Chapter 9

Indices, Surds, and Logarithms

CHAPTER HIGHLIGHTS

- *Indices*
- *Surds*
- *Rationalization of a Surd*
- *Square Root of a Surd*
- *Comparison of Surds*
- *Logarithms*

INDICES

If a number 'a' is added three times to itself, then we write it as $3a$. Instead of adding, if we multiply 'a' three times with itself, we write it as a^3.

We say that 'a' is expressed as an exponent. Here, 'a' is called the 'base' and 3 is called the 'power' or 'index' or 'exponent'.

Similarly, 'a' can be expressed to any exponent 'n' and accordingly written as a^n. This is read as 'a to the power n' or 'a raised to the power n.'

$$a^n = a \times a \times a \times a \times \cdots n \text{ times}$$

For example,

$$2^3 = 2 \times 2 \times 2 = 8 \text{ and } 3^4 = 3 \times 3 \times 3 \times 3 = 81$$

While the example taken is for a positive integer value of n, the powers can also be negative integers or positive or negative fractions. In the sections that follow, we will also see how to deal with numbers where the powers are fractions or negative integers.

If a number raised to a certain power is inside brackets and quantity is then raised to a power again {i.e. a number of the type $(a^m)^n$—read as 'a raised to the power m whole raised to the power n' or 'a raised to power m whole to the power n'}, then the number inside the brackets is evaluated first and then this number is raised to the power which is outside the brackets.

For example, to evaluate $(2^3)^2$, we first find out the value of the number inside the bracket (2^3) as 8 and now raise this to the power 2. This gives 8^2 which is equal to 64. Thus, $(2^3)^2$ is equal to 64.

If we have powers in the manner of 'steps', then such a number is evaluated by starting at the topmost of the 'steps' and coming down one 'step' in each operation.

For example, 2^{4^3} is evaluated by starting at the topmost level '3'. Thus, we first calculate 4^3 as equal to 64. Since 2 is raised to the power 4^3, we now have 2^{64}.

Similarly, 2^{3^2} is equal to '2 raised to the power 3^2' or '2 raised to the power 9' or 2^9, which is equal to 512.

There are certain basic rules/formulae for dealing with numbers having powers. These are called Laws of Indices. The important ones are listed below but you are not required to learn the proof for any of these formulae/rules. The students have to know these rules and be able to apply any of them in solving problems. Most of the problems in indices will require one or more of these formulae. These formulae should be internalized by the students to the extent that after some practice, application of these rules should come naturally and the student should not feel that he is applying some specific formula.

Table 1 Table of Rules/Laws of Indices

	Rule/Law	Example
(1)	$a^m \times a^n = a^{m+n}$	$5^2 \times 5^7 = 5^9$
(2)	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^5}{7^3} = 7^2 = 49$
(3)	$(a^m)^n = a^{mn}$	$(4^2)^3 = 4^6$
(4)	$a^{-m} = \frac{1}{a^m}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$
(5)	$\sqrt[m]{a} = a^{1/m}$	$\sqrt[3]{64} = 64^{1/3} = 4$
(6)	$(ab)^m = a^m \cdot b^m$	$(2 \times 3)^4 = 2^4 \cdot 3^4$
(7)	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$
(8)	$a^0 = 1$ (where $a \neq 0$)	$3^0 = 1$
(9)	$a^1 = a$	$4^1 = 4$

These rules/laws will help you in solving a number of problems. In addition, the student should also remember the following rules:

Rule 1: When the bases of two EQUAL numbers are equal, then their powers also will be equal. (If the bases are neither zero nor ±1.)

For example: If $2^n = 2^3$, then it means $n = 3$

Rule 2: When the powers of two equal numbers are equal (and not equal to zero), two cases arise:

1. If the power is an odd number, then the bases are equal. For example, if $a^3 = 4^3$ then $a = 4$.
2. If the powers are even numbers, then the bases are numerically equal but can have different signs. For example, if $a^4 = 3^4$ then $a = +3$ or -3.

