CBSE Class 10 Mathematics Standard

Sample Paper - 03 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

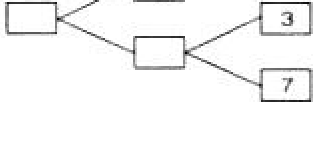
i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. Write the condition to be satisfied by q so that a rational number $\frac{p}{q}$ has a nonterminating decimal expansion.

OR

Complete the missing entries in the following factor tree.



2. State whether the following equation is quadratic equation or not ?

   $x^2 - x + 3 = 0$

3. Determine the values of m and n so that the following system of linear equations have infinite number of solutions:

   $(2m - 1)x + 3y - 5 = 0$

   $3x + (n - 1)y - 2 = 0$

4. A line touches a circle of radius 4 cm. Another line is drawn which is tangent to the circle. If the two lines are Parallel find the distance between them.

5. Find the 35th term of the A.P 20, 17, 14, 11, ...

OR

Find the 12^{th} term of the AP with first term 9 and common difference 10.

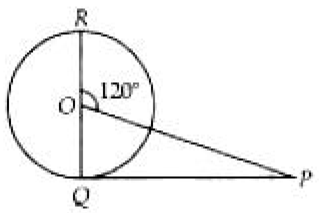
6. Find the sum of first 22 terms of the AP 8,3, - 2,...

7. If - 2 is a root of the equation $3x^2 - 5x + 2k = 0$, find the value of k.

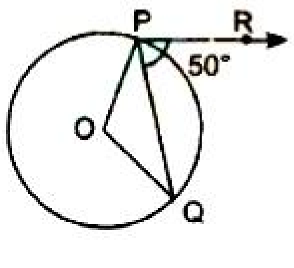
OR

Find the positive root of $\sqrt{3x^2 + 6} = 9$.

8. PQ is a tangent drawn from an external point P to a circle with centre O, QOR is the diameter of the circle. If $\angle POR = 120^o$, What is the measure of OPQ?



9. In figure, if O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$.



OR

What term will you use for a line which intersect a circle at two distinct points?

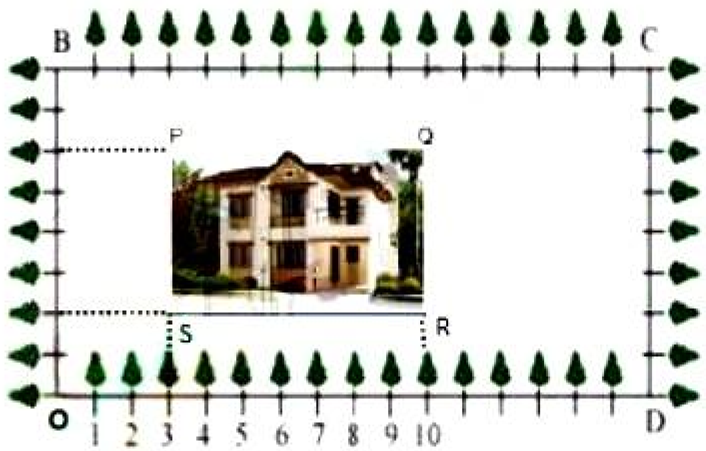
10. The sides of certain triangle are (a - 1)cm, $2\sqrt{a}$ cm, (a+1) Determine whether the triangle is a right triangle.
11. Write the n^{th} term of the A.P. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, \ldots\ldots$
12. Find the value of sin 38° - cos 52°.
13. Prove the trigonometric identity: $(\sec^2\theta - 1)\cot^2\theta = 1$
14. Two cubes each of side 4 cm are joined end to end. Find the surface area of the resulting cuboid.
15. For what value of k will the consecutive terms 2k + 1, 3k + 3 and 5k - 1 form an A.P.?
16. A card is drawn from a well shuffled deck of the 52 playing cards. Find the probability that the card will not be an ace.
17. **CASE STUDY:** CARTESIAN- PLANE

Using Cartesian Coordinates we mark a point on a graph by **how far along** and **how far up** it is.

The *left-right* (**horizontal**) direction is commonly called X-axis.

The *up-down* (**vertical**) direction is commonly called Y-axis.

In Green Park, New Delhi Ramesh is having a rectangular plot ABCD as shown in the following figure. Sapling of Gulmohar is planted on the boundary at a distance of 1m from each other. In the plot, Ramesh builds his house in the rectangular area PQRS. In the remaining part of plot, Ramesh wants to plant grass.



i. The coordinates of vertices P and S of rectangle PQRS are respectively:
   a. (2,3), (6,3)
   b. (3,2), (3,6)
   c. (6,3), (2,3)

d. (3,6), (3,2)

ii. The coordinates of mid-point of diagonal QS is given by

a. $(\frac{13}{2}, 4)$
b. $(\frac{13}{4}, 2)$
c. $(\frac{4}{13}, 2)$
d. $(\frac{2}{13}, 4)$

iii. The area of rectangle PQRS is

a. 28 m^2
b. 28 km^2
c. 28m
d. 28m^3

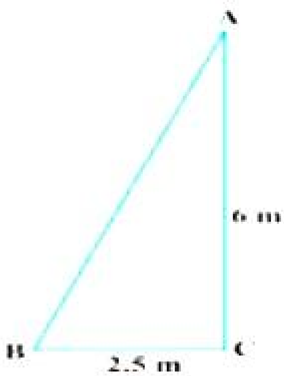
iv. The coordinates of vertices R and Q of rectangle PQRS are respectively:

a. (10, 6), (10, 2)
b. (2, 10), (10, 6)
c. (10, 2), (10, 6)
d. (2,10), (6,1)

v. The length and breadth of rectangle PQRS respectively are:

a. 4, 7
b. 7, 4
c. 6, 4
d. 4, 4

18. There is some fire incident in the house. The fireman is trying to enter the house from the window as the main door is locked. The window is 6 m above the ground. He places a ladder against the wall such that its foot is at a distance of 2.5 m from the wall and its top reaches the window.



