

CONTENTS

- Perimeter and Area of a Circle
- Area enclosed by two Concentric Circles
- Sector of a Circle
- Area of a Sector
- Segment of a Circle & its Area
- Areas of Combination of Plane Figures

➤ PERIMETER AND AREA OF A CIRCLE

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains same.

The fixed point is called the centre and the given constant distance is known as the radius of the circle.

The perimeter of a circle is known as its circumference.

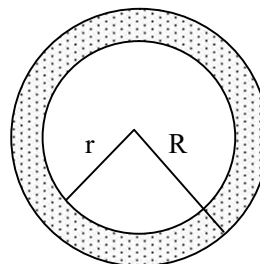
If r is the radius of a circle, then

- (i) Circumference = $2\pi r$ or πd , where $d = 2r$ is the diameter of the circle.
- (ii) Area = πr^2 or $\pi d^2/4$
- (iii) Area of semi-circle = $\frac{\pi r^2}{2}$
- (iv) Area of a quadrant of a circle = $\frac{\pi r^2}{4}$

➤ AREA ENCLOSED BY TWO CONCENTRIC CIRCLES

If R and r are radii of two con-centric circles, then area enclosed by the two circles

$$= \pi R^2 - \pi r^2 = \pi (R^2 - r^2) = \pi (R + r) (R - r)$$



Some useful results :

- (i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.
- (ii) If two circles touch externally, then the distance between their centres is equal to the sum of their radii.
- (iii) Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.
- (iv) The number of revolutions completed by a rotating wheel in one minute

$$= \frac{\text{Distance moved in one minute}}{\text{Circumference}}$$

❖ EXAMPLES ❖

Ex.1 Find the area of a circle whose circumference is 22 cm.

Sol. Let r be the radius of the circle. Then,
Circumference = 22 cm
 $\Rightarrow 2\pi r = 22$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 22$$

$$\Rightarrow r = 7/2 \text{ cm}$$

$$\therefore \text{Area of the circle} = \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$$

$$= 38.5 \text{ cm}^2$$

Ex.2 Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Let r be the radius of the circle. Then,

$$\text{Circumference} = 22 \text{ cm}$$

$$\Rightarrow 2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2} \text{ cm}$$

$$\therefore \text{Area of a quadrant} = \frac{1}{4} \pi r^2$$

$$= \left\{ \frac{1}{4} \times \frac{22}{7} \left(\frac{7}{2} \right)^2 \right\} \text{ cm}^2$$

$$= \left\{ \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \right\} \text{ cm}^2$$

$$= \frac{77}{4} \text{ cm}^2 = 9.625 \text{ cm}^2$$

Ex.3 If the perimeter of a semi-circular protractor is 66 cm, find the diameter of the protractor (Take $\pi = 22/7$).

Sol. Let the radius of the protractor be r cm. Then,

$$\text{Perimeter} = 66 \text{ cm}$$

$$\Rightarrow 1/2(2\pi r) = 66$$

$$\left[\Theta \text{ Perimeter of semi-circle} = \frac{1}{2}(2\pi r) \right]$$

$$\Rightarrow \pi r = 66 \Rightarrow 22/7 \times r = 66 \Rightarrow r = 21 \text{ cm}$$

$$\therefore \text{Diameter of the protractor} = 2r = (2 \times 21) \text{ cm}$$

$$= 42 \text{ cm}$$

Ex.4 The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

Sol. Let the radius of the circle be r cm. Then,

$$\text{Diameter} = 2r \text{ cm and Circumference} = 2\pi r \text{ cm}$$

It is given that the circumference exceeds the diameter by 16.8 cm

$$\therefore \text{Circumference} = \text{Diameter} + 16.8$$

$$\Rightarrow 2\pi r = 2r + 16.8$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 2r + 16.8 \quad \left[\Theta \pi = \frac{22}{7} \right]$$

$$\Rightarrow 44r = 14r + 16.8 \times 7$$

$$\Rightarrow 44r - 14r = 117.6 \Rightarrow 30r = 117.6$$

$$\Rightarrow r = \frac{117.6}{30} = 3.92$$

Hence, radius = 3.92 cm

Ex.5 Two circles touch externally. The sum of their areas is 130π sq. cm. and the distance between their centres is 14 cm. Find the radii of the circles.

Sol. If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

Let the radii of the two circles be r_1 cm and r_2 cm respectively.

Let C_1 and C_2 be the centres of the given circles. Then,

$$C_1C_2 = r_1 + r_2$$

$$\Rightarrow 14 = r_1 + r_2 \quad [\Theta C_1C_2 = 14 \text{ cm (given)}]$$

$$\Rightarrow r_1 + r_2 = 14 \quad \dots(i)$$

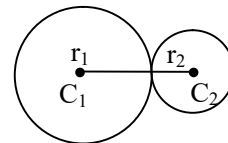
It is given that the sum of the areas of two circles is equal to 130π cm².

$$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi$$

$$\Rightarrow r_1^2 + r_2^2 = 130 \quad \dots(ii)$$

Now,

$$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$$



$$\Rightarrow 14^2 = 130 + 2r_1r_2 \quad [\text{Using (i) and (ii)}]$$

$$\Rightarrow 196 - 130 = 2r_1r_2$$

$$\Rightarrow r_1r_2 = 33 \quad \dots(iii)$$

Now,

$$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$$

$$\Rightarrow (r_1 - r_2)^2 = 130 - 2 \times 33$$

[Using (ii) and (iii)]

$$\Rightarrow (r_1 - r_2)^2 = 64 \Rightarrow r_1 - r_2 = 8 \quad \dots(\text{iv})$$

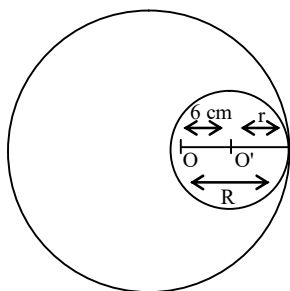
Solving (i) and (iv), we get $r_1 = 11$ cm

and $r_2 = 3$ cm.

Hence, the radii of the two circles are 11 cm and 3 cm.

Ex.6 Two circles touch internally. The sum of their areas is 116π cm² and distance between their centres is 6 cm. Find the radii of the circles.

Sol. Let R and r be the radii of the circles having centres at O and O' respectively. Then,



$$\text{Sum of areas} = 116\pi \text{ cm}^2$$

$$\Rightarrow \pi R^2 + \pi r^2 = 116\pi$$

$$\Rightarrow R^2 + r^2 = 116 \quad \dots(\text{i})$$

Distance between the centres = 6 cm

$$\Rightarrow OO' = 6 \text{ cm}$$

$$\Rightarrow R - r = 6 \quad \dots(\text{ii})$$

$$\text{Now, } (R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$$

$$\Rightarrow (R + r)^2 + 36 = 2 \times 116$$

[Using (i) and (ii)]

$$\Rightarrow (R + r)^2 = (2 \times 116 - 36) = 196$$

$$\Rightarrow R + r = 14 \quad \dots(\text{iii})$$

Solving (ii) and (iii), we get $R = 10$ and $r = 4$.

Hence, radii of the given circles are 10 cm and 4 cm respectively.

Ex.7 A copper wire, when bent in the form of a square, encloses an area of 484 cm². If the same wire is bent in the form of a circle, find the area enclosed by it (Use $\pi = 22/7$).

Sol. We have,

$$\text{Area of the square} = 484 \text{ cm}^2$$

$$\therefore \text{Side of the square} = \sqrt{484} \text{ cm} = 22 \text{ cm}$$

$$\left[\begin{array}{l} \ominus \text{Area} = (\text{Side})^2 \\ \therefore \text{Side} = \sqrt{\text{Area}} \end{array} \right]$$

So, Perimeter of the square = 4 (side)

$$= (4 \times 22) \text{ cm} = 88 \text{ cm}$$

Let r be the radius of the circle. Then,

Circumference of the circle = Perimeter of the square.

$$\Rightarrow 2\pi r = 88$$

$$\Rightarrow 2 \times 22/7 \times r = 88$$

$$\Rightarrow r = 14 \text{ cm}$$

$$\therefore \text{Area of the circle} = \pi r^2$$

$$= \left\{ \frac{22}{7} \times (14)^2 \right\} \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

Ex.8 A wire is looped in the form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of the side of the square.

Sol. We have,

Length of the wire = Circumference of the circle

$$\text{Length of the wire} = \left\{ 2 \times \frac{22}{7} \times 28 \right\} \text{ cm}$$

[Using $C = 2\pi r$]

$$\text{Length of the wire} = 176 \text{ cm} \quad \dots(\text{i})$$

Let the side of the square be x cm. Then,

Perimeter of the square = Length of the wire

$$\Rightarrow 4x = 176 \quad [\text{Using (i)}]$$

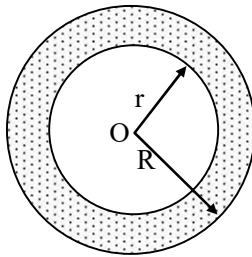
$$\Rightarrow x = 44 \text{ cm}$$

Hence, the length of the sides of the square is 44 cm.

Ex.9 A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.

Sol. Let the outer and inner radii of the ring

be R metres and r metres respectively. Then,



$$2\pi R = 396 \text{ and } 2\pi r = 352$$

$$\Rightarrow 2 \times \frac{22}{7} \times R = 396 \text{ and } 2 \times \frac{22}{7} \times r = 352$$

$$\Rightarrow R = 396 \times \frac{7}{22} \times \frac{1}{2} \text{ and } r = 352 \times \frac{7}{22} \times \frac{1}{2}$$

$$\Rightarrow R = 63 \text{ m and } r = 56 \text{ m}$$

Hence, width of the track = $(R - r)$ metres

$$= (63 - 56) \text{ metres} = 7 \text{ metres}$$

Ex.10 The inner circumference of a circular track is 220 m. The track is 7m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of ₹ 2 per metre. (Use $\pi = 22/7$)

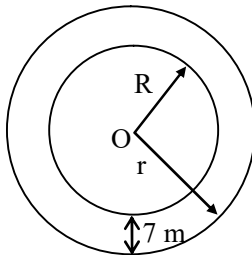
Sol. Let the inner and outer radii of the circular track be r metres and R metres respectively. Then,

$$\text{Inner circumference} = 220 \text{ metres}$$

$$\Rightarrow 2\pi r = 220 \Rightarrow 2 \times \frac{22}{7} \times r = 220 \Rightarrow r = 35 \text{ m}$$

Since the track is 7 metre wide everywhere. Therefore,

$$R = \text{Outer radius} = r + 7 = (35 + 7) \text{ m} = 42 \text{ m}$$



\therefore Outer circumference

$$= 2\pi R = 2 \times \frac{22}{7} \times 42 \text{ m} = 264 \text{ m}$$

Rate of fencing = ₹ 2 per metre

\therefore Total cost of fencing

$$= (\text{Circumference} \times \text{Rate}) = ₹ (264 \times 2) = ₹ 528$$

Ex.11 A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

Sol. Let the radius of the wheel be r cm.