The problems associated with indices are normally of THREE types:

Simplification: Here, the problem involves terms with different bases and powers, which have to be simplified using the rules/formulae discussed in the table earlier.

Solving for the value of an unknown: Here, the problem will have an equation where an unknown (like x or y) will appear in the base or in the power, and using Rule 1 and Rule 2 discussed, values of unknown are to be determined.

Comparison of numbers: Here, two or more quantities will be given—each being a number raised to a certain power. These numbers have to be compared in magnitude—either to find the largest or smallest of the quantities or to arrange the given quantities in ascending or descending order.

The following examples will make clear the different types of problems that you may be asked.

Solved Examples

Example 1

Simplify: $\left(\frac{729}{1728}\right)^{\frac{-2}{3}} \times \left(\frac{1024}{9}\right)^{\frac{1}{2}} \div \left(\frac{24}{324}\right)$

Solution

$$\left(\frac{729}{1728}\right)^{\frac{-2}{3}} \times \left(\frac{1024}{9}\right)^{\frac{1}{2}} \div \left(\frac{24}{324}\right)$$

$$= \left(\frac{9^2}{12^2}\right)^{-1} \times \left(\frac{32^2}{3^2}\right)^{\frac{1}{2}} \times \left(\frac{324}{24}\right)$$

$$\left(\frac{9^3}{12^3}\right)^{\frac{-2}{3}} \times \frac{32}{3} \times \frac{324}{24} = \frac{144}{81} \times \frac{32}{3} \times \frac{324}{24} = 256$$

Example 2

In the equation given below, solve for x

$$\sqrt[3]{\left(\frac{5}{7}\right)^{x+1}} = \frac{125}{343}$$

Solution

Given, $\left(\frac{5}{7}\right)^{x+1} = \left[\left(\frac{5}{7}\right)^3\right]^3 = \left(\frac{5}{7}\right)^9$.

By equating their indices.

$$x + 1 = 9$$

$$x = 8.$$

Example 3

If $\left(\frac{49}{2401}\right)^{4-x} = 49^{2x-6}$, find x.

Solution

$$\left(\frac{49}{2401}\right)^{4-x} = \left(49^{-1}\right)^{4-x} = 49^{x-4}$$

Given, $49^{x-4} = 49^{2x-6}$

$$x - 4 = 2x - 6$$

$$x = 2$$

Example 4

Arrange the following in ascending order 625^6, 125^7 and 25^{10}

Solution

$$625^6 = (5^4)^6 = 5^{24}$$
$$125^7 = (5^3)^7 = 5^{21}$$
$$25^{10} = (5^2)^{10} = 5^{20}$$
$$25^{10} < 125^7 < 625^6$$

Surds

Any number of the form p/q, where p and q are integers and $q \neq 0$ is called a rational number. Any real number which is not a rational number is an irrational number. Amongst irrational numbers, of particular interest to us are SURDS. Amongst surds, we will specifically be looking at 'quadratic surds'—surds of the type $a + \sqrt{b}$ and $a + \sqrt{b} + \sqrt{c}$, where the terms involve only square roots and not any higher roots. We do not need to go very deep into the area of surds—what is required is a basic understanding of some of the operations on surds.

If there is a surd of the form $(a + \sqrt{b})$, then a surd of the form $\pm(a - \sqrt{b})$ is called the conjugate of the surd $(a + \sqrt{b})$. The product of a surd and its conjugate will always be a rational number.

Rationalization of a Surd

When there is a surd of the form $\dfrac{1}{a + \sqrt{b}}$, it is difficult to perform arithmetic operations on it. Hence, the denominator is converted into a rational number, thereby facilitating ease of handling the surd. This process of converting the denominator into a rational number without changing the value of the surd is called rationalization.