i. Here, ________ be the ladder and ________ be the wall with the window.

a. CA, AB

b. AB, AC

c. AC, BC

d. AB, BC

ii. We will apply Pythagoras Theorem to find length of the ladder. It is:

a. $AB^2 = BC^2 - CA^2$

b. $CA^2 = BC^2 + AB^2$

c. $BC^2 = AB^2 + CA^2$

d. $AB^2 = BC^2 + CA^2$

iii. The length of the ladder is ________.

a. 4.5 m

b. 2.5 m

c. 6.5 m

d. 5.5 m

iv. What would be the length of the ladder if it is placed 6 m away from the wall and the window is 8 m above the ground?

a. 12 m

b. 10 m

c. 14 m

d. 8 m

v. How far should the ladder be placed if the fireman gets a 9 m long ladder?

a. 6.7 m (approx.)

b. 7.7 m (approx.)

c. 5.7 m (approx.)

d. 4.7 m (approx.)

19. 

A medical camp was held in a school to impart health education and the importance of exercise to children. During this camp, a medical check of 35 students was done and their weights were recorded as follows:

Weight (in Kg)	**Number of Students**
below 40	3
below 42	5
below 44	9
below 46	14
below 48	28
below 50	31
below 42	35

i. Modal class of given data:
   a. 46-48
   b. 48-50
   c. 44-46
   d. 50-52
ii. Compute the modal weight.
   a. 47.9 kg
   b. 46.9 kg
   c. 57.9 Kg
   d. 77.9 Kg
iii. Which of the following can not be determined graphically?
   a. Mean
   b. Median
   c. Mode
   d. None of these
iv. The mean of the following data:
   a. 55.58
   b. 50.85
   c. 45.85
   d. 35.56

v. Mode and the mean of data are 12k and 15K. The median of the data is:

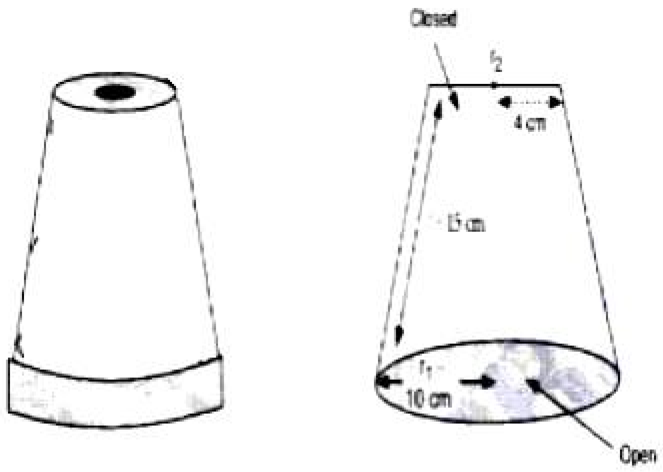
a. 12k

b. 14k

c. 15k

d. 16k

20. During the battle of Turks against the Rajputs of India, the Turk soldiers wore a costume with a metallic shield-like knee pads, buckler (elbow shield) and cap to save themselves from injuries. The headgear cap (a fez) used by these soldiers is shaped like the frustum of a cone with its radius on the open side 10 cm, and radius at the upper base as 4 cm and its slant height as 15 cm.



By using the above information, find the following:

i. The curved surface area of the cap is:

a. 650 cm^2

b. 660 cm^2

c. 606 cm^2

d. 666 cm^2

ii. Area of the closed base is:

a. 55.285

b. 50.285

c. 52.285

d. 56.285

iii. The area of the material used for making it.

a. 701.28 cm^2

b. 720.28 cm^2

c. 710.28 cm^2

d. 717.28 cm^2

iv. During the conversion of a solid from one shape to another the volume of the new shape will:

a. increase

b. remain unaltered

c. double

d. decrease

v. The formula to find the volume of the frustum of a cone is:

a. $\frac{2}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

b. $\frac{1}{3}\pi h(r_1^2 + r_2^2)$

c. $\frac{1}{3}\pi h(r_1^3 + r_2^3 + r_1 r_2)$

d. $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Part-B

21. Express $0.2\overline{54}$ as a fraction in simplest form.

22. Find the ratio in which the line segment joining the points(-3, 10) and (6, -8) is divided by (-1, 6).

OR

If point $\left(\frac{1}{2}, y\right)$ lies on the line segment joining the points A(3, -5) and B(-7, 9), then find the ratio is which P divides AB. Also find the value of y.