Distance covered by the wheel in one revolution

$$= \frac{\text{Distance moved}}{\text{Number of revolutions}} = \frac{11}{5000} \text{ km}$$

$$= \frac{11}{5000} \times 1000 \times 100 \text{ cm} = 220 \text{ cm}$$

\therefore Circumference of the wheel = 220 cm

$$\Rightarrow 2\pi r = 220 \text{ cm} \Rightarrow 2 \times \frac{22}{7} \times r = 220$$

$$\Rightarrow r = 35 \text{ cm}$$

\therefore Diameter = $2r \text{ cm} = (2 \times 35) \text{ cm} = 70 \text{ cm}$

Hence, the diameter of the wheel is 70 cm.

Ex.12 A car has wheels which are 80 cm in diameter. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Sol. We have,

Speed of the car = 66 km/hr

\therefore Distance travelled by the car in 1 hour = 66 km

\Rightarrow Distance travelled by the car in 10 min.

$$= \frac{66}{60} \times 10 \text{ km} = 11 \text{ km} = 11 \times 1000 \times 100 \text{ cm}$$

We have,

Radius of car wheels = 40 cm

\therefore Circumference of the wheels

$$= 2 \times \frac{22}{7} \times 40 \text{ cm}$$

\Rightarrow Distance travelled by the car when its wheels take one complete revolution

$$= 2 \times \frac{22}{7} \times 40 \text{ cm}$$

\therefore Number of revolutions made by the wheels in 10 minutes

$$= \frac{\text{Distance covered by the car in 10 minutes}}{\text{Distance covered by the car when its wheels make one complete revolution}}$$

$$= \frac{11 \times 1000 \times 100}{2 \times \frac{22}{7} \times 40} = \frac{11 \times 1000 \times 100 \times 7}{2 \times 22 \times 40} = 4375$$

Hence, each wheel makes 4375 revolutions in 10 minutes.

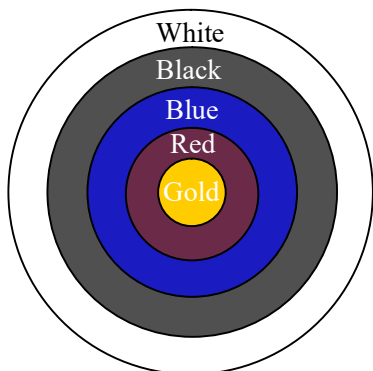
Ex.13 Fig. depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and white. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Sol. We have,

r = Radius of the region representing Gold score = 10.5 cm

$\therefore r_1$ = Radius of the region representing Gold and Red scoring areas

$$= (10.5 + 10.5) \text{ cm} = 21 \text{ cm} = 2r \text{ cm}$$



r_2 = Radius of the region representing Gold, Red and Blue scoring areas

$$= (21 + 10.5) \text{ cm} = 31.5 \text{ cm} = 3r \text{ cm}$$

r_3 = Radius of the region representing Gold, Red, Blue and Black scoring areas

$$= (31.5 + 10.5) \text{ cm} = 42 \text{ cm} = 4r \text{ cm}$$

r_4 = Radius of the region representing Gold, Red, Blue, Black and white scoring areas

$$= (42 + 10.5) \text{ cm} = 52.5 \text{ cm} = 5r \text{ cm}$$

Now, A_1 = Area of the region representing Gold scoring area

$$\begin{aligned} &= \pi r^2 = \frac{22}{7} \times (10.5)^2 = \frac{22}{7} \times 10.5 \times 10.5 \\ &= 22 \times 1.5 \times 10.5 = 346.5 \text{ cm}^2 \end{aligned}$$

A_2 = Area of the region representing Red scoring area

$$\begin{aligned} &= \pi(2r)^2 - \pi r^2 = 3\pi r^2 = 3A_1 = 3 \times 346.5 \text{ cm}^2 \\ &= 1039.5 \text{ cm}^2 \end{aligned}$$

A_3 = Area of the region representing Blue scoring area

$$\begin{aligned} &= \pi(3r)^2 - \pi(2r)^2 = 9\pi r^2 - 4\pi r^2 = 5\pi r^2 = 5A_1 \\ &= 5 \times 346.5 \text{ cm}^2 = 1732.5 \text{ cm}^2 \end{aligned}$$

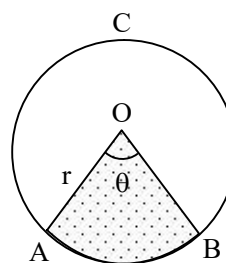
A_4 = Area of the region representing Black scoring area

$$\begin{aligned} &= \pi(4r)^2 - \pi(3r)^2 = 7\pi r^2 \\ &= 7A_1 = 7 \times 346.5 \text{ cm}^2 = 2425.5 \text{ cm}^2 \end{aligned}$$

A_5 = Area of the region representing White scoring area

$$\begin{aligned} &= \pi(5r)^2 - \pi(4r)^2 = 9\pi r^2 \\ &= 9A_1 = 9 \times 346.5 \text{ cm}^2 = 3118.5 \text{ cm}^2 \end{aligned}$$

SECTOR OF A CIRCLE



Minor sector :

A sector of a circle is called a minor sector if the minor arc of the circle is a part of its boundary

In Fig. sector OAB is the minor sector.

Major sector :

A sector of a circle is called a major sector if the major arc of the circle is a part of its boundary.

In Fig. sector OACB is the major sector.

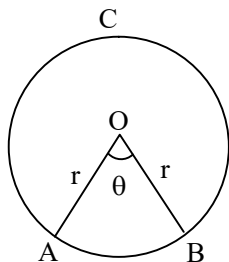
Following are some important points to remember:

- (i) A minor sector has an angle θ , subtended at the centre of the circle, whereas a major sector has no angle.
- (ii) The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.
- (iii) The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.
- (iv) The boundary of a sector consists of an arc of the circle and the two radii.

➤ AREA OF A SECTOR

If the arc subtends an angle of θ at the centre, then its arc length is

$$\frac{\theta}{180} \times \pi r$$



Hence, the arc length λ of a sector of angle θ in a circle of radius r is given by

$$\lambda = \frac{\theta}{180} \times \pi r$$

....(i)

If the arc subtends an angle θ , then area of the corresponding sector is

$$\frac{\pi r^2 \theta}{360}$$

Thus, the area A of a sector of angle θ in a circle of radius r is given by

$$\begin{aligned} A &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{\theta}{360} \times (\text{Area of the circle}) \end{aligned} \quad \text{....(ii)}$$

Some useful results to remember:

(i) Angle described by minute hand in 60 minutes = 360°

$$\therefore \text{Angle described by minute hand in one minute} = \left(\frac{360}{60}\right)^\circ = 6^\circ$$

Thus, minute hand rotates through an angle of 6° in one minute.

(ii) Angle described by hour hand in 12 hours = 360°

\therefore Angle described by hour hand in one hour

$$\begin{aligned} &= \left(\frac{360}{12}\right)^\circ \\ &= 30^\circ \end{aligned}$$

❖ EXAMPLES ❖

Ex.14 A sector is cut from a circle of radius 21 cm. The angle of the sector is 150° . Find the length of its arc and area.

Sol. The arc length l and area A of a sector of angle θ in a circle of radius r are given by

$$l = \frac{\theta}{360} \times 2\pi r \quad \text{and} \quad A = \frac{\theta}{360} \times \pi r^2 \quad \text{respectively.}$$

Here, $r = 21$ cm and $\theta = 150$

$$\therefore l = \left\{ \frac{150}{360} \times 2 \times \frac{22}{7} \times 21 \right\} \text{ cm} = 55 \text{ cm}$$

$$\begin{aligned} \text{and } A &= \left\{ \frac{150}{360} \times \frac{22}{7} \times (21)^2 \right\} \text{ cm}^2 = \frac{1155}{2} \text{ cm}^2 \\ &= 577.5 \text{ cm}^2 \end{aligned}$$

Ex.15 Find the area of the sector of a circle whose radius is 14 cm and angle of sector is 45° .

Sol. We know that the area A of a sector of angle θ in a circle of radius r is given by

$$A = \frac{\theta}{360} \times \pi r^2$$

Here, $r = 14$ cm and $\theta = 45$

$$\begin{aligned} \therefore A &= \left\{ \frac{45}{360} \times \frac{22}{7} \times (14)^2 \right\} \text{ cm}^2 \\ &= \left\{ \frac{1}{8} \times \frac{22}{7} \times 14 \times 14 \right\} \text{ cm}^2 \\ &= 77 \text{ cm}^2 \end{aligned}$$

Ex.16 In Fig. there are shown sectors of two concentric circles of radii 7 cm and 3.5 cm. Find the area of the shaded region.

(Use $\pi = 22/7$).

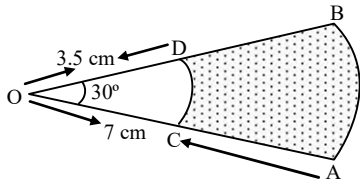
Sol. Let A_1 and A_2 be the areas of sectors OAB and OCD respectively. Then, A_1 = Area of a sector of angle 30° in a circle of radius 7 cm

$$\Rightarrow A_1 = \left\{ \frac{30}{360} \times \frac{22}{7} \times 7^2 \right\} \text{ cm}^2$$

$$\left[\text{Using: } A = \frac{\theta}{360} \times \pi r^2 \right]$$

$$\Rightarrow A_1 = 77/6 \text{ cm}^2$$

A_2 = Area of a sector of angle 30° in a circle of radius 3.5 cm.



$$\Rightarrow A_2 = \left\{ \frac{30}{360} \times \frac{22}{7} \times (3.5)^2 \right\} \text{ cm}^2$$

$$\Rightarrow A_2 = \left\{ \frac{1}{12} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \right\} \text{ cm}^2 = \frac{77}{24} \text{ cm}^2$$

\therefore Area of the shaded region

$$= A_1 - A_2 = \left(\frac{77}{6} - \frac{77}{24} \right) \text{ cm}^2$$

$$= \frac{77}{24} \times (4 - 1) \text{ cm}^2 = 77/8 \text{ cm}^2 = 9.625 \text{ cm}^2$$

Ex.17 A pendulum swings through an angle of 30° and describes an arc 8.8 cm in length. Find the length of the pendulum.

