PRACTICE SET -2

1. If $f:[1,\infty)\to[2,\infty)$ is given by $f(x)=x+\frac{1}{x}$ then $f^{-1}(x)$ equals:

a. $\frac{x+\sqrt{x^2-4}}{2}$ b. $\frac{x}{1+x^2}$

c. $\frac{x-\sqrt{x^2-4}}{2}$ d. $1+\sqrt{x^2-4}$

2. Let α and β be the roots of the equation $x^2+x+1=0$. The equation whose roots are α^{19},β^7 is

a. $x^2-x-1=0$ b. $x^2-x+1=0$

c. $x^2+x-1=0$ d. $x^2+x+1=0$

3. Let $\omega_n=\cos\left(\frac{2\pi}{n}\right)+i\sin\left(\frac{2\pi}{n}\right), i^2=-1$, then $(x+y\omega_3+z\omega_3^2)(x+y\omega_3^2+z\omega_3)$ is equal to

a. 0

b. $x^2+y^2+z^2$

c. $x^2+y^2+z^2-yz-zx-xy$

d. $x^2+y^2+z^2+yz+zx+xy$

4. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$ is

a. –1 b. 0

c. –i d. i

5. The inverse of the matrix $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ is

a. $\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$ b. $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

c. $\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$ d. $\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$

6. $1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\frac{1+2+3+4}{4!}+\ldots\infty=$

a. e b. $3e$

c. $e/2$ d. $3e/2$

7. $\log_e 2+\log_e\left(1+\frac{1}{2}\right)+\log_e\left(1+\frac{1}{3}\right)+\ldots+\log_e\left(1+\frac{1}{n-1}\right)$ is equal to

a. $\log_e 1$ b. $\log_e n$

c. $\log_e(1+n)$ d. $\log_e(1-n)$

8. If $x\sin 45^\circ\cos^2 60^\circ=\frac{\tan^2 60^\circ\operatorname{cosec} 30^\circ}{\sec 45^\circ\cot^2 30^\circ}$, then $x=$

a. 2 b. 4

c. 8 d. 16

9. If $\sin\theta+\sin 2\theta+\sin 3\theta=\sin\alpha$ and $\cos\theta+\cos 2\theta+\cos 3\theta=\cos\alpha$, then θ is equal to

a. $\alpha/2$ b. α

c. 2α d. $\alpha/6$

10. Find imaginary part of $\sin^{-1}\left(\frac{5\sqrt{7}-9i}{16}\right)$

a. log2 b. –log2

c. 0 d. None of these

11. Let $f(x)=\begin{cases}\frac{x^3+x^2-16x+20}{(x-2)^2}, & \text{if } x\neq 2\\ k, & \text{if } x=2\end{cases}$.

If $f(x)$ be continuous for all x, then $k=$

a. 7 b. –7

c. ±7 d. None of these

12. If $y=e^{x+e^{x+e^{x+\ldots\infty}}}$, then $\frac{dy}{dx}=$

a. $\frac{y}{1-y}$ b. $\frac{1}{1-y}$

c. $\frac{y}{1+y}$ d. $\frac{y}{y-1}$

13. One maximum point of $\sin^p x\cos^q x$ is

a. $x=\tan^{-1}\sqrt{(p/q)}$ b. $x=\tan^{-1}\sqrt{(q/p)}$

c. $x=\tan^{-1}(p/q)$ d. $x=\tan^{-1}(q/p)$

14. $\int\frac{1}{x(\log x)^2}dx=$

a. $\frac{1}{\log x}+c$ b. $-\frac{1}{\log x}+c$

c. $\log\log x+c$ d. $-\log\log x+c$

15. Area bounded by the parabola $y^2 = 2x$ and the ordinates $x = 1, x = 4$ is

a. $\frac{4\sqrt{2}}{3}$ sq. unit b. $\frac{28\sqrt{2}}{3}$ sq. unit

c. $\frac{56}{3}$ sq. unit d. None of these

16. The solution of the differential equation $\frac{dy}{dx} + \frac{1+x^2}{x} = 0$ is

a. $y = -\frac{1}{2}\tan^{-1}x + c$ b. $y + \log x + \frac{x^2}{2} + c = 0$

c. $y = \frac{1}{2}\tan^{-1}x + c$ d. $y - \log x - \frac{x^2}{2} = c$

17. The solution of the differential equation $\cos y \log(\sec x + \tan x)dx = \cos x \log(\sec y + \tan y)dy$ is

a. $\sec^2 x + \sec^2 y = c$ b. $\sec x + \sec y = c$

c. $\sec x - \sec y = c$ d. None of these

18. The vertex of an equilateral triangle is $(2, -1)$ and the equation of its base in $x + 2y = 1$. The length of its sides is

