Short Answer Type Questions – II

[3 marks]

Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients (Q. 1 - 2)

Que 1. $6x^2 – 3 – 7x$

Sol. We have, $p(x) = 6x^2 – 3 - 7x$

$\Rightarrow P(x) = 6x^2 – 7x – 3$ (In general form)

$= 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3)$

$= (2x – 3)(3x + 1)$

The zero of polynomial p (x) is given by

$P(x) = 0 \quad \Rightarrow \quad (2x – 3)(3x + 1) = 0 \Rightarrow \quad x = \frac{3}{2}, -\frac{1}{3}$

Thus, the zeros of $6x^2 – 7x – 3$ are $a = \frac{3}{2}$ and $b = -\frac{1}{3}$

Now, sum of the zeros = $a + b = \frac{3}{2} - \frac{1}{3} = \frac{9-2}{6} = \frac{7}{6}$

And $\frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2} = \frac{-7}{6} = \frac{7}{6}$

Therefore, sum of the zeros = $\frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2}$

Again, product of zeros = $a.b = \frac{3}{2} \times (-\frac{1}{3}) = -\frac{1}{2}$

And $\frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{-3}{6} = -\frac{1}{2}$

Therefore, product of zeros = $\frac{Constant\ term}{Coefficient\ of\ x^2}$

Que 2. $4u^2 + 8u$

Sol. We have, $p(u) = 4u^2 + 8u \quad \Rightarrow \quad p(u) = 4u(u + 2)$

The zeros of polynomial p (u) is given by

$P(u) = 0 \quad \Rightarrow \quad 4u(u + 2) = 0$

$\therefore \quad u = 0, -2$

Thus, the zeros of $4u^2 + 8u$ are $a = 0$ and $b = -2$

Now, sum of the zeros = a + b = 0 – 2 = -2

And $\frac{-(Coefficient\ of\ u)}{Coefficient\ of\ u^2} = \frac{-8}{4} = -2$

Therefore, sum of the zeros = $\frac{-(Coefficient\ of\ u)}{Coefficient\ of\ u^2}$

Again, product of the zeros = ab = 0 × (-2) = 0

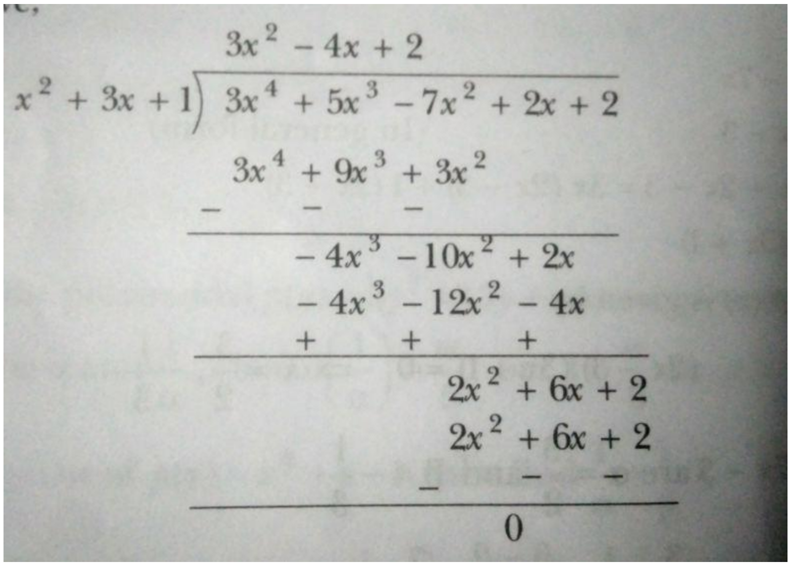
And $\frac{Constant\ term}{Coefficient\ of\ u^2} = \frac{0}{4} = 0$