Sol. Here, $\theta = 30^\circ$, $l = \text{arc} = 8.8 \text{ cm}$

$$\therefore l = \frac{\theta}{360} \times 2\pi r \Rightarrow 8.8 = \frac{30}{360} \times 2 \times \frac{22}{7} \times r$$

$$\Rightarrow r = \frac{8.8 \times 6 \times 7}{22} \text{ cm} = 16.8 \text{ cm}$$

Ex.18 The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute. (Use $\pi = 22/7$)

Sol. Clearly, minute hand of a clock describes a circle of radius equal to its length i.e., 14 cm.

Since the minute hand rotates through 6° in one minute. Therefore, area swept by the minute hand in one minute is the area of a sector of angle 6° in a circle of radius 14 cm. Hence, required area A is given by

$$A = \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow A = \left\{ \frac{6}{360} \times \frac{22}{7} \times (14)^2 \right\} \text{ cm}^2$$

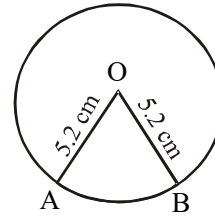
$$\Rightarrow A = \left\{ \frac{1}{60} \times \frac{22}{7} \times 14 \times 14 \right\} \text{ cm}^2 = \frac{154}{15} \text{ cm}^2$$

$$= 10.26 \text{ cm}^2$$

Ex.19 The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Sol. Let OAB be the given sector. Then,

Perimeter of sector OAB = 16.4 cm



$$\Rightarrow OA + OB + \text{arc AB} = 16.4 \text{ cm}$$

$$\Rightarrow 5.2 + 5.2 + \text{arc AB} = 16.4$$

$$\Rightarrow \text{arc AB} = 6 \text{ cm} \Rightarrow l = 6 \text{ cm}$$

$$\therefore \text{Area of sector OAB} = \frac{1}{2} lr$$

$$= \frac{1}{2} \times 6 \times 5.2 \text{ cm}^2 = 15.6 \text{ cm}^2$$

Ex.20 The minute hand of a clock is 10cm long. Find the area of the face of the clock described by the minute hand between 9 A.M. and 9.35 A.M.

Sol. We have,

Angle described by the minute hand in one minute = 6°

$$\therefore \text{Angle described by the minute hand in 35 minutes} = (6 \times 35)^\circ = 210^\circ$$

\therefore Area swept by the minute hand in 35 minutes = Area of a sector of angle 210° in a circle of radius 10 cm

$$= \left\{ \frac{210}{360} \times \frac{22}{7} \times (10)^2 \right\} \text{ cm}^2 = 183.3 \text{ cm}^2$$

$$\left[\text{Using } A = \frac{\theta}{360^\circ} \times \pi r^2 \right]$$

Ex.21 The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days.

(Take $\pi = 22/7$)

Sol. In 2 days, the short hand will complete 4 rounds.

\therefore Distance moved by its tip = 4 (Circumference of a circle of radius 4 cm)

$$= 4 \times \left(2 \times \frac{22}{7} \times 4 \right) \text{ cm} = \frac{704}{7} \text{ cm}$$

In 2 days, the long hand will complete 48 rounds.

\therefore Distance moved by its tip

= 48 (Circumference of a circle of radius 6 cm)

$$= 48 \times \left(2 \times \frac{22}{7} \times 6 \right) \text{ cm} = \frac{12672}{7} \text{ cm}$$

Hence,

Sum of the distance moved by the tips of two hands of the clock

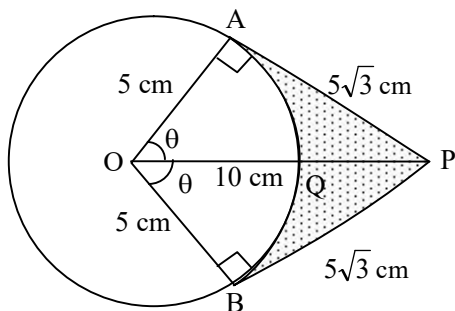
$$= \left(\frac{704}{7} + \frac{12672}{7} \right) \text{ cm} = 1910.57 \text{ cm}$$

Ex.22 An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area.

Sol. In the adjacent figure, let $\angle AOP = \angle BOP = \theta$. Clearly, portion AB of the belt is not in contact with the rim of the pulley. In right triangle OAP, we have

$$\cos \theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

$$\Rightarrow \angle AOB = 2\theta = 120^\circ$$



$$\therefore \text{Arc AB} = \frac{120^\circ \times 2 \times \pi \times 5}{360} \text{ cm} = \frac{10\pi}{3} \text{ cm}$$

$$[\text{Using } \lambda = \frac{\theta}{360} \times 2\pi r]$$

Hence, Length of the belt that is in contact with the rim of the pulley

= Circumference of the rim – Length of arc AB

$$= 2\pi \times 5 \text{ cm} - \frac{10\pi}{3} \text{ cm} = \frac{20\pi}{3} \text{ cm}$$

Now,

$$\text{Area of sector OAQB} = \frac{120}{360} \times \pi \times 5^2 \text{ cm}^2$$

$$= \frac{25\pi}{3} \text{ cm}^2 \left[\text{Using : Area} = \frac{\theta}{360} \times \pi r^2 \right]$$

Area of quadrilateral OAPB = 2 (Area of $\triangle OAP$)

$$= 2 \times \left(\frac{1}{2} \times OA \times AP \right)$$

$$= 5 \times 5\sqrt{3} \text{ cm}^2$$

$$[\because OP^2 = OA^2 + AP^2 \Rightarrow AP = \sqrt{100 - 25} = 5\sqrt{3}]$$

$$= 25\sqrt{3} \text{ cm}^2$$

Hence,

Shaded area = Area of quadrilateral

OAPB – Area of sector OAQB.

$$= \left(25\sqrt{3} - \frac{25\pi}{3} \right) \text{ cm}^2 = \frac{25}{3} (3\sqrt{3} - \pi) \text{ cm}^2$$

Ex.23 An arc of a circle is of length 5π cm and the sector it bounds has an area of 20π cm². Find the radius of the circle.

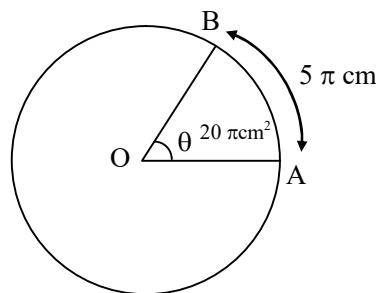
Sol. Let the radius of the circle be r cm and the arc AB of length 5π cm subtends angle θ at the centre O of the circle. Then,

Arc AB = 5π cm and

Area of sector OAB = 20π cm²

$$\Rightarrow \frac{\theta}{360} \times 2\pi r = 5\pi \text{ and } \frac{\theta}{360} \times \pi r^2 = 20\pi$$

$$\Rightarrow \frac{\frac{\theta}{360} \times \pi r^2}{\frac{\theta}{360} \times 2\pi r} = \frac{20\pi}{5\pi}$$



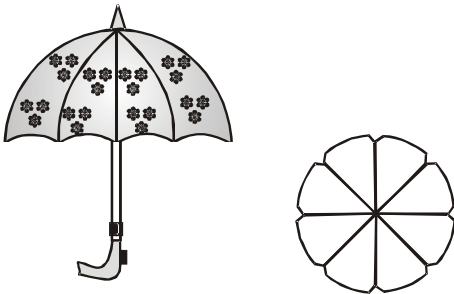
$$\Rightarrow r/2 = 4 \Rightarrow r = 8 \text{ cm}$$

ALTER : We have, Area = $\frac{1}{2}lr \Rightarrow 20\pi$

$$= \frac{1}{2} \times 5\pi \times r = 8 \text{ cm}$$

Ex.24 An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm. Find the area between the two consecutive ribs of the umbrella.

Sol. Since ribs are equally spaced. Therefore,
Angle made by two consecutive ribs at the centre = $\frac{360^\circ}{8} = 45^\circ$



Thus,

Area between two consecutive ribs

= Area of a sector of a circle of radius 45 cm and sector angle 45°

$$= \left\{ \frac{45}{360} \times \frac{22}{7} \times 45 \times 45 \right\} \text{ cm}^2$$

$$\left[\text{Using : Area} = \frac{\theta}{360} \times \pi r^2 \right]$$

$$= \frac{1}{8} \times \frac{22}{7} \times 45 \times 45 \text{ cm}^2 = 795.53 \text{ cm}^2$$

Ex.25 A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. Find:

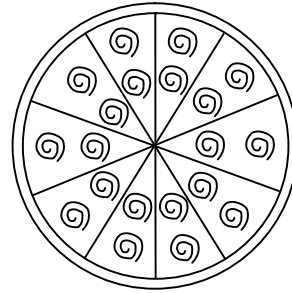
- the total length of the silver wire required
- the area of each sector of the brooch.

Sol. (i) We have,

Total length of the silver wire

= Circumference of the circle of radius

$35/2 \text{ mm} + \text{Length of five diameters}$



$$= 2\pi \times \frac{35}{2} + 5 \times 35 \text{ mm}$$

$$= \left(2 \times \frac{22}{7} \times \frac{35}{2} + 175 \right) \text{ mm}$$

$$= 285 \text{ mm}$$

(ii) The circle is divided into 10 equal sectors, Therefore, Area of each sector of the brooch

= $1/10$ (Area of the circle)

$$= 1/10 \times \pi \times (35/2)^2 \text{ cm}^2$$

$$= \frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{ mm}^2$$

$$= \frac{385}{4} \text{ mm}^2$$



SEGMENT OF A CIRCLE & ITS AREA

◆ Segment of a circle :

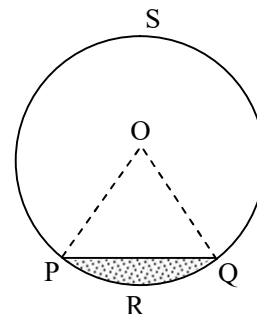
The region enclosed by an arc and a chord is called the segment of the circle.