To convert the denominator of a surd into a rational number, multiply the denominator and the numerator simultaneously with the conjugate of the surd in the denominator so that the denominator gets converted to a rational number without changing the value of the fraction. That is, if there is a surd of the type $a + \sqrt{b}$ in the denominator, then both the numerator and the denominator have to be multiplied with a surd of the form $a - \sqrt{b}$ or a surd of the type $(-a + \sqrt{b})$ to convert the denominator into a rational number.

If there is a surd of the form $(a + \sqrt{b} + \sqrt{c})$ in the denominator, then the process of multiplying the denominator with its conjugate surd has to be carried out TWICE to rationalize the denominator.

Square Root of a Surd

If there exists a square root of a surd of the type $a + \sqrt{b}$, then it will be of the form $\sqrt{x} + \sqrt{y}$. We can equate the square of $\sqrt{x} + \sqrt{y}$ to $a + \sqrt{b}$ and thus solve for x and y. Here, one point should be noted—when there is an equation with rational and irrational terms, the rational part on the left-hand side is equal to the rational part on the right-hand side and, the irrational part on the left-hand side is equal to the irrational part on the right-hand side of the equation.

However, for the problems which are expected in the entrance exams, there is no need of solving for the square root in such an elaborate manner. We will look at finding the square root of the surd in a much simpler manner. Here, first, the given surd is written in the form of $(\sqrt{x} + \sqrt{y})^2$ or $(\sqrt{x} - \sqrt{y})^2$. Then, the square root of the surd will be $(\sqrt{x} + \sqrt{y})$ or $(\sqrt{x} - \sqrt{y})$, respectively.

Comparison of Surds

Sometimes, we will need to compare two or more surds either to identify the largest one or to arrange the given surds in ascending/descending order. The surds given in such cases will be such that they will be close to each other, and, hence, we will not be able to identify the largest one by taking the approximate square root of each of the terms. In such a case, the surds can both be squared and the common rational part be subtracted. At this stage, normally,` one will be able to make out the order of the surds. If even at this stage, it is not possible to identify the larger of the two, then the numbers should be squared once more.

Example 5

Rationalize the denominator: $\dfrac{1}{1+\sqrt{6}-\sqrt{7}}$

Solution

The rationalizing factor of

$1+\sqrt{6}-\sqrt{7}$ is $1 + \sqrt{6} + \sqrt{7}$

$$\frac{1}{1+\sqrt{6}-\sqrt{7}} = \frac{(1+\sqrt{6}+\sqrt{7})}{(1+\sqrt{6}-\sqrt{7})\,(1+\sqrt{6}+\sqrt{7})}$$

$$= \frac{1+\sqrt{6}+\sqrt{7}}{(1+\sqrt{6})^2-(\sqrt{7})^2} = \frac{1+\sqrt{6}+\sqrt{7}}{2\sqrt{6}}$$

The rationalizing factor of $\sqrt{6}$ is $\sqrt{6}$

$$= \frac{\sqrt{6}+6+\sqrt{42}}{12}$$

Example 6

Find the value of $\sqrt{62+\sqrt{480}}$

Solution

Let $\sqrt{62+\sqrt{480}} = \sqrt{a}+\sqrt{b}$

Squaring both sides, $62+\sqrt{480} = a+b+2\sqrt{ab}$

$$62+\sqrt{480} = a+b+\sqrt{4ab}$$

Equating the corresponding rational and irrational parts on both sides, $a + b = 62$

$$4ab = 480 \quad \Rightarrow \quad ab = 120$$

As $a + b = 60 + 2$ and $ab = (60)$ (B) it follows that $a = 60$ and $b = 2$ or vice versa.

$$\therefore \quad \sqrt{a} + \sqrt{b} = \sqrt{60} + \sqrt{2}$$

Example 7

Which of the surds given below is greater?

$$\sqrt{3} + \sqrt{23} \text{ and } \sqrt{6} + \sqrt{19}$$

Solution

$$(\sqrt{3} + \sqrt{23})^2 = 26 + 2\sqrt{69}$$

$$\sqrt{69} \text{ lies between } \sqrt{64} \text{ and } \sqrt{81}$$

$\therefore 26 + 2\sqrt{69}$ lies between 26 + 2(8) and 26 + 2(9) i.e., 42 and 44.

