge and induced charge

→ This field is such that the tangential component are removed and has only normal components as shown below



→ The field appears to be a dipole field with negative charge called as image charge below the conductor surface

Summary:—

Every charge and its induction effects are represented by a image charge below the conductor obeying all rules of light and optics

44.

$$D_1 = 2(a_x - \sqrt{3}a_z)\ C/m^2 = \rho_s$$

$$\rho_s = |D_n| = 2\sqrt{1+3} = 4$$

45.

$$E_n = 2\ V/m$$

$$D_n = 80\epsilon_0 E_n = \rho_s$$

$$\Rightarrow \rho_s = 80 \times 8.8 \times 10^{-12} \times 2$$

$$= 1.41 \times 10^{-9}\ C/m^2$$

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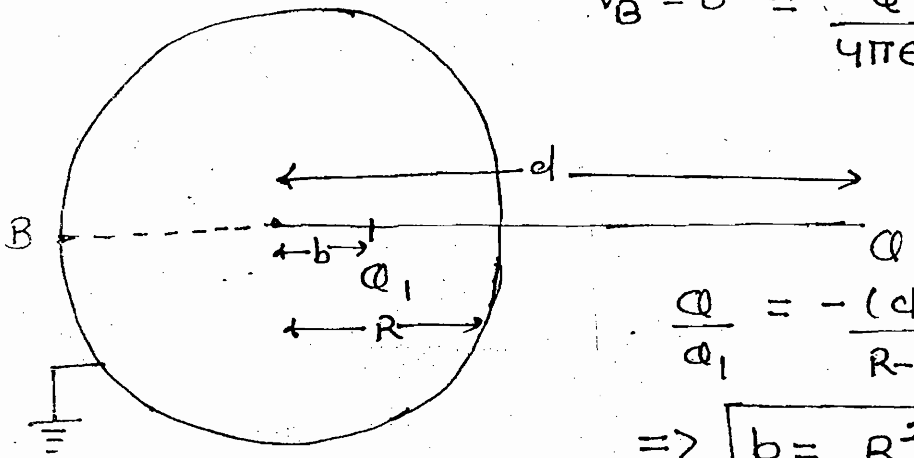
6.

$$V_A = 0 = \frac{Q}{4\pi\epsilon(d-R)} + \frac{Q_1}{4\pi\epsilon(R-b)}$$

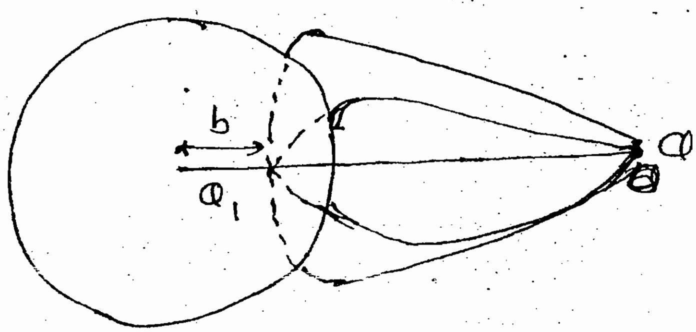
$$V_B = 0 = \frac{Q}{4\pi\epsilon(d+R)} + \frac{Q_1}{4\pi\epsilon(R+b)}$$



$$\frac{Q}{Q_1} = -\frac{(d-R)}{R-b} = -\frac{(d+R)}{(R+b)}$$

$$\Rightarrow \boxed{b = \frac{R^2}{d}} \quad \& \quad \boxed{Q_1 = -\frac{R}{d}}$$

Note:-



7. 4μC, Ans

Energy Density in Electric fields $\left(\frac{dW_E}{dV} = \frac{1}{2}\epsilon E^2\right)$:-

Consider Q_1, Q_2, Q_3 ------ Q_n point charges assemble in a region

Total energy of the Electric field = Total Energy expended in assembling the charges

