Sample Paper 13

Class IX 2022-23

Mathematics

Time: 3 Hours **Max. Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 3 Qs of 5 marks, 3 Qs of 3 marks and 2 Questions of 2 marks has been provided.
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A

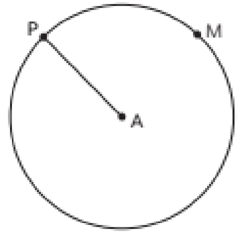
(Section A consists of 20 questions of 1 mark each.)

1. John drew a number line and pointed out two numbers $\frac{3}{8}$ and $\frac{7}{3}$ on it. He asked Neelam to find which among the following numbers is not between them. The number which does not lie between $\frac{3}{8}$ and $\frac{7}{3}$ is:

(a) $\frac{5}{4}$ (b) 2

(c) $\frac{8}{3}$ (d) $\frac{5}{12}$ **1**

2. Given below is a circular park with centre A. Madhav walks at a uniform speed of 0.5 m/s from gate P and reached the centre of the park in 150 seconds.



What is the straight line distance between the centre of the park and gate M?

(a) 300 m (b) 150 m

(c) 2.5 m (d) 75 m **1**

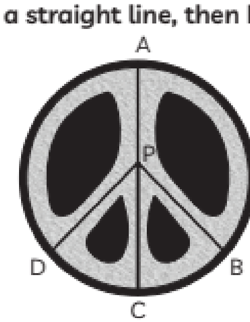
3. Shweta drew a figure named $\overline{AB}$. Which of the following best describes the figure Shweta drew?

(a) It is a ray. (b) It is an angle.

(c) It could be a line segment.

(d) It could be a point. **1**

4. Given below is the peace symbol which was designed in 1958 by Gerald Holtom, a professional artist and designer. If we join D and B by a straight line, then DB will be a:



(a) sector (b) chord

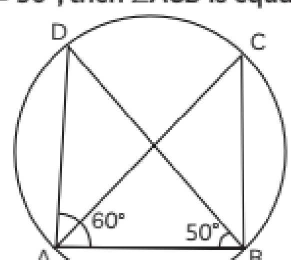
(c) diameter (d) radius **1**

5. Which of the following is a polynomial?

(a) $\frac{x^2}{2} - \frac{2}{x^2}$ (b) $\sqrt{2}\,x - 1$

(c) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$ (d) $\frac{x-1}{x+1}$ **1**

6. In the given figure, if $\angle DAB = 60°$, $\angle ABD = 50°$, then $\angle ACB$ is equal to:



(a) $60°$ (b) $50°$
(c) $70°$ (d) $80°$ 1

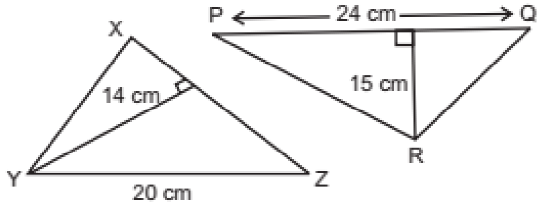
7. If $\Delta ACB \cong \Delta EDF$, then which of the following equations is/are true?

(i) AC = ED (ii) $\angle C = \angle F$

(iii) AB = EF

(a) Only (i) (b) Both (i) and (iii)
(c) Both (ii) and (iii) (d) All of these 1

8. Two triangles are shown below:



Which of the following statements is true regarding the given triangles?

(a) The area of both the triangles can be a calculated, area of $\Delta XYZ = 140\ cm^2$ and area of $\Delta PQR = 180\ cm^2$.

(b) Area of only triangle XYZ can be calculated, area of $\Delta XYZ = 140\ cm^2$.

(c) Area of only triangle PQR can be calculated, area of $\Delta PQR = 180\ cm^2$.

(d) Area of both triangles cannot be calculated. 1

9. If APB and CQD are two parallel lines, then the bisectors of the angle APQ, BPQ, CQP, and DQP form

(a) a square (b) a rhombus

(c) a rectangle

(d) any other parallelogram 1

10. The linear equation $3x - y = x - 1$ has:

(a) a unique solution
(b) two solutions
(c) infinitely many solutions
(d) no solution 1

11. The class mark of the class interval 90-120 is ....

(a) 110 (b) 105
(c) 108 (d) 115 1

12. Which of the following is not a criterion for congruence of the triangles?

(a) SAS (b) ASA
(c) SSA (d) SSS 1

13. If the radius of a sphere is $2r$ units, then its volume (in cubic units) will be:

(a) $\frac{4}{3}\pi r^3$ (b) $\frac{8\pi r^3}{3}$

(c) $4\pi r^3$ (d) $\frac{32}{3}\pi r^3$ 1

14. It is known that if P = Q and X = Y then, P – X = Q – Y. The Euclid's axiom that illustrates this statement is:

(a) first axiom (b) second axiom
(c) third axiom (d) forth axiom 1

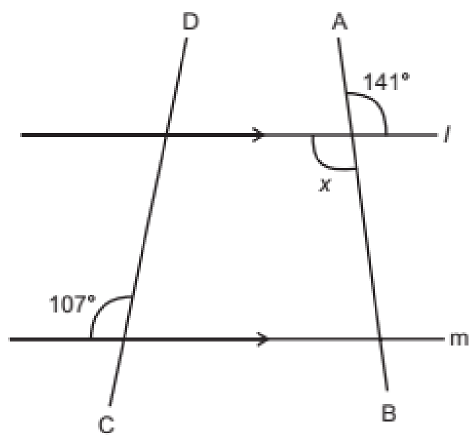
15. Monika has a chocholate in the shape of quadrilateral and she named it PQRS. She joined the mid-points of its sides (taken in order) and obtained a figure which is square only if:

(a) PQRS is a rhombus
(b) diagonals of PQRS are equal
(c) diagonals of PQRS are equal and perpendicular
(d) diagonal of PQRS are perpendicular 1

16. In a histogram, the length of the rectangle is .... to its frequency.

(a) equal
(b) not proportional
(c) proportional
(d) none of these 1

17. A check box was made by Reyashi but she wants to measure the size of the angle marked as x. Find the value of x.



(a) 141° (b) 139°
(c) 107° (d) 73° 1

18. Seneti bought some chocolates which is 2 more than thrice the number of chocolates that Rashmi does have. Find which among the following can be the correct equation.