Similarly $(\sqrt{6} + \sqrt{19})^2$ lies between 45 and 47.

$$\therefore \quad (\sqrt{3} + \sqrt{23})^2 < (\sqrt{6} + \sqrt{19})^2$$

$$\therefore \quad \sqrt{6} + \sqrt{19} > \sqrt{3} + \sqrt{23}$$

Logarithms

In the equation $a^x = N$, we are expressing N in terms of a and x. The same equation can be re-written as, $a = N^{1/x}$. Here, we are expressing a in terms of N and x. But, among a, x, and N, by normal algebraic methods known to us, we cannot express x in terms of the other two parameters a and N. This is where logarithms come into the picture. When $a^x = N$, then we say x = logarithm of N to the base a and write it as $x = \log_a N$. The definition of logarithm is given as: 'the logarithm of any number to a given base is the index or the power to which the base must be raised in order to equal the given number'.

Thus,

$$\boxed{\text{if } a^x = N \text{ then } x = \log_a N}$$

This is read as 'log N to the base a'.

In the above equation, N is a **POSITIVE NUMBER** and a is a **POSITIVE NUMBER OTHER THAN 1**.

This basic definition of logarithm is very useful in solving a number of problems on logarithms.

Example of a logarithm: $216 = 6^3$ can be expressed as $\log_6 216 = 3$.

Since logarithm of a number is a value, it will have an 'integral' part and a 'decimal' part. The integral part of the logarithm of a number is called the CHARACTERISTIC and the decimal part of the logarithm is called the MANTISSA.

Logarithms can be expressed to any base (positive number other than 1). Logarithms from one base can be converted to logarithms to any other base. (One of the formulae given below will help do this conversion.) However, there are two types of logarithms that are commonly used.

(i) **Natural Logarithms or Napierian Logarithms:** These are logarithms expressed to the base of a number called 'e'.

(ii) **Common Logarithms:** These are logarithms expressed to the base 10. For most of the problems under LOGARITHMS, it is common logarithms that we deal with. In examinations also, if logarithms are given without mentioning any base, it can normally be taken to be logarithms to the base 10.

Given below are some **important rules/formulae** in logarithms:

(i) $\log_a a = 1$ (logarithm of any number to the same base is 1)

(ii) $\log_a 1 = 0$ (log of 1 to any base other than 1 is 0)

(iii) $\log_a (mn) = \log_a m + \log_a n$

(iv) $\log_a (m/n) = \log_a m - \log_a n$

(v) $\log_a m^p = p \times \log_a m$

(vi) $\log_a b = \dfrac{1}{\log_b a}$

(vii) $\log_a m = \dfrac{\log_b m}{\log_b a}$

(viii) $\log_{a^q} m^p = \dfrac{p}{q} \log_a m$

(ix) $a^{\log_a N} = N$

(x) $a^{\log b} = b^{\log a}$

You should memorize these rules/formulae because they are very helpful in solving problems.

Like in the chapter on INDICES, in LOGARITHMS also there will be problems on

(i) Simplification using the above-listed formulae/rules and

(ii) Solving for the value of an unknown given in an equation.

In solving problems of the second type earlier, in most of the cases, we take recourse to the basic definition of logarithms (which is very important and should be memorized).

The following examples will give problems of both the above types and some problems on common logarithms.

The following rules also should be remembered while solving problems on logarithms:

Given an equation $\log_a M = \log_b N$,

(i) if $M = N$, then a will be equal to b; if $M \neq 1$ and $N \neq 1$.

(ii) if $a = b$, then M will be equal to N.

The examples that follow will explain all the above types of problems. Please note that unless otherwise specified, all the logarithms are taken to the base 10).