a. $4/\sqrt{15}$ b. $2/\sqrt{15}$

c. $4/3\sqrt{3}$ d. $1/\sqrt{5}$

19. If the equation $\frac{k(x+1)^2}{3} + \frac{(y+2)^2}{4} = 1$ represents a circle, then k =

a. 3/4 b. 1

c. 4/3 d. 12

20. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

a. $3y = 9x + 2$ b. $y = 2x + 1$

c. $2y = x + 8$ d. $y = x + 2$

21. The sum of the first five terms of the series $3 + 4\frac{1}{2} + 6\frac{3}{4} + ...$ will be

a. $39\frac{9}{16}$ b. $18\frac{3}{16}$

c. $39\frac{7}{16}$ d. $13\frac{9}{16}$

22. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + ... + a_{12}x^{12}$, then the expression $a_2 + a_4 + a_6 + ... + a_{12}$ has the value

a. 32 b. 63

c. 64 d. None of these

23. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is.

a. 10^2 b. 1023

c. 2^{10} d. 10 !

24. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals

a. $\frac{1}{2}$ b. $\frac{1}{32}$

c. $\frac{31}{32}$ d. $\frac{1}{5}$

25. An observer on the top of a tree, finds the angle of depression of a car moving towards the tree to be 30° . After 3 minutes this angle becomes 60°. After how much more time, the car will reach the tree

a. 4 min b. 4.5 min

c. 1.5 min d. 2 min

26. The variance of first 50 even natural numbers is:

a. $\frac{833}{4}$ b. 833

c. 437 d. $\frac{437}{4}$

27. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as

a. $\sin A \cos A + 1$ b. $\sec A$ coeec $A + 1$

c. $\tan A + \cot A$ d. $\sec A + \text{cosec } A$

28. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1 + \sum_{k=1}^{n}2k\right)\right)$ is

a. $\frac{23}{25}$ b. $\frac{25}{23}$

c. $\frac{23}{24}$ d. $\frac{24}{23}$

29. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point R, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is:

a. 6 b. 10 c. 20 d. 5

30. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

a. $-\frac{1}{4}$ b. $\frac{1}{2}$

c. 1 d. 2

31. Let the function $g:(-\infty, \infty)\to\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$.

Then, g is

a. even and is strictly increasing in $(0, \infty)$

b. odd and is strictly decreasing in $(-\infty, \infty)$

c. odd and is strictly increasing in $(-\infty, \infty)$

d. neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

32. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is:

a. 175 b. 162

c. 160 d. 180

33. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is:

a. 288 b. 306

c. 360 d. 310

34. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to

a. 14 b. 6 c. 4 d. 8

35. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repetition of digits allowed) is equal to:

a. 250 b. 374 c. 372 d. 375

36. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is:

a. 1365 b. 1256

c. 1465 d. 1356

37. $\sum_{i=1}^{20}\left(\frac{^{20}C_{i-1}}{^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals:

a. 200 b. 50

c. 100 d. 400

38. The number of functions f from {1, 2, 3,..., 20} onto {1, 2, 3,....., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is:

a. (15)! × 6! b. 56 × 15

c. 5! × 6! d. 65 × (15)!

39. Consider three boxes, each containing 10 balls labelled 1,2,....,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is:

a. 82 b. 240

c. 164 d. 120

40. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right);\ k = 1,2,...,9.$

Column I	Column II
(A) For each z_k there exists a z_j such $z_k . z_j = 1$	1. True
(B) There exists a $k\in\{1,2,.....,9\}$ such that $z_1 . z = z_k$ has no solution z in the set of complex numbers	2. False
(C) $\frac{\lvert 1-z_1\rvert \lvert 1-z_2\rvert .....\lvert 1-z_9\rvert}{10}$ equals	3. 1
(D) $1-\sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ equals	4. 2

a. A→ 2; B→ 1; C→ 3; D→ 4

b. A→ 3; B→ 2; C→ 1; D→ 4

c. A→ 1; B→ 2; C→ 3; D→ 4

d. A→ 1; B→ 3; C→ 4; D→ 2

41. Let (x, y) be such that $\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}$ Match the statements in Column I with the statements in Column II.