[Hint: Seneti's chocolate be x and Rashmi's chocolate be y]

(a) $2x + 3 = y$ (b) $x = 2 + 3y$
(c) $x + 2 = 3y$ (d) $3x + y = 2$ 1

Direction: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).

Choose the correct option.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. Statement A (Assertion): If $\frac{\sqrt{5}+2}{\sqrt{5}-2} = a + b\sqrt{5}$, then $a \times b = 36$.

Statement R (Reason): The values of a and b are 18 and 2 respectively. 1

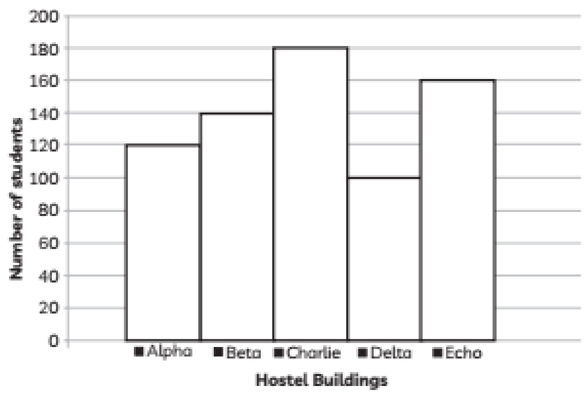
20. Statement A (Assertion): If the diagonal of a parallelogram are equal, then it is a rectangle.

Statement R (Reason): The diagonals of parallelogram bisect each other at right angles. 1

SECTION - B

(Section B consists of 5 questions of 2 marks each.)

21. The bar graph shows the number of students residing at different hostel buildings.



From the above graph, how many students reside in Charlie building. Also, find the total number of students residing in the hostel.

OR

For a particular year, following is the distribution of ages (in years) of upper primary school teachers in a district:

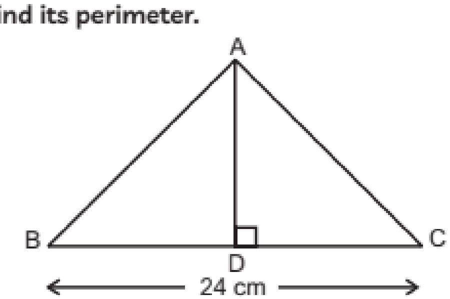
Age (in years)	Number of Teachers
15-20	10
20-25	30
25-30	50
30-35	50
35-40	30
40-45	6
45-60	4

(A) Determine the class size.

(B) Find the class mark of the class 45-50. 2

22. Sona is 7 years younger than Sam. After 2 years, Sam will be 14 years. Find the present age of Sona. 2

23. In the given figure, ABC is an isosceles triangle such that AD ⊥ BC. If the base BC of $\triangle ABC$ is 24 cm and the area of $\triangle ABC$ is 192 cm^2, then find its perimeter.



2

24. In the figure of fire escape, two landings are parallel to each other. If $\angle 1 = [17x + 9]°$, and $\angle 2 = [14x + 18]°$. Find the value of x.

OR

If one of the angles of a linear pair is 95°, then find the other angle. 2

25. To draw a histogram to represent the following frequency distribution.

Find the adjusted frequency for the class 25 - 45.

Class Interval	Frequency
5 - 10	6
10 - 15	12
15 - 25	10
25 - 45	8
45 - 75	15

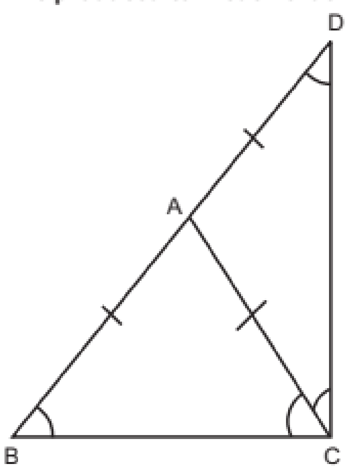
2

SECTION - C

(Section C consists of 6 questions of 3 marks each.)

26. The sum of a two-digit number and the number obtained by reversing the order of its digits is 99. If unit's and ten's digits of the number are x and y respectively, then write the linear equation representing the above statement. 3

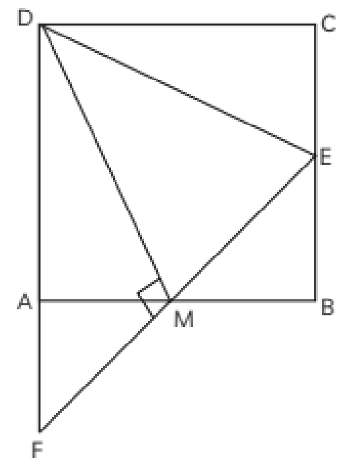
27. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB.



Show that $\angle BCD$ is a right angle. 3

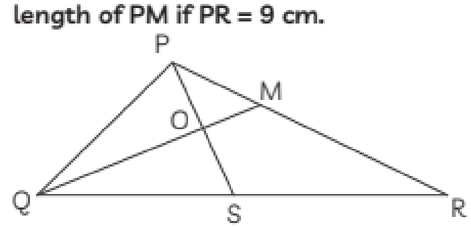
28. Rimmy was given an expression $\frac{\left[(81)^{3.6} \times (9)^{2.7}\right]^2}{(81)^{8.4} \times 27}$ and he was asked to find the first even multiple of the answer obtained. What is first even multiple? 3

29. In the following figure, ABCD is a square. M is the mid-point of AB and DM $\perp$ EF. Prove that DE = DF.



OR

In ΔPQR, PS is the median through P where, O is the mid-point of PS. Produced QO meets PR at M (as shown in the figure), then find the length of PM if PR = 9 cm.



