CBSE Class 10 Mathematics Basic

Sample Paper - 04 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. State whether $\frac{125}{441}$ will have terminating decimal expansion or a non-terminating repeating decimal expansion.

OR

Find HCF $\times$ LCM for the numbers 100 and 190.

2. Is it quadratic equation ?

$3x^2 - 2\sqrt{x} + 8 = 0$

3. For what value of k, the following system of equations represent parallel lines?

$kx - 3y + 6 = 0, 4x - 6y + 15 = 0$

4. How many tangents can a circle have?
5. Show that the progression 8, 11, 14, 17, 20, ... is an A.P. Find its first term and the common difference.

OR

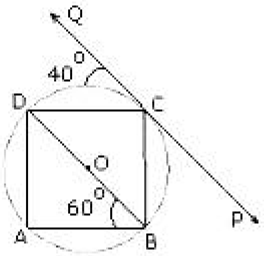
Determine the 25^{th} term of an AP whose 9^{th} term is -6 and common difference is $\frac{5}{4}$.

6. If $\frac{4}{2}$, a, 2 are in AP, find the value of a.
7. Form a quadratic equation whose roots are 2 and 3.

OR

Determine the nature of the roots of quadratic equation: $x^2 - x + 2 = 0$

8. In the given figure, ABCD is a cyclic quadrilateral and PQ is a tangent to the circle at C. If BD is a diameter, $\angle OCQ = 40^o$ and $\angle ABD = 60^o$, find $\angle BCP$



9. At which point a tangent is perpendicular to the radius?

OR

What term will you use for a line which intersect a circle at two distinct points?

10. If the altitude of two similar triangles are in the ratio 2 : 3, what is the ratio of their areas?
11. The n^{th} term of an AP is 7 - 4n. Find its common difference.
12. Prove that: $\sin 60^\circ = \frac{2 \tan 30^\circ}{1+\tan^2 30^\circ} = \frac{\sqrt{3}}{2}$
13. What happens to value of $\tan\theta$ when θ increases from 0^o to 90^o?
14. If the radii of two right circular cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3, what will be the ratio of their volumes?
15. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.
16. Red queens and black jacks are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the

drawn card is

i. a king,

ii. of red colour,

iii. a face card,

iv. a queen.

17. CARTESIAN- PLANE:

Using Cartesian Coordinates we mark a point on a graph by **how far along** and **how far up** it is.

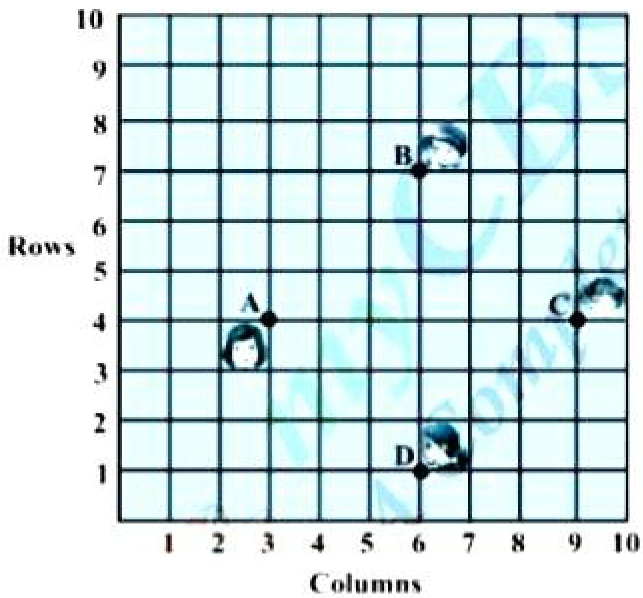
The left-right (**horizontal**) direction is commonly called X-axis.

The up-down (**vertical**) direction is commonly called Y-axis.

When we include negative values, the x and y axes divide the space up into 4 pieces.

Read the following passage and answer the questions that follow using the above information:

In a classroom, four student Sita, Gita, Rita and Anita are sitting at A(3, 4), B(6, 7), C(9, 4), D(6, 1) respectively. Then a new student Anjali joins the class.



i. Teacher tells Anjali to sit in the middle of the four students. Find the coordinates of the position where she can sit.

  a. (2, 4)

  b. (4, 4)

  c. (6, 4)

  d. (6, 5)

ii. The distance between Sita and Anita is

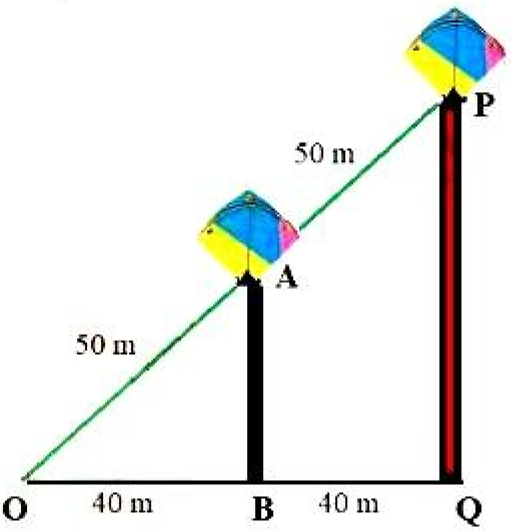
a. $3\sqrt{3}$ units
b. $3\sqrt{2}$ units
c. $2\sqrt{3}$ units
d. $3\sqrt{5}$ units

iii. Which two students are equidistant from Gita?
a. Anjali and Anita
b. Anita and Rita
c. Sita and Anita
d. Sita and Rita

iv. The geometrical figure formed after joining the points ABCD is
a. Square
b. Rectangle
c. Parallelogram
d. Rhombus

v. The distance between Sita and Rita is
a. 4 units
b. 6 units
c. $3\sqrt{2}$ units
d. $2\sqrt{3}$ units

18.



