1. Gravitation

1. Study the entries in the following table and rewrite them putting the connected items in a single row.
Ans.

I	II	III
Mass	Kg	Measure of inertia
Weight	N	Depends on height
Acceleration due to gravity	m/s^2	Zero at the centre of the earth
Gravitational constant	$N\ m^2/kg^2$	Same in the entire universe

2. Answer the following questions.
a. What is the difference between mass and weight of an object. Will the mass and weight of an object on the earth be same as their values on Mars? Why?
Ans.

Mass	Weight
1. The mass of a body is the amount of matter present in it.	1. The weight of a body is the force with which the earth attracts it.
2. It has magnitude, but not direction.	2. It has both magnitude and direction.
3. It does not change from place to place.	3. It changes from place to place.
4. It can never be zero.	4. It is zero at the centre of the earth.
5. Its SI unit is the kilogram.	5. Its SI unit is the newton.

b. What are (i) free fall. (ii) acceleration due to gravity (iii) is escape velocity (iv) centripetal force?
Ans. ***Not available***

c. Write down three given Kepler. How did they help Newton to arrive at the Inverse square law of gravity?
Ans. Kepler's first law: The orbit of a planet is an ellipse with the Sun at one of the foci.

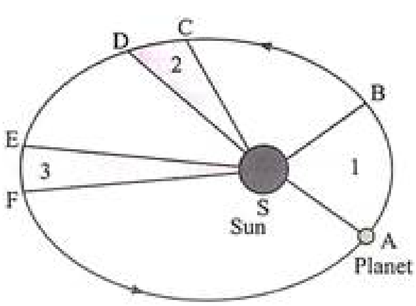


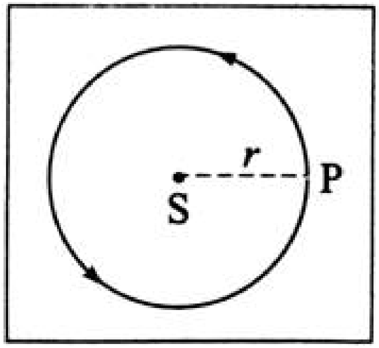
Figure 1.5 shows the elliptical orbit of a planet revolving around the Sun (S).
Kepler's second law:
The line joining the planet and the Sun sweeps equal areas in equal intervals of time.
AàB, CàD and EàF are the displacements of the planet in equal intervals of time. The straight lines AS, CS and ES sweep equal areas in equal intervals of time.

Area ASB = area CSD = area ESF.

Kepler's third law: (Rotate your phone)

The square of the period of revolution of a planet around the Sun is directly proportional to the cube of the mean distance of the planet from the Sun. Thus, if r is the average distance of the planet from the Sun and T is its period of revolution, then,

$\mathrm{T}^2(\alpha)\mathrm{r}^2, \text{i.e.,} \frac{\mathrm{T}^2}{\mathrm{r}^3}\, constant = k$



For simplicity we shall assume the orbit to be a circle. In Fig. 1.6. S denotes the position of the Sun, P denotes the position of a planet at a given instant and r denotes the radius of the orbit (= the distance of the planet from the Sun). Here, the speed of the planet is uniform. It is

$$v = \frac{\text{circumference of the circle}}{\text{period of revolution of the planet}}$$

$=\frac{2\pi r}{T}.$

If m is the mass of the planet, the centripetal force exerted on the planet by the Sun

$(\equiv gravitational\ force), F = \frac{mv^2}{r}$

$$\therefore \mathrm{F} = \frac{m(2\pi r/T)^2}{r} = \frac{4\pi^2 m r^2}{T^2 r}$$

$$= \frac{4\pi^2 m r}{T^2}$$

According to kepler's third law

$T^2 = Kr^2$

$$\therefore \ \mathrm{F} = \frac{4\pi^2 m r}{Kr^3} = \frac{4\pi^2 m}{K}\left(\frac{1}{r2}\right)$$

$$Thus, F\ \alpha \frac{1}{r^2}\ as\ \frac{4\pi^2\ m}{K}$$

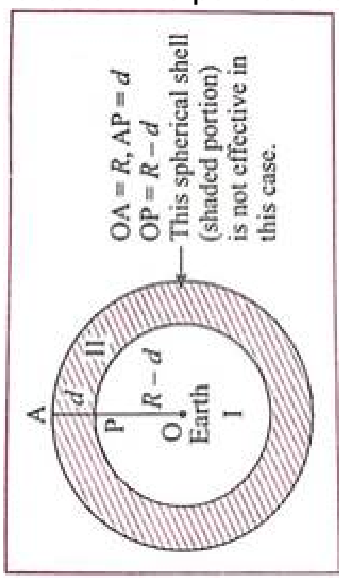
is constant in a particular case.

d. A stone thrown vertically upwards with initial velocity u reaches a height 'h before coming down. Show that the time taken to go up is same as the time taken to come down.
Ans. Not available

e. If the value of g suddenly becomes twice its value, it will become two times more difficult to pull a heavy object along the floor. Why?