◆ Minor segment :

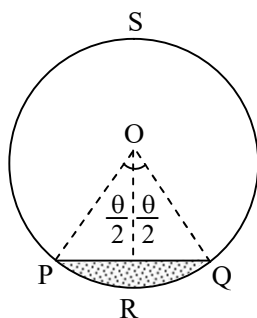
If the boundary of a segment is a minor arc of a circle, then the corresponding segment is called a minor segment.

◆ Major segment :

A segment corresponding a major arc of a circle is known as the major segment.



Area of the sector OPRQ = Area of the segment PRQ + Area of $\triangle OPQ$



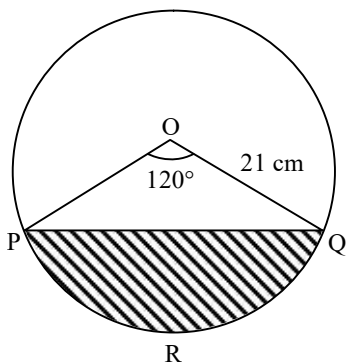
$$\Rightarrow \text{Area of segment PRQ} = \left\{ \frac{\pi}{360} \times \theta - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right\} r^2$$

Ex.26 Find the area of the segment of a circle, given that the angle of the sector is 120° and the radius of the circle is 21 cm. (Take $\pi = 22/7$)

Sol. Here, $r = 21$ cm and $\pi = 120$

\therefore Area of the segment

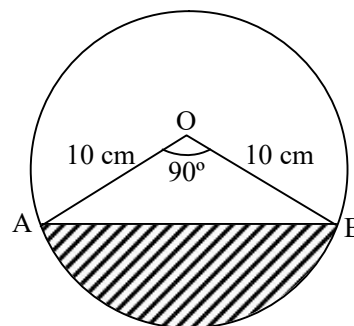
$$\begin{aligned} &= \left\{ \frac{\pi}{360} \times \theta - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right\} r^2 \\ &= \left\{ \frac{22}{7} \times \frac{120}{360} - \sin 60^\circ \cos 60^\circ \right\} (21)^2 \text{ cm}^2 \\ &= \left\{ \frac{22}{21} - \frac{1}{2} \times \frac{\sqrt{3}}{2} \right\} (21)^2 \text{ cm}^2 \end{aligned}$$



$$\begin{aligned} &= \left\{ \frac{22}{21} \times (21)^2 - (21)^2 \times \frac{\sqrt{3}}{4} \right\} \text{ cm}^2 \\ &= \left(462 - \frac{441}{4} \sqrt{3} \right) \text{ cm}^2 = \frac{21}{4} (88 - 21\sqrt{3}) \text{ cm}^2 \end{aligned}$$

Ex.27 A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of major and minor segments (Take $\pi = 3.14$)

Sol. We know that the area of a minor segment of angle θ° in a circle of radius r is given by



$$A = \left\{ \frac{\pi\theta}{360} - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right\} r^2$$

Here, $r = 10$ and $\theta = 90^\circ$

$$\therefore A = \left\{ \frac{3.14 \times 90}{4} - \sin 45^\circ \cos 45^\circ \right\} (10)^2 \text{ cm}^2$$

$$\Rightarrow A = \left\{ \frac{3.14}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \right\} 10^2 \text{ cm}^2$$

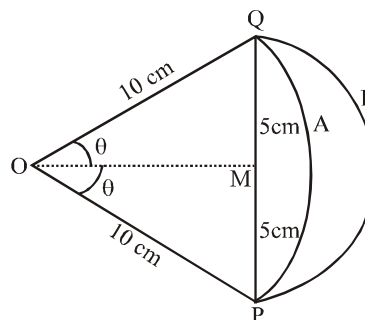
$$\Rightarrow A = \{3.14 \times 25 - 50\} \text{ cm}^2 = (78.5 - 50) \text{ cm}^2 = 28.5 \text{ cm}^2$$

Area of the major segment = Area of the circle - Area of the minor segment
 $= \{3.14 \times 10^2 - 28.5\} \text{ cm}^2$
 $= (314 - 28.5) \text{ cm}^2 = 285.5 \text{ cm}^2$

Ex.28 The diagram shows two arcs, A and B. Arc A is part of the circle with centre O and radius of PQ. Arc B is part of the circle with centre M and radius PM, where M is the mid-point of PQ. Show that the area enclosed by the two arcs is equal to $25 \left(\sqrt{3} - \frac{\pi}{6} \right) \text{ cm}^2$.

Sol. We have,

Area enclosed by arc B and chord PQ = Area of semi-circle of radius 5 cm



$$= 1/2 \times \pi \times 5^2 \text{ cm}^2 = \frac{25\pi}{2} \text{ cm}^2$$

Let $\angle MOQ = \angle MOP = \theta$

In $\triangle OMP$, we have

$$\sin \theta = \frac{PM}{OP} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ \Rightarrow \angle POQ = 2\theta = 60^\circ$$

\therefore Area enclosed by arc A and chord PQ.

= Area of segment of circle of radius 10 cm and sector containing angle 60°

$$= \left\{ \frac{\pi \times 60}{360} - \sin 30^\circ \times \cos 30^\circ \right\} \times 10^2 \text{ cm}^2$$

$$\left[\Theta A = \left\{ \frac{\pi \theta}{360} - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right\} r^2 \right]$$

$$= \left\{ \frac{50\pi}{3} - 25\sqrt{3} \right\} \text{ cm}^2$$

Hence,

$$\text{Required area} = \left\{ \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3} \right) \right\} \text{ cm}^2$$

$$\Rightarrow \text{Required area} = \left\{ 25\sqrt{3} - \frac{25\pi}{6} \right\} \text{ cm}^2$$

$$= 25 \left\{ \sqrt{3} - \frac{\pi}{6} \right\} \text{ cm}^2$$

➤ AREAS OF COMBINATIONS OF PLANE FIGURES

Ex.29 The inner and outer diameters of ring I of a dartboard are 32 cm and 34 cm respectively and those of rings II are 19 cm and 21 cm respectively. What is the total area of these two rings ?

Sol. We have,

$$\text{Area of ring I} = (\pi \times 17^2 - \pi \times 16^2) \text{ cm}^2$$

$$= \frac{22}{7} \times (17^2 - 16^2) \text{ cm}^2$$

$$= \frac{22}{7} \times (17 + 16) (17 - 16) \text{ cm}^2$$

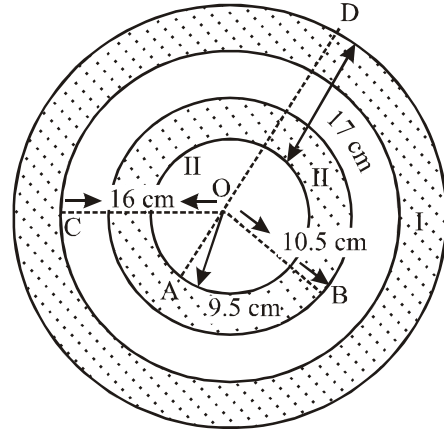
$$= \frac{22}{7} \times 33 \text{ cm}^2$$

$$\text{Area of ring II} = (\pi \times 10.5^2 - \pi \times 9.5^2) \text{ cm}^2$$

$$= \pi (10.5^2 - 9.5^2) \text{ cm}^2$$

$$= \frac{22}{7} \times (10.5 + 9.5) (10.5 - 9.5) \text{ cm}^2$$

$$= \frac{22}{7} \times 20 \text{ cm}^2$$



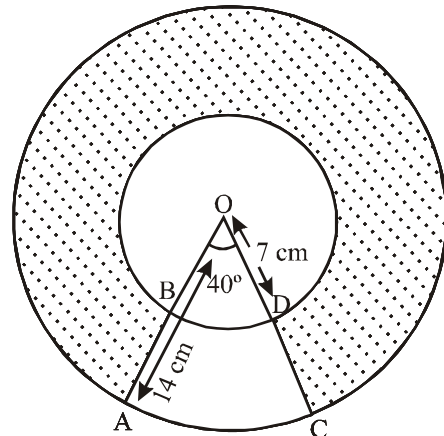
Hence,

$$\text{Total area of two rings} = \frac{22}{7} \times 33 + \frac{22}{7} \times 20 \text{ cm}^2$$

$$= \frac{22}{7} \times (33 + 20) \text{ cm}^2 = 166.57 \text{ cm}^2$$

Ex.30 Find the area of the shaded region in Fig. where radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.

Sol. We have,



Area of the region ABDC = Area of sector AOC - Area of sector BOD

$$= \left(\frac{40}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{40}{360} \times \frac{22}{7} \times 7 \times 7 \right) \text{ cm}^2$$

$$= \left(\frac{1}{9} \times 22 \times 14 \times 2 - \frac{1}{9} \times 22 \times 7 \times 1 \right) \text{ cm}^2$$

$$= \frac{22}{9} = \frac{154}{3} \times (28 - 7) \text{ cm}^2 = \text{cm}^2$$

Area of the circular ring

$$= \left(\frac{22}{7} \times 14 \times 14 - \frac{22}{7} \times 7 \times 7 \right) \text{ cm}^2$$

$$= (22 \times 14 \times 2 - 22 \times 7 \times 1) \text{ cm}^2$$

$$= 22 \times 21 \text{ cm}^2 = 462 \text{ cm}^2$$

Hence,

$$\text{Required shaded area} = \left(462 - \frac{154}{3} \right) \text{ cm}^2$$

$$= \frac{1232}{3} \text{ cm}^2 = 410.67 \text{ cm}^2$$

Ex.31 AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O. If $\angle AOB = 30^\circ$, find the area of the shaded region.