$W_1 = 0$

$W_2 = -Q_2 V_{21}$

V_{21} = Potential at 2nd due to 1st charge

$W_3 = -Q_3 V_{31} - Q_3 V_{32}$

$W_4 = -Q_4 V_{41} - Q_4 V_{43}$

⋮

$W_n = -Q_n V_{n1} - Q_n V_{n2} - \cdots Q_n V_{n\,n-1}$

$$Q_2 V_{21} = Q_2 \cdot \frac{Q_1}{4\pi\epsilon r_{21}} = Q_1 V_{12}$$

~~Substitute~~ subscripts can be interchange without change in meaning of value

$$W_1 = 0$$

$$W_2 = -Q_1 V_{12}$$

$$W_3 = -Q_1 V_{13} - Q_2 V_{23}$$

$$W_4 = -Q_1 V_{14} - Q_2 V_{24} - Q_3 V_{34}$$

$$\vdots$$

$$W_n = -Q_1 V_{1n} - Q_2 V_{2n} \cdots - Q_{n-1} V_{n-1 \cdot n}$$

$$W_E = W_1 + W_2 + W_3 + \cdots W_n$$

total energy

$$2W_E = -Q_1 V_1 - Q_2 V_2 - Q_3 V_3 \cdots\cdots Q_n V_n$$

$$\Rightarrow \quad \boxed{W_E = \frac{-1}{2} \sum_{i=1}^{n} Q_i V_i}$$

For a continous charge distribution

$$W_E = \frac{-1}{2} \int \rho_v V dv = \frac{-1}{2} \int (\nabla \cdot D) V dv = \int \frac{1}{2} D(-\nabla V) dv$$

$$\Rightarrow \quad W_E = \int \frac{1}{2} (D \cdot E) dv$$

$$\frac{dW_E}{dv} = \text{Energy density}$$

= Strength of energy at any point

$$= \frac{1}{2} (D \cdot E) = \frac{1}{2} \epsilon E^2$$

Extension :—

$$\frac{dW_H}{dv} = \text{Magnetic Energy density}$$

$$= \frac{1}{2} B \cdot H = \frac{1}{2} \mu H^2$$

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Capacitors and Inductors :-

→ Capacitance is the ability to confine Electric field in a finitely small region:

$$C = \text{Farad} = \frac{\oint D \cdot S}{\int E \cdot dl} = \frac{\epsilon \oint E \cdot ds}{\int E \cdot dl} = \frac{Q}{V}$$

→ It is the ratio with charge utilized to the potential developed by the charge.

Specific Geometries :-

→ Parallel Plates

→ cocentric cylinders

→ Cocentric sphere

Inductance :-

Inductance is the ability to confine H field in a finitely small region

$$L = \text{Henry} = \frac{\int B \cdot ds}{\oint H \cdot dl} = \frac{\mu \int H \cdot ds}{\oint H \cdot dl} = \frac{\Psi_m}{I}$$

→ It is the ratio of the flux developed to the current utilized by the flux

Specific Geometries :-

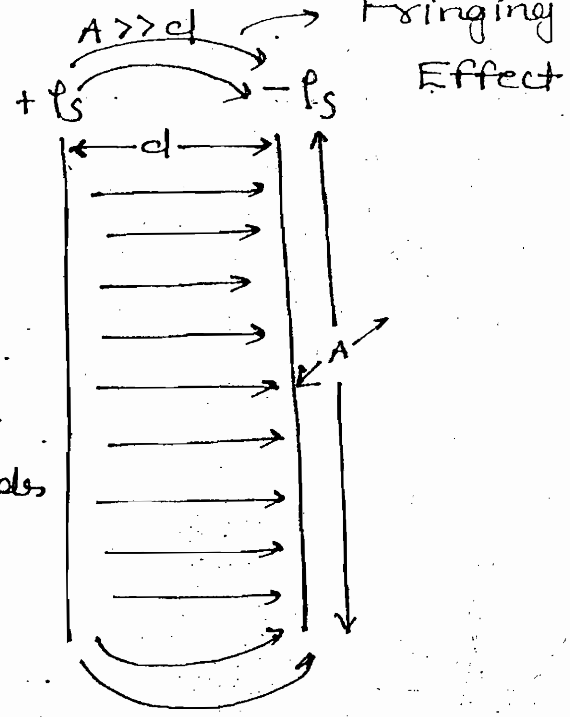
→ Solenoids

→ cocentric cylinder

→ Torroids.

Parallel Plate Capacitors :-

Parallel Plate Capacitors :-



$$E = \frac{\rho_s}{\epsilon}$$

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s d}{\epsilon}} = \frac{\cancel{\epsilon A}}{} = \frac{\epsilon A}{d}$$