Example 8

Solve for x : $\log_{10} 20x = 4$

Solution

Given that $\log_{10} 20x = 4$

$$\Rightarrow \qquad 20x = 10^4 = 10000$$

$$\therefore \qquad x = 500$$

Example 9

Solve for x : $\log (x + 3) + \log (x – 3) = \log 72$

Solution

$\log (x + 3) + \log (x – 3) = \log 72$

$$\log (x + 3)(x – 3) = \log 72$$

$$(x + 3)(x – 3) = 72$$

$$x^2 = 81$$

$x = 9$ (If $x = – 9$, $\log (x – 3)$ would be undefined)

Example 10

If log 2 = 0.301, find the value of log 1250, log 0.001250, and log 125000.

Solution

$$\log 1250 = \log \frac{10000}{8}$$

$$= 4 \log 10 – 3 \log 2 = 4 – 3 (0.3010) = 3.097$$

$$\log 0.001250 = \log \frac{1250}{10^6}$$

$$= 3.097 – 6 = –2.903$$

$$\log 125000 = \log (1250)(100)$$

$$= \log 1250 + 2 = 5.097$$

Example 11

Find the number of digits in 294^{20} given that log 6 = 0.778 and log 7 = 0.845

Solution

$\log 294^{20} = 20 \log (7^2\, 6)$

$= 20 (2 \log 7 + \log 6)$

$= 20 (2 (0.845) + 0.778) = 20 (1.69 + 0.778)\ 49.36$

Characteristic = 49.

$\therefore$ 294^{20} has 50 digits

Example 12

Find the value of $\log_{\sqrt[3]{2}} 32\sqrt[3]{16}$.

Solution

$$\log_{\sqrt[3]{2}} 32\sqrt[3]{16} = \log_{2^{1/3}} 2^5 \left(2^4\right)^{\frac{1}{3}}$$

$$= \log_{2^{1/3}} \left(2^{\frac{1}{3}}\right)^{19} = 19$$

Example 13

Find the number of zeros after the decimal point in $\left(\frac{3}{4}\right)^{500}$, given that log 3 = 0.4771 and log 2 = 0.3010.

Solution

$$\log \left(\frac{3}{4}\right)^{500} = 500\left(\log \frac{3}{4}\right)$$

$= 500 (\log 3 – 2 \log 2)$

$= 500 (0.4771 – 2 (0.3010) = – 62.4500$

$\therefore$ Number of zeros after the decimal point is 62.

EXERCISES

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. Simplify the following:

$$\left(\frac{243}{1024}\right)^{-2/5} \times \left(\frac{144}{49}\right)^{-1/2} \div \left(\frac{8}{343}\right)^{-2/3}$$

(A) $2^5 \times 3^{-1} \times 7$ (B) $2^5 \times 3^{-3} \times 7^{-2}$
(C) $2^4 \times 3^{-3} \times 7^{-1}$ (D) $2^4 \times 3^{-1} \times 7^{-2}$

2. Simplify the following:

$$\left(\frac{x^2 \cdot y^{-3}}{z^4}\right)^{-2} \times \left(\frac{x^2 \cdot y}{z^{-2}}\right)^{3} \div \left(\frac{x^{-12} \cdot y^7}{z^{-8}}\right)^{-1}$$

(A) $x^{10}. y^{16} . z^{-22}$ (B) $x^{7}. y^{-16} . z^{-22}$
(C) $x^{-7}. y^{16} . z^{-22}$ (D) $x^{-10}. y^{16} . z^{22}$

3. Simplify the following: $\dfrac{1-[1-\{1-(1+y)^{-1}\}]}{(1-y)}$

(A) $\dfrac{y}{(1-y^2)}$ (B) $\dfrac{y}{(1-y)^2}$
(C) $\dfrac{1+y}{(1-y)^2}$ (D) $\dfrac{1+y^2}{1-y^2}$

4. $(x^{a-b})^{(a^2+ab+b^2)} \times (x^{b-c})^{(b^2+bcc^2)} \times (x^{c-a})^{(c^2+ac+a^2)}$