As shown in the figure Harish is trying to measure the height of two towers AB and PQ. He is flying a kite He is having 100m thread with him, Harish found that when his half thread is open That time kite is just above the tower AB. Harish continues flying the kites, When his full thread is open that time kite reaches just above the tower PQ. Now answer the following questions:

i. What is the height of the tower AB?

a. 40m

b. 30 m

c. 50 m

d. 100m

ii. What is the height of the tower PQ?

a. 40 m

b. 30 m

c. 60 m

d. 100m

iii. What is the length of the hypotenuse in the triangle OAB?

a. 40 m

b. 50 m

c. 100 m

d. 80 m

iv. What is the length of the hypotenuse in the triangle OPQ?

a. 40 m

b. 50 m

c. 100 m

d. 80 m

v. What is the length of the Base in the triangle OPQ?

a. 40 m

b. 50 m

c. 100 m

d. 80 m

19. A survey was conducted by the Education Ministry of India. The following distribution gives the state-wise teachers-students ratio in higher secondary schools of India.

Number of students per teacher	Number of states/U.T	Number of students per teacher	Number of states/U.T
15 - 20	3	35 - 40	3
20 - 25	8	40 - 45	0
25 - 30	9	45 - 50	0
30 - 35	10	50 - 55	2

i. The modal class is
   a. 40 - 45
   b. 30 - 35
   c. 50 - 55
   d. 25 - 30

ii. The mean of this data is
   a. 19.2.
   b. 22.9
   c. 39.2
   d. 29.2

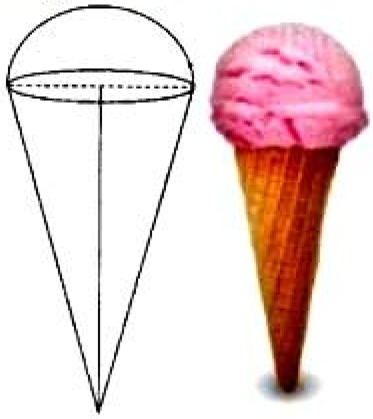
iii. The mode of the data is
   a. 36.625
   b. 30.625
   c. 32.625
   d. 31.625

iv. Half of (upper-class limit + lower class limit) is
   a. Class interval
   b. Classmark
   c. Class value
   d. Class size

v. The construction of the cumulative frequency table is useful in determining the
   a. Mean
   b. Mode
   c. Median
   d. All of the above

20. An ice-cream seller used to sell different kinds and different shapes of ice-cream like

rectangular shaped, conical shape with one end hemispherical, Rectangular shape with one end hemispherical and rectangular brick, etc. One day a child came to his shop and purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm.



By reading the above-given information, find the following:

i. Volume of only hemispherical end of the icecream is:
   a. $\frac{1357}{42}\text{cm}^3$
   b. $\frac{1375}{42}\text{cm}^3$
   c. $\frac{1575}{42}\text{cm}^3$
   d. $\frac{1373}{42}\text{cm}^3$

ii. The volume of the ice-cream without hemispherical end is:
   a. $\frac{852}{14}\text{cm}^3$
   b. $\frac{852}{41}\text{cm}^3$
   c. $\frac{825}{41}\text{cm}^3$
   d. $\frac{825}{14}\text{cm}^3$

iii. The TSA of cone is given by:
   a. $\pi rl + 2\pi r^2$
   b. $\pi rl + 2\pi r$
   c. $\pi rl + \pi r^2$
   d. $2\pi rl + \pi r^2$

iv. The volume of the whole ice-cream is:
   a. $91\frac{2}{3}\text{cm}^3$
   b. $91\frac{3}{2}\text{cm}^3$
   c. $19\frac{2}{3}\text{cm}^3$
   d. $19\frac{3}{2}\text{cm}^3$

v. During the conversion of a solid from one shape to another the volume of the new shape will:

a. increase
b. decrease
c. double
d. remain unaltered

Part-B

21. Without actual division, show that $\frac{24}{125}$ is a terminating decimal. Express the fraction in decimal form.

22. Determine the ratio in which the line 3x + y - 9 = 0 divides the segment joining the points (1, 3) and (2, 7).

OR

Find the mid-point of side BC of $\triangle$ABC, with A(1, - 4) and the mid-points of the sides through A being (2, -1) and (0, -1).

23. Form a quadratic polynomial whose zeroes are $\frac{3-\sqrt{3}}{5}$ and $\frac{3+\sqrt{3}}{5}$.

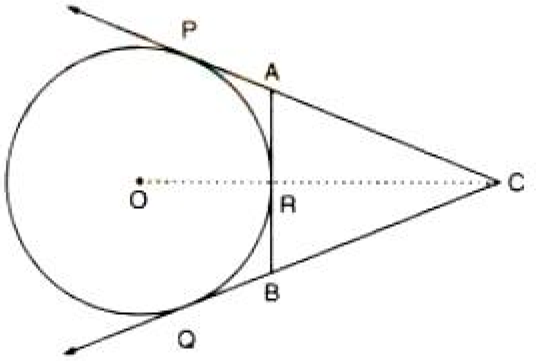
24. Find the third side of a right angled triangle whose hypotenuse is length p cm , one side of length q cm and p - q = 1.

25. Prove that $\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

OR

If $\cos\theta = \frac{3}{4}$, then find the value of 9 $\tan^2\theta$ + 9.

26. In Figure, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC



27. Prove that $(3 + 2\sqrt{5})^2$ is irrational.

28. Find the roots of the quadratic equation (if they exist) by the method of completing the square:

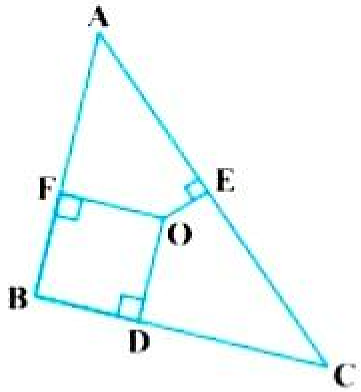
$2x^2 + x - 4 = 0$

OR

Determine the positive values of k for which the equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real roots.