Capacitance is independent of Q or V and always depends on Area and distance or physical dimensions

$$W_E = \frac{1}{2}\epsilon E^2 (Ad)$$

$$= \frac{1}{2}\frac{\epsilon A}{d}(Ed)^2$$

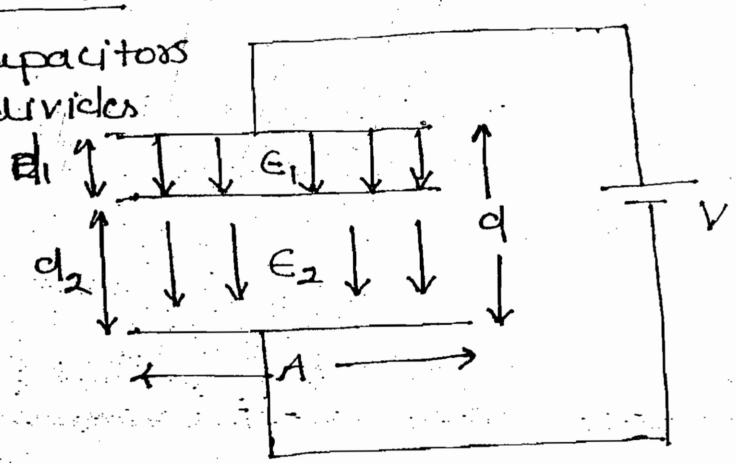
$$= \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

Multiple Dielectrics in Capacitors :-

Case-(1) :-

Equal areas of cross-section unequal width of the dielectrics :-

They are two capacitors in series as voltage divides



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\frac{\epsilon_1 A}{d_1} \cdot \frac{\epsilon_2 A}{d_2}}{\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}}$$

$$D_1 = D_2 \Rightarrow \epsilon_1 E_1 = \epsilon_2 E_2 \Rightarrow \frac{\epsilon_1 V_1}{d_1} = \frac{\epsilon_2 V_2}{d_2}$$

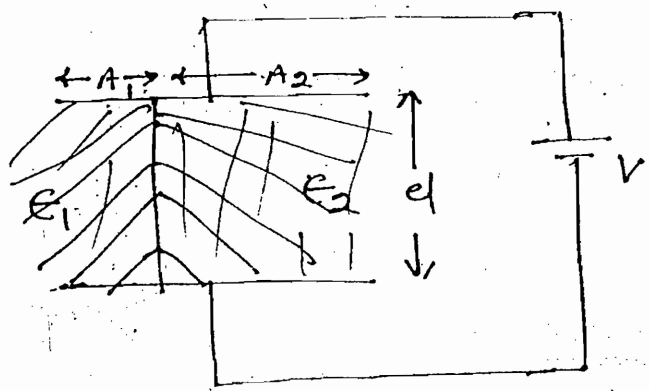
$$\Rightarrow \boxed{\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)\left(\frac{d_1}{d_2}\right)}$$

Case-(II.)

