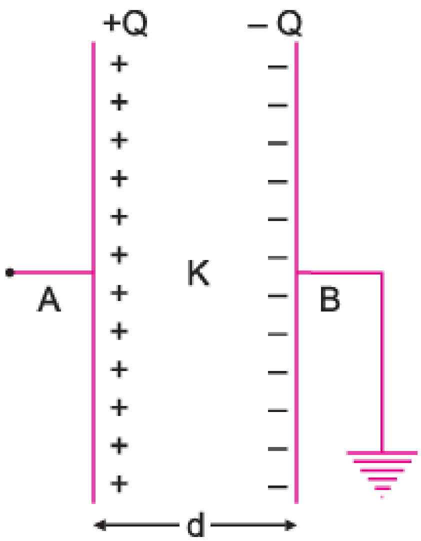
Long Answer Questions

Q. 1. Briefly explain the principle of a capacitor. Derive an expression for the capacitance of a parallel plate capacitor, whose plates are separated by a dielectric medium.

Ans. Principle of a Capacitor: A capacitor works on the principle that the capacitance of a conductor increases appreciably when an earthed conductor is brought near it.

Parallel Plate Capacitor: Consider a parallel plate capacitor having two plane metallic plates A and B, placed parallel to each other (see fig.). The plates carry equal and opposite charges +Q and –Q respectively.

In general, the electric field between the plates due to charges +Q and –Q remains uniform, but at the edges, the electric field lines deviate outward. If the separation between the plates is much smaller than the size of plates, the electric field strength between the plates may be assumed uniform.



Let A be the area of each plate, 'd' the separation between the plates, K the dielectric constant of medium between the plates. If σ is the magnitude of charge density of plates, then

$\sigma = \frac{Q}{A}$

The electric field strength between the plates

$E = \frac{\sigma}{K\varepsilon_0}$ where = ε_0 permittivity of free space. ...(i)

The potential difference between the plates, $V_{AB} = \text{Ed} = \frac{\sigma d}{K\varepsilon_0}$ ...(ii)

Putting the value of σ, we get

$V_{AB} = \frac{(Q/A)d}{K\varepsilon_0} = \frac{\sigma d}{K\varepsilon_0 A}]$

∴ Capacitance of capacitor,

$C = \frac{Q}{V_{AB}} = \frac{Q}{(\text{Qd}/K\varepsilon_0 A)}$ or $C = \frac{K\varepsilon_0 A}{d}$ ...(iii)

This is a general expression for capacitance of parallel plate capacitor. Obviously, the capacitance is directly proportional to the dielectric constant of medium between the plates.

For air capacitor (K=1); capacitance. This is expression for the capacitance $C = \frac{\varepsilon_0 A}{d}$. of a parallel plate air capacitor. It can be seen that the capacitance of parallel plate (air) capacitor is:

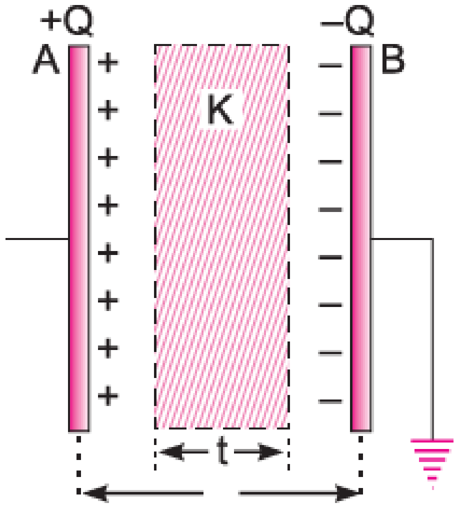
(a) Directly proportional to the area of each plate.

(b) Inversely proportional to the distance between the plates.

(c) Independent of the material of the plates.

Q. 2. Derive an expression for the capacitance of a parallel plate capacitor when a dielectric slab of dielectric constant K and thickness $t = \frac{d}{2}$ but of same area as that of the plates is inserted between the capacitor plates. (d = separation between the plates). [CBSE (F) 2010]

Ans. Consider a parallel plate capacitor, area of each plate being – A, the separation between the plates being d. Let a dielectric slab of dielectric constant K and thickness t < d be placed between the plates. The thickness of air between the plates is (d – t). If charges on plates are +Q and – Q, then surface charge density



$\sigma = \frac{Q}{A}$

The electric field between the plates in air, $E_1 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

The electric field between the plates in slab, $E_2 = \frac{\sigma}{K\varepsilon_0} = \frac{Q}{K\varepsilon_0 A}$