29. If α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$, then, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

30. In figure, O is a point in the interior of a triangle ABC, $OD \perp BC, OE \perp AC$ and $OF \perp AB$, show that $OA^2 + OB^2 + OC^2 - OD^2 - OE - OF^2 = AF^2 + BD^2 + CE^2$



OR

In ΔPQR, M and N are points on sides PQ and PR respectively such that PM= 15 cm and NR = 8 cm. If PQ = 25 cm and PR = 20 cm state whether MN || QR.

31. A piggy bank contains hundred 50-p coins, fifty Rs.1 coins, twenty Rs.2 and ten Rs.5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, find the probability the coin falling out will be (i) a 50-p coin, (ii) of value more than Rs.1, (iii) of value less than Rs.5 (iv) a Rs.1 or Rs. 2 coin.

32. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30° respectively. Find the height of the balloon above the ground.

33. The arithmetic mean of the following frequency distribution is 50.

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50

Frequency	16	p	30	32	14

Find the value of p.

34. The sum of the radii of two circles is 7 cm, and the difference of their circumferences is 8 cm. Find the circumferences of the circles.

35. Solve the following equations by using cross multiplication method:

$ax + by = \frac{a+b}{2}$

3x + 5y =4

36. The angle of elevation of the top of a tower as observed from a point in a horizontal plane through the foot of the tower is 30°. When the observer moves towards the tower a distance of 100 m, he finds the angle of elevation of the top to be 60°. Find the height of the tower and the distance of first position from the tower.

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Solution

Part-A

1. According to the question,

   The given number is $\frac{125}{441}$

   Here, $441 = 3^2 \times 7^2$ and none of 3 and 7 is a factor of 125.

   The given number is in its simplest form.

   Now, $441 = 3^2 \times 7^2$ is not of the form $2^m \times 5^n$

   The given number has a non-terminating repeating decimal expansion.

   OR

   HCF $\times$ LCM = one number $\times$ another number

   = 100 $\times$ 190 = 19000

2. $3x^2 - 2\sqrt{x} + 8$ is not of the form $ax^2 + bx + c = 0$.

   $\therefore 3x^2 - 2\sqrt{x} + 8$ is not a quadratic equation.

3. For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

   then, $\frac{k}{4} = \frac{-3}{-6} \neq \frac{6}{15} \Rightarrow \frac{k}{4} = \frac{1}{2} \Rightarrow k = \frac{4}{2} = 2$

4. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

5. The given progression is 8,11,14,17, 20,....

   Clearly, (11-8) = (14-11) = (17 -14) = (20 -17) = 3 (constant).

   Thus, each term differs from its preceding term by 3.

   So, the given progression is an A.P.

   Its first term= 8 and common difference = 3.

   OR

   Let 1^{st} term = a

   Common difference, d = $\frac{5}{4}$ (Given)

   Also, $a_9 = -6$

   $\Rightarrow a + 8d = -6 \Rightarrow a + 8 \times \frac{5}{4} = -6 \Rightarrow a = -16$

Now, $a_{25} = a + 24d = -16 + 24 \times \frac{5}{4} = 14$

6. Middle term of an AP is arithmetic mean of AP.

   $\Rightarrow$ a is the arithmetic mean.

   Arithmetic mean of an AP

   $= (4/5 + 2)/2 = 7/5$

   $\Rightarrow a = \frac{7}{5}$.

7. We have, x = 2 and x = 3.

   Then,

   x -2 = 0 and x - 3 = 0

   $\Rightarrow (x - 2)(x - 3) = 0$

   $\Rightarrow x^2 - 3x - 2x + 6 = 0$

   $\Rightarrow x^2 - 5x + 6 = 0$

   This is the required quadratic equation.

OR

We have the following equation,

$x^2 - x + 2 = 0$

where a = 1, b = -1 and c = 2

$\therefore D = b^2 - 4ac$

$= (-1)^2 - 4(1)(2)$

$= 1 - 8$

$= -7 > 0$

Hence, the roots of the given equation has no real roots.

8. $\because$ BD is a diameter

   $\therefore \angle BCD = 90^o$ [Angle in the semi-circle]

   $\therefore \angle BCP = 180^o - 90^o - 40^o = 50^o$

9. A line which intersects a circle at any one point is called the tangent. The tangent at any point of a circle is perpendicular to the radius through the all point of contact.

OR

A line that interests a circle at two points in a circle is called a Secant.

10. We know that the ratio of areas of two similar triangles is equal to the square of the ratio of corresponding altitude.

Ratio of their areas = (ratio of their altitudes)^2

$= \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

= 4 : 9

11. $a_n = 7 - 4n \Rightarrow a_1 = 7 - 4 \times 1 = 3$

$a_2 = 7 - 4 \times 2 = -1$ and $a_3 = 7 - 4 \times 3 = -5$

$\therefore$ Common difference = $a_2 - a_1 = -1 - 3 = -4$.