Therefore, product of zeros = $\frac{Constant\ term}{Coefficient\ of\ u^2}$

Que 3. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

- **(i)** **$X^2 + 3x + 1, 3x^4 + 5x^3 – 7x^2 + 2x + 2$**
- **(ii)** **t2 – 3, $2t^4 + 3t^3 – 2t^2 – 9t – 12$**

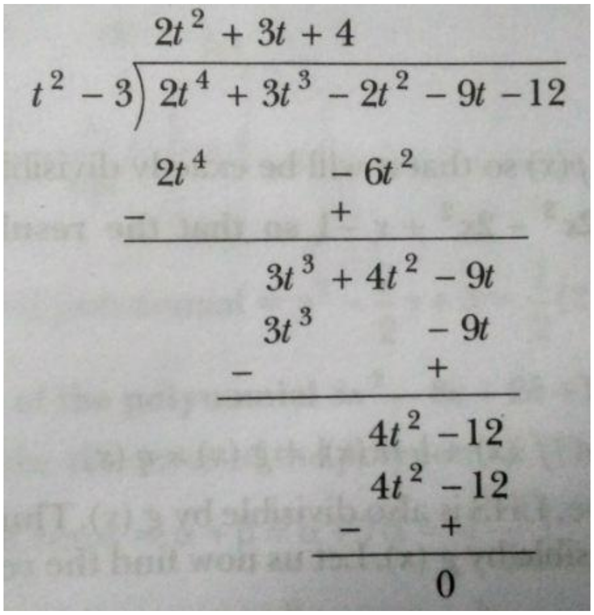
Sol. (i) we have,



$$\begin{array}{r|l} & 3x^2 - 4x + 2 \\ x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\ & 3x^4 + 9x^3 + 3x^2 \\ & - \quad - \quad - \\ \hline & -4x^3 - 10x^2 + 2x \\ & -4x^3 - 12x^2 - 4x \\ & + \quad + \quad + \\ \hline & 2x^2 + 6x + 2 \\ & 2x^2 + 6x + 2 \\ & - \quad - \quad - \\ \hline & 0 \end{array}$$

Clearly, remainder is zero, so $x^2 + 3x + 1$ is a factor of polynomial $3x^4 + 5x^3 – 7x^2 + 2x +2$.

(ii) We have,



$$\begin{array}{rl}
 & 2t^2 + 3t + 4 \\
t^2 - 3 \big) & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 \qquad\quad - 6t^2 \\
 & - \qquad\qquad\;\; + \\
\hline
 & \qquad 3t^3 + 4t^2 - 9t \\
 & \qquad 3t^3 \qquad\;\; - 9t \\
 & \qquad - \qquad\quad\;\; + \\
\hline
 & \qquad\qquad 4t^2 - 12 \\
 & \qquad\qquad 4t^2 - 12 \\
 & \qquad\qquad - \quad\;\; + \\
\hline
 & \qquad\qquad\qquad 0
\end{array}$$

Clearly, remainder is zero, so $t^2 - 3$ is a factor of polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

Que 4. If a and B are the zeros of the quadratic polynomial f(x) = $2x^2 - 5x + 7$, find a polynomial whose zeros are 2a + 3b and 3a + 2b.

Sol. Since a and b are the zeros of the quadratic polynomial f(x) = $2x^2 - 5x + 7$

∴ a + b = $= \frac{-(5)}{2} = \frac{5}{2}$ $and\ ab = \frac{7}{2}$

Let s and p denote respectively the sum and product of the zeros of the required polynomial.