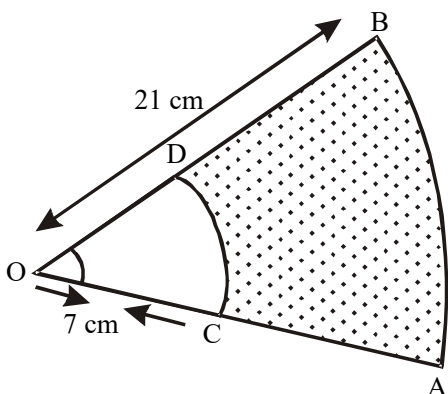
Sol. We have,

Shaded area = Area of sector OAB – Area of sector OCD

\Rightarrow Shaded area

$$= \left(\frac{30}{360} \times \frac{22}{7} \times 21 \times 21 - \frac{30}{360} \times \frac{22}{7} \times 7 \times 7 \right) \text{ cm}^2$$

$$= \frac{30}{360} \times \frac{22}{7} \times (21 \times 21 - 7 \times 7) \text{ cm}^2$$



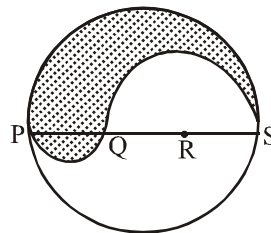
$$= \frac{11}{42} \times (21 + 7) \times (21 - 7) \text{ cm}^2$$

$$= \frac{11}{42} \times 28 \times 14 \text{ cm}^2 = 102.67 \text{ cm}^2$$

Ex.32 PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region.

Sol. We have,

PS = Diameter of a circle of radius 6 cm = 12 cm



$$\therefore PQ = QR = RS = 12/3 = 4 \text{ cm}$$

$$QS = QR + RS = (4 + 4) \text{ cm} = 8 \text{ cm}$$

Hence, required perimeter

$$= \text{Arc of semi-circle of radius 6 cm}$$

$$+ \text{Arc of semi-circle of radius 4 cm}$$

$$+ \text{Arc of semi-circle of radius 2 cm}$$

$$= (\pi \times 6 + \pi \times 4 + \pi \times 2) \text{ cm} = 12\pi \text{ cm}$$

Required area = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter – Area of semi-circle with QS as diameter.

$$= \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$$

$$= \frac{1}{2} \times \frac{22}{7} (6^2 + 2^2 - 4^2)$$

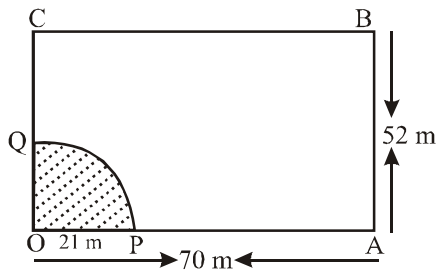
$$= \frac{1}{2} \times \frac{22}{7} \times 24$$

$$= \frac{264}{7} = 37.71 \text{ cm}^2$$

Ex.33 A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze ?

Sol. Shaded portion indicates the area which the horse can graze. Clearly, shaded area is the area of a quadrant of a circle of radius $r = 21$ m.

$$\therefore \text{Required area} = \frac{1}{4} \pi r^2$$



$$\Rightarrow \text{Required area} = \left\{ \frac{1}{4} \times \frac{22}{7} \times (21)^2 \right\} \text{ cm}^2$$

$$= \frac{693}{2} \text{ cm}^2 = 346.5 \text{ cm}^2$$

Ex.34 A paper is in the form of a rectangle ABCD in which AB = 20 cm and BC = 14 cm. A semi-circular portion with BC as diameter is cut off. Find the area of a remaining part.

Sol. We have,

Length of the rectangle ABCD = AB = 20 cm

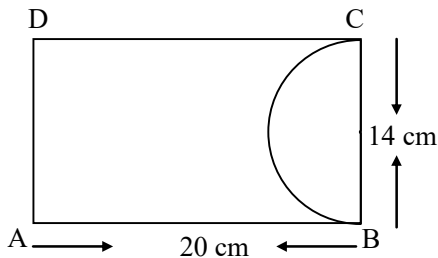
Breadth of the rectangle ABCD = BC = 14 cm

$$\therefore \text{Area of rectangle ABCD} = (20 \times 14) \text{ cm}^2$$

$$= 280 \text{ cm}^2$$

Diameter of the semi-circle = BC = 14 cm

\therefore Radius of the semi-circle = 7 cm



Area of the semi-circular portion cut off from the rectangle ABCD

$$= \frac{1}{2} (\pi r^2) = \left(\frac{1}{2} \times \frac{22}{7} \times 7^2 \right) \text{ cm}^2 = 77 \text{ cm}^2$$

\therefore Area of the remaining part

= Area of rectangle ABCD – Area of semi-circle

$$= (280 - 77) \text{ cm}^2 = 203 \text{ cm}^2$$

Ex.35 In figure, find the area of the shaded region
[Use $\pi = 3.14$]

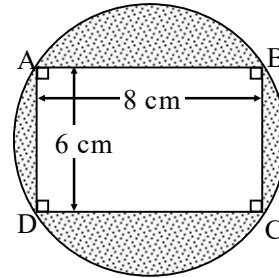
Sol. Clearly, Diameter of the circle = Diagonal BD of rectangle ABCD

$$\therefore \text{Diameter} = BD = \sqrt{BC^2 + CD^2}$$

$$= \sqrt{6^2 + 8^2} \text{ cm} = 10 \text{ cm}$$

Let r be the radius of the circle. Then,

$$r = (10/2) \text{ cm} = 5 \text{ cm}$$



Area of rectangle ABCD = AB \times BC

$$= (8 \times 6) \text{ cm}^2 = 48 \text{ cm}^2$$

Area of the circle = $\pi r^2 = 3.14 \times (5)^2 \text{ cm}^2$

$$= 78.50 \text{ cm}^2$$

Hence,

Area of the shaded region = Area of the circle – Area of rectangle ABCD

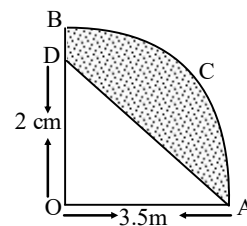
$$= (78.50 - 48) \text{ cm}^2 = 30.50 \text{ cm}^2$$

Ex.36 In Fig. AOBCA represents a quadrant of a circle of radius 3.5 cm with centre O. Calculate the area of the shaded portion

(Take $\pi = 22/7$).

Sol. We have,

Area of quadrant AOB



$$= \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{27}{8} \text{ cm}^2 = 9.625 \text{ cm}^2$$

Area of $\triangle AOD = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} (OA \times OD) = \frac{1}{2} [3.5 \times 2] \text{ cm}^2 = 3.5 \text{ cm}^2$$

Hence,

Area of the shaded portion

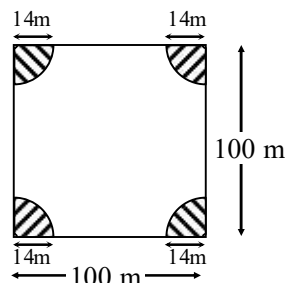
= Area of quadrant – Area of $\triangle AOD$

$$= (9.625 - 3.5) \text{ cm}^2 = 6.125 \text{ cm}^2$$

Ex.37 A square park has each side of 100 m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 14 m as shown in Fig. Find the area of the remaining part of the park (Use $\pi = 22/7$).

Sol. We have,

Area of each quadrant of a circle of radius 14 m



$$= \frac{1}{4} (\pi r^2) = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 \quad [\Theta r = 14]$$

$$= 154 \text{ m}^2$$

$$\therefore \text{Area of 4 quadrants} = (4 \times 154) \text{ m}^2 = 616 \text{ m}^2$$

Area of square park having side 100 m long

$$= (100 \times 100) \text{ m}^2 = 10,000 \text{ m}^2$$

Hence,

Area of the remaining part of the park

$$= 10,000 - 616 = 9384 \text{ m}^2$$

Ex.38 ABCD is a flower bed. If OA = 21 cm and OC = 14 m, find the area of the bed.

[Take $\pi = 22/7$]

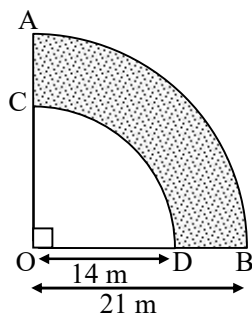
Sol. We have,

OA = R = 21 m and OC = r = 14 m

\therefore Area of the flower bed

= Area of a quadrant of a circle of radius R

– Area of a quadrant of a circle of radius r



$$= \frac{1}{4} \pi R^2 - \frac{1}{4} \pi r^2 = \frac{\pi}{4} (R^2 - r^2)$$

$$= \frac{1}{4} \times \frac{22}{7} (21^2 - 14^2) \text{ cm}^2$$

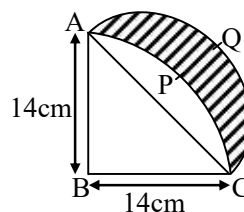
[$\Theta R = 21 \text{ m}$ and $r = 14 \text{ m}$]

$$= \left\{ \frac{1}{4} \times \frac{22}{7} \times (21 + 14) (21 - 14) \right\} \text{ m}^2$$

$$= \left\{ \frac{1}{4} \times \frac{22}{7} \times 35 \times 7 \right\} \text{ m}^2 = 192.5 \text{ m}^2$$

Ex.39 ABCP is a quadrant of a circle of radius 14 cm. With AC as diameter, a semi-circle is drawn. Find the area of the shaded portion.

Sol. In the right-angled triangle ABC, we have



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 14^2 + 14^2 \Rightarrow AC = \sqrt{2 \times 14^2} = 14\sqrt{2} \text{ cm}$$

$$\Rightarrow \frac{AC}{2} = \frac{14\sqrt{2}}{2} \text{ cm} = 7\sqrt{2} \text{ cm}$$

Now,

Required area

$$= \text{Area APCQA}$$

$$= \text{Area ACQA} - \text{Area ACPA}$$

$$= \text{Area ACQA} - (\text{Area ABCPA} - \text{Area of } \triangle ABC)$$

$$= (\text{Area of semi-circle with AC as diameter})$$

$$- [\text{Area of a quadrant of a circle with AB as radius} - \text{Area of } \triangle ABC]$$

$$= \left[\frac{1}{2} \left\{ \frac{22}{7} \times (7\sqrt{2})^2 \right\} - \left\{ \frac{1}{4} \times \frac{22}{7} \times 14^2 - \frac{1}{2} \times 14 \times 14 \right\} \right]$$

$$= \left\{ \frac{1}{2} \times \frac{22}{7} \times 49 \times 2 - \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 + \frac{1}{2} \times 14 \times 14 \right\} \text{ cm}^2$$

$$= (154 - 154 + 98) \text{ cm}^2 = 98 \text{ cm}^2$$

Ex.40 It is proposed to add to a square lawn measuring 58 cm on a side, two circular ends. The centre of each circle being the point of intersection of the diagonals of the square. Find the area of the whole lawn.