23. Find the zeroes of the quadratic polynomial $2x^2 - 25$.

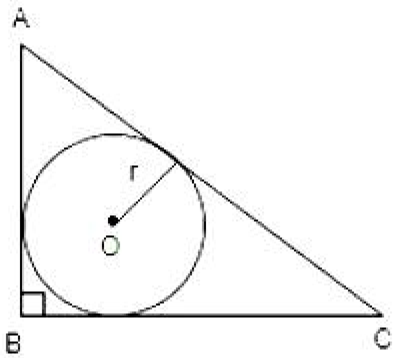
24. Draw a circle of radius 1.5 cm. Take a point P outside it. Without using the centre draw two tangents to the circle from the point P.

25. Evaluate: $\sin^2 30° \cos^2 45° + 4\tan^2 30° + \frac{1}{2}\sin^2 90° + \frac{1}{8}\cot^2 60°$.

OR

Prov that: $\frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\sec A + 1}{\sec A - 1}$

26. In the adjoining figure, a right angled $\triangle ABC$, circumscribes a circle of radius r. If AB and BC are of lengths 8 cm and 6 cm respectively, then find the value of r.



27. Prove that $\sqrt{2}$ is an irrational number.
28. In a class test, the sum of the marks obtained by a student in mathematics and science is 28. Had he got 3 marks more in mathematics and 4 marks less in science, the product of the marks would have been 180. Find his marks in two subjects.

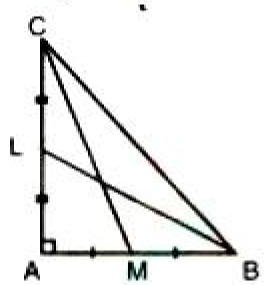
OR

Find the value of k so that the quadratic equation $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$ has equal roots.

29. If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate: $\frac{1}{\alpha} - \frac{1}{\beta}$
30. In an isosceles $\triangle ABC$, AB = AC and $BD \perp AC$. Prove that $(BD^2 - CD^2) = 2CD \cdot AD$

OR

In the given figure, BL and CM are medians of $\triangle ABC$, right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.



31. Cards bearing numbers 1,3,5,...,35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing
    i. a prime number less than 15,
    ii. a number divisible by 3 and 5.

32. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the sun is 60^o. Find the angle of elevation of the sun at the time of the longer shadow.

33. If the median for the following frequency distribution is 28.5, find the value of x and y:

Class	Frequencies
0 - 10	5
10 -20	x
20 -30	20
30 -40	15
40 -50	y
50 -60	5
Total	60

34. Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use π = 22/7).

35. Draw the graph of x - y + 1 = 0 and 2x + y - 10 = 0.

Shade the region bounded by these lines and x-axis. Find the area of the shaded region

36. A boy standing on a horizontal plane finds a bird flying at a distance of 100m from him at an elevation of 30°. A girl standing on the roof of a 20-m-high building, finds the angle of elevation of the same bird to be 45°. The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. [Given $\sqrt{2}$=141.]

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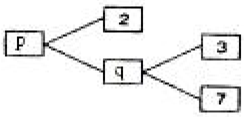
Solution

Part-A

1. If the prime factorization of q is not of the form $2^m \times 5^n$, where m, n are non-negative integers. Then given decimal is non-terminating decimal expansion.

OR

Here, we have to fill the entries p and q



Here, q = 3 × 7

q = 21

Therefroe, p = 2 × q = 2 × 21

p = 42

2. We have, $x^2 - x + 3 = 0$.

Clearly, it is in the form of $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

3. We have to determine the values of m and n so that the following system of linear equations have infinite number of solutions:

(2m - 1)x + 3y - 5 = 0

3x + (n - 1)y - 2 = 0

It is given that (2m - 1)x + 3y - 5 = 0 ...(i)

where $a_1 = 2m - 1, b_1 = 3, c_1 = -5$

3x + (n - 1)y - 2 = 0 ...(ii)

where $a_2 = 3, b_2 = n - 1, c_2 = 2$

For a pair of linear equations to have an infinite number of solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

or $\frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$

$\frac{2m-1}{3} = \frac{5}{2}$

or 2(2m -1) = 15

or, 4m - 2 = 15

or, 4m = 17

$m = \frac{17}{4}$

and $\frac{3}{n-1} = \frac{5}{2}$

or, 5(n - 1) = 6

or, 5n - 5 = 6

or, 5n = 11

or, $n = \frac{11}{5}$

Hence, $m = \frac{17}{4}, n = \frac{11}{5}$

4. Given, A line touches a circle of radius 4 cm.

   Distance between two parallel tangents = Diameter of the circle = 2 × 4 = 8 cm.

5. The given A.P is 20, 17, 14, 11, ...

   First term, a = 20

   Common difference, d = - 3

   nth term of the A.P, a_n = a + (n - 1)d = 20 + (n - 1) × (- 3)

   ∴ 35th term of the A.P, a_{35} = 20 + (35 - 1) × (-3) = 20 - 102 = - 82

OR

a = 9, d = 10

∴ a_{12} = a + 11d

= 9 + 11(10) = 119

6. Here $a = 8,\ d = 3 - 8 = -5.$

   So, $S_{22} = \frac{22}{2}$ [2a + (22 - 1)d]

   $\Rightarrow S_{22} = 11(16 - 105) = -979$

7. We have the following equation,

   $3x^2 - 5x + 2k = 0$

   Putting x = -2

   $3(-2)^2 - 5(-2) + 2k = 0$

   ⇒ 12 + 10 + 2k = 0

$\Rightarrow 22 + 2k = 0$

$\Rightarrow k = \frac{-22}{2}$

$\Rightarrow k = -11$

OR

$\sqrt{3x^2+6} = 9$

$3x^2 + 6 = 81$

$3x^2 = 81 - 6 = 75$

$x^2 = 25$

$\therefore x = \pm 5$

Hence, Positive root = 5

8. PQ is a tangent drawn from an external point P to a circle with centre O, QOR is the diameter of the circle. If $\angle POR = 120^o$, we need to find the measure of OPQ.