Unequal areas of cross-section and equal width of the dielectrics :-

$$C_{eq} = C_1 + C_2$$

$$= \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{d}$$



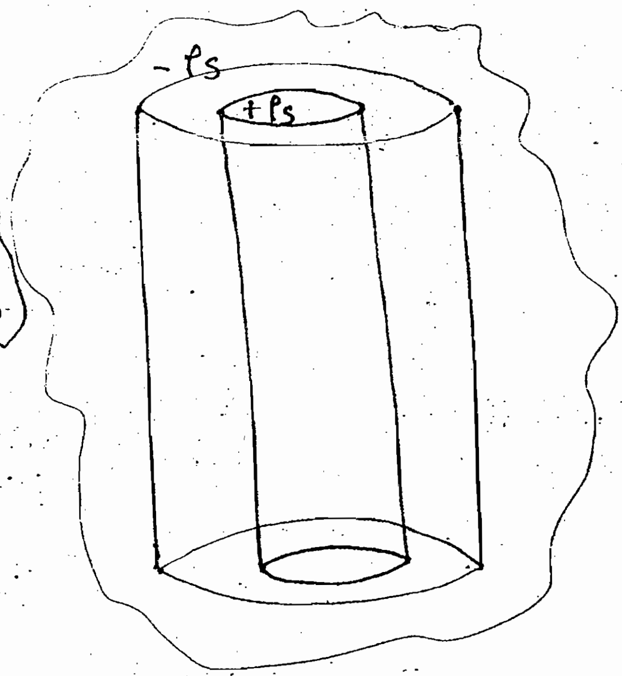
Cocentric Cylinder Capacitance :-

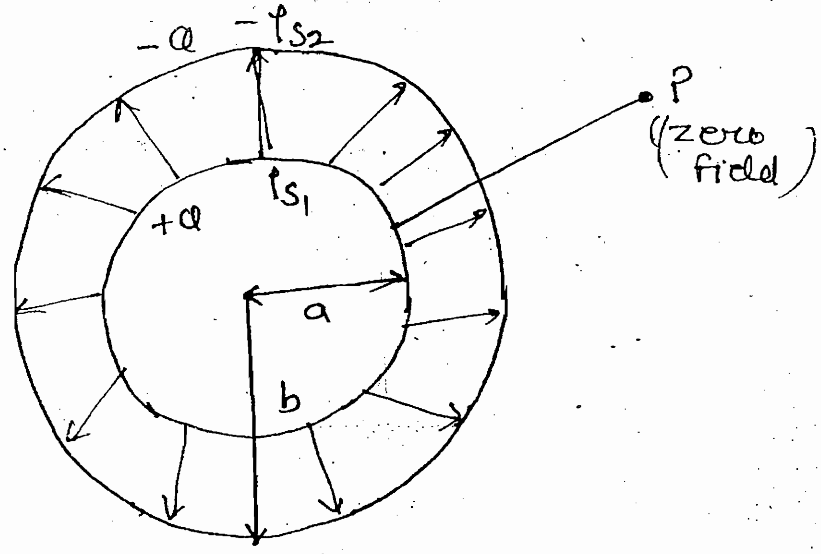
Two cocentric cylinders with equal and opposite charge density may not have confined flux but two equal and opposite charges on concentric cylinders has flux confined only b/w the cylinders

$$Q_{net} = Q_{out} + Q_{in}$$

$$= -\rho_s 2\pi bh + \rho_s 2\pi ah$$

$$\neq 0$$





$D \propto \rho_S \propto \frac{1}{\rho}$

→ The field of a sheet of cylindrical charge obeys same geometry as a line charge

$$C = \frac{Q}{V} = \frac{\rho_L h}{\frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow \boxed{C = \frac{2\pi\epsilon h}{\ln(b/a)}}$$

Extension:-

Cocentric sphere capacitance

$$C = \frac{Q}{V}$$

$$\boxed{C = \frac{Q}{\frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\epsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}}$$

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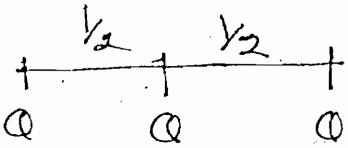
$$W_{E1} = 0 + Q\cdot\frac{Q}{4\pi\epsilon \, l/2} + Q\cdot\frac{Q}{4\pi\epsilon \, l/2} + Q\cdot\frac{Q}{4\pi\epsilon\cdot l} = \frac{5Q^2}{4\pi\epsilon}$$



$$\boxed{W_{E_2} = \frac{5Q^2}{8\pi\epsilon} = \frac{W_{E1}}{2}}$$

Note :-

$$W_E \propto QV \propto \frac{1}{r}$$

31. $$\frac{1}{2}(J\cdot A) = \frac{\text{Amp}}{m^2} \times \frac{\text{Joules}}{\text{Amp-m}} = \frac{\text{Joules}}{m^3}$$

OR

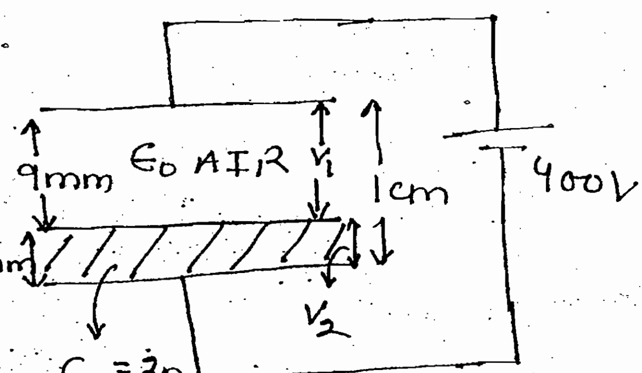
$$\Rightarrow \frac{1}{2}(J\cdot A) = \frac{1}{2}((\nabla\times H)\cdot A) = \frac{1}{2}(H(\nabla\times A))$$

$$= \frac{1}{2} B\cdot H$$

32. $$E_{AIR} = \frac{V_{AIR}}{9mm}$$

$$V_1 + V_2 = 400 \quad \text{—(I)}$$

$$\frac{V_1}{V_2} = \left(\frac{\epsilon_2}{\epsilon_1}\right)\left(\frac{d_1}{d_2}\right) \quad \text{—(II)}$$



$$\frac{V_1}{V_2} = \frac{20\epsilon_0}{\epsilon_0} \times \frac{9mm}{1mm} = 180$$

$$V_1 = 180V_2$$

Ans → V_1 = 44 kV/m

4. Circulation $= \oint H\cdot dl = I$

$= I + I + I - (-I$

$= 4I$, Ans.