∴ The potential difference between the plates

VAB = work done in carrying unit positive charge from one plate to another

= ΣEx (as field between the plates is not constant).

$$= E_1(d-t)+E_2 t = \frac{Q}{\varepsilon_0 A}(d-t)+\frac{Q}{K\varepsilon_0 A}t$$

$$\therefore \quad \therefore \qquad V_{AB} = \frac{Q}{\varepsilon_0 A}\left[d - t + \frac{t}{K}\right]$$

$$\therefore \text{ Capacitance of capacitor, } C = \frac{Q}{V_{AB}} = \frac{Q}{\frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{K}\right)}$$

$$\text{or,} \qquad C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

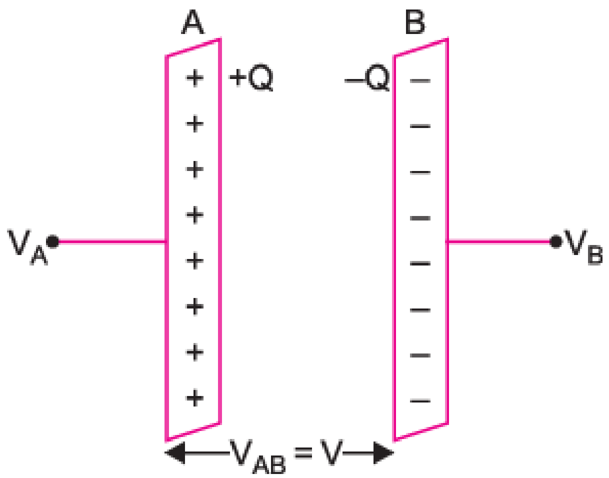
$$\text{Here, } t = \frac{d}{2} \qquad \therefore \qquad C = \frac{\varepsilon_0 A}{d - \frac{d}{2}\left(1 - \frac{1}{K}\right)} = \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 - \frac{1}{K}\right)}$$

Q. 3. Derive an expression for the energy stored in a parallel plate capacitor C, charged to a potential difference V. Hence derive an expression for the energy density of a capacitor. [CBSE (AI) 2012, (F) 2013, Allahabad 2015]

OR

Obtain an expression for the energy stored per unit volume in a charged parallel plate capacitor.

b. Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2 so that the energy stored in the two cases becomes the same.
[CBSE Central 2016]



Ans. (a) When a capacitor is charged by a battery, work is done by the charging battery at the expense of its chemical energy. This work is stored in the capacitor in the form of electrostatic potential energy.

Consider a capacitor of capacitance C. Initial charge on capacitor is zero. Initial potential difference between capacitor plates is zero. Let a charge Q be given to it in small steps. When charge is given to capacitor, the potential difference between its plates increases. Let at any instant when charge on capacitor be q, the potential difference between its plates $V = \frac{q}{C}$

Now work done in giving an additional infinitesimal charge dq to capacitor.

$$\mathrm{dW} = V\,\mathrm{dq} = \frac{q}{C}\mathrm{dq}$$

The total work done in giving charge from 0 to Q will be equal to the sum of all such infinitesimal works, which may be obtained by integration. Therefore total work

$$W=\int_0^Q V\ \mathrm{dq}=\int_0^Q \frac{q}{C}\mathrm{dq}=\frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q=\frac{1}{C}\left(\frac{Q^2}{2}-\frac{0}{2}\right)=\frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then Q=CV

$$\therefore \qquad W=\frac{(\mathrm{CV})^2}{2C}=\frac{1}{2}\mathrm{CV}^2=\frac{1}{2}\mathrm{QV}$$

This work is stored as electrostatic potential energy of capacitor i.e.,

$$\text{Electrostatic potential energy, } U=\frac{Q^2}{2C}=\frac{1}{2}\mathrm{CV}^2=\frac{1}{2}\mathrm{QV}$$

Energy density: Consider a parallel plate capacitor consisting of plates, each of area A, separated by a distance d. If space between the plates is filled with a medium of dielectric constant K, then

$$\text{Capacitance of capacitor, } C=\frac{K\varepsilon_0 A}{d}$$

If σ is the surface charge density of plates, then electric field strength between the plates.

$$E=\frac{\sigma}{K\varepsilon_0}\Rightarrow\sigma=K\varepsilon_0 E$$

$$\text{Charge on each plate of capacitor, } Q=\sigma A=K\varepsilon_0\,\mathrm{EA}$$

$$\text{Energy stored by capacitor, } U=\frac{Q^2}{2C}=\frac{(K\varepsilon_0\,\mathrm{EA})^2}{2(K\varepsilon_0 A/d)}=\frac{1}{2}K\varepsilon_0 E^2\,\mathrm{Ad}$$

But Ad = volume of space between capacitor plates

$$\therefore \text{Energy stored, } U=\frac{1}{2}K\varepsilon_0 E^2\,\mathrm{Ad}$$

$$\text{Electrostatic Energy stored per unit volume, } u_e=\frac{U}{\mathrm{Ad}}=\frac{1}{2}K\varepsilon_0 E^2$$

This is expression for electrostatic energy density in medium of dielectric constant K.

