# Congruence of Triangles

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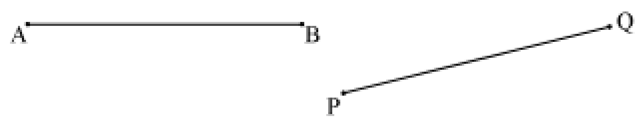
**Difference Between Similarity and Congruence**

Similar figures and congruent figures may appear to be closely related concepts, but there is an important difference between them.

**Congruency of line segments:**

"***Two line segments are congruent to each other if their lengths are equal".***

Consider the following line segments.




Here, the line segments AB and PQ will be congruent to each other, if they are of equal length.

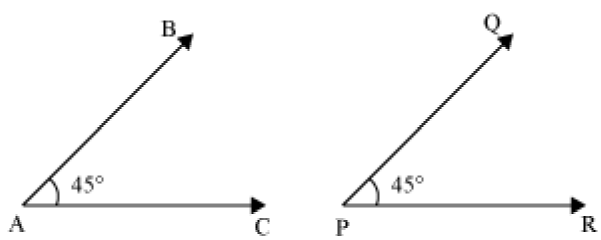
Conversely, we can say that, ***"Two line segments are of equal length if they are congruent to each other".***

i.e. if $\overline{AB} \cong \overline{PQ}$, then AB = PQ.

**Congruency of angles:**

"***Two angles are said to be congruent to each other if they have the same measure".***

The angles shown in the following figures are congruent to each other as both the angles are of the same measure 45°.



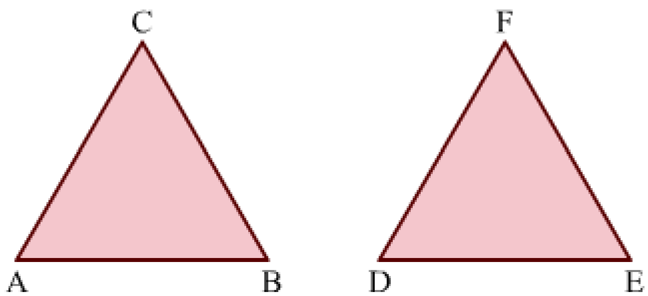

Thus, we can write ∠BAC ≅ ∠QPR.

Its converse is also true.

"***If two angles are congruent to each other, then their measures are also equal".***

There is one special thing about congruent figures that their corresponding parts are always equal.

For example, if two triangles are congruent then their corresponding sides will be equal. Also, their corresponding angles will be equal.
Look at the following triangles.



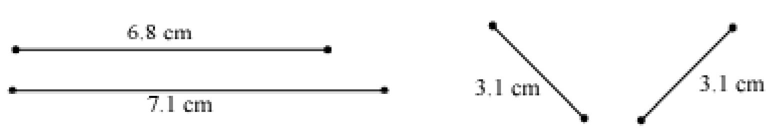

Here, △ABC ≅ △DEF under the correspondence △ABC ↔ △DEF. This correspondence rule represents that in given triangles, AB ↔ DE (AB corresponds to DE), BC ↔ EF, CA ↔ FD, ∠A ↔ ∠D, ∠B ↔ ∠E, ∠C ↔ ∠F. These are **corresponding parts of congruent triangles** (CPCT), △ABC and △DEF.
Since △ABC and △DEF are congruent, their corresponding parts are equal.
Therefore, AB = DE, BC = EF, CA = FD
And, ∠A = ∠D, ∠B = ∠E, ∠C = ∠F

Similarly, we can apply the method of CPCT on other congruent triangles also.

Let us now try and apply what we have just learnt in some examples.

**Example 1:**

**Find which of the pairs of line segments are congruent.**



(i) (ii)

**Solution:**

**(i)** Lengths of the two line segments are not same. Therefore, they are not congruent.

**(ii)** Each of the line segments is of length 3.1 cm, i.e. they are equal. Therefore, they are congruent.

**Example 2:**

**If $\overline{AB} \cong \overline{PQ}$ and $\overline{PQ}$ = 9 cm, then find the length of $\overline{AB}$.**

**Solution:**

Since $\overline{AB} \cong \overline{PQ}$, i.e. line segment AB is congruent to line segment PQ, therefore, $\overline{AB}$ and $\overline{PQ}$ are of equal length.

∴ $\overline{AB}$ = 9 cm

**Example 3:**

**If ∠ABC ≅∠PQR and ∠PQR = $75^o$, then find the measure of ∠ABC.**

**Solution:**

If two angles are congruent, then their measures are equal.

