Congruency - Congruent Triangles

- **Similar and Congruent Figures**
  - Two geometric figures having the same shape and size are said to be congruent figures.
  - Two geometric figures having the same shape, but not necessarily the same size, are called similar figures.

  Example:
  (1) All circles are similar.
  (2) All equilateral triangles are similar.
  (3) All congruent figures are similar. However, the converse is not true.

- **Similarity of Polygons**

  Two polygons with the same number of sides are similar, if

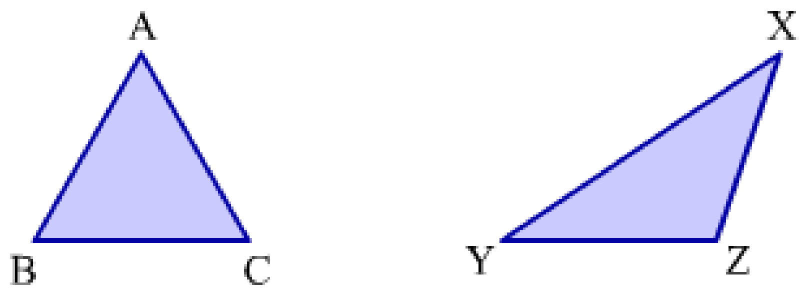
  - their corresponding angles are equal
  - their corresponding sides are in the same ratio (or proportion)

- Two lines segments are congruent, if they are equal in length.
- Two angles are congruent, if they have the same measure.
- **CPCT:**

CPCT stands for Corresponding Parts of Congruent Triangles.

If $\Delta ABC \cong \Delta PQR$, then corresponding sides are equal i.e., AB = PQ, BC = QR, and CA = RP and corresponding angles are equal i.e., $\angle A = \angle P$, $\angle B = \angle Q$, and $\angle C = \angle R$.

1. In triangles, ΔABC and ΔXYZ, if vertices A and X corresponds to each other, then it is symbolized as **'A ↔ X'** and read as **'There is one to one correspondence between A and X'**.

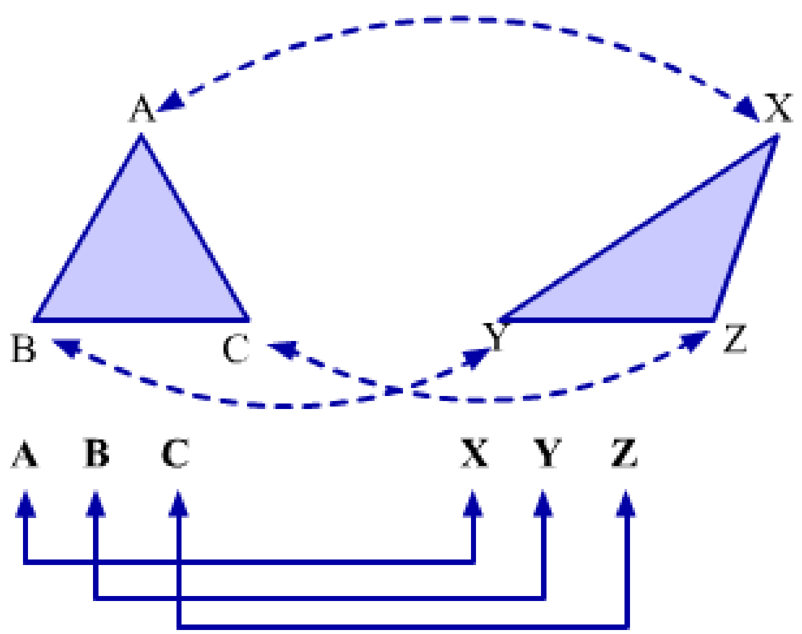


2. All of the correspondences can be represented together as **'ABC ↔ XYZ'**.

3. All the possible correspondences between ΔABC and ΔXYZ are:

The correspondence between vertices	The correspondence written together
(1) A ↔ X, B ↔ Y, C ↔ Z	ABC ↔ XYZ
(2) A ↔ X, B ↔ Z, C ↔ Y	ABC ↔ XZY
(3) A ↔ Y, B ↔ X, C ↔ Z	ABC ↔ YXZ
(4) A ↔ Y, B ↔ Z, C ↔ X	ABC ↔ YZX
(5) A ↔ Z, B ↔ X, C ↔ Y	ABC ↔ ZXY
(6) A ↔ Z, B ↔ Y, C ↔ X	ABC ↔ ZYX

4. The arrow diagram to show the correspondence between the given triangles can be shown as follows:



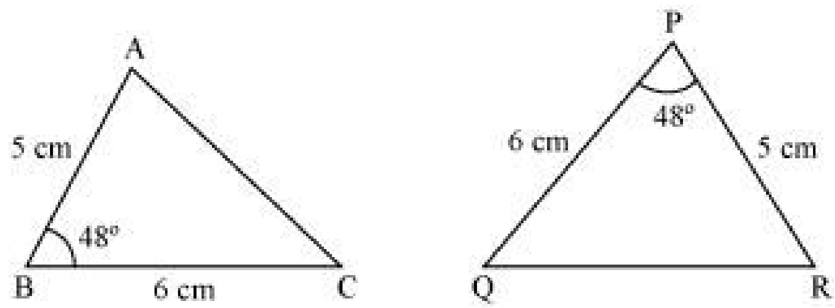
5. The correspondence between sides and angles of the given triangles can be written as follows:

Pairs of corresponding sides	Pairs of corresponding angles
(1) AB ↔ XY	(1) ∠A ↔ ∠X
(2) BC ↔ YZ	(2) ∠B ↔ ∠Y
(3) AC ↔ XZ	(3) ∠C ↔ ∠Z

- **SAS congruence rule**

If two sides of a triangle and the angle included between them are equal to the corresponding two sides and included angle of another triangle, then the triangles are congruent by SAS congruence rule.

