

## Pair of Linear Equations in Two Variables

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### Case Study Based Questions

#### Case Study 1

Sanjeev a student of class X, goes to Yamuna river with his friends. When he saw a boat in the river, then he wants to sit in boat. So his all friends are ready to sit with him. In this order, Sanjeev is sitting on a boat which upstream at a speed of 8 km/h and downstream at a speed of 16 km/h. When Sanjeev is in the boat, some questions are arises in the mind, then answer the given questions.



Based on the above information, solve the following questions:

**Q1. The speed of the boat in still water is:**

- a. 8 km/h
- b. 10 km/h
- c. 12 km/h
- d. 14 km/h

**Q2. The speed of stream is:**

- a. 3 km/h
- b. 4 km/h
- d. 6 km/h
- c. 5 km/h

**Q3. Which mathematical concept is used in above problem?**

- a. Pair of linear equations
- b. Cross-multiplication method

- c. Factorisation method
- d. None of the above

**Q4. The direction in which the speed is maximum, is:**

- a. upstream
- b. downstream
- c. both have equal speed
- d. None of the above

**Q5. The average speed of stream and boat in still water is:**

- a. 7 km/h
- b. 10 km/h
- c. 12 km/h
- d. 5 km/h

### Solutions

1. Let the speed of the boat in still water be  $x$  km/h and speed of the stream be  $y$  km/h.

Then,  $x - y = 6$  ...(1)

$x + y = 14$  ...(2)

On adding eqs. (1) and (2), we get

$$2x = 20$$

$$= x = 10$$

.. Speed of the boat in still water is 10 km/h

So, option (b) is correct.

2. On putting the value of  $x$  in eq. (2), we get

$$10 + y = 14$$

$$= y = 14 - 10 = 4$$

.. Speed of the stream is 4 km/h.

So, option (b) is correct.

3. Pair of linear equation concept is used in above problem.

So, option (a) is correct.

4. In downstream, the speed is maximum because in downstream, the speed is  $(x + y)$  km/h and in upstream, the speed is  $(x - y)$  km/h. So, option (b) is correct.

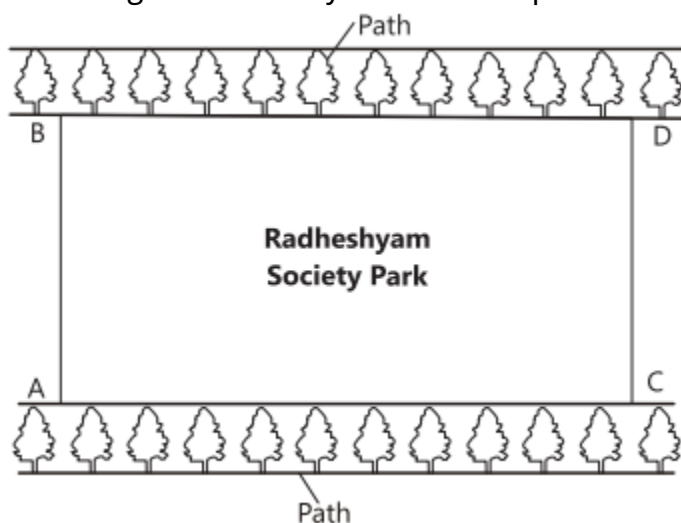
## 5. Average speed of stream and boat in still water

$$= \frac{y+x}{2} = \frac{4+10}{2} = \frac{14}{2} = 7 \text{ km/h}$$

So, option (a) is correct.

### Case Study 2

The resident welfare association of a Radheshyam society decided to build two straight paths in their neighbourhood park such that they do not cross each other and also plant trees along the boundary lines of each path.



One of the members of association Shyam Lal suggested that the paths should be constructed represented by the two linear equations  $x-3y = 2$  and  $-2x+6y = 5$ .

Based on the above information, solve the following questions:

Q1. If the pair of equations  $ax + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  has infinitely solutions, then condition is:

a.  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

b.  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

c.  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

d. None of these

Q2. If pair of lines are parallel, then pair of linear equations is:

a. inconsistent

b. consistent

- c. consistent or inconsistent
- d. None of the above

**Q3. Check whether the two paths will cross each other or not.**

- a. yes
- b. no
- c. can't say
- d. None of these

**Q4. How many point(s) lie on the line  $x-3y = 2$ ?**

- a. one
- b. two
- c. three
- d. infinitely

**Q5. If the line  $2x+6y= 5$  intersect the X-axis, then find its coordinate.**

- a. (-2.5, 0)
- b. (2.5, 0)
- c. (0,2.5)
- d. (0, -2.5)

## Solutions

1. If the pair of equation  $ax + by + G_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  has infinitely solutions, then

condition is  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, option (b) is correct.

2. If pair of lines are parallel, then pair of linear equation is consistent.

So, option (b) is correct.

3. Given, equation of paths are

$$x - 3y = 2 \quad \dots(1)$$

$$\text{and } -2x + 6y = 5 \quad \dots(2)$$

$$\text{Here, } a_1 = 1, b_1 = -3, c_1 = -2$$

$$\text{and } a_2 = -2, b_2 = 6, c_2 = -5$$

$$\therefore \frac{a_1}{a_2} = \frac{1}{-2}, \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the paths represented by the equations are parallel i.e., do not intersect each other.  
So, option (b) is correct.