Since ∠ABC ≅ ∠PQR,

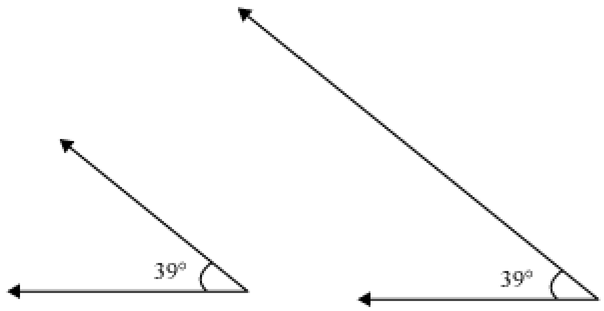
∴ ∠ABC = ∠PQR

Therefore, ∠ABC = $75^o$

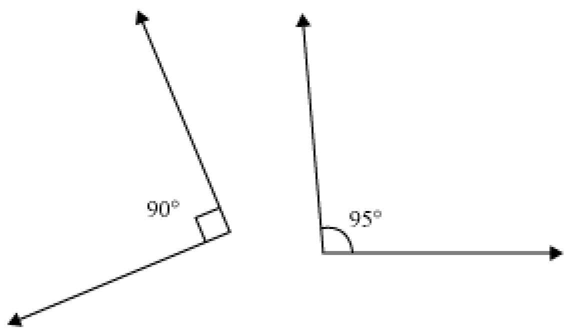
**Example 4:**

**Which of the following pairs of angles are congruent?**

**(i)**




**(ii)**




**Solution:**

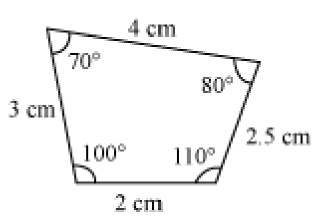
**(i)** The measure of both the angles is the same. Therefore, they are congruent**.**

**(ii)** The measures of the two angles are different. Therefore, they are not congruent.

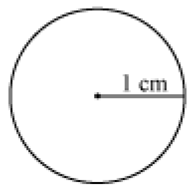
**Example 5:**

**Identify the pairs of similar and congruent figures from the following.**

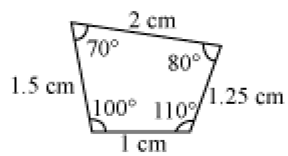
**(i)**



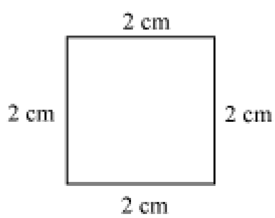

**(ii)**



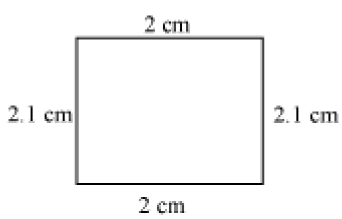

**(iii)**



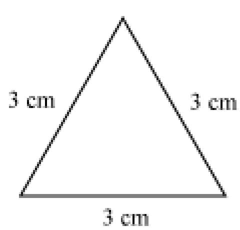

**(iv)**



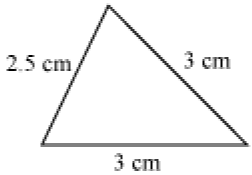

**(v)**



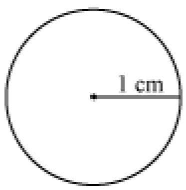

**(vi)**




**(vii)**




**(viii)**




**Solution:**

Figures **(i)** and **(iii)** are similar because their corresponding angles are equal and their corresponding sides are in the same ratio. However, these figures are not congruent as they are of different sizes.

Figures **(ii)** and **(viii)** are congruent as they are of the same shape and size (circles with radius 1 cm each).

**Example 6:**

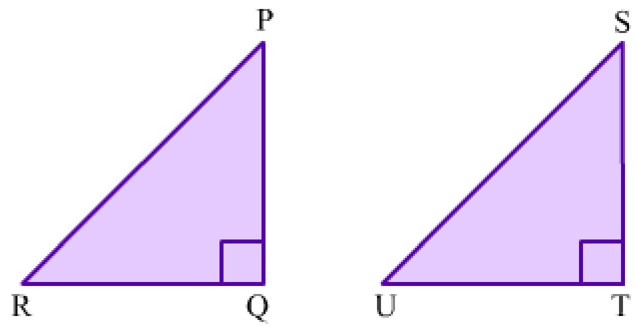
**Are the following figures similar or congruent?**

**Solution:**

The two given figures show two one-rupee coins. As both the figures represent the same coin in two different sizes, they are similar to each other. However, the pictures are not congruent because of their different sizes.