Sol. We have,

Length of the diagonal of the square

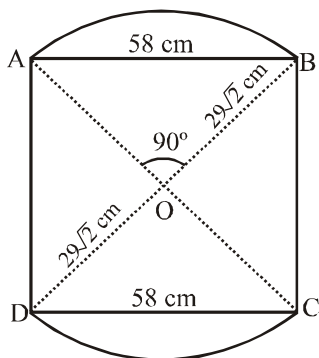
$$= \sqrt{58^2 + 58^2} = 58\sqrt{2} \text{ cm}$$

So, radius of the circle having centre at the point of intersection of diagonals is $29\sqrt{2}$ cm.

Now,

Area of one circular end

= Area of a segment of angle 90° in a circle of radius $29\sqrt{2}$ cm



$$= \left\{ \frac{22}{7} \times \frac{90}{360} - \sin 45^\circ \cos 45^\circ \right\} \times (29\sqrt{2})^2 \text{ cm}^2$$

$$\left[\ominus \text{Area} = \left\{ \frac{\pi\theta}{360} - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right\} r^2 \right]$$

$$= \left(\frac{11}{14} - \frac{1}{2} \right) \times 29 \times 29 \times 2 \text{ cm}^2$$

$$= 29 \times 29 \times 2 \times \frac{4}{14} \text{ cm}^2 = \frac{3364}{7} \text{ cm}^2$$

\therefore Area of the whole lawn

= Area of the square + 2 (Area of a circular end)

$$= \left\{ 58 \times 58 + 2 \times \frac{3364}{7} \right\} \text{ cm}^2$$

$$= \left\{ 3364 + 2 \times \frac{3364}{7} \right\} \text{ cm}^2$$

$$= 3364 \left(1 + \frac{2}{7} \right) \text{ cm}^2 = 3364 \times \frac{9}{7} \text{ cm}^2$$

$$= 4325.14 \text{ cm}^2$$

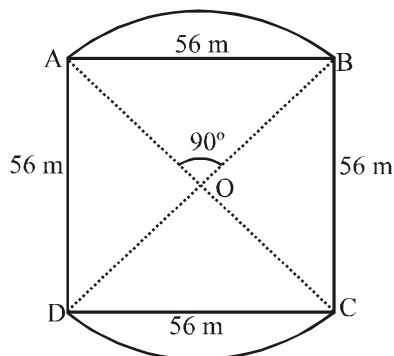
Ex.41 In Fig. two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 cm. If the centre of each circular flower bed is the point of intersection of the

diagonals of the square lawn, find the sum of the areas of the lawns and the flower beds.

Sol.

We have,

$$AC = BD = \sqrt{56^2 + 56^2} = 56\sqrt{2} \text{ m}$$



$$\therefore OA = OB = \frac{1}{2} AC = 28\sqrt{2} \text{ m}$$

So, radius of the circle having centre at the point of intersection of diagonals is $28\sqrt{2}$ cm.

Now,

Area of one circular end = Area of a segment of angle 90° in a circle of radius $28\sqrt{2}$ m.

$$= \left\{ \frac{22}{7} \times \frac{90}{360} - \sin 45^\circ \cos 45^\circ \right\} \times (28\sqrt{2})^2 \text{ cm}^2$$

$$\left[\text{Substituting } r = 28\sqrt{2} \text{ and } \theta = 90^\circ \right]$$

$$\left[\text{in Area} = \left(\frac{\pi\theta}{360} - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right) r^2 \right]$$

$$= \left\{ \frac{11}{14} - \frac{1}{2} \right\} \times 28 \times 28 \times 2 \text{ cm}^2$$

$$= 28 \times 28 \times 2 \times \frac{4}{14} \text{ cm}^2$$

$$= 448 \text{ cm}^2$$

\therefore Area of two flower beds

$$= 2 \times 448 \text{ m}^2 = 896 \text{ m}^2$$

Area of the square lawn

$$= 56 \times 56 \text{ m}^2 = 3136 \text{ m}^2$$

Hence,

$$\text{Total area} = (3136 + 896) \text{ m}^2$$

$$= 4032 \text{ m}^2$$

Ex.42 Find the area of the shaded region in Fig. where ABCD is a square side 10 cm. (Use $\pi = 3.14$)

Sol. Let us mark the four unshaded regions as R_1 , R_2 , R_3 and R_4 .

We have,

Area of R_1 + Area of R_3

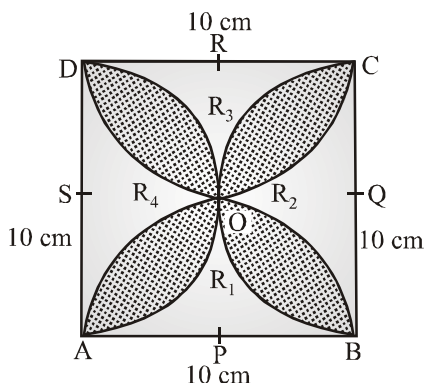
= Area of square ABCD – Area of two semi-circles having centres at Q and S

$$= \left(10 \times 10 - 2 \times \frac{1}{2} \times 3.14 \times 5^2 \right) \text{cm}^2$$

$$[\Theta \text{ Radius} = AP = 5 \text{ cm}]$$

$$= (100 - 3.14 \times 25) \text{cm}^2 = (100 - 78.5) \text{cm}^2$$

$$= 21.5 \text{ cm}^2$$



Similarly, we have

$$\text{Area of } R_2 + \text{Area of } R_4 = 21.5 \text{ cm}^2$$

\therefore Area of the shaded region

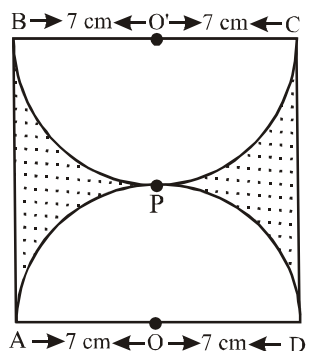
= Area of square ABCD –

(Area of R_1 + Area of R_2 + Area of R_3 + Area of R_4)

$$= (100 - 2 \times 21.5) \text{ cm}^2 = 57 \text{ cm}^2$$

Ex.43 Find the area of the shaded region in Fig. If ABCD is a square of side 14 cm and APD and BPC are semi-circles.

Sol. We have,



Area of the shaded region

= Area of square ABCD – Area of two semi-circles

$$= 14 \times 14 \text{ cm}^2 - 2 \left(\frac{1}{2} \times \frac{22}{7} \times 7^2 \right) \text{cm}^2$$

$$= 196 \text{ cm}^2 - 154 \text{ cm}^2 = 42 \text{ cm}^2$$

Ex.44 ABCD is a field in the shape of a trapezium.

$AB \parallel DC$ and $\angle ABC = 90^\circ$, $\angle DAB = 60^\circ$. Four sectors are formed with centers A, B, C and D. The radius of each sector is 17.5 m. Find the

- total area of the four sectors.
- area of remaining portion given that $AB = 75\text{m}$ and $CD = 50\text{m}$.

Sol. Since $AB \parallel CD$ and $\angle ABC = 90^\circ$. Therefore

$$\angle BCD = 90^\circ. \text{ Also, } \angle BAD = 60^\circ$$

$$\therefore \angle CDA = 180^\circ - 60^\circ = 120^\circ$$

[Co-interior angles]

- We have,

Total area of the four sectors

= Area of sector at A + Area of sector at B

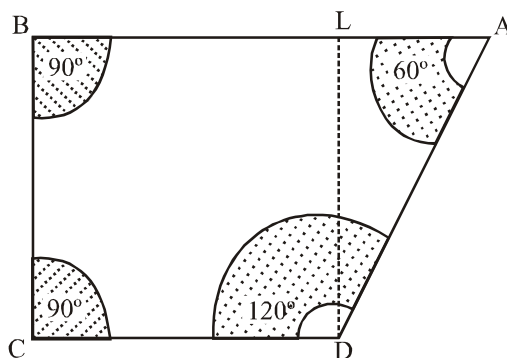
+ Area of sector at C + Area of sector at D.

$$= \frac{60}{360} \times \pi \times (17.5)^2 + \frac{90}{360} \times \pi \times (17.5)^2$$

$$+ \frac{90}{360} \times \pi \times (17.5)^2 + \frac{120}{360} \times \pi \times (17.5)^2$$

$$= \left\{ \left(\frac{1}{6} + \frac{1}{4} + \frac{1}{4} + \frac{1}{3} \right) \times \pi \times (17.5)^2 \right\} \text{m}^2$$

$$= \pi \times \left(\frac{35}{2} \right)^2 \text{m}^2 = \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{m}^2 = 962.5 \text{m}^2$$



(ii) Let DL be perpendicular drawn from D on AB. Then,

$$AL = AB - BL = AB - CD = (75 - 50) \text{ m} = 25 \text{ m}$$

In $\triangle ALD$, we have

$$\tan 60^\circ = \frac{DL}{AL} \Rightarrow \sqrt{3} = \frac{DL}{25} \Rightarrow DL = 25\sqrt{3} \text{ m}$$

\therefore Area of trapezium ABCD

$$= \frac{1}{2} (AB + CD) \times DL$$

$$= \frac{1}{2} (75 + 50) \times 25\sqrt{3} \text{ m}^2$$

$$= 1562.5 \times 1.732 \text{ m}^2 = 2706.25 \text{ m}^2$$

Hence,

Area of the remaining portion

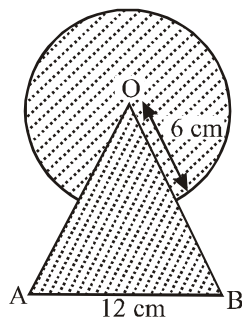
$$= \text{Area of trapezium ABCD} - \text{Area of 4 sectors}$$

$$= 2706.25 \text{ m}^2 - 962.5 \text{ m}^2 = 1743.75 \text{ m}^2$$

Ex.45 Find the area of the shaded region in Fig. where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

Sol. We have,

Required Area = Area of $\triangle OAB$ + Area of the circle – Area of a sector of a circle of radius 6 cm and of angle 60°



\Rightarrow Required Area

$$= \left\{ \frac{\sqrt{3}}{4} \times 12^2 + \pi \times 6^2 - \frac{60}{360} \times \pi \times 6^2 \right\} \text{ cm}^2$$

$$= (36\sqrt{3} + 36\pi - 6\pi) \text{ cm}^2$$

$$= \left(36\sqrt{3} + 30 \times \frac{22}{7} \right) \text{ cm}^2 = \left(\frac{660}{7} + 36\sqrt{3} \right) \text{ cm}^2$$

Ex.46 On a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. Find the area of the design (shaded region).