Then, S = (2a + 3b) + (3a + 2b) = 5(a + b) = 5 × $\frac{5}{2} = \frac{25}{2}$

And p = (2a + 3b) (3a + 2b)

⇒ p = $6a^2 + 6b^2 + 13ab = 6a^2 + 6b^2 + 12ab + ab$

$= 6(a^2 + b^2 + 2ab) + ab = 6\,(a + b)^2 + ab$

⇒ p = $6 \times \left(\frac{5}{2}\right)^2 + \frac{7}{2} = \frac{75}{2} + \frac{7}{2} = 41$

Hence, the required polynomial g(x) is given by

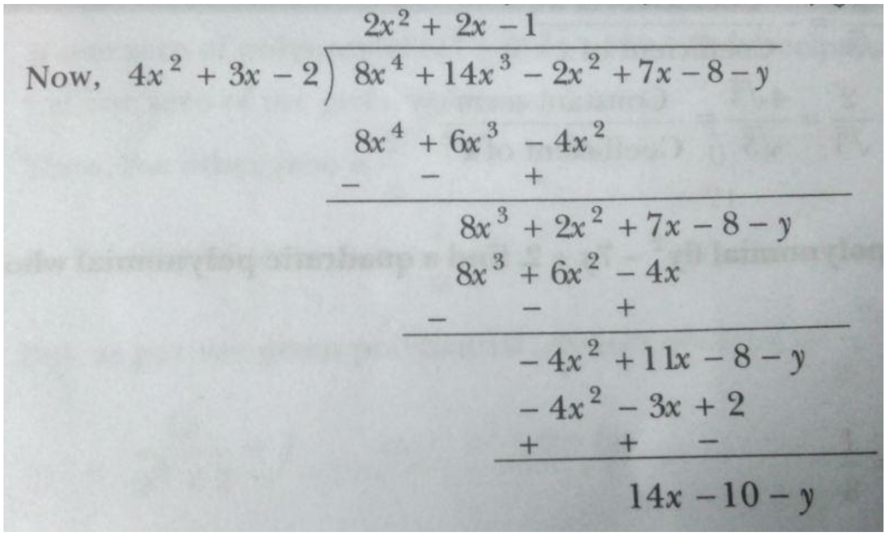
g(x) = $k(x^2 - Sx + p)$

Or g(x) = k $\left(x^2 - \frac{25}{2}x + 41\right)$, where k is any non-zero real number.

Que 5. What must be subtracted from p(x) = $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by g(x) = $4x^2 + 3x - 2$?

Sol. Let y be subtracted from p(x)

∴ $8x^4 + 14x^3 - 2x^2 + 7x - 8y$ is exactly divisible by g(x)



$$
\begin{array}{rl}
 & 2x^2 + 2x - 1 \\
\text{Now, } 4x^2 + 3x - 2 & \overline{\big)\, 8x^4 + 14x^3 - 2x^2 + 7x - 8 - y} \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & - \quad - \quad + \\
\hline
 & 8x^3 + 2x^2 + 7x - 8 - y \\
 & 8x^3 + 6x^2 - 4x \\
 & - \quad - \quad + \\
\hline
 & -4x^2 + 11x - 8 - y \\
 & -4x^2 - 3x + 2 \\
 & + \quad + \quad - \\
\hline
 & 14x - 10 - y
\end{array}
$$

∵ Remainder should be 0.

∴ $14x - 10 - y = 0$

Or $14x - 10 = y$ or $y = 14x - 10$

∴ (14x – 10) should be subtracted from p(x) so that it will be exactly divisible by g(x).

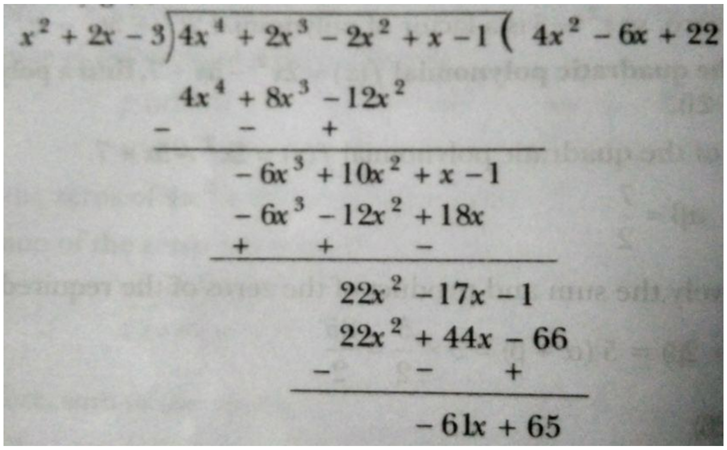
Que 6. What must be added to f(x) = $4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by g(x) = $x^2 + 2x - 3$?

