7.5 Ellipse

Semimajor axis: a
Semiminor axis: b
Foci: $F_1(-c,0)$, $F_2(c,0)$
Distance between the foci: 2c
Eccentricity: e
Real numbers: A, B, C, D, E, F, t
Perimeter: L
Area: S

645. Equation of an Ellipse (Standard Form)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

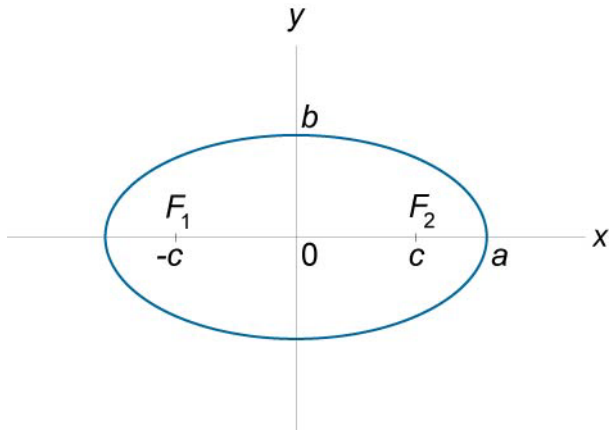

Figure 115.

646. $r_1+r_2=2a$,
where r_1, r_2 are distances from any point $P(x,y)$ on the ellipse to the two foci.

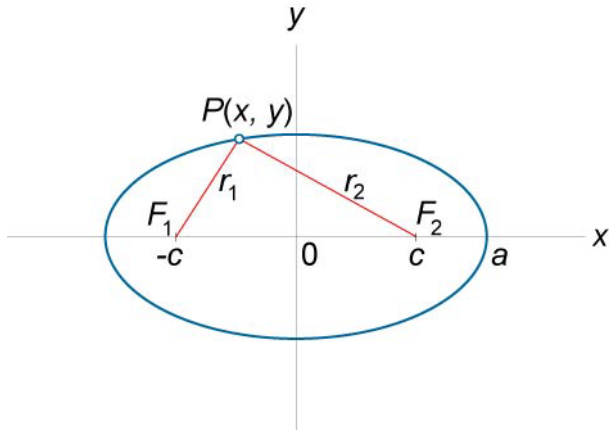

Figure 116.

647. $a^2 = b^2 + c^2$

648. Eccentricity

$$e = \frac{c}{a} < 1$$

649. Equations of Directrices

$$x = \pm\frac{a}{e} = \pm\frac{a^2}{c}$$

650. Parametric Form

$$\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}, \quad 0 \le t \le 2\pi.$$

651. General Form

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

where $B^2 - 4AC < 0$.

652. General Form with Axes Parallel to the Coordinate Axes

$Ax^2 + Cy^2 + Dx + Ey + F = 0$,

where $AC > 0$.

653. Circumference

$L = 4aE(e)$,

where the function E is the complete elliptic integral of the second kind.

654. Approximate Formulas of the Circumference

$L = \pi\left(1.5(a+b) - \sqrt{ab}\right)$,

$L = \pi\sqrt{2\left(a^2 + b^2\right)}$.

655. $S = \pi ab$