(A) 0
(B) 1
(C) $x^{a^3+b^3+c^3}$
(D) $x^{3(a^2+b^2+c^2+ac+bc+ca)}$

5. $343^{0.12} \times 2401^{0.08} \times 49^{0.01} \times 7^{0.1} =$
(A) 7 (B) $7^{4/5}$ (C) 7^8 (D) $7^{3/5}$

6. Solve for x: $9^{2x+1} = 27^{5x-3}$
(A) 1 (B) 2 (C) –1 (D) –2

7. If $\frac{p}{q} = \frac{r}{s}$ and $p^a = q^b = r^c = s^d$, then $\frac{1}{a} - \frac{1}{b} =$
(A) $\frac{1}{d} - \frac{1}{c}$ (B) $\frac{1}{c} + \frac{1}{d}$
(C) $\frac{1}{c} - \frac{1}{d}$ (D) $-\left(\frac{1}{c} + \frac{1}{d}\right)$

8. Which of the following is the largest in value?
(A) $6^{1/2}$ (B) $7^{1/3}$ (C) $8^{1/4}$ (D) $9^{1/5}$

9. $2\sqrt{\frac{5}{2}} - 5\sqrt{\frac{2}{5}} + \sqrt{10} + \sqrt{1000} =$
(A) $9\sqrt{10}$ (B) $8\sqrt{10}$
(C) $8\sqrt{10}$ (D) $11\sqrt{10}$

10. $\left(\frac{\sqrt{p} - \sqrt[4]{pq}}{\sqrt[4]{pq} - \sqrt{q}}\right)^{-4} =$
(A) $\frac{p}{q}$ (B) $\sqrt{\frac{p}{q}}$ (C) $\sqrt{\frac{q}{p}}$ (D) $\frac{q}{p}$

11. If $y = 12 + 2\sqrt{35}$, then $\sqrt{y} - \frac{1}{\sqrt{y}} =$
(A) $\frac{\sqrt{7}+\sqrt{5}}{2}$ (B) $\frac{3\sqrt{5}-\sqrt{7}}{2}$
(C) $\frac{2\sqrt{5}+\sqrt{7}}{2}$ (D) $\frac{3\sqrt{5}+\sqrt{7}}{2}$

12. Arrange the following in ascending order.
$a = \sqrt{2} + \sqrt{11}$, $b = \sqrt{6} + \sqrt{7}$,
$c = \sqrt{3} + \sqrt{10}$ and $d = \sqrt{5} + \sqrt{8}$
(A) *abcd* (B) *abdc*
(C) *acdb* (D) *acbd*

13. Arrange the following in descending order.
$a = \sqrt{13} + \sqrt{11}$, $b = \sqrt{15} + \sqrt{9}$, $c = \sqrt{18} + \sqrt{6}$,
$d = \sqrt{7} + \sqrt{17}$.
(A) *abdc* (B) *dcab*
(C) *adcb* (D) *acdb*

14. Solve for x and y:
$3.5^x + 2^{y+2} = 107$, $5^{x+1} + 8.2^y = 189$
(A) 3, 2 (B) 5, 7 (C) 7, 5 (D) 2, 3

15. Solve for x, if $\left(5\sqrt{7}\right)^{5x-4} = (35)^3 (25)^{3/2}$.
(A) 2 (B) 5/4 (C) 7/2 (D) 3

16. If $5^{x+3} - 5^{x-3} = 78120$, find x.
(A) 4 (B) 3 (C) 5 (D) 6

17. If $a^a \cdot b^b \cdot c^c = a^b \cdot b^c \cdot c^a = a^c \cdot b^a \cdot c^b$ and a, b, c are positive integers greater than 1, then which of the following can NOT be true for any of the possible values of a, b, c?
(A) $abc = 8$ (B) $a + b + c = 8$
(C) $abc = 27$ (D) $a + b + c = 27$