Sol. By division algorithm, we have

$f(x) = g(x) \times q(x) + r(x)$

$\Rightarrow f(x) - r(x) = g(x) \times q(x)$ $\Rightarrow f(x) + \{-r(x)\} = g(x) \times q(x)$

Clearly, RHS is divisible by g(x). Therefore, LHS is also divisible by g(x). Thus, if we add -r(x) to f(x), then the resulting polynomial is divisible by g(x). Let us now find the remainder when f(x) is divided by g(x).



$x^2 + 2x - 3 \overline{) 4x^4 + 2x^3 - 2x^2 + x - 1} (4x^2 - 6x + 22$

$4x^4 + 8x^3 - 12x^2$

$- \quad - \quad +$

$-6x^3 + 10x^2 + x - 1$

$-6x^3 - 12x^2 + 18x$

$+ \quad + \quad -$

$22x^2 - 17x - 1$

$22x^2 + 44x - 66$

$- \quad - \quad +$

$-61x + 65$

∴ r(x) = -61x + 65 or -r(x) = 61x – 65

Hence, we should add -r(x) = 61x – 65 to f(x) so that the resulting polynomial is divisible by g(x).

Que 7. Obtain the zeros of quadratic polynomial $\sqrt{3x^2}$ – 8x + 4$\sqrt{3}$ and verify the relation between its zeros and coefficients.

Sol. We have,

f(x) = $\sqrt{3}x^2 - 8x + 4\sqrt{3}$

= $\sqrt{3x^2} - 6x - 2x + 4\sqrt{3} = \sqrt{3x}(x - 2\sqrt{3}) - 2(x - 2\sqrt{3}$

⇒ $(x - 2\sqrt{3})(\sqrt{3x} - 2) = 0$

$x = 2\sqrt{3} \qquad or \qquad x = \frac{2}{\sqrt{3}}$.

So, the zeros of $f(x)$ are $2\sqrt{3}$ and $\frac{2}{\sqrt{3}}$.

Sum of zeros = $2\sqrt{3} + \frac{2}{\sqrt{3}} = \frac{8}{\sqrt{3}} = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$

Product of the zeros = $2\sqrt{3} \times \frac{2}{\sqrt{3}} = \frac{4\sqrt{3}}{3} = \frac{Constant\ term}{Coefficient\ of\ x^2}$

Hence verified.

Que 8. If a and b are the zeros of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeros are $\frac{1}{a}$ and $\frac{1}{b}$.

Sol. Let $p(y) = 6y^2 - 7y + 2$

$$a + b = -\left(\frac{-7}{6}\right) = \frac{7}{6}; ab = \frac{2}{6} = \frac{1}{3}$$

Now, $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{7}{6 \times \frac{1}{3}} = \frac{7}{2}$

$$\frac{1}{a} \times \frac{1}{b} = \frac{1}{ab} = \frac{1}{\frac{1}{3}} = 3$$

The required polynomial = $y^2 - \frac{7}{2}y + 3 = \frac{1}{2}(2y^2 - 7y + 6)$

Que 9. If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Sol. Let a and b be the zeros of the polynomial. Then as per question b = 7a

Now sum of zeros = $a + b = a + 7a = -\left(\frac{-8}{3}\right)$

$$= 8a = \frac{8}{3}\ or = a = \frac{1}{3}$$

And $\quad a \times b = a \times 7a = \frac{2k+1}{3}$

$\Rightarrow \quad 7a^2 = \frac{2k+1}{3} \Rightarrow 7\left(\frac{1}{2}\right)^2 = \frac{2k+1}{3} \quad \left(\because a = \frac{1}{3}\right)$

$\Rightarrow \quad \frac{7}{9} = \frac{2k+1}{3} \quad \Rightarrow \quad \frac{7}{3} = 2k + 1$

$\Rightarrow \quad \frac{7}{3} - 1 = 2k \Rightarrow \quad k = \frac{2}{3}$

Que 10. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and other zero.