   In $\triangle OQP, \angle POR = \angle OQP + \angle OPQ$ (Exterior angle)

   $\therefore \angle OPQ = \angle POR - \angle OQP$

   $= 120^\circ - 90^\circ$

   $= 30^\circ$

9. OP $\perp$ PR [$\because$ Tangent and radius are $\perp$ to each other at the point of contact]

   $\angle OPQ = 90^o - 50^o = 40^o$

   $OP = OQ$ [By isosceles triangle's property]

   $\angle OPQ = \angle OQP = 40^o$

   In $\triangle$ OPQ,

   $\Rightarrow \angle O + \angle P + \angle Q = 180^\circ$

   $\Rightarrow \angle O + 40^o + 40^o = 180^o$

   $\angle O = 180^o - 80^o = 100^o$.

OR

A line that interests a circle at two points in a circle is called a Secant.

10. Since, we know that by the converse of the Pythagoras theorem, if the square of the length of the longest side of the triangle is equal to the sum of the square of other two

sides, then the triangle is a right triangle.

Let ABC be a triangle and let AB = (a-1) cm, BC = $2\sqrt{a}$ cm and CA = (a+1) cm

Then,

$AB^2 = (a-1)^2 = a^2 + 1 - 2a$

$BC^2 = (2\sqrt{a})^2$ = 4a

$CA^2 = (a+1)^2$ = $a^2 + 1 + 2a$

Thus, $AB^2 + BC^2$ = a^2 + 1- 2a + 4a = a^2 + 1 + 2a =$(a+1)^2$ =AC^2

Hence,

$AB^2 + BC^2 = CA^2$

Thus, the given triangle ABC is a right-angled triangle and right angles at B.

11. We have, a = $\frac{1}{m}$

d = $\frac{1+m}{m} - \frac{1}{m}$ = $\frac{1+m-1}{m}$ = $\frac{m}{m}$ = 1

$\therefore$ a_n = $\frac{1}{m}$ +$(n-1)1$

Hence, a_n = $\frac{1}{m}$ +$n - 1$

12. sin 38° - cos 52°

= sin 38° - sin (90° - 52°) [$\because$ cos A = sin(90° - A]

= sin 38° - sin 38° = 0

13. LHS = $(\sec^2\theta - 1)\cot^2\theta$ [$\because$ $\sec^2\theta - 1 = \tan^2\theta$]

= $\tan^2\theta \times \cot^2\theta$

= $\tan^2\theta \times \frac{1}{\tan^2\theta}$ = 1 = RHS

Therefore, LHS = RHS

14. Length of resulting cuboid, l = 4 cm + 4 cm = 8 cm, breadth b = 4 cm, & height h = 4 cm

We know that,

Surface area of cuboid

= 2 (lb + bh + hl)

= 2 (8×4 + 4×4 + 4×8)

= 2 × 80 = 160 cm^2.

15. If $2k+1, 3k+3, 5k-1$ are in A.P.

then $(5k-1)-(3k+3)=(3k+3)-(2k+1)$

or, $5k-1-3k-3=3k+3-2k-1$

or, $2k-4=k+2$

or, $2k - k = 4 + 2$

or $k = 6$

16. Let A be the event of that the card will not be an ace.

Total no. of cards = 52

So total no. of outcomes = 52

No. of aces in one pack of cards = 4

The number of outcomes favorable to A = 52 - 4 = 48

P(A) = $\frac{\text{Number of outcomes favorable to A}}{\text{Number of total outcomes}} = \frac{48}{52} = \frac{12}{13}$

17. i. (d) The perpendiculars from P, Q, R and S intersect the x-axis at 3, 10, 10 and 3 respectively.

Also, the perpendiculars from P, Q, R and S intersect the y-axis at 6, 6, 2, 2 respectively.

Hence coordinates of P and S are P(3, 6) and S(3, 2).

ii. (b) Let M be the mid-point of QS.

So, using mid-point formula, Coordinates of M are $\left(\frac{3+10}{2}, \frac{2+6}{2}\right) = \left(\frac{13}{2}, 4\right)$

iii. (a) Now, $PQ = \sqrt{(10-3)^2 + (6-6)^2} = \sqrt{49} = 7 \text{ m}$

$PS = \sqrt{(3-3)^2 + (2-6)^2} = \sqrt{16} = 4 \text{ m}$

Hence, area of rectangle PQRS = PQ $\times$ PS = 7 $\times$ 4 = 28 m^2

iv. (c) R(10, 2), Q(10, 6)

v. (b) 7, 4

18. i. (b) AB, AC

ii. (d) $AB^2 = BC^2 + CA^2$

iii. (c) 6.5 m

iv. (b) 10 m

v. (a) 6.7 m (approx.)