12. We have,

LHS = $\sin 60^\circ = \frac{\sqrt{3}}{2}$

RHS = $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

$= \frac{2\times\frac{1}{\sqrt{3}}}{1+(1/\sqrt{3})}$

$= \frac{2/\sqrt{3}}{1+\frac{1}{3}}$

$= \frac{2/\sqrt{3}}{4/3}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$

$= \frac{\sqrt{3}}{2}$

$\therefore$ LHS = RHS

Hence Proved

13. Value of $\tan\theta$ increases when θ increases from 0^o to 90^o. $(0, \frac{1}{\sqrt{3}}, 1, \sqrt{3},$ not defined at $90^\circ)$

14. Let the radius of 1^{st} cylinder be 'r_1'

let the height of 1^{st} cylinder be 'h_1'

let the radius of 2^{nd} cylinder be 'r_2'

let the height of 2^{nd} cylinder be 'h_2'

$\frac{\text{Voume of } 1^{\text{st}} \text{ cylinder}}{\text{Voume of } 2^{\text{nd}} \text{ cylinder}} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$

$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$

$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3}$

$= \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$

= 20 : 27

15. $(a^2 + b^2) - (a - b)^2 = a^2 + b^2 - (a^2 + b^2 - 2ab) = a^2 + b^2 - a^2 - b^2 + 2ab = 2ab$

$(a+b)^2 - (a^2 + b^2) = (a^2 + b^2 + 2ab) - a^2 - b^2 = a^2 + b^2 + 2ab - a^2 - b^2 = 2ab$

Since $(a^2 + b^2) - (a - b)^2 = (a + b)^2 - (a^2 + b^2)$,

$(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.

16. After removing 2 red queens and 2 black jacks, the number of remaining cards = 52 - (2 + 2) = 48.

   i. Out of 48 cards, there are 4 kings.

   $\therefore$ P(getting a king)$= \frac{4}{48} = \frac{1}{12}$

   ii. Number of cards of red colour = 26 - 2 = 24.

   Total number of cards = 48.

   $\therefore$ P(getting a card of red colour)$= \frac{24}{48} = \frac{1}{2}$

   iii. Number of face cards = 12 - (2 + 2) = 8.

   Total number of cards = 48.

   $\therefore$ P(getting a face card)$= \frac{8}{48} = \frac{1}{6}$

   iv. Number of queens in 48 cards = 4-2 = 2.

   $\therefore$ P(getting a queen)$= \frac{2}{48} = \frac{1}{24}$.

17. i. (c) Given: A(3, 4), B(6, 7), C(9, 4), D(6, 1)

   Using distance formula,

   AB = $\sqrt{(6-3)^2 + (7-4)^2} = 3\sqrt{2}$ units

   BC = $\sqrt{(9-6)^2 + (4-7)^2} = 3\sqrt{2}$ units

   CD = $\sqrt{(6-9)^2 + (1-4)^2} = 3\sqrt{2}$ units

   DA = $\sqrt{(3-6)^2 + (4-1)^2} = 3\sqrt{2}$ units

   AC = $\sqrt{(9-3)^2 + (4-4)^2}$ = 6 units

   BD = $\sqrt{(6-6)^2 + (1-7)^2}$ = 6 units

   As sides AB = BC = CD = DA, and diagonals AC and BD are equal, so ABCD is a square.

   Now as diagonals of a square bisect each other, so midpoint of the diagonal gives the position of Anjali to sit in the middle of the four students.

   Here diagonal is AC or BD.

   So, mid-point of AC = $\left(\frac{3+9}{2}, \frac{4+4}{2}\right)$ = (6, 4)

   So, position of Anjali is (6, 4).

ii. (b) Position of Sita is at point A i.e. (3, 4) and Position of Anita is at point D i.e. (6, 1).
So, distance between Sita and Anita, AD = $\sqrt{(6-3)^2+(1-4)^2}=3\sqrt{2}$ units
iii. (d) Now, Gita is at position B and as BA and BC are equal and equidistant from point B.
So, we can say Sita and Rita are the two students who are equidistant from Gita.
iv. (a) Square
v. (b) 6 units

18. i. (a) 30 m
ii. (c) 60 m
iii. (b) 50 m
iv. (d) 100 m
v. (d) 80 m

19. We may observe from the given data that maximum class frequency is 10 belonging to class interval 30 - 35.

So, modal class = 30 - 35

Class size (h) = 5

Lower limit (l) of modal class = 30

Frequency (f) of modal class = 10

Frequency (f_1) of class preceding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

Mode $= l+\frac{f-f_1}{2f-f_1-f_2}\times h$
$=30+\frac{10-9}{2\times10-9-3}\times 5$
$=30+\frac{1}{20-12}\times 5$
$=30+\frac{5}{8}$
= 30.625

Mode = 30.6

It represents that most of states / U.T have a teacher- student ratio as 30.6

Now we may find class marks by using the relation

Class mark $=\frac{\text{upper class limit }+\text{lower class limit}}{2}$

Now taking 32.5 as assumed mean (a) we may calculate d_i, u_i, and f_iu_i as following

Number of students per teacher	Number of states/U.T (f_i)	x_i	$d_i = x_i$ - 32.5	U_i	f_iu_i
15 – 20	3	17.5	-15	-3	-9

20 – 25	8	22.5	-10	-2	-16
25 – 30	9	27.5	-5	-1	-9
30 – 35	10	32.5	0	0	0
35 – 40	3	37.5	5	1	3
40 – 45	0	42.5	10	2	0
45 – 50	0	47.5	15	3	0
50 – 55	2	52.5	20	4	8
Total	35				-23

Now, Mean $\overline{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 32.5 + \frac{-23}{35} \times 5$