4. Infinitely point lies on the line  $x-3y=2$ . So, option (d) is correct.

5. The y-coordinate on X-axis is zero.

Put  $y = 0$  in  $2x+6y=5$ , we get

$$2x+6(0) = 5$$

$$\Rightarrow x = \frac{5}{2} = 2.5$$

Hence, the coordinates of X-axis is (2.5,0). So, option (b) is correct.

### Case Study 3

Akhila went to a fair in her village. She wanted to enjoy rides on the giant wheel and play hoopla (a game in which you throw a ring on the items kept in a stall and if the ring covers any object completely you get it). The number of times she played hoopla is half the number of times she rides the giant wheel. If each ride costs ₹ 3 and a game of hoopla costs 4 and she spent 20 in the fair.



Based on the above information, solve the following questions:

Q1. Write the representation of given statement algebraically.

Q2. Find the intersection point of two lines.

Q3. Find the intersection points of the line  $x-2y=0$  on X and Y-axes.

OR

Intersection points of the line  $3x + 4y = 20$  on X and Y-axes.

### Solutions

1. Let  $x$  and  $y$  be the number of rides on the giant wheel and number of hoopla respectively played by Akhila.

Then, according to the given condition,

$$y = \frac{x}{2} \text{ and } 3x + 4y = 20$$

.. The given situation can be algebraically represented by the following pair of the linear equations are

$$x-2y=0 \dots(1)$$

$$\text{and } 3x+4y=20\dots(2)$$

2. Put  $x=2y$  in eq. (2), we get

$$3(2y) + 4y = 20$$

$$= 10y=20$$

$$= y=2$$

$$\therefore x=2 \times 2=4$$

Hence, intersection point of two lines is  $(4, 2)$ .

3. Table for equation  $x-2y=0$  is:

$x$	0
$y = \frac{x}{2}$	0
Points	$(0, 0)$

i.e., the lines passes through the origin.

Or

Table for equation  $3x + 4y = 20$  is:

x	0	$\frac{20}{3}$
$y = \frac{20-3x}{4}$	0	0
Points	(0, 5)	( $\frac{20}{3}$ , 0)

i.e., the intersection points of the line on X and Y-axes are  $\left(\frac{20}{3}, 0\right)$  and (0, 5).

#### Case Study 4

The residents of a housing society at Jaipur decided to build a rectangular garden to beautify the garden.



One of the members of the society made some calculations and informed that if the length of the rectangular garden is increased by 2m and the breadth reduced by 2 m, the area gets reduced by 12 sq. m. However, when the length is decreased by 1 m and breadth increased by 3m, the area of the rectangular garden is increased by 21 sq. m. Based on the above information, solve the following questions:

**Q1. Find the coordinates of the points on X-axis, where the two lines, plotted on a graph paper intersect the X-axis.**

**Q2. Find the value of k for which the system of equations  $x + y - 4 = 0$  and  $2x + ky = 3$  has no solution.**

**Q3. Find the dimensions of the rectangle.**

OR

If the graphs of the equations in the given situation are plotted on the same graph paper, then lines will intersect. Check whether given statement is True/False.

## Solutions

1. The points where the two lines will intersect the X-axis can be found by putting  $y = 0$  in both the equations, as the y-coordinate of all points lying on the X-axis is zero.

Putting  $y = 0$  in the equations  $x - y = 4$  and  $3x - y = 24$ , we get  $x = 4$  and  $x = 8$ , respectively.  
Therefore, the points are  $(4, 0)$  and  $(8, 0)$ .

2. For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$$
$$\Rightarrow k = 2$$

3. Let the length and breadth of the rectangular garden be denoted by  $x$  m and  $y$  m respectively. The area of the rectangular garden =  $xy$  sq. m. According to the question,  
 $(x+2)(y-2) = xy - 12$

Simplifying the above equations, we get

$$xy - 2x + 2y - 4 = xy - 12$$

$$= -2x + 2y = -8$$

$$= x - y = 4 \dots (1)$$

$$\text{And } (x-1)(y+3) = xy + 21$$

$$xy + 3x - y - 3 = xy + 21$$

$$= 3x - y = 24 \dots (2)$$

Let us now solve the eqs. (1) and (2) by the method of substitution.

$$\text{From eq. (1). } x = y + 4 \dots (3)$$

Substituting in eq. (2).

$$3(y+4) - y = 24$$

$$= 3y + 12 - y = 24$$

$$= 2y = 12 \Rightarrow y = 6$$

$$\text{Substituting } y = 6 \text{ in eq. (3), } x = 6 + 4 = 10$$

Therefore, length = 10 m and breadth = 6 m.

Let us now solve the eqs. (1) and (2) by the method of substitution.

From eq. (1).  $x = y + 4$  ... (3)

Substituting in eq. (2).

$$3(y+4)-y=24$$

$$= 3y+12-y=24$$

$$= 2y=12 \quad y=6$$

Substituting  $y = 6$  in eq. (3),  $x=6+4=10$

Therefore, length = 10 m and breadth = 6 m.

OR

We can find whether lines will be intersecting, coincident or parallel, by calculating the ratios of the coefficients of the pair of linear equations. As the two equations are given by

$$x-y=4$$

$$\text{and } 3x-y=24$$

Let us calculate the ratios of their coefficients

### Case Study 5

Gagan went to a fair. He ate several rural delicacies such as jalebis, chaat etc. He also wanted to play the ring game in which a ring is thrown on the items displayed on the table and the balloon shooting game.



The cost of three balloon shooting games exceeds the cost of four ring games by 4. Also, the total cost of three balloon shooting games and four ring games is 20.

Based on the above information, solve the following questions:

Q1. Taking the cost of one ring game to be  $x$  and that of one balloon game as  $y$ , find the pair of linear equations describing the given statement.