Ans. To pull an object along the floor, it is necessary to do work against the force of friction between the object and the surface of the floor. This force of friction is proportional to the weight, mg, of the object. If the value of g becomes twice its value, the weight of the object and hence the force of friction will become double. Therefore, it will become two times more difficult to pull a heavy object along the floor.

3. Explain why the value of g is zero at the centre of the earth.
Ans. The value of g changes while going deep inside the earth. It goes on decreasing as we go from the earth's surface towards the earth's centre. We shall treat the earth as a sphere of uniform density. If we consider a particle of mass m at point P at a distance (R-d) from the earth's centre, where R is the radius of the earth and d is the depth below the earth's surface, the gravitational force on the particle due to the earth is



$$F = \frac{GmM'}{(R-d)^2},$$

Where M' is the mass of the sphere of radius (R - D)

$$M = \frac{4}{3}\pi\,(R-D)^3 \times \frac{M}{\frac{4}{3}\pi R^3} = \frac{M\,(R-D)^3}{R^2}.$$

This is because the outer spherical shell is not effective (fig. 1.10). In this case, the acceleration due to gravity is surface $\left(\frac{1}{r2}\right)$.

At the earth's centre, d = R ∴ g = 0

4. Let the period of revolution of a planet at a distance R from a star be T. Prove that if it was at a distance of 2R from the star, its period of revolution will be √81.

Ans. $T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$,

where T= period of revolution of a planet around the Sun, M= mass of the Sun, G = gravitational constant and r=radius of the orbit assumed to be circular = distance of the planet from the Sun.

For r = R, T = T_1

∴ $T_1 = \frac{2\pi}{\sqrt{GM}} R^{\frac{3}{2}}$

For r = 2R, T = T_1

$\therefore T_2 = \frac{2\pi}{\sqrt{GM}} (2R)^{\frac{3}{2}}$

$= \frac{2\pi}{\sqrt{GM}} R^{3/2} \times 2^{3/2} = T_1\, 2^{3/2}$

$\therefore T_2 = T_1\sqrt{8}\, T.$

5. Solve the following examples.

a. An object takes 5s to reach the grond from a height of 5 m on a planet. What is the value of g on the planet?

Solution: Data u = 0m/s, s = 5m, t = 5s, g =?

$\therefore s = \frac{1}{2}\, gt^2$

$\therefore 5m = \frac{1}{2} g \times (5s)^2$

$= \frac{1}{2}\, g \times 5s \times 5s$

$\therefore g = \frac{1}{5}\, m/s^2 = \mathbf{0.4\, m/s^2\, (on\ the\ planet)}$

b. The radius of planet A is half the radius of planet B. If the mass of Ais M, what must be the mass of B so that the value of g on B is half that of its value on A?

Solution: Data: $R_A = R_B/2, g_B = \frac{1}{2} g_A, M_B = ?$

$g = \frac{GM}{R^2}$

$\therefore g_A = \frac{GM_A}{R_A^2}\, and\ g_B = \frac{GM_B}{R_B^2}$

$\therefore \frac{g_B}{g^A} \left(\frac{M_B}{M_A}\right) (\frac{R_A}{R_B})^2$

$\therefore \frac{1}{2} = \left(\frac{M_B}{M_A}\right) (\frac{1}{2})^2 = \frac{1}{4}\left(\frac{M_B}{M_A}\right)$

$\therefore \frac{M_B}{M_A} = \frac{4}{2} = 2$

$\therefore M_B = 2M_A$

c. The mass and weight of an object on earth are 5 kg and 49 N respectively. What will be their values on the moon? Assume that the acceleration due to gravity on the moon is 1/6th of that on the earth.

Solution: Data: m = 5kg, W(on the moon)= ?