3

30. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord. 3

31. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

OR

Factorise: $x^{24} - y^{24}$. 3

SECTION - D

(Section D consists of 4 questions of 5 marks each.)

32. If A = 0.36363636...... and B = 0.636363..... then what is the value of $\frac{1}{A}+\frac{1}{B}$? 5

33. Write any four solutions for the equation, $\pi x + y = 9$. 5

34. PQRS is a rhombus and A, B, C and D are the mid-points of PQ, QR, RS and SP respectively. Prove that the quadrilateral ABCD is a rectangle.

OR

If the bisectors of ∠A and ∠B of quadrilateral ABCD intersect each other at P, bisectors of ∠B and ∠C at Q, bisectors of ∠C and ∠D at R and bisectors of ∠D and ∠A at S, then prove that PQRS is a quadrilateral whose opposite angles are supplementary. 5

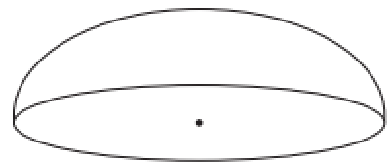
35. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and a height of 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs.12 per m^2, then what will be the cost of painting all these cones? (Use π = 3.14 and take $\sqrt{1.04} = 1.02$)

OR

A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹4989.60. If the cost of white-washing is ₹20 per square meter, find the:

(A) inside surface area of the dome,

(B) volume of the air inside the dome.



5

SECTION - E

(Case study based questions are compulsory.)

36. *An object which is thrown or projected into the air, subject to only the acceleration of gravity is called a projectile and its path is called its trajectory. The curved path a projectile follows was shown by Galileo to be a parabola. Parabola is represented by a polynomial. If the polynomial to represent the distance covered is: $p(x) = -6x^2 + 48x + 24$.*

(A) What is the degree of the polynomial? 1

(B) Find the height of the projectile 5 seconds after it's launched. 1

(C) Find the value of k, if $x - 3$ is a factor of $k^2x^3 - x^2 + 3x - 1$.

OR

If the polynomial to represent the distance covered is given by, $p(x) = -6x^2 + 48x + 24$, then find the value of $p(4)$. 2

37. *Makemytrip.com organized a trekking trip to mountains in the Himalayas. A group of 4 college students decided to go for the same trekking trip. As the journey was long so they took a halt in Chandigarh. But due to peak season, they did not get a proper hotel in the city. So they decided to make a conical tent at the park of sector- 17. They were carrying a 1960 sq. m cloth with them. They made a tent with a height of 35 m (as one among four friends was taller than usual) and a diameter of 56 m. The remaining cloth was used for the floor.*

(A) Find the volume of the tent. 1

(B) Find the curved surface of tent. 1

(C) Find the total surface area of the tent.

OR

If the volume and base area of the tent are 48π cubic cm and 12π sq.cm respectively, then find the height of the cone. 2

38. *Swastik and Samar are classmates and they have participated in Inter-School Sports Carnival. This event has been organized on a rectangular ground PQRS. In the ground PQRS, lines have been drawn at a distance of 1 m each and 200 plants have also been planted at a distance of 1 m from each other along PQ. Swastik runs $\frac{1}{5}$ of the distance PQ on the second line and throws a Javelin and Samar runs $\frac{1}{4}$ of the distance PQ on the 7th line and throws the Javelin.*

(A) Find the coordinates at which Swastik stands to throw Javelin. **1**

(B) Find the coordinates at which Samar stands to throw Javelin. **1**

(C) Find the quadrant in which Swastik's Javelin lie.

OR

If the coordinates of two points A(2, 90) and B(4, 60), then find (Abscissa of B) – (Abscissa of A). **2**

SOLUTION

SAMPLE PAPER - 3

SECTION - A

1. *(c)* $\frac{8}{3}$

Explanation: Write the equivalent fractions of the given fractions, with denominator 24 (LCM of 8 and 3), we get,

$$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$$

And, $$\frac{7}{3} \times \frac{8}{8} = \frac{56}{24}$$

So, the rational number should be lie in between 0.375 to 2.33....

Hence, $\frac{8}{3} = 2.66 > 2.33...$ does not lie in between the given numbers.

2. *(d) 75 m*

Explanation: As we know,

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$= 0.5 \text{ m/s} \times 150 \text{ seconds}$$
$$\text{Distance} = 75 \text{ m}$$

So, $$AP = 75 \text{ m}$$

Hence, the straight line distance between the centre of the park to gate M *i.e.*, AM

$$AM = AP = 75 \text{ m}$$

3. *(c) It could be a line segment.*

Explanation: $\overline{AB}$ is a line segment.

4. *(b) Chord*

Explanation: The line segment joining any two points on the circle is called chord. DB is a chord

5. *(c)* $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$

Explanation: $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}} = x^2 + 3x^{\frac{3}{2}-\frac{1}{2}} = x^2 + 3x^{\frac{2}{2}}$

$= x^2 + 3x$

Therefore, it is a polynomial because power x is a whole number but in other options, either the power of x is not a whole number or the function is rational.

6. *(c) 70°*

Explanation: In ΔADB,

$\angle DAB + \angle ABD + \angle ADB = 180^\circ$

$\Rightarrow \quad 60^\circ + 50^\circ + \angle ADB = 180^\circ$

$\Rightarrow \quad \angle ADB = 180^\circ - 60^\circ - 50^\circ$

$\Rightarrow \quad \angle ADB = 70^\circ$

Angles in the same segment of a circle are equal.

Therefore, $\angle ACB = \angle ADB = 70^\circ$

7. *(b) Both (i) and (iii)*

Explanation: Since, $\Delta ACB \cong \Delta EDF$

$\therefore \quad AC = ED$

$CB = DF$

$AB = EF$

And $\angle A = \angle E$, $\angle C = \angle D$ and $\angle B = \angle F$

Therefore, statements (i) and (iii) are true.