19. i. (a) 46-48

ii. (b) 46.9 kg

iii. (a) Mean

iv. (c) 45.85

v. (b) 14K

20. Clearly, the fez is in the shape of a frustum of a cone with radii of one base as r_1 = 10 cm and radii of another base as r_2 = 4 cm and slant height l = 15 cm. Then,

i. (b) The curved surface area of cap = $\pi\left(r_1+r_2\right) l$

$\Rightarrow \quad A=\frac{22}{7} \times(10+4) \times 15$ = 660 cm^2

ii. (b) 50.285

iii. (a) Let A be the area of the material used:

A = Curved surface area + Area of the closed base

$\Rightarrow A=660+\pi r_2^2$

$\Rightarrow A=660+\frac{22}{7} \times 4^2$

= 710.28 cm^2

iv. (b) remains unaltered

v. (d) $\frac{1}{3} \pi h\left(r_1^2+r_2^2+r_1 r_2\right)$

Part-B

21. Let $x=0.2\overline{54}$.

Then, x = 0.2545454... ...(i) (multiply 10 on both sides)

∴ 10x = 2.545454... ...(ii)

and 1000x = 254.545454... ... (iii)

Subtract (ii) from (iii), we get

990x = 252 $\Rightarrow x=\frac{252}{990}=\frac{126}{495}=\frac{42}{165}=\frac{14}{55}$

$\therefore \quad 0.2\overline{54}=\frac{14}{55}$

22. Let (-1, 6) divides the line segment joining the points (-3, 10) and (6, -8) in k:1.

Using Section formula, we get

$-1=\frac{(-3) \times 1+6 \times k}{k+1}$

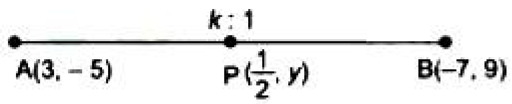
$\Rightarrow-k-1=(-3+6 k)$

$\Rightarrow -7k = -2 \Rightarrow k = \frac{2}{7}$

Therefore, the ratio is $\frac{2}{7}: 1$ which is equivalent to 2:7.

Therefore, (-1, 6) divides the line segment joining the points (-3, 10) and (6, -8) in 2:7.

OR



Let P divides AB in the ratio k:1.

$\therefore \quad\left(\frac{-7 k+3}{k+1}, \frac{9 k-5}{k+1}\right)=\left(\frac{1}{2}, y\right)$

$\Rightarrow \quad \frac{-7 k+3}{k+1}=\frac{1}{2}$

$\Rightarrow$ -14k + 6 = k + 1

$\Rightarrow$ -15k = - 5

$\Rightarrow$ k = $\frac{1}{3}$

$\therefore$ Ratio is k : 1, i.e., $\frac{1}{3}$: 1

$\Rightarrow$ 1 : 3

and y = $\frac{9k-5}{k+1} = \frac{9\times\frac{1}{3}-5}{\frac{1}{3}+1} = \frac{-6}{4} = \frac{-3}{2}$

23. Let p(x) = $2x^2 - 25$

For zeros of p(x), we put, p(x) = 0, then,

$2x^2 - 25 = 0$

$2x^2 = 25$

$x^2 = \frac{25}{2}$

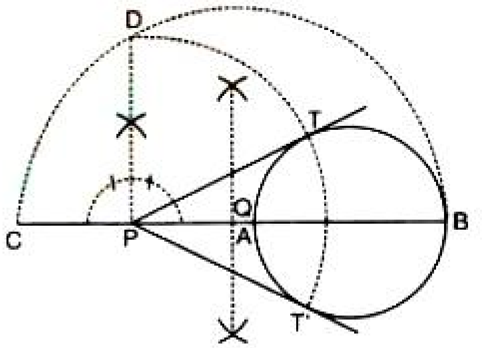
x = $\pm\sqrt{\frac{25}{2}} = \pm\frac{5}{\sqrt{2}}$

So the zeros of p(x) are $\frac{5}{\sqrt{2}}, -\frac{5}{\sqrt{2}}$

24. Steps of construction :

i. Draw a circle of radius 1.5 cm. Take a point P outside it.
ii. Through P draw a secant PAB to meet the circle at A and B.
iii. Produce AP to C such that PC = PA. Bisect CB at Q.
iv. With CB as diameter and centre as Q, draw a semicircle.
v. Draw PD $\perp$ CB, to meet semi-circle at point D.
vi. Intersect P as centre and PD as radius draw an arc to interest the circle at T and T'. PT and PT' are the required tangents.
vii. Join P to T and P to T'

Hence



25. Given, $sin^2 30^o cos^2 45^o + 4tan^2 30^o + \frac{1}{2}\sin^2 90^\circ + \frac{1}{8}\cot^2 60^\circ$

$= \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + 4 \times \left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2} \times (1)^2 + \frac{1}{8} \times \left(\frac{1}{\sqrt{3}}\right)^2$

$= \left(\frac{1}{4} \times \frac{1}{2} + 4 \times \frac{1}{3} + \frac{1}{2} + \frac{1}{8} \times \frac{1}{3}\right)$

$= \left(\frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}\right)$

$= \frac{48}{24}$

= 2

OR

We have

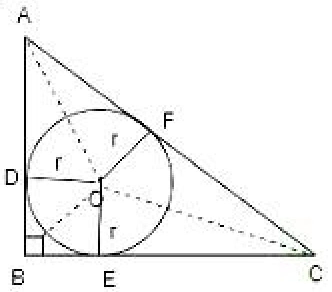
LHS = $\frac{\tan A + \sin A}{\tan A - \sin A}$