In air or free space (K=1) therefore energy density, $u_e=\frac{1}{2}\varepsilon_0 E^2$

b. $U_S = \frac{1}{2} C_S {V_S}^2 \quad \Rightarrow \quad U_p = \frac{1}{2} C_P {V_P}^2$

$$\text{Also}, \frac{C_1}{C_2} = \frac{1}{2} \,(\text{ given }) \Rightarrow C_2 = 2C_1$$

$$\Rightarrow \qquad \frac{V_{\text{series}}}{V_{\text{parallel}}} = \sqrt{\frac{C_{\text{equivalent} \quad \text{parallel}}}{C_{\text{equivalent} \quad \text{series}}}}$$

$$= \sqrt{\frac{\frac{C_1 + C_2}{C_1 C_2}}{C_1 + C_2}}$$

$$= \frac{C_1 + C_2}{\sqrt{C_1 C_2}} = \frac{3C_1}{\sqrt{2{C_1}^2}} = \frac{3}{\sqrt{2}}$$

Q. 4. Find the expression for the energy stored in the capacitor. Also find the energy lost when the charged capacitor is disconnected from the source and connected in parallel with the uncharged capacitor. Where does this loss of energy appear?
[CBSE Sample Paper 2017]

Ans.

$Q = Q_1 + Q_2$

$V_1 = V_2$ potential of both capacitors after they are connected with each other.

$$\therefore \qquad \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \qquad \Rightarrow \qquad Q = \left(\frac{C_1}{C_2} + 1\right) Q_2$$

$$Q_2 = \frac{QC_2}{C_1 + C_2} \qquad Q_1 = \frac{QC_1}{C_1 + C_2}$$

$$V_2 = V_1 = \frac{Q}{C_1 + C_2} = \frac{Q_2}{C_2} = \frac{Q_1}{C_1}$$

$$U_f = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = \frac{1}{2}(C_1 + C_2)\frac{Q^2}{(C_1 + C_2)^2} = \frac{Q^2}{2(C_1 + C_2)}$$

$$U_i = \frac{Q^2}{2C_1}$$

$$U_i - U_f = \frac{Q^2}{2C_1} - \frac{Q^2}{2(C_1 + C_2)} = \frac{Q^2(C_2)}{(C_1)(C_1 + C_2)}$$

The lost energy appears in the form of heat.

Q. 5. Answer the following questions

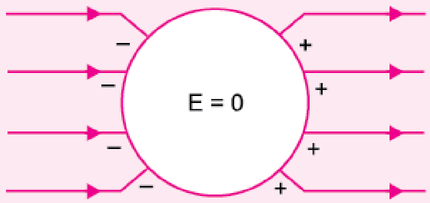
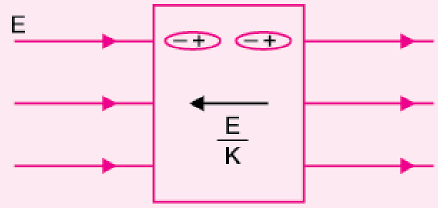
(i) Distinguish, with the help of a suitable diagram, the difference in the behaviour of a conductor and a dielectric placed in an external electric field. How does polarised dielectric modify the original external field?

(ii) A capacitor of capacitance C is charged fully by connecting it to a battery of emf E. It is then disconnected from the battery. If the separation between the plates of the capacitor is now doubled, how will the following change?

(a) Charge stored by the capacitor.
(b) Field strength between the plates.
(c) Energy stored by the capacitor.

Justify your answer in each case. [CBSE North 2016]

Ans. (i)

	Conductor		Dielectric
			 where K is dielectric constant
1.	No electric field lines travel inside conductor.	1.	Alignment of atoms takes place due to electric field.
2.	Electric field inside a conductor is zero.	2.	This results in a small electric field inside dielectric in opposite direction. Net field inside the dielectric is $\frac{E}{K}$.

Induced electric field, due to polarisation of dielectric, is in opposite direction to the applied field.