8. *(c) Area of only triangle PQR can be calculated, area of $\Delta PQR = 180\ cm^2$.*

Explanation: We have, area of triangle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

From the above figure, area of ΔXYZ cannot be calculated as the base of the triangle is not given.

In ΔPQR,

$$\text{area of } \Delta PQR = \frac{1}{2} \times 15 \times 24\ cm^2$$

$$= 180\ cm^2$$

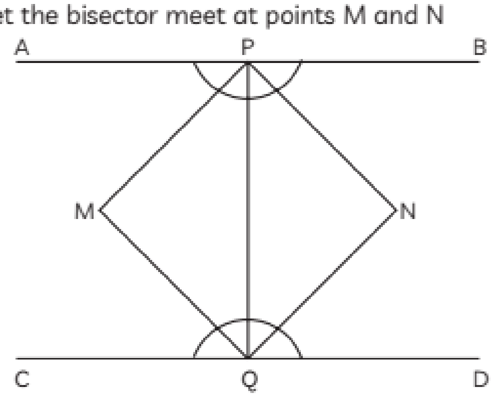
9. *(c) a rectangle*

Explanation:

Given: APB and CQD are two parallel lines.

Construction: Draw the bisectors of the angles APQ, BPQ, CQP and DQP.

Let the bisector meet at points M and N



Since, APB || CQD

$\angle APQ = \angle PQD$ (Alternate angles)

and

$2\angle MPQ = 2\angle NQP$

$\angle MPQ = \angle NQP$ (Alternate interior angles)

$\therefore$ PM || QN

Similarly, $\angle BPQ = \angle CQP$ (Alternate angles)

So, PN || QM

So, quadrilateral PMQN is a parallelogram

Since, CQD is a line

$\therefore \quad \angle CQD = 180°$

$\Rightarrow \quad \angle CQP + \angle DQP = 180°$

$\Rightarrow 2\angle MQP + 2\angle NQP = 180°$

$\Rightarrow 2(\angle MQP + \angle NQP) = 180°$

$\Rightarrow \quad \angle MQP + \angle NQP = 90°$

$\Rightarrow \quad \angle MQN = 90°$

Hence, PMQN is a rectangle.

10. *(c) infinitely many solutions*

Explanation: Given linear equation is:

$$3x - y = x - 1$$

Rewrite the equation as,

$$3x - x = y - 1$$

$\Rightarrow \quad 2x = y - 1$

$\Rightarrow \quad y = 2x + 1$

This is a linear equation in two variables and it has infinitely many solutions.

11. *(b) 105*

Explanation: We have

$$\text{Classmark} = \frac{\text{Upper limit} + \text{lower limit}}{2}$$

$$= \frac{120 + 90}{2}$$

$$= \frac{210}{2}$$

$$= 105$$

12. *(c) SSA*

Explanation:

(a) SAS – If two pairs of corresponding sides of two triangles are equal in length and the corresponding included angles are equal in measurement, then the triangles are congruent.

(b) ASA – If two pairs of corresponding angles of two triangles are equal in measurement, and the corresponding included sides are equal in length, then the triangles are congruent.

(c) SSA – The SSA condition (Side-Side-Angle) which specifies two sides and a non-included angle (also known as ASS, or Angle-Side-Side) does not by itself prove congruence.

(d) SSS – If three pairs of corresponding sides of two triangles are equal in length, then the triangles are congruent.

13. *(d) $\frac{32}{3}\pi r^3$*

Explanation: The radius of sphere = $2r$ [given]

The volume of sphere = $\frac{4}{3}\pi r^3$

Here, r is the radius of the sphere

New radius = $2r$

$\therefore$ Volume $= \frac{4}{3}\pi(2r)^3$

$= \frac{4}{3}\pi(8r^3)$

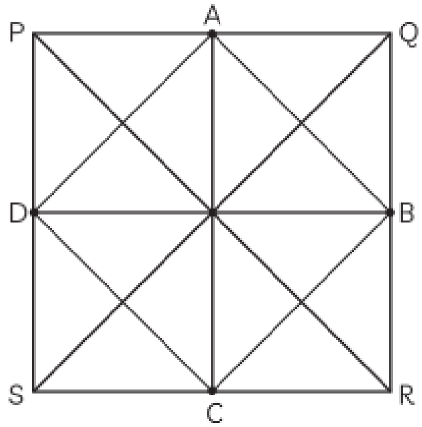
$= \frac{32}{3}\pi r^3$

14. *(c) Third Axiom*

Explanation: If equals are subtracted from equals, the remainders are equal.

15. *(c) diagonals of PQRS are equal and perpendicular*

Explanation: In given figure,



PQRS is a quadrilateral and A, B, C and D are mid-points of sides PQ, QR, RS and SP respectively. Then, ABCD is a square.

$$AB = BC = CD = DA \quad \text{...... (i)}$$

and $AC = DB$

But $AC = PS = QR$ and $DB = PQ = SR$

$\therefore$ $PQ = QR = RS = SP$

Thus, all sides of quadrilaterals PQRS are equal.

Now, in ΔPSQ,

By using mid-point theorem,

$$DA \parallel SQ \text{ and } DA = \frac{1}{2} SQ \quad \text{...... (ii)}$$

Similarly, in ΔPQR,

$$AB \parallel PR \text{ and } AB = \frac{1}{2} PR \quad \text{...... (iii)}$$

From eq. (i),

$$DA = AB$$

From (ii) and (iii),

$$\frac{1}{2} SQ = \frac{1}{2} PR$$

$\therefore$ $SQ = PR$

Thus, diagonals of PQRS are equal and therefore PQRS is a square. So, diagonals of quadrilateral also perpendicular.

16. *(c) proportional*

Explanation: A histogram is a graph where a set of rectangles are represented as per the class intervals and the frequencies. In a histogram, the area of the rectangle is proportional to its frequency. Thus, we can say that the lengths of the rectangles are proportional to the frequencies.