$= 32.5 - \frac{23}{7}$

= 32.5 - 3.28

= 29.22

So the mean of data is 29.2

It represents that on an average teacher-student ratio was 29.2

i. (b) 30 - 35
ii. (d) 29.2
iii. (b) 30.625
iv. (b) Classmark
v. (c) Median

20. For cone, Radius of the base (r)

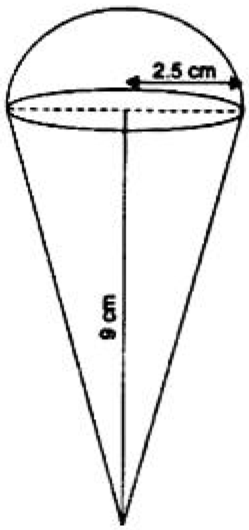
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

i. (a) $\frac{1357}{42}\text{cm}^3$

ii. (d) The volume of the ice-cream without hemispherical end = Volume of the cone $= \frac{825}{14}\text{cm}^3$

iii. (c) The TSA of the cone is given by:
$\pi rl + \pi r^2$

iv. (a) Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere
$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$
$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

v. (d) remain unaltered

Part-B

21. Here the given number:
$\frac{24}{125} = \frac{4\times6}{5\times5\times5} = \frac{8\times6}{2\times5^3}$
$.= \frac{8\times6}{2\times5^3} = \frac{48}{2^1\times5^3}$
Therefore the denominator of the given number is of the form $2^m \times 5^n$.
So, the given number is a terminating decimal.
Now
$\frac{24}{125} = \frac{24}{5^3} = \frac{24\times2^3}{2^3\times5^3}$
$= \frac{24\times8}{10^3} = \frac{192}{1000} = 0.192$

22. Suppose the line 3x + y - 9 = 0 divides the line segment joining A (1, 3) and B (2, 7) in the

ratio k : 1 at point C. Then, the coordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

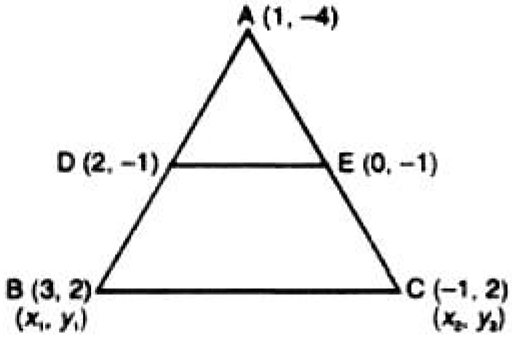
But, C lies on 3x + y - 9 = 0. Therefore,

$3\left(\frac{2k+1}{k+1}\right)+\frac{7k+3}{k+1}-9=0$

⇒ 6k + 3 + 7k + 3 - 9k - 9 = 0 ⇒ $k=\frac{3}{4}$

So, the required ratio is 3 : 4 internally.

OR



Let Co-ordinates of B are (x_1, y_1)

$2=\frac{1+x_1}{2} \therefore x_1=3$

$-1=\frac{-4+y_1}{2} \therefore y_1=2$

∴ The coordinate of B is (x_1,y_1) = (3, 2).

Let the coordinates of C are (x_2,y_2)

$0=\frac{1+x_2}{2} \Rightarrow x_2=-1$

$-1=\frac{-4+y_2}{2} \Rightarrow y_2=2$

So, coordinates of mid - point of BC are = $\left(\frac{3-1}{2}, \frac{2+2}{2}\right)$ = (1, 2)

23. Let α= $\frac{3-\sqrt{3}}{5}$ and β=$\frac{3+\sqrt{3}}{5}$

Given α+β=$\frac{3-\sqrt{3}}{5}$ + $\frac{3+\sqrt{3}}{5}$ = $\frac{6}{5}$

Product of zeroes,

αβ = $\left(\frac{3-\sqrt{3}}{5}\right)\times\left(\frac{3+\sqrt{3}}{5}\right)$ = $\frac{3^2-(\sqrt{3})^2}{5\times5}=\frac{6}{25}$

Polynomial

p(x) = x^2 - (α+β)x)+αβ

= x^2 - $\frac{6}{5}x+\frac{6}{25}$

p(x)= $25x^2$ - 30x + 6

24. Given, the length of the hypotenuse of triangle = p cm & the length of one side be q cm

Let, the length of third side be x cm

By using the Pythagoras theorem it is,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Altitude})^2$

$\Rightarrow p^2 = q^2 + x^2$

$\Rightarrow x^2 = p^2 - q^2$

$\Rightarrow x^2 = (p+q)(p-q)$

$\Rightarrow x^2 = p + q$ [Given, $p - q = 1$]

$\Rightarrow x = \sqrt{p+q}$

Hence, the length of the third side is $\sqrt{p+q}$.

25. $\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

$\text{L.HS.} = \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

$= \frac{\sin^2 A \cos^2 B - \cos^2 A \sin^2 B}{\cos^2 A \cos^2 B} = \frac{\sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B}{\cos^2 A \cos^2 B}$

$\left[\because \sin^2 A + \cos^2 A 1 \text{ or } \cos^2 A = 1 - \sin^2 A\right]$

$= \frac{\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B}{\cos^2 A \cos^2 B}$

$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} \therefore \text{L. HS.} = \text{R. HS}$

OR

Given:

$\cos\theta = \frac{3}{4}$

$\Rightarrow \frac{1}{\cos\theta} = \frac{4}{3}$

$\Rightarrow \sec\theta = \frac{4}{3}$

We know that

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \left(\frac{4}{3}\right)^2 - \tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{16}{9} - 1$

$\Rightarrow \tan^2\theta = \frac{7}{9}$

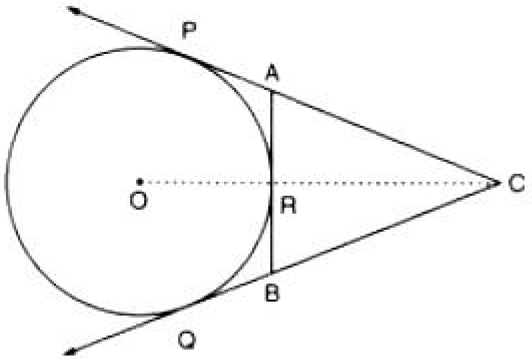
Therefore,

$9\tan^2\theta + 9 = 9 \times \frac{7}{9} + 9$

= 7 + 9

= 16

26. We are given the following figure



From the figure, we have

BC = CQ − BQ...... (1)

Let us now find out the values of CQ and BQ separately.