Column I	Column II
(A) If $a = 1$ and $b = 0$, then (x, y)	1. lies on the circle $x^2 + y^2 = 0$
(B) If $a = 1$ and $b = 1$, then (x, y)	2. lies on $(x^2-1)(y^2-1) = 0$
(C) If $a = 1$ and $b = 2$, then (x, y)	3. lies on $y = x$
(D) If $a = 2$ and $b = 2$, then (x, y)	4. lies on $(4x^2-1)(y^2-1) = 0$

a. A→ 1; B→ 2; C→ 1; D→ 4

b. A→ 4; B→ 2; C→ 1; D→ 3

c. A→ 1; B→ 2; C→ 3; D→ 4

d. A→ 1; B→ 3; C→ 4; D→ 2

42. Match the Statements/Expressions in Column I with the Statements/Expressions in Column II.

Column I	Column II
(A) The minimum value of $\frac{x^2+2x+4}{x+2}$ is	1. 0
(B) Let A and B be 3 × 3 matrices of real numbers, where A is symmetric, B is skew symmetric, and $(A+B)(A-B) = (A-B)(A+B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	2. 1
(C) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	3. 2
(D) If $\sin\theta + \cos\phi$, then the possible value of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are	4. 3

a. A→3; B→2,4; C→3,4; D→1,3
b. A→2; B→2,3; C→2,4; D→1,2
c. A→3; B→2,3; C→1,2; D→1,3
d. A→2; B→2,4; C→1,4; D→1,2

43. In the following $[x]$ denotes the greatest integer less than or equal to x. Match the functions in Column I with the properties Column II.

Column I	Column II
(A) $x\|x\|$	1. continuous in $(-1,1)$
(B) $\sqrt{\|x\|}$	2. differentiable in $(-1,1)$
(C) $x+[x]$	3. strictly increasing in $(-1,1)$
(D) $\|x-1\|+\|x+1\|$	4. not differentiable at least at one point in $(-1,1)$

a. A→1,2; B→2,4; C→3,4; D→1,3
b. A→1,2,3; B→1,4; C→3,4; D→1,2
c. A→3; B→2,3; C→1,2,3; D→1,3
d. A→2; B→2,4; C→1,4; D→1,2

44. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is

a. $\frac{1}{\sin 2°}$
b. $\frac{1}{3\sin 2°}$
c. $\frac{1}{2\sin 2°}$
d. $\frac{1}{4\sin 2°}$

45. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to:

a. $(2, -1)$
b. $(-2, 1)$
c. $(2, 1)$
d. $(-2, -1)$

46. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,..... is

a. $\frac{7}{81}(179-10^{-20})$
b. $\frac{7}{9}(99-10^{-20})$
c. $\frac{7}{81}(179+10^{-20})$
d. $\frac{7}{9}(99+10^{-20})$

47. Coefficient of x^{11} in the expansion of $(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$ is

a. 1051
b. 1106
c. 1113
d. 1120

48. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

a. 55
b. 66
c. 77
d. 88

49. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations values 3, 4 and 5 are added to the data, then the mean of the resultant data, is

a. 16.8
b. 16.0
c. 15.8
d. 14.0

50. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the

men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is:

a. 9 b. 11
c. 12 d. 7

Answer and Solutions

1. (a) Use the identity $f(f^{-1}(x)) = x$

replace x by $f^{-1}(x)$, in the given function we get

$$f(f^{-1}(x)) = f^{-1}(x) + \frac{1}{f^{-1}(x)}$$

$\Rightarrow$ $x = f^{-1}(x) + \frac{1}{f^{-1}(x)}$, solve to find $f^{-1}x$

2. (d) Given $x^2 + x + 1 = 0$

$\therefore$ $x = \frac{1}{2}[-1 \pm i\sqrt{3}] = \frac{1}{2}(-1 + i\sqrt{3}), \frac{1}{2}(-1 - i\sqrt{3}) = \omega, \omega^2$

But $\alpha^{19} = \omega^{19} = \omega$ and $\beta^7 = \omega^{14} = \omega^2$.

Hence the equation will be same.

3. (c) $\omega_n = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right)$

$\Rightarrow$ $\omega_3 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{i\sqrt{3}}{2} = \omega$

and $\omega_3^2 = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^2 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$= -\frac{1}{2} - \frac{i\sqrt{3}}{2} = \omega^2$.

$\therefore$ $(x + y\omega_3 + z\omega_3^2)(x + y\omega_3^2 + z\omega_3)$

$= (x + y\omega + z\omega^2)(x + y\omega^2 + z\omega)$

$= x^2 + y^2 + z^2 - xy - yz - zx$.