Example:



Are ΔABC and ΔRPQ congruent?

Solution:

In ΔABC and ΔRPQ,

AB = RP

∠ABC = ∠RPQ

BC = PQ

∴ ΔABC ≅ ΔRPQ (By SAS congruence rule)

- **CPCT**

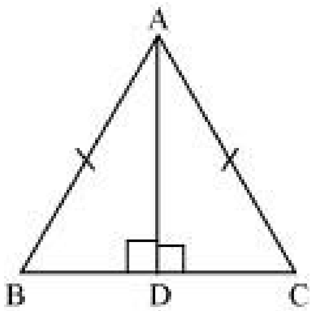
CPCT stands for 'corresponding parts of congruent triangles'. 'Corresponding parts' means corresponding sides and angles of triangles. According to CPCT, if two or more triangles are congruent to one another, then all of their corresponding parts are equal.

- **ASA congruence rule**

  If two angles and included side of a triangle are equal to the two corresponding angles and the included side of another triangle, then the triangles are congruent by ASA congruence rule.

  Example:

  In the following figure, AD is the median of ΔABC.



  Are ΔABD and ΔACD congruent?

  Solution:

  In ΔABC,

  AB = AC (Given)

  ∴∠ACB = ∠ABC (Base angles of an isosceles triangle have equal measures)

  Now, in ΔABD and ΔACD,

  ∠ABD = ∠ACD

  BD = CD (AD is the median)

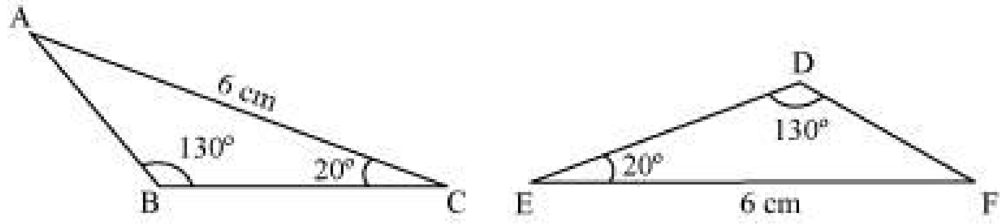
  ∠ADB = ∠ADC = 90°

  ∴ ΔABD ≅ ΔACD (By ASA congruence rule)

- **AAS congruence rule**

If two angles and one side of a triangle are equal to the corresponding angles and side of the other triangle then the two triangles are congruent to each other. This criterion is known as the **AAS congruence rule.**

For example, in the given triangles, ∠B = ∠D = 130°, ∠C = ∠E = 20° and AC = EF = 6 cm.

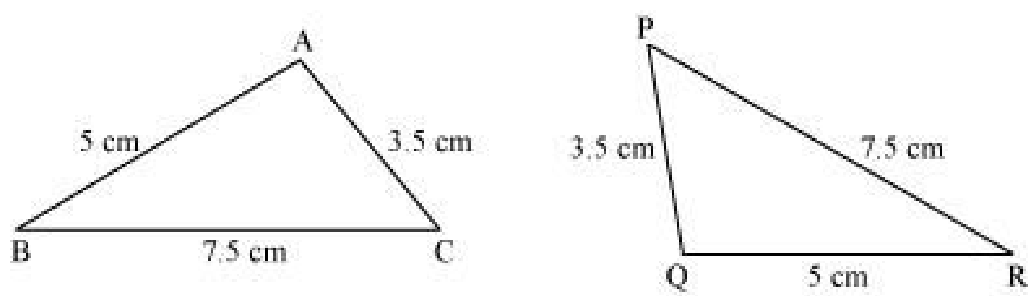


∴ By AAS congruence rule, ΔABC ≅ ΔFDE

- **SSS congruence rule**

  If three sides of a triangle are equal to the three sides of the other triangle, then the two triangles are congruent by SSS congruence rule.

Example:



Are ΔABC and ΔQRP congruent?

Solution:

In ΔABC and ΔQRP

AB = QR = 5 cm

BC = PR = 7.5 cm

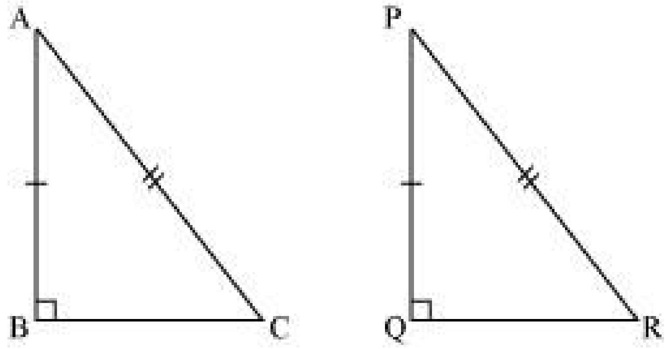
AC = PQ = 3.5 cm

∴ ΔABC ≅ ΔQRP (By SSS congruence rule)

- **RHS congruence rule**

  If the hypotenuse and one side of a right triangle are equal to the hypotenuse and one side of the other right triangle, then the two triangles are congruent to each other by RHS congruence rule.

  Example:



If in the given figure, ∠B = ∠Q = 90°, AC = PR, and AB = PQ,
∴ By RHS congruence rule, ΔABC ≅ ΔPQR