From the property of tangents we know that length of two tangents drawn from the same external point will be equal. Therefore,

CP = CQ

It is given in the problem that,

CP = 11

Therefore,

CQ = 11

Now let us find out the value of BQ.

Again from the same property of tangents, we know that length of two tangents drawn from the same external point will be equal. Therefore,

BR = BQ

It is given in the problem that,

BR = 4 cm

Therefore,

BQ = 4 cm

Now let us substitute the values of CQ and BQ in equation (1). We have,

BC = 11 − 4

BC = 7

Therefore, length of BC is 7 cm.

27. Let take that $3 + 2\sqrt{5}$ is a rational number.

So we can write this number as

$3 + 2\sqrt{5} = \frac{a}{b}$

Here a and b are two co-prime numbers and b is not equal to 0.

Subtract 3 both sides we get,

$2\sqrt{5} = \frac{a}{b} - 3$

$2\sqrt{5} = \frac{a-3b}{b}$

Now divide by 2 we get

$\sqrt{5} = \frac{a-3b}{2b}$

Here a and b are an integer so $\frac{a-3b}{2b}$ is a rational number so $\sqrt{5}$ should be a rational number but $\sqrt{5}$ is an irrational number so it contradicts the fact.

Hence the result is $3 + 2\sqrt{5}$ is an irrational number

Now its square will again contain an irrational number.

Hence the given number is an irrational number.

28. Given that,

$2x^2 + x - 4 = 0$

$4x^2 + 2x - 8 = 0$ [multiplying both sides by 2]

$4x^2 + 2x = 8$

$\Rightarrow (2x)^2 + 2 \times 2x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 = 8 + \left(\frac{1}{2}\right)^2$ [adding $\left(\frac{1}{2}\right)^2$ on both sides]

$\Rightarrow \left(2x + \frac{1}{2}\right)^2 = \left(8 + \frac{1}{4}\right) = \frac{33}{4} = \left(\frac{\sqrt{33}}{2}\right)^2$

$\Rightarrow$ 2x + $\frac{1}{2} = \pm\left(\frac{\sqrt{33}}{2}\right)$ [taking square root on both sides]

$\Rightarrow 2x + \frac{1}{2} = \frac{\sqrt{33}}{2}$ or 2x + $\frac{1}{2} = \frac{-\sqrt{33}}{2}$

$\Rightarrow 2x = \frac{\sqrt{33}}{2} - \frac{1}{2}\ or\ 2x = -\frac{\sqrt{33}}{2} - \frac{1}{2} \Rightarrow x = \frac{\sqrt{33}-1}{4}\ \ or\ \ x = \frac{-(\sqrt{33}+1)}{4}$

OR

First quadratic equation is $x^2 + kx + 64 = 0$

Here, a = 1, b = k, c = 64

Therefore, discriminant $(D_1) = b^2 - 4ac$

If the first quadratic equation has real roots, then

$D_1 \geq 0$

$\Rightarrow k^2 - 256 \geq 0 \Rightarrow k^2 \geq 256$

$\Rightarrow k \geq 16 \ldots . (1) \because k > 0$

Second quadratic equation is

x - 8x + k = 0

Here, a = 1, b = -8, c = k

Therefore, discriminant (D_2) = b^2 - 4ac

= $(-8)^2$ - 4(1)(k) = 64 - 4k

If the second quadratic equation has real roots, then

$D_2 \geq 0$

$\Rightarrow 64 - 4k \geq 0 \Rightarrow 4k \leq 64 \Rightarrow k \leq 16$ ....(ii)

We see that k = 16 satisfies (1) and (2) both.

Hence, both the equations will have real roots for k = 16.

29. $f(x) = 6x^2 + x - 2$

a = 6, b = 1, c = -2

Let zeroes be α and β.Then

Sum of zeroes= $\alpha + \beta = -\frac{b}{a} = -\frac{1}{6}$

Product of zeroes $\alpha \times \beta = \frac{c}{a} = \frac{-2}{6} = -\frac{1}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\left[\because (\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta\right]$

$= \frac{\left[-\frac{1}{6}\right]^2-2\left[-\frac{1}{3}\right]}{\left[-\frac{1}{3}\right]}$

$= \frac{\frac{1}{36}+\frac{2}{3}}{-\frac{1}{3}}$

$= \frac{\frac{1+24}{36}}{-\frac{1}{3}}$

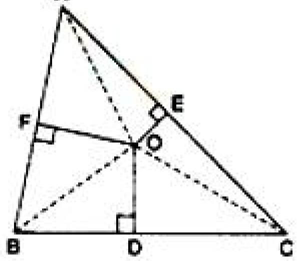
$= \frac{25}{36} \times \frac{-3}{1}$

$= \frac{-25}{12}$

30. In right triangle OAF,

$OA^2 = OF^2 + AF^2$ ....[By Pythagoras theorem]

$\Rightarrow AF^2 = OF^2 + AF^2$.........(1)



In right triangle OBD,

$OB^2 = OD^2 + BD^2$..........[By Pythagoras theorem]

$\Rightarrow BD^2 = OB^2 - OD^2$........(2)

In right triangle OCE,

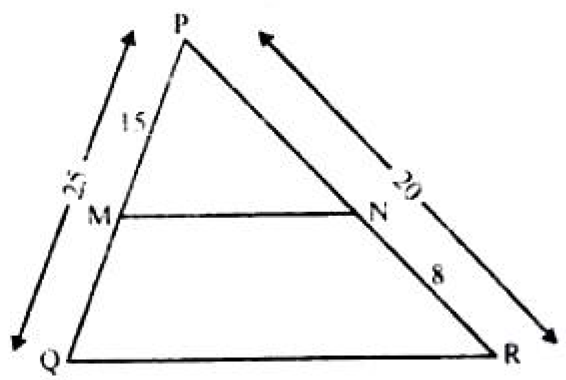
$OC^2 = OE^2 + CE^2$........By Pythagoras theorem