18. The ascending order of $16^{\frac{7}{12}}$, $81^{\frac{3}{8}}$, $625^{\frac{2}{3}}$ is ______.
(A) $16^{\frac{7}{12}}, 81^{\frac{3}{8}}, 625^{\frac{2}{3}}$ (B) $16^{\frac{7}{12}}, 625^{\frac{2}{3}}, 81^{\frac{3}{8}}$,
(C) $625^{\frac{2}{3}}, 16^{\frac{7}{12}}, 81^{\frac{3}{8}}$ (D) $81^{\frac{3}{8}}, 16^{\frac{7}{12}}, 625^{\frac{2}{3}}$

19. If $2\sqrt{2} + \sqrt{3} = x$, what is the value of $\frac{11+4\sqrt{6}}{2\sqrt{2}-\sqrt{3}}$ in terms of x?
(A) $\frac{x^2}{\sqrt{2}}$ (B) x^3 (C) $\frac{x^3}{8}$ (D) $\frac{x^3}{5}$

20. Simplify: $\sqrt{(a+b+c) + 2\sqrt{ac+bc}}$.
(A) $\sqrt{a} + \sqrt{b} + \sqrt{c}$ (B) $\sqrt{a+b} + \sqrt{c}$
(C) $\sqrt{ab+bc}$ (D) $\sqrt{abc}$

21. Find the value of $x^2 - y^2$, if $\log_y(x-1) + \log_y(x+1) = 2$.
(A) 2 (B) $2y$ (C) 1 (D) $2xy$

22. If $a > 1$, $\log_a a + \log_{a^{\frac{1}{2}}} a + \log_{a^{\frac{1}{3}}} a + \cdots + \log_{a^{\frac{1}{20}}} a =$
(A) 420 (B) 210 (C) 380 (D) 190

23. If $\log_7(x-7) + \log_7(x^2 + 7x + 49) = 4$, then $x =$
(A) 196 (B) 7 (C) 49 (D) 14

24. If $\frac{\log a}{5} = \frac{\log b}{6} = \frac{\log c}{7}$, then $b^2 =$
(A) ac (B) a^2 (C) bc (D) ab

25. What is the value of $\log_{(1/5)} 0.0000128$?
(A) –7 (B) –5 (C) 5 (D) 7

26. If $(\log\tan 5°)(\log\tan 10°)(\log\tan 15°)\ldots(\log\tan 60°) = x$, what is the value of x?
(A) $\log(\sin 5°)^{12}$ (B) 1
(C) 0 (D) $\log(\cos 60°)$

27. Solve for x, if $\log_x[\log_5(\sqrt{x+5} + \sqrt{x})] = 0$.
(A) 1 (B) 9 (C) 12 (D) 4

28. If a, b, c are distinct values, what is the value of abc if $(\log_b a)(\log_c a) + (\log_a b)(\log_c b) + (\log_a c)(\log_b c) - 3 = 0$?
(A) 2 (B) 1
(C) $1 - \log a - \log b - \text{loc } c$ (D) 0

29. If $\log_6 161 = a$, $\log_6 23 = b$, what is the value of $\log_7 6$ in terms of a and b?
(A) a/b (B) $a + b$
(C) $1/(a - b)$ (D) b/a

30. $x = y^2 = z^3 = w^4 = u^5$, then find the value of $\log_x xyzwu$.
(A) $1\frac{47}{60}$ (B) $\frac{111}{120}$ (C) $2\frac{17}{60}$ (D) $2\frac{13}{60}$

Answer Keys

1. C	**2.** D	**3.** A	**4.** B	**5.** B	**6.** A	**7.** C	**8.** A	**9.** D	**10.** D
11. D	**12.** C	**13.** A	**14.** D	**15.** A	**16.** A	**17.** B	**18.** A	**19.** D	**20.** B
21. C	**22.** B	**23.** D	**24.** A	**25.** D	**26.** C	**27.** D	**28.** B	**29.** C	**30.** C