17. *(a) 141°*

Explanation: $x = 141°$ (vertically opposite angles)

18. *(b) $x = 2 + 3y$*

Explanation: Let Seneti has x chocolates, and Rashmi has y chocolates.

According to question,

$$x = 3y + 2$$

or, $x = 2 + 3y$

19. *(c) Assertion (A) is true but reason (R) is false.*

Explanation:

Given, $\frac{\sqrt{5}+2}{\sqrt{5}-2} = a + b\sqrt{5}$

$\Rightarrow$ $a + b\sqrt{5} = \frac{\sqrt{5}+2}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$

[Rationalize the denominator]

$\Rightarrow$ $a + b\sqrt{5} = \frac{(\sqrt{5}+2)^2}{(\sqrt{5})^2 - (2)^2}$

[$(a + b)(a - b) = a^2 - b^2$]

$\Rightarrow$ $a + b\sqrt{5} = \frac{9 + 4\sqrt{5}}{5 - 4}$

[$(a + b)^2 = a^2 + b^2 + 2ab$]

$\Rightarrow$ $a + b\sqrt{5} = 9 + 4\sqrt{5}$

Now, comparing both sides, we get

$$a = 9$$

$$b = 4$$

$\Rightarrow$ $a \times b = 36$

⚠ Caution

➥ *While rationalising the denominator always remember to change the sign + to – or – to + in the middle of two terms in binomial expression.*

20. (c) *Assertion (A) is true, but reason (R) is false.*

Explanation: A rectangle is a parallelogrom whose diogonals are equal and bisect each other. But diagonals of parallelogram not bisect each other.

SECTION - B

21. From the given bar graph,

Charlie building has 180 students residing.

Also, total number of students residing in the hostel = 120 + 140 + 180 + 100 + 160 = 700

OR

(A) Class size = upper limit of each class interval – Lower limit of each class interval

Her, class size = 20 – 05

= 5

(B) Class mark of the class 45-50

$$= \frac{45+50}{2}$$

$$= \frac{95}{2}$$

= 47.5

22. Let the present age of Sam = x years

And, the present age of Sona = y years

But according to the question,

$$y = (x - 7) \quad \text{... (i)}$$

Now, the age of Sam after two years = $(x + 2)$ years

$\therefore \quad x + 2 = 14$

$\Rightarrow \quad x = 12$ years

Now, from equation (i),

$y = x - 7 = 12 - 7$

$y = 5$ years

So, the present age of Sona is 5 years.

23. In isosceles ΔABC,

BC = 24 cm, AB = AC and AD⊥BC

We have, Area of a triangle = $\frac{1}{2}$ × base × height

Therefore, $\frac{1}{2} \times AD \times BC = 192$

[Since, area of ΔABC = 192 cm^2, given]

$\Rightarrow \quad \frac{1}{2} \times AD \times 24 = 192$

$\Rightarrow \quad AD = \frac{192 \times 2}{24}$ cm = 16 cm

Let $\quad AB = AC = a$

In ΔADB,

$$BD = \frac{24}{2} = 12 \text{ cm}$$

Now, $\quad AB^2 = AD^2 + BD^2$

[By Pythagoras theorem]

$$a = \sqrt{(16)^2 + (12)^2}$$

$$= \sqrt{256 + 144} = \sqrt{400}$$

$$a = 20 \text{ cm}$$

Thus, the perimeter of the isosceles triangle ABC = 20 + 20 + 24 = 64 cm

⚠ Caution

➥ *In an isosceles triangle, the perpendicular drawn from the vertex angle to the base bisects both the vertex angle and the base of the triangle.*

24. Since, the two landings are parallel and the ladder placed as a transversal. Then,

$\angle 1 = \angle 2$

[Pair of alternate angles]

$\Rightarrow \quad [17x + 9]^\circ = [14x + 18]^\circ$

$\Rightarrow \quad 17x - 14x = 18 - 9$

$\Rightarrow \quad 3x = 9$

$\Rightarrow \quad x = 3$

OR

For a linear pair of angles, let one of the given angle be ∠A = 95° and other angle be ∠B. Therefore,

$\angle A + \angle B = 180^\circ$

$\Rightarrow \quad 95^\circ + \angle B = 180^\circ$

$\Rightarrow \quad \angle B = 180^\circ - 95^\circ$

$\Rightarrow \quad \angle B = 85^\circ$

25. We have, adjusted frequency of a class

$$= \frac{\text{Minimum class width} \times \text{Frequency}}{\text{Class size of given class}}$$

$$= \frac{(10-5) \times 8}{(45-25)}$$

$$= \frac{5 \times 8}{20} = 2$$

SECTION - C

26. Given that:

Unit's digit is x and ten's digit is y.

And, the number after the reversing the digits

$= 10x + y$.

According to question,

$(10y + x) + (10x + y) = 99.$

$\Rightarrow \quad 11x + 11y = 99$

$$\Rightarrow \quad x + y = \frac{99}{11}$$

$$\Rightarrow \quad x + y = 9$$

Caution

- *The number formed by two digits is given $10x + y$, where x is ten's digit any y is units's digit.*

27. **Given:** In ΔABC,

$$AB = AC$$

Also, $AD = AB$

i.e., $AC = AB = AD$

To prove: $\angle BCD = 90^o$

Proof: In ΔABC,

$$AB = AC$$

$\Rightarrow \quad \angle ACB = \angle ABC$ ... (i)

[Angles opposite to equal sides are equal]

In ΔACD,

$AC = AD$ [As AC = AB]

$\Rightarrow \quad \angle ADC = \angle ACD$ ... (ii)

[Angles opposite to equal sides are equal]

In ΔBCD,

$\angle DBC + \angle BCD + \angle BDC = 180^o$ ... (iii)

[Angle sum property of triangle]

Now, $\angle DBC = \angle ABC$, $\angle BCD = \angle ACB + \angle ACD$, $\angle ACB = \angle ABC$, $\angle BDC = \angle ADC$ and $\angle ADC = \angle ACD$

Thus, $\angle ACB + \angle ACB + \angle ACD + \angle ACD = 180^o$

[From ...(iii)]

$\Rightarrow \quad \angle BCD + \angle BCD = 180^o$

$\Rightarrow \quad 2\angle BCD = 180^o$

$$\Rightarrow \quad \angle BCD = \frac{180^o}{2}$$

$\Rightarrow \quad \angle BCD = 90^o$

Hence proved.