Q2. Find the total cost of five ring games and eight balloon games.

Q3. Find the cost of one balloon game.

OR

Cost of which game is more and by how much?

### Solutions

1. Given, the cost of one ring game =  $x$  and cost of one balloon game =  $y$ .

According to the question,

$$3y = 4x + 4 \text{ or } 4x - 3y = -4 \dots(1)$$

$$\text{and } 4x + 3y = 20 \dots(2)$$

2. Total cost of five ring games and eight balloon

$$\text{games} = 5x + 8y$$

$$= 5 \times 2 + 8 \times 4$$

$$= 10 + 32$$

$$= 42.$$

3. Solving the equations  $4x - 3y = -4$  and  $4x + 3y = 20$

by the method of substitution.

$$\text{From eq. (1). } 4x = 3y - 4 \dots(3)$$

Substituting the value of  $4x$  in eq. (2),

$$(3y - 4) + 3y = 20$$

$$= 6y - 4 = 20$$

$$= 6y = 24$$

$$= y = 4$$

.. Cost of one balloon game = Rs 4.

OR

Now, substituting  $y = 4$  in eq. (3).

$$4x = 3 \times 4 - 4 - 8 \Rightarrow x = 2$$

Therefore, cost of one ring game = Rs2

Thus cost of one balloon game is more and by  
Rs  $(4-2)$  = Rs2

## Solutions for Questions 6 to 15 are Given Below

### Case Study 6

#### Hostel Monthly Charges

A part of monthly hostel charges in a college is fixed and the remaining depends on the number of days one has taken food in the mess. When a student Anu takes food for 25 days, she has to pay ₹ 4500 as hostel charges, whereas another student Bindu who takes food for 30 days, has to pay ₹ 5200 as hostel charges.



Considering the fixed charges per month by ₹  $x$  and the cost of food per day by ₹  $y$ , then answer the following questions.

- (i) Represent algebraically the situation faced by both Anu and Bindu.
- (a)  $x + 25y = 4500, x + 30y = 5200$       (b)  $25x + y = 4500, 30x + y = 5200$   
(c)  $x - 25y = 4500, x - 30y = 5200$       (d)  $25x - y = 4500, 30x - y = 5200$
- (ii) The system of linear equations, represented by above situations has
- (a) No solution      (b) Unique solution  
(c) Infinitely many solutions      (d) None of these
- (iii) The cost of food per day is
- (a) ₹ 120      (b) ₹ 130      (c) ₹ 140      (d) ₹ 1300
- (iv) The fixed charges per month for the hostel is
- (a) ₹ 1500      (b) ₹ 1200      (c) ₹ 1000      (d) ₹ 1300
- (v) If Bindu takes food for 20 days, then what amount she has to pay?
- (a) ₹ 4000      (b) ₹ 3500      (c) ₹ 3600      (d) ₹ 3800

## Case Study 7

### Ticket Counter on Bus Stand

From Bengaluru bus stand, if Riddhima buys 2 tickets to Malleswaram and 3 tickets to Yeswanthpur, then total cost is ₹ 46; but if she buys 3 tickets to Malleswaram and 5 tickets to Yeswanthpur, then total cost is ₹ 74.



Consider the fares from Bengaluru to Malleswaram and that to Yeswanthpur as ₹  $x$  and ₹  $y$  respectively and answer the following questions.

- (i) 1<sup>st</sup> situation can be represented algebraically as
- (a)  $3x - 5y = 74$                       (b)  $2x + 5y = 74$                       (c)  $2x - 3y = 46$                       (d)  $2x + 3y = 46$
- (ii) 2<sup>nd</sup> situation can be represented algebraically as
- (a)  $5x + 3y = 74$                       (b)  $5x - 3y = 74$                       (c)  $3x + 5y = 74$                       (d)  $3x - 5y = 74$
- (iii) Fare from Bengaluru to Malleswaram is
- (a) ₹ 6                                      (b) ₹ 8                                      (c) ₹ 10                                      (d) ₹ 2
- (iv) Fare from Bengaluru to Yeswanthpur is
- (a) ₹ 10                                      (b) ₹ 12                                      (c) ₹ 14                                      (d) ₹ 16
- (v) The system of linear equations represented by both situations has
- (a) infinitely many solutions                      (b) no solution
- (c) unique solution                                      (d) none of these

## Case Study 8

### National Highway

Points  $A$  and  $B$  representing Chandigarh and Kurukshetra respectively are almost 90 km apart from each other on the highway. A car starts from Chandigarh and another from Kurukshetra at the same time. If these cars go in the same direction, they meet in 9 hours and if these cars go in opposite direction they meet in  $9/7$  hours. Let  $X$  and  $Y$  be two cars starting from points  $A$  and  $B$  respectively and their speed be  $x$  km/hr and  $y$  km/hr respectively.



Then, answer the following questions.

- (i) When both cars move in the same direction, then the situation can be represented algebraically as  
(a)  $x - y = 10$  (b)  $x + y = 10$  (c)  $x + y = 9$  (d)  $x - y = 9$
- (ii) When both cars move in opposite direction, then the situation can be represented algebraically as  
(a)  $x - y = 70$  (b)  $x + y = 90$  (c)  $x + y = 70$  (d)  $x + y = 10$
- (iii) Speed of car X is  
(a) 30 km/hr (b) 40 km/hr (c) 50 km/hr (d) 60 km/hr
- (iv) Speed of car Y is  
(a) 50 km/hr (b) 40 km/hr (c) 30 km/hr (d) 60 km/hr
- (v) If speed of car X and car Y, each is increased by 10 km/hr, and cars are moving in opposite direction, then after how much time they will meet?  
(a) 5 hrs (b) 4 hrs (c) 2 hrs (d) 1 hr

## Case Study 9

### Lunch Party

Mr Manoj Jindal arranged a lunch party for some of his friends. The expense of the lunch are partly constant and partly proportional to the number of guests. The expenses amount to ₹ 650 for 7 guests and ₹ 970 for 11 guests.