$E_{net} = E_o - E_p$

(ii) (a) Charge remains same, as after disconnecting capacitor no transfer of charge take place.

(b) Electric field, $E = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A}$ remain same, as there is no change in charge.

(c)
$$\text{Energy stored} = \frac{q^2}{2C} = \frac{q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)} = \frac{q^2 d}{2\varepsilon_o A}$$

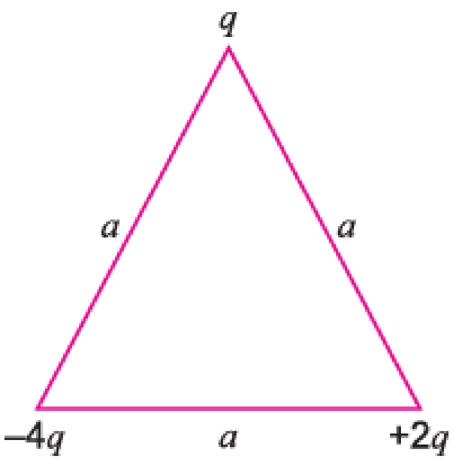
Energy will be doubled as separation between the plates (d) is doubled.

Q. 6. Answer the following questions

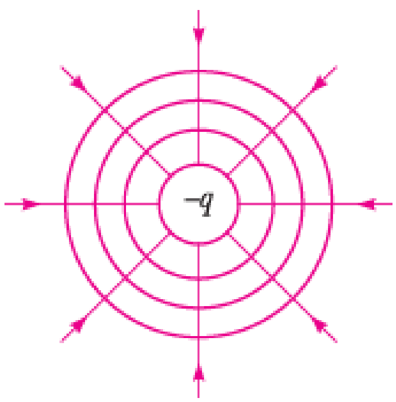
(i) Explain why, for any charge configuration, the equipotential surface through a point is normal to the electric field at that point.

Draw a sketch of equipotential surfaces due to a single charge (– q), depicting the electric field lines due to the charge.

(ii) Obtain an expression for the work done to dissociate the system of three charges placed at the vertices of an equilateral triangle of side ‘a’ as shown alongside.
[CBSE North 2016]



Ans. The work done in moving a charge from one point to another on an equipotential surface is zero. If the field is not normal to an equipotential surface, it would have a non-zero component along the surface. This would imply that work would have to be done to move a charge on the surface which is contradictory to the definition of equipotential surface.



Mathematically

Work done to move a charge dq, on a surface, can be expressed as

$$dW = dq\,(\vec{E}\,.\,\vec{dr})$$

But dW = 0 on an equipotential surface

$$\therefore \qquad \vec{E} \perp \vec{dr}$$

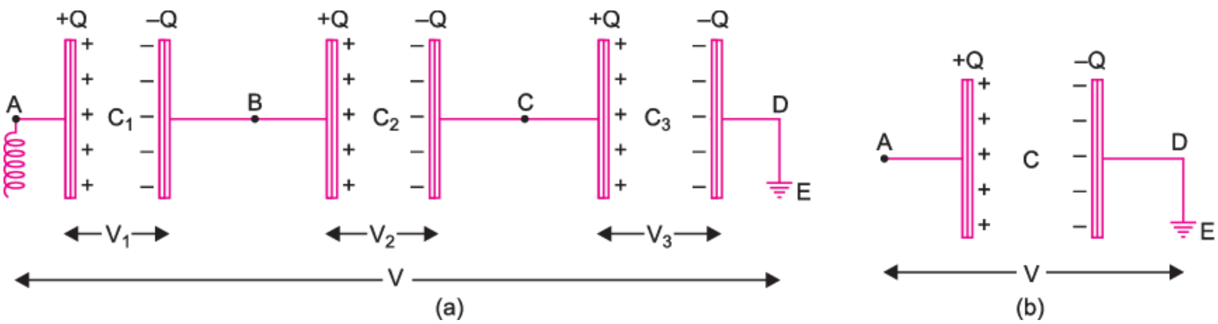
Equipotential surfaces for a charge –q is shown alongside.

Q. 7. Answer the following questions.

(i) Derive an expression for equivalent capacitance of three capacitors when connected in series

(ii) Derive an expression for equivalent capacitance of three capacitors when connected in parallel.

Ans. In fig. (a) Three capacitors of capacitances C_1, C_2, C_3 are connected in series between points A and D.



In series' first plate of each capacitor has charge +Q and second plate of each capacitor has charge –Q i.e., charge on each capacitor is Q.