$= \frac{\frac{\sin A}{\cos A} + \sin A}{\frac{\sin A}{\cos A} - \sin A} = \frac{\sin A\left(\frac{1}{\cos A} + 1\right)}{\sin A\left(\frac{1}{\cos A} - 1\right)}$

$= \frac{\left(\frac{1}{\cos A} + 1\right)}{\left(\frac{1}{\cos A} - 1\right)} = \frac{\sec A + 1}{\sec A - 1} = \mathbf{RHS}$

$\therefore$ LHS = RHS

26. Let D, E and F are points where the in-circle touches the sides AB, BC and CA respectively. Join OA, OB and OC.



In $\triangle ABC, AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$= 8^2 + 6^2$

= 64 + 36

= 100

$\therefore AC = \sqrt{1}00 = 10\ cm$ [taking the positive square root, as length cannot be negative]

Now, $ar(\triangle OAB) = \frac{1}{2} \times OD \times AB = \frac{1}{2} \times r \times 8 = \frac{8r}{2} = 4rcm^2$

$ar(\triangle OBC) = \frac{1}{2} \times OE \times BC = \frac{1}{2} \times r \times 6 = \frac{6r}{2} = 3rcm^2$

and $ar(\triangle OAC) = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times r \times 10 = \frac{10r}{2} = 5rcm^2$

$\therefore ar(\triangle ABC) = ar(\triangle OAB) + ar(\triangle OBC) + ar(\triangle OAC)$

$\Rightarrow \frac{1}{2} \times AB \times BC = 4r + 3r + 5r = 12r$
$\Rightarrow \frac{1}{2} \times 8 \times 6 = 12r$
$\Rightarrow 24 = 12r$
$\Rightarrow r = \frac{24}{12}$
$\Rightarrow r = 2\ cm$

The value of r is 2 cm.

27. We have to prove that $\sqrt{2}$ is an irrational number.

Let $\sqrt{2}$ be a rational number.

$\therefore \quad \sqrt{2} = \frac{p}{q}$

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get,

or, $2 = \frac{p^2}{q^2}$

or, $p^2 = 2q^2$

$\therefore$ p^2 is divisible by 2.

p is divisible by 2........(i)

Let p = 2r for some integer r

or, $p^2 = 4r^2$

$2q^2 = 4r^2$ [$\because p^2 = 2q^2$]

or, $q^2 = 2r^2$

or, q^2 is divisible by 2.

$\therefore$ q is divisible by 2..........(ii)

From (i) and (ii)

p and q are divisible by 2, which contradicts the fact that p and q are co-primes.

Hence, our assumption is wrong.

$\therefore$ $\sqrt{2}$ is an irrational number.

28. Marks obtained in maths = x

Marks obtained in science = 28 - x

$\therefore$ (x + 3)(28 - x - 4) = 180

or, (x + 3)(24 - x) = 180

or, $24x - x^2 + 72 - 3x = 180$

or, $x^2 - 21x + 108 = 0$

or, (x - 9)(x -12) = 0

x = 9 or x = 12

Case I:

Marks obtained in maths = 9

Marks obtained in science = 19

Case ii :

Marks obtained in maths = 12

Marks obtained in science = 16

OR

The given equation is

$x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$

or $x^2 - 2(1 + 3k)x + 7(3 + 2k) = 0$

It is given that the given quadratic equation has equal roots,

$\Rightarrow b^2 - 4ac = 0$

In the given equation we have,

a = 1, b = - 2(1 + 3k) and c = 7(3 + 2k)

Now, $b^2 - 4ac = 0$

$\Rightarrow [-2(1 + 3k)]^2 - 4 \times 1 \times 7(3 + 2k) = 0$

$\Rightarrow 4 \times (1 + 3k)^2 - 4 \times 7(3 + 2k) = 0$

$\Rightarrow 4(1 + 9k^2 + 6k) - 4(21 + 14k) = 0$

$\Rightarrow (1 + 9k^2 + 6k) - (21 + 14k) = 0$

$\Rightarrow 1 + 9k^2 + 6k - 21 - 14k = 0$

$\Rightarrow 9k^2 - 8k - 20 = 0$

Factorize above equation we get

$9k^2 - 18k + 10k - 20 = 0$

$\Rightarrow 9k(k - 2) + 10(k - 2) = 0$

Therefore, either 9k +10=0 and k - 2 = 0

⇒ k = -10/9 and k = 2

Hence the values of k are -10/9, 2

29. The quadratic polynomial $ax^2 + bx + c = f(x)$

α and β are the zeroes of an equation.

$\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\frac{\beta-\alpha}{\alpha\beta}=\frac{-(\alpha-\beta)}{\alpha\beta} \ldots\ldots (i)$$

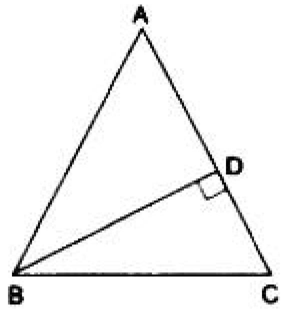
consider,

$$(\alpha-\beta)^2=(\alpha+\beta)^2+4\alpha\beta$$

$$(\alpha-\beta)^2=\left(\frac{-b}{a}\right)^2+\frac{4c}{a}$$

$$\alpha-\beta=\sqrt{\frac{b^2}{a^2}+\frac{4c}{a}}=\sqrt{\frac{b^2+4ac}{a^2}}=\frac{\sqrt{b^2+4ac}}{a}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\frac{-(\alpha-\beta)}{\alpha\beta}=\frac{\frac{\sqrt{b^2+4ac}}{a}}{\frac{c}{a}}=\frac{\sqrt{b^2+4ac}}{c}$$

30. 