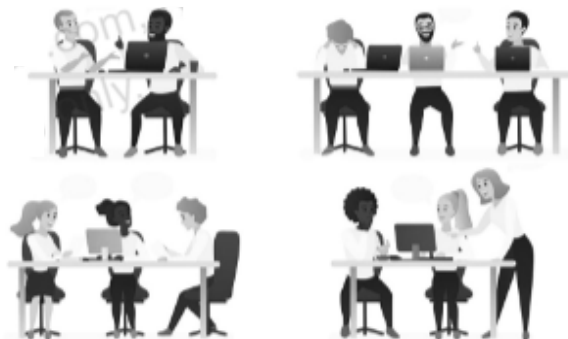
Denote the constant expense by ₹  $x$  and proportional expense per person by ₹  $y$  and answer the following questions.

- (i) Represent both the situations algebraically.  
(a)  $x + 7y = 650, x + 11y = 970$  (b)  $x - 7y = 650, x - 11y = 970$   
(c)  $x + 11y = 650, x + 7y = 970$  (d)  $11x + 7y = 650, 11x - 7y = 970$
- (ii) Proportional expense for each person is  
(a) ₹ 50 (b) ₹ 80 (c) ₹ 90 (d) ₹ 100
- (iii) The fixed (or constant) expense for the party is  
(a) ₹ 50 (b) ₹ 80 (c) ₹ 90 (d) ₹ 100
- (iv) If there would be 15 guests at the lunch party, then what amount Mr Jindal has to pay?  
(a) ₹ 1500 (b) ₹ 1300 (c) ₹ 1200 (d) ₹ 1290
- (v) The system of linear equations representing both the situations will have  
(a) unique solution (b) no solution  
(c) infinitely many solutions (d) none of these

## Case Study 10

### Office Work

In a office, 8 men and 12 women together can finish a piece of work in 10 days, while 6 men and 8 women together can finish it in 14 days. Let one day's work of a man be  $1/x$  and one day's work of a woman be  $1/y$ .



Based on the above information, answer the following questions.

(i) 1<sup>st</sup> situation can be represented algebraically as

(a)  $\frac{80}{x} - \frac{120}{y} = 1$

(b)  $\frac{120}{x} - \frac{80}{y} = 1$

(c)  $\frac{120}{x} + \frac{80}{y} = 1$

(d)  $\frac{80}{x} + \frac{120}{y} = 1$

(ii) 2<sup>nd</sup> situation can be represented algebraically as

(a)  $\frac{112}{x} - \frac{84}{y} = 1$

(b)  $\frac{84}{x} - \frac{112}{y} = 1$

(c)  $\frac{84}{x} + \frac{112}{y} = 1$

(d)  $\frac{112}{x} + \frac{84}{y} = 1$

(iii) One woman alone can finish the work in

(a) 220 days

(b) 140 days

(c) 280 days

(d) 160 days

(iv) One man alone can finish the work in

(a) 140 days

(b) 220 days

(c) 160 days

(d) 280 days

(v) If 14 men and 28 women work together, then in what time, the work will be completed?

(a) 2 days

(b) 3 days

(c) 4 days

(d) 5 days

## Case Study 11

### Book Store

From a shop, Sudhir bought 2 books of Mathematics and 3 books of Physics of class X for ₹ 850 and Suman bought 3 books of Mathematics and 2 books of Physics of class X for ₹ 900. Consider the price of one Mathematics book and that of one Physics book be ₹  $x$  and ₹  $y$  respectively.



Based on the above information, answer the following questions.

(i) Represent the situation faced by Sudhir, algebraically.

- (a)  $2x + 3y = 850$  (b)  $3x + 2y = 850$  (c)  $2x - 3y = 850$  (d)  $3x - 2y = 850$

(ii) Represent the situation faced by Suman, algebraically.

- (a)  $2x + 3y = 90$  (b)  $3x + 2y = 900$  (c)  $2x - 3y = 900$  (d)  $3x - 2y = 900$

(iii) The price of one Physics book is

- (a) ₹ 80 (b) ₹ 100 (c) ₹ 150 (d) ₹ 200

(iv) The price of one Mathematics book is

- (a) ₹ 80 (b) ₹ 100 (c) ₹ 150 (d) ₹ 200

(v) The system of linear equations represented by above situation, has

- (a) unique solution (b) no solution  
(c) infinitely many solutions (d) none of these

## Case Study 12

### Boating in River

A boat in the river Ganga near Rishikesh covers 24 km upstream and 36 km downstream in 6 hours while it covers 36 km upstream and 24 km downstream in  $6\frac{1}{2}$  hours. Consider speed of the boat in still water be  $x$  km/hr and speed of the stream be  $y$  km/hr and answer the following questions.



(i) Represent the 1<sup>st</sup> situation algebraically.

- (a)  $\frac{24}{x-y} + \frac{36}{x+y} = 6$  (b)  $\frac{24}{x+y} + \frac{36}{x-y} = 6$  (c)  $24x + 36y = 6$  (d)  $24x - 36y = 6$

(ii) Represent the 2<sup>nd</sup> situation algebraically.