Sol. In $\triangle OBD$, we have

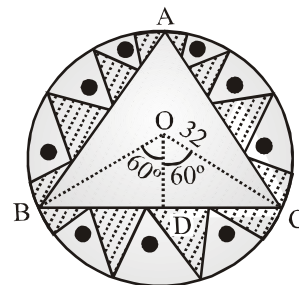
$$\cos 60^\circ = \frac{OD}{OB} \text{ and } \sin 60^\circ = \frac{BD}{OB}$$

$$\Rightarrow \frac{1}{2} = \frac{OD}{32} \text{ and } \frac{\sqrt{3}}{2} = \frac{BD}{32}$$

$$\Rightarrow OD = 16 \text{ and } BD = 16\sqrt{3}$$

$$\Rightarrow BC = 2 BD = 32\sqrt{3}$$

Area of the shaded region



$$= \text{Area of the circle} - \text{Area of } \triangle ABC$$

$$= \left\{ \pi \times 32^2 - \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2 \right\} \text{ cm}^2$$

$$= \left\{ \frac{22}{7} \times 32 \times 32 - 768\sqrt{3} \right\} \text{ cm}^2$$

$$= \left\{ \frac{22528}{7} - 768\sqrt{3} \right\} \text{ cm}^2$$

Ex.47 In Fig. AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of shaded region.

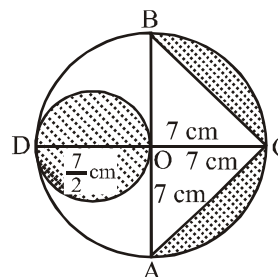
Sol. We have,

Area of the shaded region

$$= (\text{Area of circle with OD (= 7 cm) as diameter})$$

$$+ \text{Area of semi-circle with AB as diameter}$$

$$- \text{Area of } \triangle ABC$$



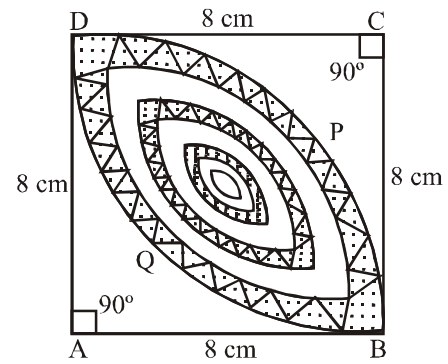
$$\begin{aligned}
&= \pi \times \left(\frac{7}{2}\right)^2 + 1/2 \times \pi \times (7)^2 - 1/2 \times AB \times OC \\
&= \left\{ \frac{\pi}{4} \times 49 + \frac{\pi}{2} \times 49 - \frac{1}{2} \times 14 \times 7 \right\} \text{cm}^2 \\
&= \left(\frac{3\pi}{4} \times 49 - 49 \right) \text{cm}^2 = \left(\frac{3}{4} \times \frac{22}{7} \times 49 - 49 \right) \text{cm}^2 \\
&= \frac{231 - 98}{2} \text{cm}^2 = 66.5 \text{cm}^2
\end{aligned}$$

Ex.48 Calculate the area of the designed region in Fig. common between two quadrants of circles of radius 8 cm each.

Sol. We have,

Area of the designed region

$$= 2(\text{Area of quadrant ABCD} - \text{Area of } \triangle ABD)$$



$$\begin{aligned}
&= 2 \left\{ \frac{\pi}{4} \times (8)^2 - \frac{1}{2} \times 8 \times 8 \right\} \text{cm}^2 \\
&= 2 \left\{ \frac{22}{7} \times \frac{1}{4} \times 64 - 32 \right\} \text{cm}^2 \\
&= 2 \left\{ \frac{22 \times 16}{7} - 32 \right\} \text{cm}^2 \\
&= 2 \left(\frac{352 - 224}{7} \right) \text{cm}^2 = \frac{256}{7} \text{cm}^2
\end{aligned}$$

IMPORTANT POINTS TO BE REMEMBERED

◆ For a circle of a radius r , we have

(i) Circumference = $2\pi r$

(ii) Area = πr^2

(iii) Area of semi-circle = $\frac{\pi r^2}{2}$

(iv) Area of a quadrant = $\frac{\pi r^2}{4}$

◆ If R and r are the radii of two concentric circles such that $R > r$ then area enclosed by the two circles

$$= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$$

◆ If a sector of a circle of radius r contains an angle of θ° . Then,

(i) Length of the arc of the sector

$$= \frac{\theta}{360} \times 2\pi r$$

$$= \frac{\theta}{360} \times (\text{Circumference of the circle})$$

(ii) Perimeter of the sector = $2r + \frac{\theta}{360} \times 2\pi r$

(iii) Area of the sector

$$= \frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360} \times (\text{Area of the circle})$$

(iv) Area of the segment

$$= \text{Area of the corresponding sector}$$

$$- \text{Area of the corresponding triangle}$$

$$= \frac{\theta}{360} \times \pi r^2 - r^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$= \left\{ \frac{\pi\theta}{360} - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right\} r$$

EXERCISE # 1

Q.1 Find the circumference and area of a circle of radius 4.2 cm.

Q.2 Find the circumference of a circle whose area is 301.84 cm^2 .

Q.3 Find the area of a circle whose circumference is 44 cm.

Q.4 The circumference of a circle exceeds the diameter by 16.8 cm. Find the circumference of the circle.

Q.5 A horse is tied to a pole with 28 m long string. Find the area where the horse can graze. (Take $\pi = 22/7$).

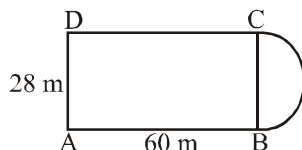
Q.6 A steel wire when bent in the form of a square encloses an area of 121 cm^2 . If the same wire is bent in the form of a circle, find the area of the circle.

Q.7 A sector of a circle of radius 4 cm contains an angle of 30° . Find the area of the sector.

Q.8 A sector of a circle of radius 8 cm contains an angle of 135° . Find the area of the sector.

Q.9 The area of a sector of a circle of radius 2 cm is $\pi \text{ cm}^2$. Find the angle contained by the sector.

Q.10 A plot is in the form of a rectangle ABCD having semi-circle on BC as shown in Fig. If $AB = 60 \text{ m}$ and $BC = 28 \text{ m}$, find the area of the plot.



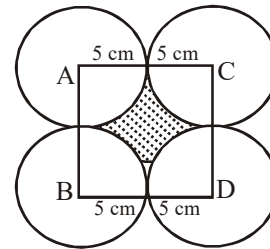
Q.11 A play ground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of

the rectangle are 36 m and 24.5 m, find the area of the playground. (Take $\pi = 22/7$).

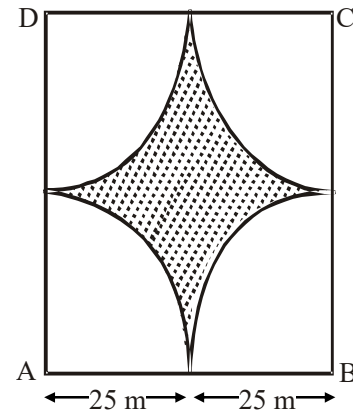
Q.12 The outer circumference of a circular race-track is 528 m. The track is everywhere 14m wide. Calculate the cost of levelling the track at the rate of 50 paise per square metre (Use $\pi = 22/7$).

Q.13 A rectangular piece is 20 m long and 15 m wide. From its four corners, quadrants of radii 3.5 m have been cut. Find the area of the remaining part.

Q.14 Four equal circles, each of radius 5 cm, touch each other as shown in Fig. Find the area included between them (Take $\pi = 3.14$).



Q.15 Four cows are tethered at four corners of a square plot of side 50 m, so that they just cannot reach one another. What area will be left ungrazed ?



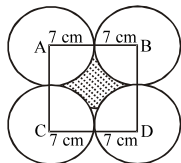
- Q.16** The circumference of two circles are in the ratio 2 : 3. Find the ratio of their areas.
- Q.17** The side of a square is 10 cm. Find the area of circumscribed and inscribed circles.
- Q.18** The sum of the radii of two circles is 140 cm and the difference of their circumferences is 88 cm. Find the diameters of the circles.
- Q.19** The area of a circle inscribed in an equilateral triangle is 154 cm^2 . Find the perimeter of the triangle. [Use $\pi = 22/7$ and $\sqrt{3} = 1.73$]
- Q.20** A field is in the form of a circle. A fence is to be erected around the field. The cost of fencing would be ₹ 2640 at the rate of ₹ 12 per metre. Then, the field is to be thoroughly ploughed at the cost of ₹ 0.50 per m^2 . What is the amount required to plough the field ? [Take $\pi = 22/7$].
- Q.21** If a square is inscribed in a circle, find the ratio of the areas of the circle and the square.
- Q.22** A park is in the form of a rectangle $120\text{m} \times 100 \text{ m}$. At the centre of the park there is a circular lawn. The area of park excluding lawn is 8700 m^2 . Find the radius of the circular lawn. (Use $\pi = 22/7$)
- Q.23** The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its area equal to the sum of the areas of the two circles.
- Q.24** The radii of two circles are 19 cm and 9 cm respectively. Find the radius and area of the circle which has its circumference equal to the sum of the circumferences of the two circles.
- Q.25** A car travels 1 kilometre distance in which each wheel makes 450 complete revolutions. Find the radius of the its wheels.
- Q.26** The area enclosed between two concentric circles is 770 cm^2 . If the radius of the outer circle is 21 cm, find the radius of the inner circle.
- Q.27** Find, in terms of π , the length of the arc that subtends an angle of 30° at the centre of a circle of radius 4 cm.
- Q.28** Find the angle subtended at the centre of a circle of radius 5 cm by an arc of length $(5\pi/3) \text{ cm}$.
- Q.29** An arc of length $20\pi \text{ cm}$ subtends an angle of 144° at the centre of a circle. Find in terms of π , the radius of the circle.
- Q.30** An arc of length 15 cm subtends an angle of 45° at the centre of a circle. Find the radius of the circle.
- Q.31** Find the angle subtended at the centre of a circle of radius 'a' by an arc of length $(a\pi/4) \text{ cm}$.
- Q.32** A chord AB of a circle of radius 15 cm makes an angle of 60° at the centre of the circle. Find the area of the major and minor segment. (Take $\pi = 3.14$, $\sqrt{3} = 1.73$)
- Q.33** In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
- length of the arc
 - area of the sector formed by the arc
 - area of the segment formed by the corresponding chord of the arc.