4. (d) $\sum_{k=1}^{6}\left(\sin\frac{2k\pi}{7} - i\cos\frac{2k\pi}{7}\right)$

$= \sum_{k=1}^{6} -i\left(\cos\frac{2k\pi}{7} + i\sin\frac{2k\pi}{7}\right) = -i\left\{\sum_{k=1}^{6} e^{\frac{i2k\pi}{7}}\right\}$

$= -i\{e^{i2\pi/7} + e^{i4\pi/7} + e^{i6\pi/7} + e^{i8\pi/7} + e^{i10\pi/7} + e^{i12\pi/7}\}$

$= -i\left\{e^{i2\pi/7}\frac{(1 - e^{i12\pi/7})}{1 - e^{i2\pi/7}}\right\} = -i\left\{\frac{e^{i2\pi/7} - e^{i14\pi/7}}{1 - e^{i2\pi/7}}\right\}$

$(\because e^{i14\pi/7} = 1)$ $= -i\left\{\frac{e^{i2\pi/7} - 1}{1 - e^{i2\pi/7}}\right\} = i$

5. (b) It is understandable.

6. (d) $T_n = \frac{\Sigma n}{n!} = \frac{n(n+1)}{2(n)!}$

$= \frac{1}{2}\left[\frac{(n+1)}{(n-1)!}\right] = \frac{1}{2}\left[\frac{n-1}{(n-1)!} + \frac{2}{(n-1)!}\right]$

$= \frac{1}{2}\left[\frac{1}{(n-2)!} + \frac{2}{(n-1)!}\right] = \frac{(e + 2e)}{2} = \frac{3e}{2}$.

7. (b) The given series reduces to

$\log_e 2 + \log_e\left(\frac{3}{2}\right) + \log_e\left(\frac{4}{3}\right) + \ldots + \log_e\left(\frac{n}{n-1}\right)$

$= \log_e 2 + \log_e 3 - \log_e 2 + \log_e 4 - \log_e 3 + \ldots$

$+ \log_e(n) - \log_e(n-1) = \log_e n$.

8. (c) $x.\frac{1}{\sqrt{2}}.\frac{1}{4} = \frac{3.2}{\sqrt{2}.3} \Rightarrow \frac{x}{4\sqrt{2}} = \sqrt{2} \Rightarrow x = 8$.

9. (a) $\sin\theta + \sin 3\theta + \sin 2\theta = \sin\theta\alpha$

$\Rightarrow$ $2\sin 2\theta\cos\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow$ $\sin 2\theta(2\cos\theta + 1) = \sin\alpha$ . . .(i)

Now, $\cos\theta + \cos 3\theta + \cos 2\theta = \cos\alpha$

$2\cos 2\theta\cos\theta + \cos 2\theta = \cos\alpha$

$\Rightarrow$ $\cos 2\theta(2\cos\theta + 1) = \cos\alpha$ . . .(ii)

From (i) and (ii),

$\Rightarrow$ $\tan 2\theta = \tan\alpha \Rightarrow 2\theta = \alpha \Rightarrow \theta = \alpha/2$.

10. (b) Imaginary part of $\left[\sin^{-1}\left(\frac{5\sqrt{7}}{16} - \frac{9i}{16}\right)\right]$

$= -\log\left[\sqrt{\frac{9}{16}} + \sqrt{1 + \frac{9}{16}}\right] = -\log(2)$.

11. (a) For continuous $\lim_{x\to 2} f(x) = f(2) = k$

$\Rightarrow$ $k = \lim_{x\to 2}\frac{x^3 + x^2 - 16x + 20}{(x-2)^2}$

$= \lim_{x\to 2}\frac{(x^2 - 4x + 4)(x + 5)}{(x-2)^2} = 7$.

12. (a) $y = e^{x+y} \Rightarrow \log y = (x + y)\log e$

$\Rightarrow$ $\frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{y}{1-y}$.