- (a)  $\frac{36}{x+y} + \frac{24}{x-y} = \frac{13}{2}$  (b)  $\frac{36}{x-y} + \frac{24}{x+y} = \frac{13}{2}$  (c)  $36x - 24y = \frac{13}{2}$  (d)  $36x + 24y = \frac{13}{2}$

(iii) If  $u = \frac{1}{x-y}$  and  $v = \frac{1}{x+y}$ , then  $u =$

- (a)  $\frac{1}{4}$  (b)  $\frac{1}{12}$  (c)  $\frac{1}{8}$  (d)  $\frac{1}{6}$

(iv) Speed of boat in still water is

- (a) 4 km/hr (b) 6 km/hr (c) 8 km/hr (d) 10 km/hr

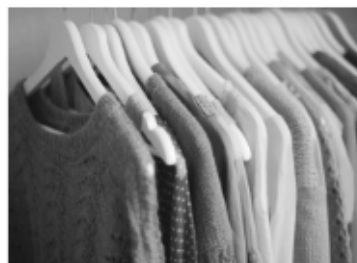
(v) Speed of stream is

- (a) 3 km/hr (b) 4 km/hr (c) 2 km/hr (d) 5 km/hr

## Case Study 13

### Profit and Loss

Piyush sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum of ₹ 1008. If he had sold the saree at 10% profit and the sweater at 8% discount, he would have got ₹ 1028.



Denote the cost price of the saree and the list price (price before discount) of the sweater by ₹  $x$  and ₹  $y$  respectively and answer the following questions.

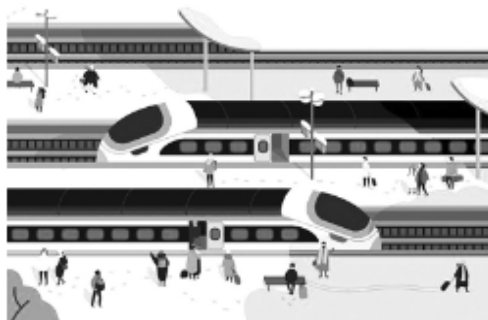
- (i) The 1<sup>st</sup> situation can be represented algebraically as  
(a)  $2.08x + 1.9y = 2008$  (b)  $1.08x + 0.9y = 1008$  (c)  $10x + 8y = 1008$  (d)  $8x + 10y = 1008$
- (ii) The 2<sup>nd</sup> situation can be represented algebraically as  
(a)  $10x + 8y = 1028$  (b)  $2.1x + 1.92y = 1028$  (c)  $1.1x + 0.92y = 1028$  (d)  $8x + 10y = 1028$
- (iii) Linear equation represented by 1<sup>st</sup> situation intersect the  $x$ -axis at  
(a)  $(2800, 0)$  (b)  $(2500, 0)$  (c)  $\left(\frac{2500}{3}, 0\right)$  (d)  $\left(\frac{2800}{3}, 0\right)$
- (iv) Linear equation represented by 2<sup>nd</sup> situation intersect the  $y$ -axis at  
(a)  $\left(0, \frac{25700}{23}\right)$  (b)  $(0, 25700)$  (c)  $\left(0, \frac{25800}{23}\right)$  (d)  $(0, 26800)$
- (v) Both linear equations represented by situation 1<sup>st</sup> and 2<sup>nd</sup> intersect each other at  
(a)  $(400, 600)$  (b)  $(600, 400)$  (c)  $(200, 200)$  (d)  $(800, 600)$

## Case Study 14

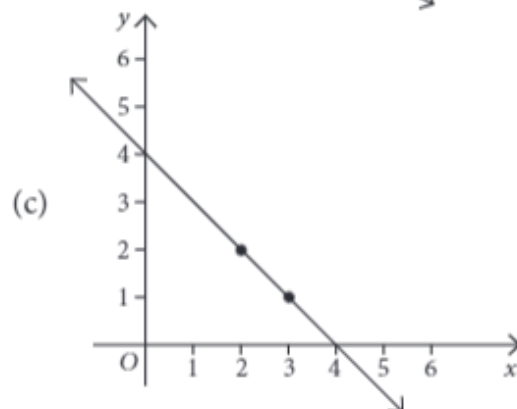
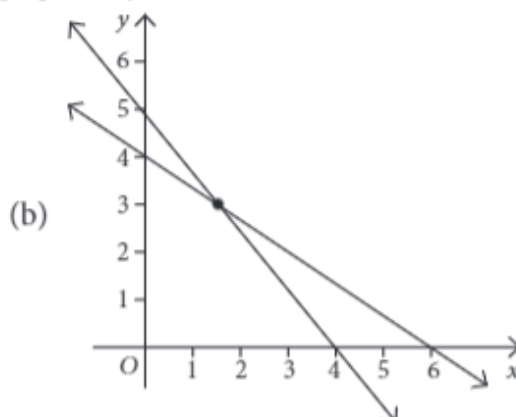
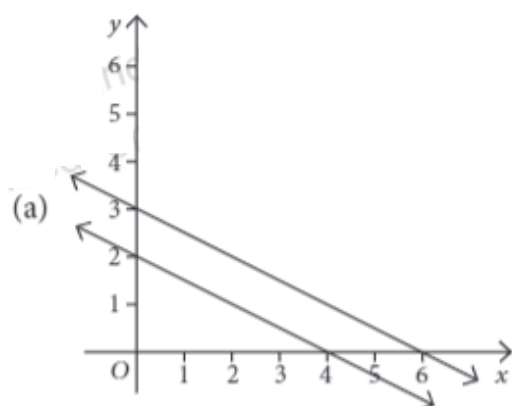
### Metro Track

Puneet went for shopping in the evening by metro with his father who is an expert in mathematics. He told Puneet that path of metro A is given by the equation  $2x + 4y = 8$  and path of metro B is given by the equation  $3x + 6y = 18$ . His father put some questions to Puneet.