Given an isosceles $\triangle ABC$ in which AB = AC and $BD \perp AC$.

To Prove $(BD^2 - CD^2) = 2CD \cdot AD$

Proof In right angle triangle $\triangle ADB$, we have $\angle ADB = 90^\circ$, So using Pythagoras Theorem

$AB^2 = AD^2 + BD^2$

$\Rightarrow AC^2 = AD^2 + BD^2$ [∵ AB = AC, Given]

$\Rightarrow (CD + AD)^2 = AD^2 + BD^2$ [∵ AC = CD + AD, as BD intersects AC at D]

$$\Rightarrow CD^2+AD^2+2CD\cdot AD=AD^2+BD^2$$

$$\Rightarrow \quad \left(BD^2-CD^2\right)=2CD\cdot AD$$

Hence, $\left(BD^2-CD^2\right)=2CD\cdot AD$

Hence Proved

OR

Given: $\triangle$ABC, right-angled at A.

BL and CM are medians.

In $\triangle ABL$,

$BL^2 = AB^2 + AL^2$

$= AB^2 + \left(\frac{AC}{2}\right)^2$

= $AB^2 + \frac{1}{4}AC^2$

$\Rightarrow 4BL^2 = 4AB^2 + AC^2 \ldots..(1)$

$In \triangle ACM$

$CM^2 = AC^2 + AM^2 = AC^2 + \frac{1}{4}AB^2$

$4CM^2 = 4AC^2 + AB^2 \ldots..(2)$

From equations (1) and (2), we get

$4(BL^2 + CM^2) = 4(AB^2 + AC^2) + AB^2 + AC^2$

$= 4BC^2 + BC^2 = 5BC^2$ (Since$AB^2 + AC^2 = BC^2$)

Hence proved.

31. Given numbers 1, 3, 5, ..., 35 form an AP with a = 1 and d = 2.

Let T_n = 35. Then,

$1 + (n-1)2 = 35$

$\Rightarrow 1 + 2n - 2 = 35$

$\Rightarrow 2n = 36$

$\Rightarrow$ n = 18

Thus, total number of outcomes = 18.

i. Let E_1 be the event of getting a prime number less than 15.

   among these numbers, prime numbers less than 15 are 3, 5, 7, 11 and 13.

   The number of favorable outcomes = 5.

   Therefore, P(getting a prime number less than 15) = P(E_1) =

   $\frac{\text{Number of outcomes favorable } to E_1}{\text{Number of all possible outcomes}} = \frac{5}{18}$

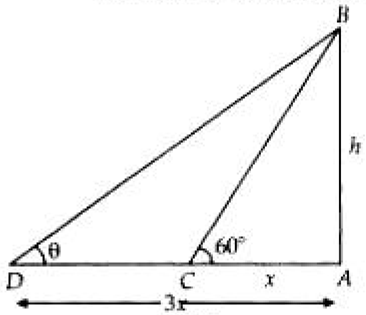
ii. Let E_2 be the event of getting a number divisible by 3 and 5.

   among these numbers, the number divisible by 3 and 5 means number divisible by 15 is 15.

   The number of favorable outcomes = 1.

Therefore, P(getting a number divisible by 3 and 5) = P(E_2) =

$\frac{\text{Number of outcomes favorable to } E_2}{\text{Number of all possible outcomes}} = \frac{1}{18}$

32. 

In $\triangle$ABC, $\frac{AB}{AC} = \tan 60^\circ$

$\frac{h}{x} = \sqrt{3}$

$\Rightarrow \quad h = x\sqrt{3}$

In $\triangle$ABD

$\tan\theta = \frac{AB}{AD} = \frac{h}{3x}$

$\tan\theta = \frac{\sqrt{3}x}{3x} = \frac{1}{\sqrt{3}}$

Hence $\theta = 30^\circ$

33.

C I.	f	c.f.
0 -10	5	5
10-20	x	x + 5
20 -30	20	x + 25
30 -40	15	x + 40
40 -50	y	x + y + 40
50 -60	5	x + y + 45
	Σf = 60	

From table, N = 60 = x + y + 45

$\Rightarrow$x + y = 60 - 45

$\Rightarrow$x + y = 15..........(1)

Since, Median = 28.5, which lies between 20-30

$\therefore$Median class = 20 - 30

$l=20, N=60, C.f.=x+5, f=20$ and $h=10$

Median = $l+\frac{\left(\frac{N}{2}-c.f.\right)}{f}\times h$

$\Rightarrow \quad 28.5=20+\frac{[\frac{60}{2}-(x+5)]}{20}\times 10$

$\Rightarrow \quad 28.5=20+\frac{[30-(x+5)]}{20}\times 10$

$\Rightarrow \quad 28.5-20=\frac{[30-(x+5)]}{2}$

$\Rightarrow \quad 8.5=\frac{[30-x-5]}{2}$

$\Rightarrow \quad 8.5=\frac{25-x}{2}$

$\Rightarrow 25-x=17$

$\Rightarrow x=25-17$

$\Rightarrow x=8$

From (1), $x+y=15$

$\Rightarrow 8+y=15$

$\Rightarrow y=15-8=7$

Hence, x = 8 and y = 7

34. We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

For the first triangle, we have a = 35, b = 53 and c = 66.