28. Given, $\dfrac{\left[(81)^{3.6} \times (9)^{2.7}\right]^2}{(81)^{8.4} \times 27}$

We know, $(a^m)^n = a^{mn}$, $a^m \times a^n = a^{(m+n)}$ and $\dfrac{a^m}{a^n} = a^{m-n}$

$$= \frac{\left[\left(3^4\right)^{3.6} \times \left(3^2\right)^{2.7}\right]^2}{(3^4)^{8.4} \times 3^3}$$

$$= \frac{\left[(3)^{4 \times 3.6 + 2 \times 2.7}\right]^2}{(3)^{4 \times 8.4 + 3}}$$

$$= \frac{3^{39.6}}{3^{36.6}}$$

$$= (3)^{39.6 - 36.6} = (3)^3 = 27$$

Therefore, the first even multiple of 27 is $27 \times 2 = 54$.

29. ABCD is a square, M is the mid-point of AB.

Thus, AM = MB and DM $\perp$ EF

Therefore, $\angle DMF = \angle DME = 90°$

To prove: DE = DF

Proof: In ΔBME and ΔAMF

AM = MB [Given]

$\angle BME = \angle AMF$

[Vertically opposite angles]

$\angle EBM = \angle FAM$ [Each 90°]

Therefore, $\Delta BME \cong \Delta AMF$

[By ASA Congruency Rule]

EM = MF [By CPCT] .... (i)

Now, In ΔDME and ΔDMF

$\angle DME = \angle DMF$ [Each 90°]

DM = DM [Common]

EM = MF [From (i)]

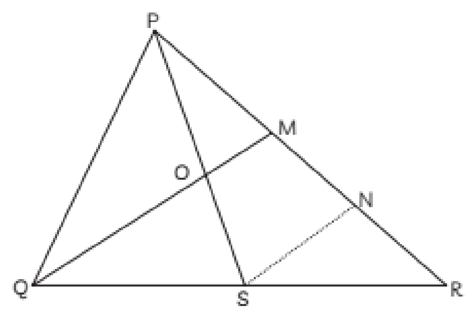
So, $\Delta DME \cong \Delta DMF$ [By SAS Congruency Rule]

Thus, DE = DF [By CPCT]

Hence proved.

OR

Through S, draw SN || QM. In ΔPSN, O is the mid-point of PS and OM || SN.



Therefore, M is the mid-point of PN

PM = MN ....(i)

In ΔQRM, S is the mid-point of QR and SN || QM

Therefore, N is the mid-point of MR

MN = NR ....(ii)

From (i) and (ii)

PM = MN = NR

Now, PR = PM + MN + NR

$\Rightarrow$ PR = PM + PM + PM

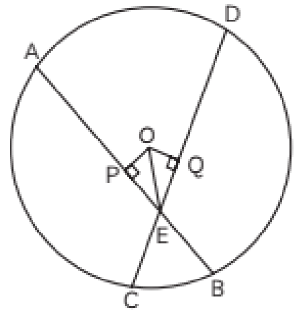
$\Rightarrow$ PR = 3PM

Now, PR = 9 cm [Given]

Thus, $PM = \dfrac{PR}{3} = \dfrac{9}{3} = 3$ cm

30. **Given:** A circle with centre O.

AB = CD, and chords AB and CD intersect at E.



To prove: AE = DE and CE = BE

Construction: Draw OP ⊥ AB and OQ ⊥ CD

Join OE.

Proof: In ΔOPE and ΔOQE,

OP = OQ

(Equal chords of a circle are equidistant from the centre)

∠OPE = ∠OQE [90^o each]

OE = OE [Common]

Therefore, ΔOPE ≅ ΔOQE [By RHS Congurence Rule]

So, PE = QE [By CPCT]

Now, $AP = PB = \frac{1}{2}$ AB

$DQ = QC = \frac{1}{2}$ CD

⇒ AP + PE = DQ + QE

⇒ AE = DE

Also AB – AE = CD – DE [AE = DE]

⇒ EB = CE

Hence proved.

31. $R.H.S = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Using $(a - b)^2 = a^2 + b^2 - 2ab$

$= \frac{1}{2}(x + y + z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (z^2 + x^2 - 2zx)]$

$= \frac{1}{2}(x + y + z)[(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$

$= \frac{1}{2} \times 2(x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$

$= (x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$

[We know $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$]

$= x^3 + y^3 + z^3 - 3xyz$

= L.H.S.

Hence proved.

OR

$x^{24} - y^{24} = (x^{12})^2 - (y^{12})^2$

⇒ $= (x^{12} + y^{12})(x^{12} - y^{12})$

$= [(x^4)^3 + (y^4)^3][(x^6)^2 - (y^6)^2]$

$= (x^4 + y^4)(x^8 + y^8 - x^4y^4)[(x^6 + y^6)(x^6 - y^6)]$

$= (x^4 + y^4)(x^8 + y^8 - x^4y^4)(x^2 + y^2)[(x^2)^2 - x^2 y^2 + (y^2)^2][(x^3 + y^3)(x^3 - y^3)]$

$= (x^4 + y^4)(x^8 + y^8 - x^4y^4)(x^2 + y^2)[x^4 - x^2 y^2 + y^4][(x + y)[(x^2 - xy + y^2](x - y)(x^2 + xy + y^2)$

SECTION - D

32. If $A = 0.363636....$

$A = 0.\overline{36}$ ... (i)

Multiply by 100

$100A = 36.\overline{36}$ ... (ii)

Subtract (i) from (ii)

$100A - A = 36.\overline{36} - 0.\overline{36}$

⇒ $99A = 36$

⇒ $A = \frac{36}{99}$

⇒ $A = \frac{4}{11}$

Now, $B = 0.636363........$

$B = 0.\overline{63}$ ... (iii)

Multiply by 100

$100B = 63.\overline{63}$ ... (iv)

Subtract (iii) from (iv)

$100B - B = 63.\overline{63} - 0.\overline{63}$

⇒ $99B = 63$

⇒ $B = \frac{63}{99}$

$B = \frac{7}{11}$

Now, $\frac{1}{A} + \frac{1}{B} = \frac{11}{4} + \frac{11}{7}$

$= \frac{77 + 44}{28}$

$= \frac{121}{28}$

33. **Given:** *equation is* $\pi x + y = 9$.