Help Puneet to solve the questions.



- (i) Equation  $2x + 4y = 8$  intersects the  $x$ -axis and  $y$ -axis respectively at  
 (a)  $(4, 0), (0, 2)$  (b)  $(0, 4), (2, 0)$  (c)  $(4, 0), (2, 0)$  (d)  $(0, 4), (0, 2)$
- (ii) Equation  $3x + 6y = 18$  intersects the  $x$ -axis and  $y$ -axis respectively at  
 (a)  $(6, 0), (0, 8)$  (b)  $(0, 6), (0, 8)$  (c)  $(6, 0), (0, 3)$  (d)  $(0, 6), (0, 3)$
- (iii) Coordinates of point of intersection of two given equations are  
 (a)  $(1, 2)$  (b)  $(2, 4)$  (c)  $(3, 7)$  (d) does not exist
- (iv) Represent the equations,  $2x + 4y = 8$  and  $3x + 6y = 18$  graphically.



(d) None of these

- (v) System of linear equations represented by two given lines is  
 (a) inconsistent (b) having infinitely many solutions  
 (c) consistent (d) overlapping each other

## Case Study 15

### Dry Fruit Shop

Raman usually go to a dry fruit shop with his mother. He observes the following two situations.

On 1<sup>st</sup> day : The cost of 2 kg of almonds and 1 kg of cashew was ₹ 1600.

On 2<sup>nd</sup> day : The cost of 4 kg of almonds and 2 kg of cashew was ₹ 3000.

Denoting the cost of 1 kg almonds by ₹  $x$  and cost of 1 kg cashew by ₹  $y$ , answer the following questions.

- (i) Represent algebraically the situation of day-I.  
 (a)  $x + 2y = 1000$  (b)  $2x + y = 1600$  (c)  $x - 2y = 1000$  (d)  $2x - y = 1000$



(ii) Represent algebraically the situation of day-II.

(a)  $2x + y = 1500$

(b)  $2x - y = 1500$

(c)  $x + 2y = 1500$

(d)  $2x + y = 750$

(iii) The linear equation represented by day-I, intersect the  $x$  axis at

(a)  $(0, 800)$

(b)  $(0, -800)$

(c)  $(800, 0)$

(d)  $(-800, 0)$

(iv) The linear equation represented by day-II, intersect the  $y$ -axis at

(a)  $(1500, 0)$

(b)  $(0, -1500)$

(c)  $(-1500, 0)$

(d)  $(0, 1500)$

(v) Linear equations represented by day-I and day-II situations, are

(a) non parallel

(b) parallel

(c) intersect at one point

(d) overlapping each other.

## HINTS & EXPLANATIONS

6. (i) (a): For student Anu:

Fixed charge + cost of food for 25 days = ₹ 4500

i.e.,  $x + 25y = 4500$

For student Bindu:

Fixed charges + cost of food for 30 days = ₹ 5200

i.e.,  $x + 30y = 5200$

(ii) (b): From above, we have  $a_1 = 1, b_1 = 25,$

$c_1 = -4500$  and  $a_2 = 1, b_2 = 30, c_2 = -5200$

$$\therefore \frac{a_1}{a_2} = 1, \frac{b_1}{b_2} = \frac{25}{30} = \frac{5}{6}, \frac{c_1}{c_2} = \frac{-4500}{-5200} = \frac{45}{52}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, system of linear equations has unique solution.

(iii) (c): We have,  $x + 25y = 4500$  ... (i)

and  $x + 30y = 5200$  ... (ii)

Subtracting (i) from (ii), we get

$$5y = 700 \Rightarrow y = 140$$

$\therefore$  Cost of food per day is ₹ 140

(iv) (c): We have,  $x + 25y = 4500$

$$\Rightarrow x = 4500 - 25 \times 140$$

$$\Rightarrow x = 4500 - 3500 = 1000$$

$\therefore$  Fixed charges per month for the hostel is ₹ 1000

(v) (d): We have,  $x = 1000, y = 140$  and Bindu takes food for 20 days.

$\therefore$  Amount that Bindu has to pay

$$= ₹ (1000 + 20 \times 140) = ₹ 3800$$

7. (i) (d): 1<sup>st</sup> situation can be represented algebraically as

$$2x + 3y = 46$$

(ii) (c): 2<sup>nd</sup> situation can be represented algebraically as  $3x + 5y = 74$

(iii) (b): We have,  $2x + 3y = 46$  ... (i)

$$3x + 5y = 74 \quad \dots (ii)$$

Multiplying (i) by 5 and (ii) by 3 and then subtracting, we get

$$10x - 9x = 230 - 222 \Rightarrow x = 8$$

$\therefore$  Fare from Bengaluru to Malleswaram is ₹ 8.

(iv) (a): Putting the value of  $x$  in equation (i), we get

$$3y = 46 - 2 \times 8 = 30 \Rightarrow y = 10$$

$\therefore$  Fare from Bengaluru to Yeswanthpur is ₹ 10.