$\Rightarrow CE^2 = OC^2 - OE^2$...........(3)

Adding (1), (2) and (3), we get

$AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OA^2 - OE^2 - OF^2$

OR



In $\triangle$ PQR , P and Q are points on PQ and PR such that

PM = 15 cm, NR = 8 cm PQ = 25 cm

and PR = 20 cm, PN = PR - NR = 20 - 8 = 12

Now in $\triangle$PMN and $\triangle$PQR,

$\frac{PM}{PQ} = \frac{15}{25} = \frac{3}{5}$ and

$\frac{PN}{NR} = \frac{12}{20} = \frac{3}{5}$

$\because \frac{\text{PM}}{\text{PQ}} = \frac{\text{PN}}{\text{NR}}$

$\therefore$ MN || QR

31. Total number of coins = (100 + 50 + 20 +10) = 180.

So, the number of all possible outcomes is 180.

i. Let E_1 be the event of getting a 50-p coin.

   Then, the number of favourable outcomes = 100.

   $\therefore$ P(getting a 50-p coin)=$P(E_1) = \frac{100}{180} = \frac{3}{9}$

ii. Let E_2 be the event of getting a coin of value more than Rs. 1. Then, it can be Rs.2 or Rs. 5 coin.

   Number of all such coins = 20 + 10 = 30.

P(getting a coin of value more than Rs.1)

$= P(E_2) = \frac{30}{180} = \frac{1}{6}$

iii. Let E_3 be the event of getting a coin of value less than Rs.5. Then, it can be 50-p or Rs.1 or Rs.2 coin.

Number of favourable outcomes

= number of all coins of 50-p, Rs.1 and Rs.2

= (100 + 50 + 20 = 170.

∴ P(getting a coin of value less than Rs.5)

$= P(E_3) = \frac{170}{180} = \frac{17}{18}$

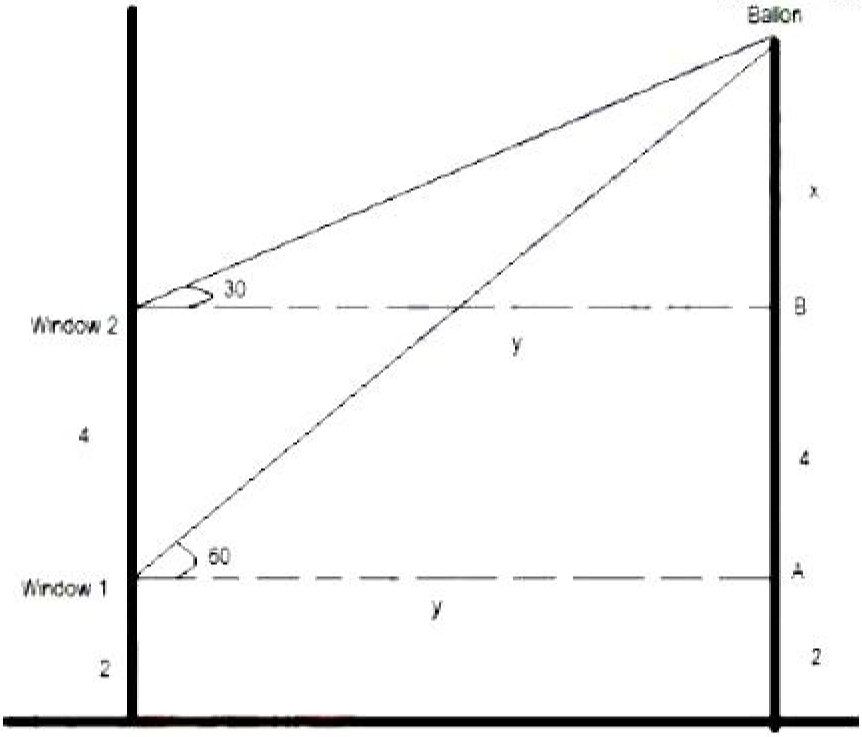
iv. Let E_4 be the event of getting an Rs.1 or Rs.2 coin.

Number of all such coins = 50 + 20 = 70.

Number of favourable outcomes = 70.

∴ p(getting a Rs.1 or Rs.2 coin)=$P(E_4) = \frac{70}{180} = \frac{7}{18}$

32.



From the figure,

let the height of balloon is = x + 4 + 2 = x + 6

$\tan 30^\circ = \frac{x}{y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y}$

$\Rightarrow y = \sqrt{3}x$ .... (i)

And, $\tan 60^\circ = \frac{x+4}{y}$

$\Rightarrow \tan 60^\circ = \frac{x+4}{y}$

$\Rightarrow \sqrt{3} = \frac{x+4}{y}$

$\Rightarrow \sqrt{3} = \frac{x+4}{\sqrt{3}x}$

$\Rightarrow 3x = x + 4$

$\Rightarrow 2x = 4 \Rightarrow x = 2$

Thus, height of balloon = x + 4 + 2 $\Rightarrow$ 2 + 4 + 2 = 8m

33. We have,

Class Interval	Frequency f_i	Mid- value x_i	$f_i x_i$
0 - 10	16	5	80
10 - 20	p	15	15p
20 - 30	30	25	750
30 - 40	32	35	1120
40 - 50	14	45	630
	$\Sigma f_i = 92 + p$		$\sum f_i x_i = 2580 + 15p$

Now, mean = $\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 25 = \frac{2580+15p}{92+p}$