13. (a) Let $y = \sin^p x\cos^q x$

$\frac{dy}{dx} = p\sin^{p-1}x\cos x\cos^q x + q\cos^{q-1}x(-\sin x)\sin^p x$

$\frac{dy}{dx} = p\sin^{p-1}x\cos^{q+1}x - q\cos^{q-1}x\sin^{p+1}x$

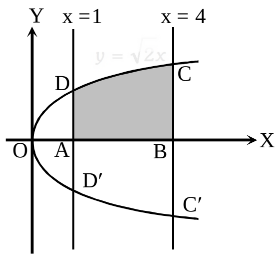
Put $\frac{dy}{dx}=0$, $\therefore \tan^2 x=\frac{p}{q}$ $\Rightarrow$ $\tan x=\pm\sqrt{\frac{p}{q}}$

$\therefore$ Point of maxima $x=\tan^{-1}\sqrt{\frac{p}{q}}$.

14. (b) Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$, then

$$\int \frac{1}{x(\log x)^2}dx=\int \frac{1}{t^2}dt=-\frac{1}{t}+c=-\frac{1}{\log x}+c.$$

15. (b)



Required area $=CDD'C'=2\times ABCD$

$$=2\int_1^4 \sqrt{2}\,x^{1/2}\,dx=\frac{28\sqrt{2}}{3}\text{ sq. unit.}$$

16. (b) $\frac{dy}{dx}+\frac{1+x^2}{x}=0 \Rightarrow dy+\left(\frac{1}{x}+x\right)dx=0$

On integrating, we get $y+\log x+\frac{x^2}{2}+c=0$.

17. (d) $\cos y\log(\sec x+\tan x)dx=\cos x\log(\sec y+\tan y)dy$

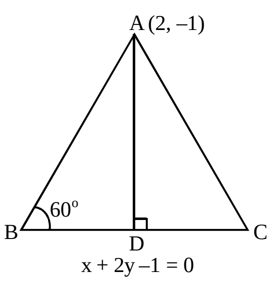
$\Rightarrow \int \sec y\log(\sec y+\tan y)dy$

$=\int \sec x\log(\sec x+\tan x)dx$

Put $\log(\sec x+\tan x)=t$ and $\log(\sec y+\tan y)=z$

$$\frac{[\log(\sec x+\tan x)]^2}{2}=\frac{[\log(\sec y+\tan y)]^2}{2}+c.$$

18. (b)



$$|AD|=\left|\frac{2-2-1}{\sqrt{1^2+2^2}}\right|=\frac{1}{\sqrt{5}}$$

$\Rightarrow$ $\tan 60^\circ=\frac{AD}{BD}$ $\quad\Rightarrow \sqrt{3}=\frac{1/\sqrt{5}}{BD}$

$\Rightarrow$ $BD=\frac{1}{\sqrt{15}}$ $\quad\Rightarrow BC=2BD=2/\sqrt{15}$.

19. (a) It represents a circle, if $a=b$

$\Rightarrow \frac{k}{3}=\frac{1}{4} \Rightarrow k=\frac{3}{4}$.

20. (d) Tangent to the curve $y^2=8x$ is $y=mx+2/m$.

So, it must satisfy $xy=-1$.

$\Rightarrow x\left(mx+\frac{2}{m}\right)=-1 \Rightarrow mx^2+\frac{2}{m}x+1=0$

Since, it has equal roots. $\therefore$ $D=0$

$\Rightarrow \frac{4}{m^2}-4m=0 \Rightarrow m^3=1$

$\Rightarrow$ $m=1$ Hence, equation of common tangent is $y=x+2$.

21. (a) Given series is $3+4\frac{1}{2}+6\frac{3}{4}+\dots\dots = 3+\frac{9}{2}+\frac{27}{4}+\dots$

$=3+\frac{3^2}{2}+\frac{3^3}{4}+\frac{3^4}{8}+\frac{3^5}{16}+\dots$ (in G.P.)

Here $a=3, r=\frac{3}{2}$, then sum of the five terms

$$S_5=\frac{a(r^n-1)}{r-1}=\frac{3\left[\left(\frac{3}{2}\right)^5-1\right]}{\frac{3}{2}-1}=\frac{1\left[\frac{3^5}{32}-1\right]}{\frac{1}{2}}$$

$$=6\left[\frac{243-32}{32}\right]=\frac{211\times 3}{16}=\frac{633}{16}=39\frac{9}{16}.$$

22. (d) $(1+x-2x^2)^6=1+a_1x+a_2x^2+\dots+a_{12}x^{12}$.

Putting $x=1$ and $x=-1$ and adding the results

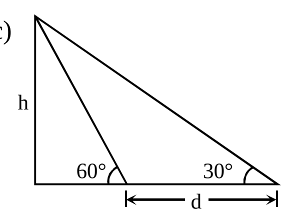
$64=2(1+a_2+a_4+\dots)$ $\therefore$ $a_2+a_4+a_6+\dots+a_{12}=31$.