(v) (c): We have,  $a_1 = 2, b_1 = 3, c_1 = -46$  and

$$a_2 = 3, b_2 = 5, c_2 = -74$$

$$\therefore \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{3}{5}, \frac{c_1}{c_2} = \frac{-46}{-74} = \frac{23}{37} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus system of linear equations has unique solution.

8. (i) (a): Suppose two cars meet at point Q. Then,

Distance travelled by car X = AQ,

Distance travelled by car Y = BQ.

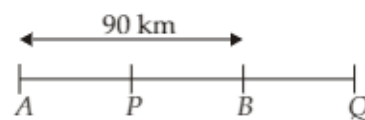
It is given that two cars meet in 9 hours.

$\therefore$  Distance travelled by car X in 9 hours =  $9x$  km

$$\Rightarrow AQ = 9x$$

Distance travelled by car Y in 9 hours =  $9y$  km

$$\Rightarrow BQ = 9y$$



Clearly,  $AQ - BQ = AB$

$$\Rightarrow 9x - 9y = 90$$

$$\Rightarrow x - y = 10$$

(ii) (c) : Suppose two cars meet at point  $P$ . Then

Distance travelled by car  $X = AP$  and

Distance travelled by car  $Y = BP$ .

In this case, two cars meet in  $9/7$  hours.

$\therefore$  Distance travelled by car  $X$  in  $9/7$  hours  $= \frac{9}{7}x$  km

$$\Rightarrow AP = \frac{9}{7}x$$

Distance travelled by car  $Y$  in  $9/7$  hours  $= \frac{9}{7}y$  km

$$\Rightarrow BP = \frac{9}{7}y$$

Clearly,  $AP + BP = AB$

$$\Rightarrow \frac{9}{7}x + \frac{9}{7}y = 90 \Rightarrow \frac{9}{7}(x + y) = 90 \Rightarrow x + y = 70$$

(iii) (b) : We have  $x - y = 10$  ... (i)

$$\Rightarrow x + y = 70 \quad \dots (ii)$$

Adding equations (i) and (ii), we get

$$2x = 80 \Rightarrow x = 40$$

Hence, speed of car  $X$  is 40 km/hr.

(iv) (c) : We have  $x - y = 10$

$$\Rightarrow 40 - y = 10 \Rightarrow y = 30$$

Hence, speed of car  $Y$  is 30 km/hr

(v) (d)

9. (i) (a) : 1<sup>st</sup> situation can be represented as

$$x + 7y = 650 \quad \dots (i)$$

and 2<sup>nd</sup> situation can be represented as

$$x + 11y = 970 \quad \dots (ii)$$

(ii) (b) : Subtracting equations (i) from (ii), we get

$$4y = 320 \Rightarrow y = 80$$

$\therefore$  Proportional expense for each person is ₹ 80.

(iii) (c) : Putting  $y = 80$  in equation (i), we get

$$x + 7 \times 80 = 650 \Rightarrow x = 650 - 560 = 90$$

$\therefore$  Fixed expense for the party is ₹ 90

(iv) (d) : If there will be 15 guests, then amount that

Mr Jindal has to pay = ₹  $(90 + 15 \times 80) = ₹ 1290$

(v) (a) : We have  $a_1 = 1, b_1 = 7, c_1 = -650$  and

$$a_2 = 1, b_2 = 11, c_2 = -970$$

$$\therefore \frac{a_1}{a_2} = 1, \frac{b_1}{b_2} = \frac{7}{11}, \frac{c_1}{c_2} = \frac{-650}{-970} = \frac{65}{97}$$

$$\text{Here, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, system of linear equations has unique solution.

10. (i) (d) : Since 8 men and 12 women can finish the work in 10 days.

$$\therefore \left( \frac{8}{x} + \frac{12}{y} \right) = \frac{1}{10} \Rightarrow \frac{80}{x} + \frac{120}{y} = 1$$

(ii) (c) : Since 6 men and 8 women can finish a piece of work in 14 days.

$$\therefore \left( \frac{6}{x} + \frac{8}{y} \right) = \frac{1}{14} \Rightarrow \frac{84}{x} + \frac{112}{y} = 1$$

(iii) (c) : Let  $\frac{1}{x} = u, \frac{1}{y} = v$

Thus, we have

$$80u + 120v = 1 \quad \text{and} \quad 84u + 112v = 1$$

Solving above two equations, we get

$$v = \frac{1}{280} \Rightarrow \frac{1}{y} = \frac{1}{280} \Rightarrow y = 280$$

Thus one woman alone can finish the work in 280 days.

$$(iv) (a) : \text{We have } \frac{80}{x} + \frac{120}{y} = 1 \Rightarrow \frac{80}{x} + \frac{120}{280} = 1$$

$$\Rightarrow \frac{80}{x} = 1 - \frac{3}{7} \Rightarrow \frac{80}{x} = \frac{4}{7} \Rightarrow x = 140$$

Thus one man alone can finish the work in 140 days.

(v) (d) : We have,  $x = 140$  and  $y = 280$

One day's work of 14 men and 28 women

$$= \frac{14}{140} + \frac{28}{280} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$$

Thus, work will be finished in 5 days.

11. (i) (a) : Situation faced by Sudhir can be represented algebraically as

$$2x + 3y = 850$$

(ii) (b) : Situation faced by Suman can be represented algebraically as

$$3x + 2y = 900$$

(iii) (c) : We have

$$2x + 3y = 850 \quad \dots (i)$$

$$\text{and } 3x + 2y = 900 \quad \dots (ii)$$

Multiplying (i) by 3 and (ii) by 2 and subtracting, we get

$$5y = 750 \Rightarrow y = 150$$

Thus, price of one Physics book is ₹ 150.