Sol. Let $p(x) = 2x^2 + 3x + \boldsymbol{\lambda}$

Its one zero is $\frac{1}{2}\ so\ p\left(\frac{1}{2}\right) = 0$

$$p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \frac{1}{2} + \boldsymbol{\lambda} = \mathbf{0}$$

$\Rightarrow \quad \frac{1}{2} + \frac{-3}{2} + \lambda = 0 \quad \Rightarrow \quad \frac{4}{2} + \lambda = 0$

$\Rightarrow \quad 2 + \lambda = 0 \quad \Rightarrow \quad \lambda = -2$

Let the other zero be a.

Then $a + \frac{1}{2} = \frac{-3}{2} + \frac{1}{2} = -2$

Que 11. If one zero of polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other, find the value of a.

Sol. Let one zero of the given polynomial be a.

Then, the other zero is $\frac{1}{a}$

$\therefore \ Product\ of\ zeros = a \times \frac{1}{a} = 1$

But, as per the given polynomial product of zeros = $\frac{6a}{a^2+9}$

$\therefore \quad \frac{6a}{a^2+9} = 1 \quad \Rightarrow \quad a^2 + 9 = 6a$

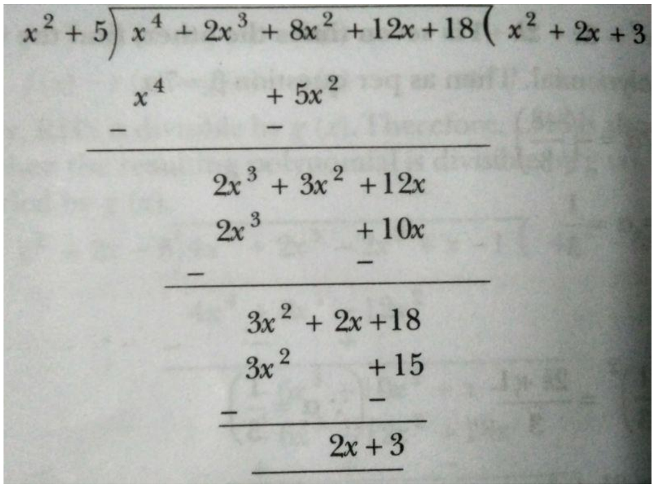
$\Rightarrow \quad a^2 - 6a + 9 = 0 \quad \Rightarrow \quad (a-3)^2 = 0$

$\Rightarrow \quad a - 3 = 0 \quad \Rightarrow \quad a = 3$

Que 12. If the polynomial $(x^4 + 2x^3 + 8x^2 + 12x + 18)$ is divided by another polynomial $(x^2 + 5)$, the remainder comes out to be $(px + q)$. Find value of p and q.

Sol. Let $f(x) = (x^4 + 2x^3 + 8x^2 + 12x + 18)$ and $g(x) = (x^2 + 5)$.

On dividing f(x) by g(x), we get



$x^2 + 5 \overline{)\, x^4 + 2x^3 + 8x^2 + 12x + 18} \left(x^2 + 2x + 3 \right.$

$x^4 \quad + 5x^2$

$- \quad -$

$2x^3 + 3x^2 + 12x$

$2x^3 \quad + 10x$

$- \quad -$

$3x^2 + 2x + 18$

$3x^2 \quad + 15$

$- \quad -$

$2x + 3$

Now, $px + q = 2x + 3 \quad \Rightarrow \quad p = 2, q = 3$ (By comparing the coefficient of x and constant term).