$\Rightarrow$ 25(92 + p) = 2580 + 15p

$\Rightarrow$ 2300 + 25p = 2580 + 15p

$\Rightarrow$ 10p = 280

$\Rightarrow$ p = 28

34. Let the radii of the two circles be r_1 cm and r_2 cm.

Now,

Sum of the radii of the two circles = 7 cm

$r_1 + r_2 = 7$ ....(i)

Difference of the circumferences of the two circles = 8 cm

$\Rightarrow 2\pi r_1 - 2\pi r_2 = 8$

$\Rightarrow 2\pi(r_1 - r_2) = 8$

$\Rightarrow (r_1 - r_2) = \frac{8}{2\pi}$

$\Rightarrow r_1 - r_2 = \frac{8}{2\times\frac{22}{7}}$

$\Rightarrow r_1 - r_2 = \frac{8\times7}{44}$

$\Rightarrow r_1 - r_2 = \frac{56}{44}$

$\Rightarrow r_1 - r_2 = \frac{14}{11}$ ......(ii)

Adding (i) and (ii), we get,

$2r_1 = \frac{91}{11}$

$r_1 = \frac{91}{22}$

$\therefore$ Circumference of the first circle = $2\pi r_1$

$= 2 \times \frac{22}{7} \times \frac{91}{22} = 26$ cm

Also, $r_1 - r_2 = \frac{14}{11}$

$\frac{91}{22} - r_2 = \frac{14}{11}$

$\frac{91}{22} - \frac{14}{11} = r_2$

$r_2 = \frac{63}{22}$

$\therefore$ Circumference of the second circle = $2\pi r_2 = 2 \times \frac{22}{7} \times \frac{63}{22} = 18$ cm

Hence, circumferences of the first and second circles are 18 cm and 26 cm, respectively.

35. The given system of equations is

$ax + by = \frac{a+b}{2}$ ...(1)

$3x + 5y = 4$ ...(2)

From (1), we get

$2(ax + by) = a + b$

$\Rightarrow 2ax + 2by - (a + b) = 0$ ...(3)

From (2), we get; $3x + 5y - 4 = 0$ ...(4)

We know that if,

$a_1x + b_1y + c_1 = 0$...........(i)

$a_2x + b_2y + c_2 = 0$............(ii)

then by cross multiplication method,

$\frac{x}{b_1c_2-c_1b_2} = -\frac{y}{a_1c_2-c_1a_2} = \frac{1}{a_1b_2-a_2b_1}$

Here,comparing equation (3) & (4) with the above formula, we get:-

$a_1 = 2a, b_1 = 2b, c_1 = -(a + b)$

$a_2 = 3, b_2 = 5, c_2 = -4$

Now applying cross-multiplication method, we have ;

$\frac{x}{2b\times(-4)-[-(a+b)\times5]}=\frac{-y}{2a\times(-4)-[-(a+b)]\times3}=\frac{1}{2a\times5-2b\times3}$

$\Rightarrow \frac{x}{-8b+5(a+b)}=\frac{-y}{-8a+3(a+b)}=\frac{1}{10a-6b}$

$\Rightarrow \frac{x}{-8b+5a+5b}=\frac{-y}{-8a+3a+3b}=\frac{1}{10a-6b}$

Now, from 1st & 3rd part; we have

$\frac{x}{5a-3b}=\frac{1}{10a-6b}$

$\Rightarrow x=\frac{5a-3b}{10a-6b}=\frac{5a-3b}{2(5a-3b)}=\frac{1}{2}$

Again, from 2nd & 3rd part; we have

$\frac{-y}{-5a+3b}=\frac{1}{10a-6b}$

$\Rightarrow -y=\frac{-5a+3b}{2(5a-3b)}$

$\Rightarrow y=\frac{-(-5a+3b)}{2(5a-3b)}$

$=\frac{5a-3b}{2(5a-3b)}$

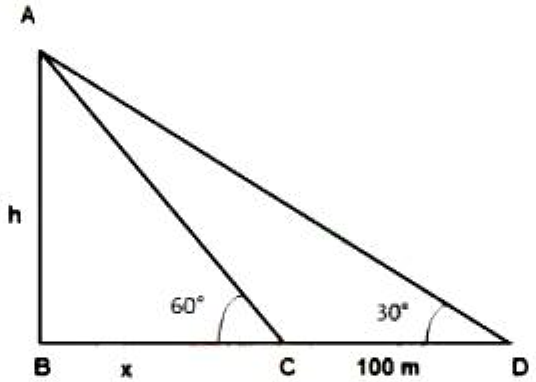
$\Rightarrow y=\frac{1}{2}$

Hence, $x=\frac{1}{2},\ y=\frac{1}{2}$ is the solution to the given system of equations.

36. As shown in figure AB is the tower.

Initial position of observer is at D and 2nd position is at C

Given that CD = 100 m.



Let h is the height of tower and BC = x

Now in ΔABC,

tan 60° = $\frac{\mathrm{AB}}{\mathrm{BC}}=\frac{\mathrm{h}}{\mathrm{x}}$

$\sqrt{3}=\frac{h}{x}$

$x=\frac{\mathbf{h}}{\sqrt{3}}$ .... (i)

Now in $\triangle$ABD,

tan 30° = $\frac{\mathrm{AB}}{\mathrm{BD}}=\frac{\mathrm{h}}{\mathrm{x+100}}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+100}$

x + 100 = $h\sqrt{3}$

$\frac{\text{h}}{\sqrt{3}} + 100 = \text{h}\sqrt{3}$

h + $100\sqrt{3}$ = 3h

2h = $100\sqrt{3}$

h = $50\sqrt{3}$ = 50 $\times$ 1.732 = 86.6 m

So, height of the tower is 86.6 m.

The distance of first position from the tower = BD = x + 100 = 50 + 100 = 150 m