(iv) (d) : From equation (i) we have,  $2x + 3 \times 150 = 850$

$$\Rightarrow 2x = 850 - 450 = 400 \Rightarrow x = 200$$

Hence, cost of one Mathematics book = ₹ 200

(v) (a) : From above, we have

$$a_1 = 2, b_1 = 3, c_1 = -850$$

$$\text{and } a_2 = 3, b_2 = 2, c_2 = -900$$

$$\therefore \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{3}{2}, \frac{c_1}{c_2} = \frac{-850}{-900} = \frac{17}{18} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus system of linear equations has unique solution.

**12.** Speed of boat in upstream =  $(x - y)$  km/hr and speed of boat in downstream =  $(x + y)$  km/hr.

(i) (a): 1<sup>st</sup> situation can be represented algebraically

$$\text{as } \frac{24}{x - y} + \frac{36}{x + y} = 6$$

(ii) (b): 2<sup>nd</sup> situation can be represented algebraically

$$\text{as } \frac{36}{x - y} + \frac{24}{x + y} = \frac{13}{2}$$

(iii) (c): Putting  $\frac{1}{x - y} = u$  and  $\frac{1}{x + y} = v$ , we get

$$24u + 36v = 6 \text{ and } 36u + 24v = 13/2$$

Solving the above equations, we get  $u = \frac{1}{8}, v = \frac{1}{12}$

$$(iv) (d): \because u = \frac{1}{8} = \frac{1}{x - y} \Rightarrow x - y = 8 \quad \dots(i)$$

$$\text{and } v = \frac{1}{12} = \frac{1}{x + y} \Rightarrow x + y = 12 \quad \dots(ii)$$

Adding equations (i) from (ii), we get  $2x = 20 \Rightarrow x = 10$

$\therefore$  Speed of boat in still water = 10 km/hr

(v) (c): From equation (i),  $10 - y = 8 \Rightarrow y = 2$

$\therefore$  Speed of stream = 2 km/hr

**13.** (i) (b): Piyush sells a saree at 8% profit + sells a

sweater at 10% discount = ₹ 1008

$$\Rightarrow (100 + 8)\% \text{ of } x + (100 - 10)\% \text{ of } y = 1008$$

$$\Rightarrow 108\% \text{ of } x + 90\% \text{ of } y = 1008$$

...(i)

(ii) (c): Piyush sold the saree at 10% profit + sold the

sweater at 8% discount = ₹ 1028

$$\Rightarrow (100 + 10)\% \text{ of } x + (100 - 8)\% \text{ of } y = 1028$$

$$\Rightarrow 110\% \text{ of } x + 92\% \text{ of } y = 1028$$

$$\Rightarrow 1.1x + 0.92y = 1028$$

...(ii)

(iii) (d): At  $x$ -axis,  $y = 0$

$$\Rightarrow 1.08x = 1008 \Rightarrow x = \frac{1008}{1.08} = \frac{2800}{3}$$

(iv) (a): At  $y$ -axis,  $x = 0$

$$\Rightarrow 0.92y = 1028 \Rightarrow y = \frac{1028}{0.92} = \frac{25700}{23}$$

(v) (b): Solving equations (i) and (ii), we get

$$x = 600 \text{ and } y = 400$$

Hence both linear equations intersect at (600, 400).

**14.** (i) (a): At  $x$ -axis,  $y = 0$

$$\therefore 2x + 4y = 8 \Rightarrow x = 4$$

At  $y$ -axis,  $x = 0$

$$\therefore 2x + 4y = 8 \Rightarrow y = 2$$

$\therefore$  Required coordinates are (4, 0), (0, 2).

(ii) (c): At  $x$ -axis,  $y = 0$

$$\therefore 3x + 6y = 18 \Rightarrow 3x = 18 \Rightarrow x = 6$$

At  $y$ -axis,  $x = 0$

$$\therefore 3x + 6y = 18 \Rightarrow 6y = 18 \Rightarrow y = 3$$

$\therefore$  Required coordinates are (6, 0), (0, 3).

(iii) (d): Since, lines are parallel.

So, point of intersection of these lines does not exist.

(iv) (a)

(v) (a): Since the lines are parallel.

$\therefore$  These equations have no solution i.e., the given system of linear equations is inconsistent.

**15.** (i) (b): Algebraic representation of situation of day-I is  $2x + y = 1600$ .

(ii) (a): Algebraic representation of situation of day-II is  $4x + 2y = 3000 \Rightarrow 2x + y = 1500$ .

(iii) (c): At  $x$ -axis,  $y = 0$

$$\therefore \text{At } y = 0, 2x + y = 1600 \text{ becomes } 2x = 1600$$

$$\Rightarrow x = 800$$

$\therefore$  Linear equation represented by day-I intersect the  $x$ -axis at (800, 0).

(iv) (d): At  $y$ -axis,  $x = 0$

$$\therefore 2x + y = 1500 \Rightarrow y = 1500$$

$\therefore$  Linear equation represented by day-II intersect the  $y$ -axis at (0, 1500).

(v) (b): We have,  $2x + y = 1600$

and  $2x + y = 1500$

$$\text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \text{ i.e., } \frac{1}{1} = \frac{1}{1} \neq \frac{16}{15}$$

$\therefore$  System of equations have no solution.

$\therefore$  Lines are parallel.