(i) The mass of the object on the moon=

The mass of the object on the earth **= 5kg**

(ii) W = mg

$\therefore \frac{W_M}{W^E} = \frac{mg^M}{mg^E} = \frac{g^M}{g^E} = \frac{1}{6}$

$\therefore W_E \frac{W_E}{6} = \frac{49N}{6} = 8.167\, N$

(Weight of the object on the moon)

d. An object thrown vertically upwards reaches a height of 500 m. What was its initial velocity? How long will the object take to come back to the earth? Assumed g = 10 $\mathbf{m/s^2} =$

Solution: Data: h $= 500$ m,g $= 10m/s^2$.

v = 0 m/s, u = ?, t (for the object going up) + t (foe the object coming down) =?

As the object moves upwards,

$v^2 = u^2 + 2as$

$= u^2 + 2(-g)h\,(\therefore a = -g)$

Now, v = 0 m/s

$\therefore u^2 = 2gh = 2 \times \frac{10m}{s^2} \times 500m$

$\therefore u^2 = (100 \times 100)m/s^2$

∴ u = 100 m/s (initial velocity of the body)

Also, v = u + at = u- gt

For v = 0m/s, u = gt

∴100m/s= 10m/$s^2 \times t$

∴ t (for the object going up)=10s

Now, t (for the object coming down) =

t (for the going up) 10s

∴ t (for the object going up) +

= 10s + 10s = 20s

It will take 20s for the object to come back to the earth,

e. A ball falls off a table and reaches the ground in 1 $\mathbf{s.\,Assuming\,g - 10\,\frac{m^2}{s}}$,

Calculate its on speed reaching the ground and the height of the table.

Solution:

$\mathbf{Data: t = 1s,}$

$\mathbf{g = \frac{10m}{s^2},}$

$\boldsymbol{u = 0\,m/s,}$

S =?, v =?

(i) $s = ut + \frac{1}{2}gt^2$

$= \frac{1}{2}gt^2 for\ u = \frac{0m}{s}$

$\therefore\ s = \frac{1}{2} \times \frac{10m}{s^2} \times (1s)^2$

=5m

∴ The height of the table = 5m

(iii) v = u + at = u + gt

= 0m/s + 10m/$s^2 \times 1s$

= 10m/s

∴ **The velocity of the ball on reaching the ground = 10 m/s.**

f. The masses of the earth and moon are

6×10^{24} kg and 7.4×10^{22} kg,

respectively. The distance between

them is 3.84×10^{5} km.

Calculate the gravitational force of attraction between the two?

Use G = $6.7 \text{ x } 10^{-11} \text{N m}^2 \, kg^2$

***Solution*: Data:** $T_1 = 6 \times \mathbf{10^{22}} \boldsymbol{kg},$

$m_2 = 7.2 \times 10^{22} kg,$

$r = 3.84 \times 10^5 \, km = 3.84 \times 10^8 m,$

$G = 6.7 \times 10^{-11} \, N.m^2 kg^{-2}, F = ?$

$$F = \frac{Gm_1}{r^2}$$

(Rotate your phone)

$$\frac{6.7 \times 10^{-11} \, N.m^2 kg^{-2} \times 6 \times 10^{24} kg \times 7.4 \times 10^{22} \, kg}{(3.84 \times 10^8 \, m)^8}$$

$$= \frac{6.7 \times 6 \times 7.4 \times 10^{35}}{3.84 \times 3.84 \times 10^{16}} \, N$$

$= 2.017 \times 10^{22} kg$

This is the (the magnitude of) the gravitational force between the earth and the moon.

g. The mass of the earth is 6×10^{24} kg. The distance between the earth and the Sun is 1.5×10^{11} m. If the gravitational force between the two is $3.5\ 10^{22}$ N, what is the mass of the Sun?

Use $G = 6.7 \times 10^{-11}\ N\,m^2\,kg$?

Solution: Data: $m_1 = 6 \times 10^{24} kg$

$R = 1.5 \times 10^{11} m, F = 3.5 \times 10^{24} N.$

$G = 6.7 \times 10^{-11}\ N.m^2 kg^{-2}, m_2 = ?$

$$F = \frac{GM_1 GM_2}{R^2}$$

$$\therefore M_2 = \frac{FR^2}{Gm_1}$$

$$= \frac{3.5 \times 10^{22} N \times (1.5 \times 10^{11} m)^2}{6.7 \times 10^{-11} N.m^2 kg^{-2} \times 10^{24} kg}$$

$$\therefore \frac{3.5 \times 1.5 \times 1.5 \times 10^{44}}{6.7 \times 6 \times 10^{13}} kg$$

$= 1.96 \times 10^{30} kg$ (mass of the sun).