23. (b) $2^{10}-1=1023$, corresponds to none of the lamps is being switched ON.

24. (a) The event that the fifth toss results in a head is independent the event that the first four tosses result in tails.

$\therefore$ Probability of the required event $=\frac{1}{2}$

25. (c)



$d=h\cot 30^\circ-h\cot 60^\circ$ and time = 3 min.

$\therefore$ Speed $=\frac{h(\cot 30^\circ-\cot 60^\circ)}{3}$ per minute

It will travel distance $h\cot 60^\circ$ in

$$\frac{h\cot 60^\circ\times 3}{h(\cot 30^\circ-\cot 60^\circ)}=1.5\text{ minute.}$$

26. (b) Variance $= \frac{\sum x_i^2}{n} - \left(\bar{x}\right)^2$

⇒ $\sigma^2 \left(\frac{2^2+4^2+6^2+...+100^2}{50}\right) - \left(\frac{2+4+...+100}{50}\right)^2$

⇒ $\sigma^2 = 3434 - 2601 = 833$

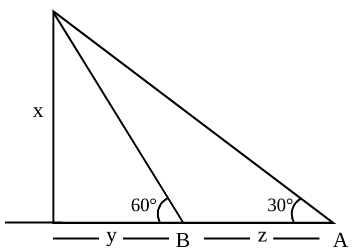
27. (b) Exp. $= \frac{\tan^2 A}{\tan A - 1} + \frac{1}{\tan A - \tan^2 A} = \frac{1}{\tan A - 1}\left[\tan^2 A - \frac{1}{\tan A}\right]$

$= \frac{\tan^2 A + \tan A + 1}{\tan A} = \tan A + \cot A + 1 = \sec A . \operatorname{cosec} A + 1$

28. (b) $\cot\left(\sum_{n=1}^{23}\cot^{-1}(n^2+n+1)\right)$ $\cot\left(\sum_{n=1}^{23}\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)\right)$

⇒ $\cot\left(\tan^{-1}\left(\frac{23}{325}\right)\right) = \frac{25}{23}$.

29. (d) $\tan 30° = \frac{x}{y+z} = \frac{1}{\sqrt{3}}$



⇒ $\sqrt{3}x = y + z$

⇒ $\tan 60° = \frac{x}{y} = \sqrt{3}$

⇒ $x = \sqrt{3}y = y + z$

$3y = y + z$

⇒ $2y = z$

For 2y distance time = 10 min.

So For y distance time = 5 min.

30. (d) $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$

$= \lim_{x\to 0}\frac{(2\sin^2 x)(3+\cos x)}{x\left(\frac{\tan 4x}{4x}\right)\times 4x}$

$= \lim_{x\to 0}\frac{2\sin^2 x(3+\cos x)}{4x^2} = \frac{2}{4}(3+1) = 2$

31. (c) $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$

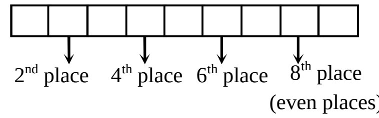
$= 2\tan^{-1}e^u - \tan^{-1}e^u - \cot^{-1}e^u = \tan^{-1}e^u - \cot^{-1}e^u$

$g(-x) = -g(x)$

⇒ $g(x)$ is odd and $g'(x) > 0$

⇒ increasing.

32. (d)



Number of such numbers $= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{2!4!} = 180$

33. (d) The number of four -digit numbers Starting with 5 is equal to $6^3 = 216$

Starting with 44 and 55 is equal to 36 × 2 = 72

Starting with 433, 434 and 435 is equal to 6 × 3 = 18

Remaining numbers are 4322, 4323, 4324, 4325 is equal to 4

So total numbers are 216 + 72 + 18 + 4 = 310

34. (d) $\frac{2^{403}}{15} = \frac{23(2^4)^{100}}{15} = \frac{8}{15}(15+1)^{100}$

$= \frac{8}{15}(15\lambda + 1) = 8\lambda + \frac{8}{15}$

∵ 8λ is integer

fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15} \Rightarrow k = 8$

35. (b)

a_1	a_2	a_3

Number of numbers = $5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Number of numbers = 2×5^3

Required number = $5^3 + 2 \times 5^3 - 1 = 374$

36. (d) $\sum_{r=2}^{13}(7r+2) = 7 \cdot \frac{2+13}{2} \times 6 + 2 \times 12$

$= 7 \times 90 + 24 = 654$

$\sum_{r=2}^{13}(7r+5) = 7\left(\frac{1+13}{2}\right) \times 13 + 5 \times 13 = 702$