$\therefore \quad s=\frac{a+b+c}{2}=\frac{35+53+66}{2}=77\text{cm}$

Let Δ_1 be the area of the first triangle. Then,

$\Delta_1=\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_1=\sqrt{77(77-35)(77-53)(77-66)}=\sqrt{77\times 42\times 24\times 11}$

$\Rightarrow \quad \Delta_1=\sqrt{7\times 11\times 7\times 6\times 6\times 4\times 11}=$

$\sqrt{7^2\times 11^2\times 6^2\times 2^2}=7\times 11\times 6\times 2=924\text{cm}^2$ ...(i)

For the second triangle, we have a = 33,b = 56,c = 65

$\therefore \quad s=\frac{a+b+c}{2}=\frac{33+56+65}{2}=77\text{cm}$

Let Δ_2 be the area of the second triangle. Then,

$\Delta_2=\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_2=\sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow \quad \Delta_2=\sqrt{77\times 44\times 21\times 12}=$

$\sqrt{7\times 11\times 4\times 11\times 3\times 7\times 3\times 4}=\sqrt{7^2\times 11^2\times 4^2\times 3^2}$

$\Rightarrow \quad \Delta_2 = 7 \times 11 \times 4 \times 3 = 924\text{cm}^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2 = \Delta_1 + \Delta_2$

$\Rightarrow \quad \pi r^2 = 924 + 924$

$\Rightarrow \quad \frac{22}{7} \times r^2 = 1848$

$\Rightarrow \quad r^2 = 1848 \times \frac{7}{22} = 3 \times 4 \times 7 \times 7 \Rightarrow$

$r = \sqrt{3 \times 2^2 \times 7^2} = 2 \times 7 \times \sqrt{3} = 14\sqrt{3}\text{cm}$

35. $x - y + 1 = 0$

$\Rightarrow y = x + 1$

x	-1	1	0
y	0	2	1

$2x + y - 10 = 0$

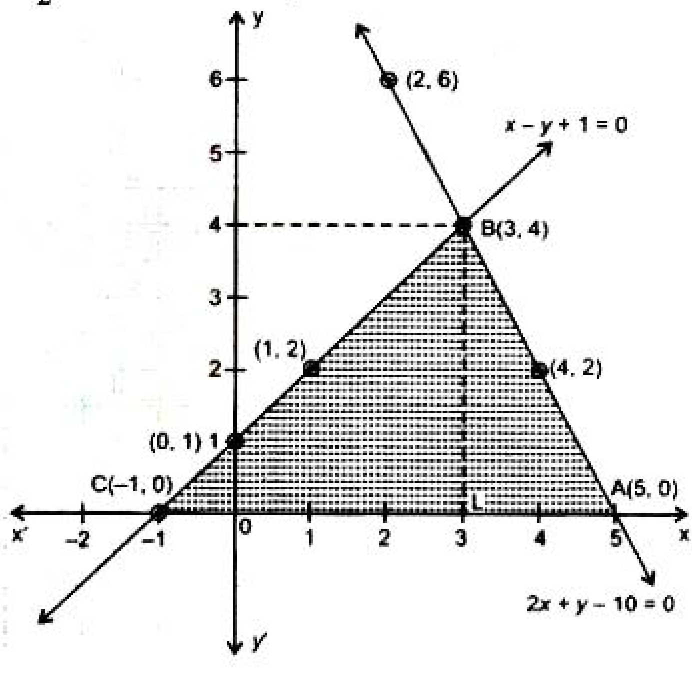
$\Rightarrow y = 10 - 2x$

x	2	4	5
y	6	2	0

Shaded portion is the area bounded by the lines and x-axis.

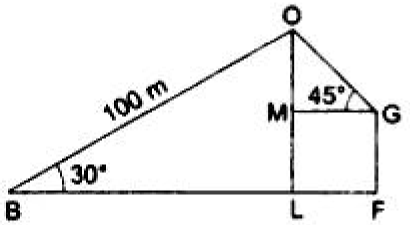
Area = $\frac{1}{2} AC \times BL$

= $\frac{1}{2} \times 6 \times 4$ = 12sq. units



36. According to the question,

Let O be the position of the bird, B be the position of the boy and FG be the building at which G is the position of the girl.



$OL \perp BF \text{ and } GM \perp OL.$

BO = 100m,

$\angle OBL = 30°,$

FG = 20m and

$\angle OGM = 45°.$

From right ΔOLB, we have

$\frac{OL}{BO} = \sin 30° \Rightarrow \frac{OL}{100\text{m}} = \frac{1}{2}$

$\Rightarrow OL = 100\text{m} \times \frac{1}{2} = 50\text{m}$

$OM = OL - ML$

$= OL - FG$

$= 50m - 20m$

$= 30m.$

From right ΔOMG, we have

$\frac{OM}{OG} = \sin 45° = \frac{1}{\sqrt{2}}$

$\Rightarrow OG = \sqrt{2} \times OM$

$= \sqrt{2} \times 30\text{m}$

$= 30 \times 1.41\text{m}$

$= 42.3\text{m}.$

The distance of the bird from the girl = 42.3m.