Putting $x = 0$

⇒ $\pi (0) + y = 9$

$\Rightarrow \quad 0 + y = 9$

$\Rightarrow \quad y = 9$

So, (0, 9) is a solution.

Putting $y = 0$

$\pi x + 0 = 9$

$\pi x = 9$

$x = \frac{9}{\pi}$

So $(\frac{9}{\pi}, 0)$ is a solution.

Putting $x = 1$

$\Rightarrow \quad \pi(1) + y = 9$

$\Rightarrow \quad \pi + y = 9$

$\Rightarrow \quad y = 9 - \pi$

So, $(1, 9 - \pi)$ is a solution.

Putting $y = 1$

$\Rightarrow \quad \pi x + 1 = 9$

$\Rightarrow \quad \pi x = 9 - 1$

$\Rightarrow \quad \pi x = 8$

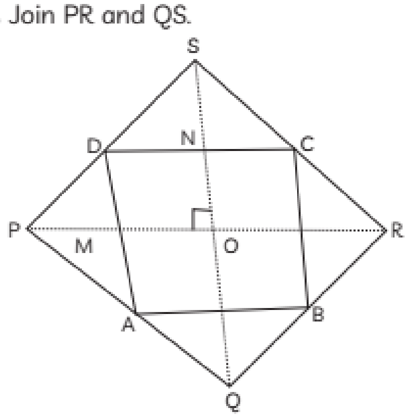
$\Rightarrow \quad x = \frac{8}{\pi}$

So, $\left(\frac{8}{\pi}, 1\right)$ is a solution.

Thus, the four solutions of the given equation are:

x	0	$\frac{9}{\pi}$	1	$\frac{8}{\pi}$
y	9	0	$9 - \pi$	1

34. Join PR and QS.



In ΔSPR, D and C are the mid-points of *PS* and *SR* respectively.

Therefore, $DC = \frac{1}{2}$ PR and DC || PR ....(i)

Also, in ΔQPR, A and B are the mid-points of PQ and QR respectively.

So, $AB = \frac{1}{2}$ PR and AB || PR ....(ii)

From (i) and (ii),

ABCD is a parallelogram

Since, ABCD is a parallelogram

Therefore, AD || BC

Or, MD || ON ....(iii)

Also, AB || CD

Or, OM || DN ....(iv)

From (iii) and (iv) we get MDNO, is a parallelogram

Since, PQRS is a rhombus, so

$\angle SOP = 90°$

$\angle MDN = 90°$

[Opposite angles of parallelogrom MDNO]

Hence, ABCD is a rectangle

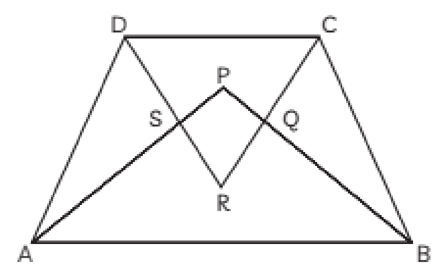
Hence proved.

Caution

➥ *Each angle of a rectangle is of measure 90° but its diagonals donot intersect at 90°.*

OR

Given: ABCD is a quadrilateral.



$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

So, we have $\frac{1}{2}(\angle A + \angle B + \angle C + \angle D) = \frac{1}{2} \times 360°$

$= 180°$ ....(i)

Since, AP, PB, RC and RD are bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$

Then, $\angle PAB + \angle ABP + \angle RCD + \angle RDC = 180°$ ...(ii)

Now, Sum of all angles of a triangle = 180°

In ΔPAB

$\angle PAB + \angle APB + \angle ABP = 180°$

$\Rightarrow \quad \angle PAB + \angle ABP = 180° - \angle APB$ .... (iii)

Similarly, In ΔRCD

$\therefore \quad \angle RDC + \angle RCD + \angle CRD = 180°$

$\Rightarrow \quad \angle RDC + \angle RCD = 180° - \angle CRD$ .... (iv)

Substituting the value of equations (iii) and (iv) in equation (ii).

$180° - \angle APB + 180° - \angle CRD = 180°$

$\Rightarrow \quad 360° - \angle APB - \angle CRD = 180°$

$\Rightarrow \quad \angle APB + \angle CRD = 360° - 180°$

$\Rightarrow$ $\angle APB + \angle CRD = 180°$ .... (v)

Now,

$\angle SPQ = \angle APB$ [Same angles]

$\angle SRQ = \angle DRC$ [Same angles]

Substitute in equation (v),

$\angle SPQ + \angle SRQ = 180°$

Hence, PQRS is a quadrilateral whose opposite angles are supplementary.

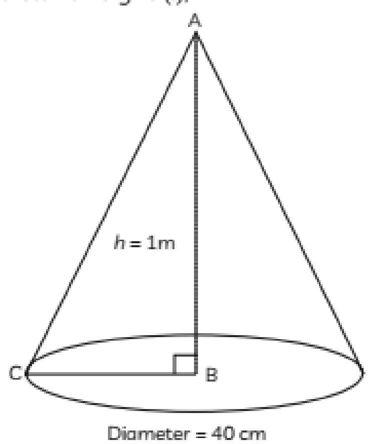
35. Only the surface area of the cone will be painted.