Total 654 + 702 = 1356

37. (c) $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$

⇒ $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i}\right)^3 = \frac{k}{21}$

⇒ $\sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{k}{21}$

⇒ $\frac{1}{(21)^3}\left[\frac{20(21)}{2}\right]^2 = \frac{k}{21}$

⇒ 100 = k

38. **(a)** $f(k) = 3m$ (3,6,9,12,15,18)
for $k = 4,8,12,16,20$
6.5.4.3.2 ways
For rest numbers 15! ways
Total ways = (15)! × 6!

39. **(d)** No. of ways $= 10C_3 = 120$

40. **(c)** A→ 1; B→ 2; C→ 3; D→ 4

(A) z_k is 10^{th} root of unity $\Rightarrow \overline{z}_k$ will also be 10^{th} root of unity. Take z_j as $\overline{z}_k$.

(B) $z_1 \neq 0$ take $z = \frac{z_k}{z_1}$, we can always find z.

(C) $z^{10} - 1 = (z - _1)(z - z_1)...(z - z_9)$

$\Rightarrow$ $(z - z_1)(z - z_2)...(z - z_9) = 1 + z + z^2 + ... + z^9 \forall z \in$ complex number.

Put $z = 1$

$(1 - z_1)(1 - z_2)...(1 - z_9) = 10.$

(D) $1 + z_1 + z_2 + ... + z_9 = 0$

$\Rightarrow$ $\text{Re}(1) + \text{Re}(z_1) + ... + \text{Re}(z_9) = 0$

$\Rightarrow$ $\text{Re}(z_1) + \text{Re}(z_2) + ... + \text{Re}(z_9) = -1.$

$\Rightarrow$ $1 - \sum_{k=1}^{9} \cos\frac{2k\pi}{10} = 2.$

41. **(a)** A→ 1; B→ 2; C→ 1; D→ 4

(A) If $a = 1, b = 0$ then $\sin^{-1} x + \cos^{-1} y = 0$

$\Rightarrow$ $\sin^{-1} x = -\cos^{-1} y \Rightarrow x^2 + y^2 = 1.$

(B) If $a = 1, b = 1$ then $\sin^{-1} x + \cos^{-1} y + \cos^{-1} xy = \frac{\pi}{2}$

$\Rightarrow$ $\cos^{-1} x - \cos^{-1} y = \cos^{-1} xy$

$\Rightarrow$ $xy + \sqrt{1 - x^2}\sqrt{1 - y^2} = xy$ (taking sine on both the sides)

(C) If $a = 1, b = 2$ then $\sin^{-1} x + \cos^{-1} y + \cos^{-1}(2xy) = \frac{\pi}{2}$

$\Rightarrow$ $\sin^{-1} x + \cos^{-1} y = \sin^{-1}(2xy)$

$\Rightarrow$ $xy + \sqrt{1 - x^2}\sqrt{1 - y^2} = 2xy \Rightarrow x^2 + y^2 = 1$ (on squaring)

(D) If $a = 2, b = 2$ then $\sin^{-1}(2x) + \cos^{-1}(y) + \cos^{-1}(2xy) = \frac{\pi}{2}$

$\Rightarrow$ $2xy + \sqrt{1 - 4x^2}\sqrt{1 - y^2} = 2xy \Rightarrow (4x^2 - 1)(y^2 - 1) = 0$

42. **(a)** A→3; B→2,4; C→3,4; D→1,3

(A) $y = \frac{x^2 + 2x + 4}{x + 2}$

$\Rightarrow$ $x^2 + (2 - y)x + 4 - 2y = 0$

$\Rightarrow$ $y^2 + 4y - 12 \geq 0$ $y \leq -6$ or $y \geq 2$

Minimum value is 2.

(B) $(A + B)(A - B) = (A - B)(A + B)$

$\Rightarrow$ $AB = BA$ as A is symmetric and B is skew symmetric

$\Rightarrow$ $(AB)^t = -AB$ $\Rightarrow$ $k = 1$ and $k = 3$

(C) $a = \log_3 \log_3 2 \Rightarrow 3^{-a} = \log_2 3$

Now $1 < 2^{-k + \log_2^3} < 2$

$\Rightarrow$ $1 < 3.2^{-k} < 2$

$\Rightarrow$ $\log_2\left(\frac{3}{2}\right) < k < \log_2(3)$

$\Rightarrow$ $k = 1$ or $k < 2$ and $k < 3$.