Let the potential differences across the capacitors C_1, C_2, C_3 be V_1, V_2, V_3 respectively. As the second plate of first capacitor C_1 and first plate of second capacitor C_2 are connected together, their potentials are equal. Let this common potential be VB. Similarly the common potential of second plate of C_2 and first plate of C3 is VC. The second plate of capacitor C_3 is connected to earth, therefore its potential VD=0. As charge flows from higher potential to lower potential, therefore $V_A > V_B > V_C > V_D$.

For the first capacitor, $V_1 = V_A - V_B = \frac{Q}{C_1}$ ...(i)

For the second capacitor, $V_2 = V_B - V_C = \frac{Q}{C_2}$ ...(ii)

For the third capacitor, $V_3 = V_C - V_D = \frac{Q}{C_3}$ ...(iii)

Adding (i), (ii) and (iii), we get

$$V_1 + V_2 + V_3 = V_A - V_D = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad ...(iv)$$

If V be the potential difference between A and D, then

$$V_A - V_D = V$$

$\therefore$ From (iv), we get

$$V = (V_1 + V_2 + V_3) = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad ...(v)$$

If in place of all the three capacitors, only one capacitor is placed between A and D such that on giving it charge Q, the potential difference between its plates become V, then it will be called equivalent capacitor. If its capacitance is C, then

$$V = \frac{Q}{C} \quad ...(vi)$$

Comparing (v) and (vi), we get

$$\frac{Q}{C} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad \text{or} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad ...(viii)$$

Q. 8. Answer the following Questions

(i) If two similar large plates, each of area A having surface charge densities +σ and –σ are separated by a distance d in air, find the expressions for

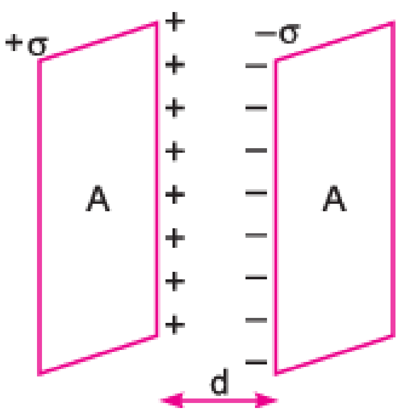
(ii) Field at points between the two plates and on outer side of the plates. Specify the direction of the field in each case.

(iii) The potential difference between the plates.

(iv) The capacitance of the capacitor so formed. **[CBSE Central 2016]**

Ans. (i) Let the two large plates each of area A having surface charge densities+σ and –σ are separated by a distance d in air. We know that electric field due to plate having charge density σ is

$$E = \frac{\sigma}{2\varepsilon_0}$$



If σ is positive, then electric field will be outward and if σ is negative, then electric field will be inward.

Q. 9. Answer the following questions.

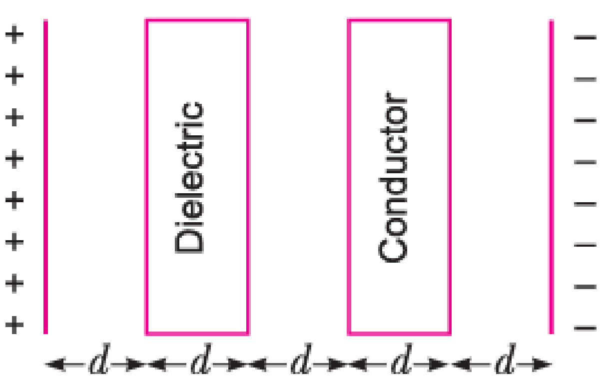
(i) Compare the individual dipole moment and the specimen dipole moment for H2O molecule and O2 molecule when placed in

(a) Absence of external electric field

(b) Presence of external eclectic field. Justify your answer.

(ii) Given two parallel conducting plates of area A and charge densities +σ and –σ . A dielectric slab of constant K and a conducting slab of thickness d each are inserted in between them as shown.

$_2$O



(a) Find the potential difference between the plates.

(b) Plot E versus x graph, taking x = 0 at positive plate and x = 5d at negative plate.
[CBSE Sample Paper 2016]

Ans. (i)

	Non-Polar (O_2)	Polar (H_2O)
(a) Absence of electric field		
Individual	No dipole moment exists	Dipole moment exists
Specimen	No dipole moment exists	Dipole are randomly oriented. Net P = 0
(b) Presence of electric field		
Individual	Dipole moment exists (molecules become polarised)	Torque acts on the molecules to align them parallel to $\vec{E}$
Specimen	Dipole moment exists	Net dipole moment exists parallel to dipole moment exists $\vec{E}$

(ii) (a) The potential difference between the plates is given by

(b) E versus x graph