Curved area of 1 cone = πrl

Given, Radius $= 20 \text{ cm} = \frac{20}{100} \text{ m} = 0.2 \text{ m}$

Height = 1 m

Now find slant height (l),



In ΔABC,

$$l^2 = h^2 + r^2$$

$\Rightarrow$ $l^2 = (1)^2 + (0.2)^2$

$\Rightarrow$ $l^2 = 1 + 0.04$

$\Rightarrow$ $l^2 = 1.04$

$\Rightarrow$ $l = \sqrt{1.04}$

Curved surface area of 1 cone

$= \pi rl$

$= 3.14 \times 0.2 \times 1.02) \text{ m}^2$

$= 0.64056 \text{ m}^2$

Curved surface area of 50 cones

$= 50 \times 0.64056$

$= 32.028 \text{ m}^2$

Cost of painting 1m^2= Rs 12

Cost of painting 32.028m^2 = ₹ 12 × 32.028

= ₹ 384.336

= ₹ 384.34 (approx)

Hence, the cost of painting 50 cones is ₹ 384.34

Caution

Always convert all the parameters in same units.

OR

(A) Area to be whitewashed × cost of whitewash per sq. m = Total cost

Area to be whitewashed × ₹ $20/\text{m}^2$ = ₹ 4989.60

Area to be whitewashed = $\frac{4989.60}{20/\text{m}^2}$

Area to be whitewashed = 249.48 m^2

As only walls are whitewashed, not floor.

So, the area to whitewash = curved surface area of hemisphere.

$\therefore$ Inside surface area of the dome = 249.48 m^2

(B) The volume of air inside dome= volume of

hemisphere $= \frac{2}{3}\pi r^3$ ... (i)

But the surface area of the dome = 249.48 m^2

$\Rightarrow$ $2\pi r^2 = 249.48$

$\Rightarrow$ $2 \times \frac{22}{7} \times r^2 = 249.48$

$\Rightarrow$ $r^2 = \frac{249.48 \times 7}{22 \times 2}$

$\Rightarrow$ $r^2 = 39.69$

$\Rightarrow$ $r = \sqrt{39.69}$

$\therefore$ $r = 6.3 \text{ m}$

Now put the volue of r in eq. (i)

The volume of air inside the dome

$= \frac{2}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3$

$= 523.908 \text{ m}^3$ (approx)

SECTION - E

36. (A) The degree of the polynomial is 2.

(B) $p(x) = -6x^2 + 48x + 24$

To calculate height, put $x = 5$

Height of projectile,

$p(5) = -6 \times (5)^2 + 48 \times 5 + 24$

$= -6 \times 25 + 240 + 24$

$= 114$

So, the height of projectile reaches is 114 m.

(C) Let $p(x) = k^2x^3 - x^2 + 3x - 1$.

If $(x-3)$ is a factor of $p(x)$, then $p(3) = 0$.

$\therefore p(3) = k^2(3)^3 - (3)^2 + 3(3) - 1$

$= k^2(27) - 9 + 9 - 1$

$= 27k^2 - 1$

$\Rightarrow$ $k^2 = \frac{1}{27}$

$\Rightarrow$ $k = \pm\frac{1}{3\sqrt{3}}$

OR

$$p(x) = -6x^2 + 48x + 24$$

Put $x = 4$

$\therefore$ $p(4) = -6(4)^2 + 48(4) + 24$

$= -96 + 192 + 24$

$= 120$

Hence, the value of $p(4)$ is 120.

37. (A) Here, diameter $= 56$ m

Radius $= \frac{56}{2} = 28$ m

$h = 35$ m

The volume of tent $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 28 \times 28 \times 35$

$= 28746.66 \text{ m}^3$

(B) Here, $r = 28$ m

$h = 35$ m

Curved surface area $= \pi rl$

We know that,

$\Rightarrow$ $l^2 = r^2 + h^2$

$\Rightarrow$ $l^2 = (28)^2 + (35)^2$

$\Rightarrow$ $l^2 = 784 + 1225$

$\Rightarrow$ $l^2 = 2009$

$\Rightarrow$ $l = \sqrt{2009}$

$\Rightarrow$ $l = 44.82$ m

Curved surface area $= \pi rl$

$= \frac{22}{7} \times 28 \times 44.82$

$= 3944.16 \text{ m}^2$

(C) Here, $r = 28$

$h = 35$

$l = 44.82$ m [From above]

Total surface area $= \pi r(r + l)$

$= \frac{22}{7} \times 28\,(28 + 44.82)$

$= 88(72.82)$

$= 6408.16 \text{ m}^2$

OR

Base area, $\pi r^2 = 12\pi$

$\Rightarrow$ $r^2 = 12$

Volume, $\frac{1}{3}\pi r^2 h = 48\pi$

$\Rightarrow$ $\frac{1}{3} \times 12 \times h = 48$

$\Rightarrow$ $h = 12$ cm.

38. (A) In the rectangular ground PQRS, vertical lines are drawn at a distance of 1m each. Also, 200 plants are planted along PQ at a distance of 1 m each i.e., Total length of PQ is 200 m.

Now, we consider PS as x-axis and PQ as y-axis.

Since, Swastik runs $\frac{1}{5}$ of the distance PQ on the second line and threw Javelin.

$\therefore$ Position of Swastik's Javelin throw

$= 2, \frac{1}{5} \times 200$

$= (2, 40)$

(B) For Samar, the coordinates of the point

$= (7, \frac{1}{4} \times 200)$

$= (7, 50)$

(C) If a point lies in the I quadrant, then the point will be of the form (+, +). So, Swastik's Javelin lie in I guadrant.

OR

Given point A(2, 90), *i.e.*, abscissa of A = 2

And point B(4, 60) *i.e.*, Abscissa of B = 4

$\therefore$ Abscissa of B – Abscissa of A

$= 4 - 2$

$= 2$

Caution

Students always remember that abscissa of a point is its distance from y-axis.