(D) $\sin \quad = \cos \quad \Rightarrow \cos\left(\frac{\quad}{2} - \quad\right) = \cos$

$\frac{\quad}{2} \qquad 2n$

$-\left(\quad \pm \quad \frac{\quad}{2}\right) = -2n$ $\Rightarrow$ 0 and 2 are possible.

43. **(b)** A→1,2,3; B→1,4; C→3,4; D→1,2

(A) $x|x|$ is continuous, differentiable and strictly increasing in $(-1, 1)$.

(B) $\sqrt{|x|}$ is continuous in $(-1, 1)$ and not differentiable at $x = 0$.

(C) $x + [x]$ is strictly increasing in $(-1, 1)$ and discontinuous at $x = 0$

$\Rightarrow$ not differentiable at $x = 0$.

(D) $|x - 1| + |x + 1| = 2$ in $(-1, 1)$.

$\Rightarrow$ the function is continuous and differentiable in $(-1, 1)$.

44. **(d)** $X \quad \sin \quad \sin 3 \quad ... \quad \sin 29$

$2(\sin \quad)X \quad 1 \quad \cos 2 \quad \cos 2 \quad \cos 4 \quad ... \quad \cos 28 \quad \cos 30$

$\Rightarrow$ $X \quad \frac{1 \quad \cos 30}{2\sin} \quad \frac{1}{4\sin 2}$

45. **(d)** $AA^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9I$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a + 4 + 2b \\ 0 & 9 & 2a + 2 - 2b \\ a + 4 + 2b & 2a + 2 - 2b & a^2 + 4 + b^2 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Equation $a + 4 + 2b = 0$

$\Rightarrow$ $a + 2b = -4$ $\quad ...(i)$

and $2a+2-2b=0$

$\Rightarrow$ $2a-2b=-2$ . . .(ii)

$a^2+4+b^2=0$

$\Rightarrow$ $a^2+b^2=5$ . . .(iii)

Solving $a=-2, b=-1$

46. (c) $0.7+0.77+0.777+.....+0.777...7$

$$=\frac{7}{9}[0.9+0.99+0.999+...+0.999...9]$$

$$=\frac{7}{9}[(1-0.1)+(1-0.01)+(1-0.001...1)+...+(1-0.000...1)]$$

$$=\frac{7}{9}\left[20-\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+...+\frac{1}{10^{20}}\right)\right]$$

$$=\frac{7}{9}\left[20-\frac{1}{10}\cdot\frac{1-\frac{1}{10^{20}}}{1-\frac{1}{10}}\right]=\frac{7}{9}\left[20-\frac{1}{9}\cdot\left(\frac{10^{20}-1}{10^{20}}\right)\right]$$

$$\frac{7}{81}\left[180-\left(1-\frac{1}{10^{20}}\right)\right]=\frac{7}{81}[179+10^{-20}]$$

47. (c) $2x_1+3x_2+4x_3=11$

Possibilities are (0, 1, 2); (1, 3, 0); (2, 1, 1); (4, 1, 0).

$\therefore$ Required coefficients

$$=({}^4C_0\times{}^7C_1\times{}^{12}C_2)+({}^4C_1\times{}^7C_3\times{}^{12}C_0)+({}^4C_2\times{}^7C_1\times{}^{12}C_1)+({}^4C_4\times{}^7C_1\times 1)$$

$$=(1\times7\times66)+(4\times35\times1)+(6\times7\times12)+(1\times7)$$

$$=462+140+504+7=1113.$$

48. (c) Coefficient of x^{10} in $(x+x^2+x^3)^7$

Coefficient of x^3 in $(1+x+x^2)^7$

Coefficient of x^3 in $(1-x^3)^7(1-x)^{-7}$ $={}^{7+3+7}C_3-7$

$$={}^9C_3-7=\frac{9\times8\times7}{6}-7=77$$

49. (d) $\frac{\Sigma x_i}{16}=16 \Rightarrow \Sigma x_i=256$

$$\frac{(\Sigma x_i)-16+3+4+5}{18}=\frac{252}{18}=14$$

50. (c) Let m-men, 2-women

${}^mC_2\times{}^2C_2={}^mC_1\,{}^2C_1\cdot2+84$

$m^2-5m-84=0$

$\Rightarrow$ $(m-12)(m+7)=0$

$\Rightarrow$ $m=12$

❑❑❑