Q.34 A chord of a circle of radius 10 cm subtends a right angle at the centre. Find:

- area of the minor sector
- area of the minor segment
- area of the major sector
- area of the major segment (Use $\pi = 3.14$)

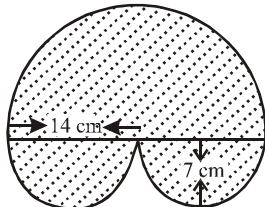
Q.35 A circular grassy plot of land, 42 m in diameter, has a path 3.5 m wide running round it on the outside. Find the cost of gravelling the path at ₹ 4 per square metre.

Q.36 Four equal circles are described about the four corners of a square so that each touches two of the others as shown in Fig. Find the area of the shaded region, each side of the square measuring 14 cm.



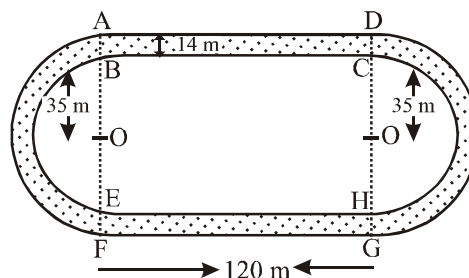
Q.37 Find to the three places of decimals the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

Q.38 Find the areas of the shaded region in the Fig.

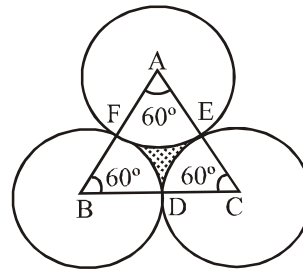


Q.39 In an equilateral triangle of side 24 cm, a circle is inscribed touching its sides. Find the area of the remaining portion of the triangle (Take $\sqrt{3} = 1.732$).

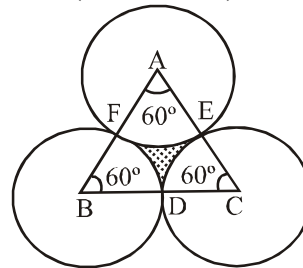
Q.40 An athletic track 14 m wide consists of two straight sections 120 m long joining semi-circular ends whose inner radius is 35 m. Calculate the area of the shaded region.



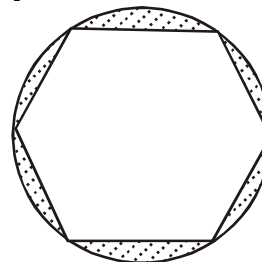
Q.41 The area of an equilateral triangle is $49\sqrt{3}$ cm². Taking each angular point as centre, a circle is described with radius equal to half the length of the side of the triangle as shown in Fig. Find the area of the triangle not included in the circle.



Q.42 The area of an equilateral triangle is 1732.05 cm². About each angular point as centre, a circle is described with radius equal to half the length of the side of the triangle. Find the area of the triangle not included in the circles (Use $\pi = 3.14$)



Q.43 A round table cover has six equal designs as shown in Fig. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 3.50 per cm². (Use $\sqrt{3} = 1.7$)



Answer Key

- | | | | | |
|--|--|-----------------------------|---------------------------------|---------------------------|
| 1. 26.4 cm, 55.44 cm ² | 2. 61.6 cm | 3. 154 cm ² | 4. 24.64 cm | 5. 2464 m ² |
| 6. 154 cm ² | 7. cm ² | 8. 24π cm ² | 9. 90° | 10. 1988 m ² |
| 11. 1353.625 m ² | 12. j- 3388 | 13. 261.5 m ² | 14. 21.5 m ² | 15. 535.71 m ² |
| 16. 4 : 9 | 17. 157m ² , 78.5 m ² | 18. 154 cm, 126 cm | 19. 72.7 cm | 20. j- 1925 |
| 21. $\pi : 2$ | 22. 21. 41m | 23. 10 cm | 24. 28 cm, 2464 cm ² | 25. 35.35 cm |
| 26. 14 cm | 27. cm | 28. 60° | 29. 25 cm | 30. cm |
| 31. 45° | 32. 20.295 cm ² , 686.205 cm ² | | | |
| 33. (i) 22cm | (ii) 231 cm ² | (iii) 40.05 cm ² | | |
| 34. (i) 78.5cm ² (ii) 28.5cm ² (iii) 235.5cm ² (iv) 285.5 cm ² | | | 35. 500.5 m ² | 36. 154 cm ² |
| 37. 14 cm | 38. 462 cm ² | 39. 98.55 cm ² | 40. 7056 m ² | 41. 7.77 cm ² |
| 42. 162. 01 cm ² | 43. j- 1626.80 | | | |
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EXERCISE # 2

Q.1 Find the circumference and the area of a circle of diameter 35 cm.

Q.2 The circumference of a circle is 39.6 cm. Find its area.

Q.3 The area of a circle is 301.84 cm^2 . Find its circumference.

Q.4 A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3} \text{ cm}^2$. The same wire is bent to form a circle. Find the area enclosed by the circle.

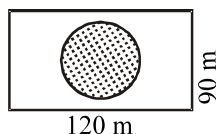
Q.5 The length of a chain used as the boundary of a semicircular park is 90 m. Find the area of the park.

Q.6 The sum of the radii of two circles is 7 cm, and the difference of their circumferences is 8 cm. Find the circumferences of the circles.

Q.7 Find the area of a ring whose outer and inner radii are respectively 23 cm and 12 cm.

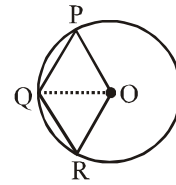
Q.8 A path of 8 m width runs around the outside of a circular park whose radius is 17 m. Find the area of the path.

Q.9 A park is in the form of a rectangle 120 m by 90 m. At the centre of the park there is a circular lawn as shown in the figure. The area of the park excluding the lawn is 2950 m^2 . Find the radius of the circular lawn. (Given: $\pi = 3.14$)



Q.10 In the given figure, OPQR is a rhombus, three of whose vertices lie on a circle with centre

O. If the area of the rhombus is $32\sqrt{3} \text{ cm}^2$, find the radius of the circle.



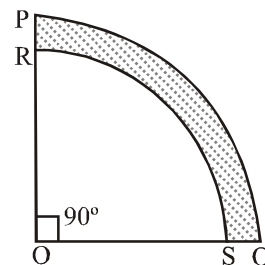
Q.11 Find the area of a quadrant of a circle whose circumference is 22 cm.

Q.12 A horse is placed for grazing inside a rectangular field 70 m by 52 m. It is tethered to one corner by a rope 21 m long. On how much area can it graze ? How much area is left ungrazed ?

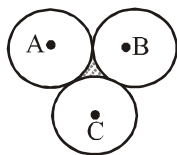
Q.13 A horse is tethered to one corner of a field which is in the shape of an equilateral triangle of side 12 m. If the length of the rope is 7 m, find the area of the field which the horse cannot graze. Take $\sqrt{3} = 1.732$. Write the answer correct to 2 places of decimal.

Q.14 Four cows are tethered at the four corners of a square field of side 50 m such that each can graze the maximum unshared area. What area will be left ungrazed ? Take $\pi = 3.14$.

Q.15 In the given figure, PQSR represents a flower bed. If $OP = 21 \text{ m}$ and $OR = 14 \text{ m}$, find the area of the flower bed.

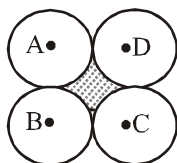


- Q.16** Three equal circles, each of radius 6 cm, touch one another as shown in the figure. Find the area enclosed between them. Take $\pi = 3.14$ and $\sqrt{3} = 1.732$.



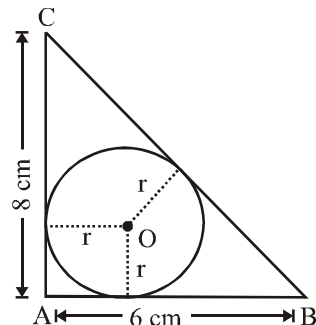
- Q.17** If three circles of radius a each, are drawn such that each touches the other two, then find the area included between them. (Take $\pi = 3.14$ and $\sqrt{3} = 1.732$)

- Q.18** Four equal circles, each of radius 5 cm, touch each other, as shown in the figure. Find the area included between them. Take $\pi = 3.14$.



- Q.19** Four equal circles, each of radius a units, touch each other. Find the area between them.
- Q.20** A rope by which a cow is tethered is increased from 16 m to 23 m. How much additional ground does it have now to graze ?

- Q.21** In the given figure, $\triangle ABC$ is right angled at A, with $AB = 6$ cm and $AC = 8$ cm. A circle with centre O had been inscribed inside the triangle. Find the value of r the radius of the inscribed circle.



- Q.22** A pendulum swings through an angle of 30° and describes an arc 8.8 cm in length. Find the length of the pendulum.
- Q.23** A circular disc of radius 6 cm is divided into three sectors with central angles 90° , 120° and 150° . What part of the whole circle is the sector with central angle 150° ? Also, calculate the ratio of the areas of the three sectors.

Answer Key

1. 110 cm, 962.5 cm²

2. 134.74 cm²

3. 61.6 cm

4. 346.5 cm²

5. 481.25 cm²

6. 26 cm, 18 cm

7. 1210 cm²

8. 1056 m²

9. 50 m

10. 8 cm

11. 9.625 cm²

12. 346.5 m², 3293.5 m²

13. 36.69 m²

14. 537.5 m²

15. 192.5 m²

16. 5.76 cm²

17. $\frac{4}{25}a^2$

18. 21.5 cm²

19. $\frac{6}{7}a^2$

20. 858 m²

21. $r = 2$ cm

22. 16.8 cm

23. 5/12; 3 : 4 : 5