**Example 7:**

**In the following figure, ΔPQR and ΔSTU are congruent.**




**If PQ = 8 cm, QR = 6 cm then find the perimeter of ΔSTU.**

**Solution:**

In ΔPQR, we have

PQ = 8 cm, QR = 6 cm and ∠Q = 90°

Applying Pythagoras theorem in ΔPQR, we obtain

$RP^2 = PQ^2 + QR^2$

$\Rightarrow RP^2 = 8^2 + 6^2$

$\Rightarrow RP^2 = 64 + 36$

$\Rightarrow RP^2 = 100$

⇒ RP = 10 cm

Since ΔPQR and ΔSTU are congruent, their corresponding parts will be equal.

Therefore,

PQ = 8 cm = ST (CPCT)

QR = 6 cm = TU and (CPCT)

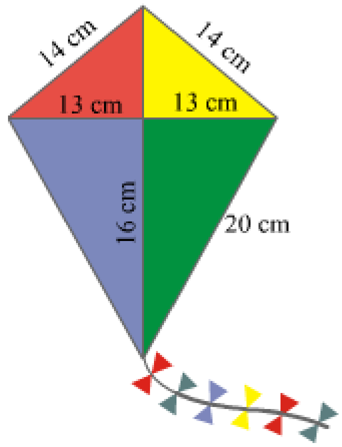
RP = 10 cm = US (CPCT)

∴ Perimeter of ΔSTU = ST + TU + US = 8 cm + 6 cm + 10 cm = 24 cm

**SSS Congruence Rule**

**Relation between the Congruency of Triangles and Their Sides**

Consider the **kite** shown below.




It can be seen that the red and yellow coloured triangles have equal sides. On the basis of this information, can we say that the two triangles are congruent? Or, to rephrase the question, do the sides of triangles determine the congruency of the triangles? Yes, they do, and this is why we have the SSS (Side-Side-Side) congruence rule.

In this lesson, we will discuss the SSS congruence rule and its proof. We will also crack some problems based on it.

**SSS Congruence Rule**

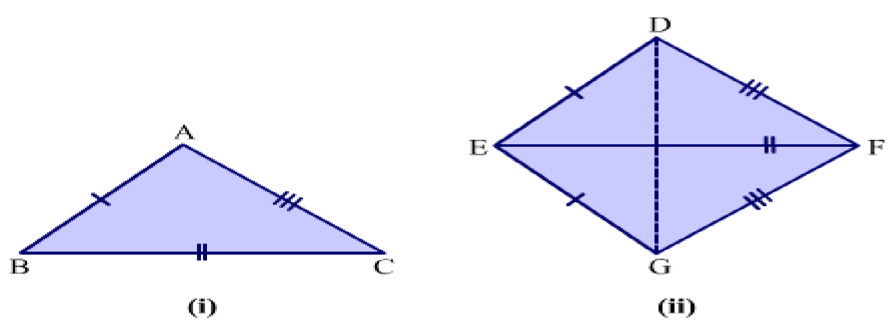
**Proving the SSS Congruence Rule**

**Statement**: Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.

**Given:** ΔABC and ΔDEF such that AB = DE, BC = EF and AC = DF

**To prove**: ΔABC ≅ ΔDEF

***Construction***: Suppose BC and EF are the longest sides of the two triangles. Draw EG such that ∠GEF = ∠ABC and GE = AB. Join point G to points F and D.



**Proof:** In ΔABC and ΔGEF, we have:

BC = EF (Given)

AB = GE (By construction)

∠ABC = ∠GEF (By construction)

So, by the SAS congruence rule, we have:

ΔABC ≅ ΔGEF

⇒ ∠BAC = ∠EGF and AC = GF (By CPCT)

Now, AB = DE and AB = GE

⇒ DE = GE … (1)

Similarly, AC = DF and AC = GF

⇒ DF = GF … (2)

In ΔDEG, we have:

DE = GE (From equation 1)

⇒ ∠EDG = ∠EGD … (3)

In ΔDFG, we have:

DF = GF (From equation 2)

⇒ ∠FDG = ∠FGD… (4)

On adding equations 3 and 4, we get:

∠EDG + ∠FDG = ∠EGD + ∠FGD

⇒ ∠EDF = ∠EGF

We know that ∠EGF = ∠BAC (Proved above)

∴ ∠BAC = ∠EDF … (5)

Thus, in ΔABC and ΔDEF, we have:

AB = DE (Given)

∠BAC = ∠EDF (From equation 5)

AC = DF (Given)

So, by the SAS congruence rule, we obtain:

ΔABC ≅ ΔDEF

Hence, the SSS congruence rule holds true.

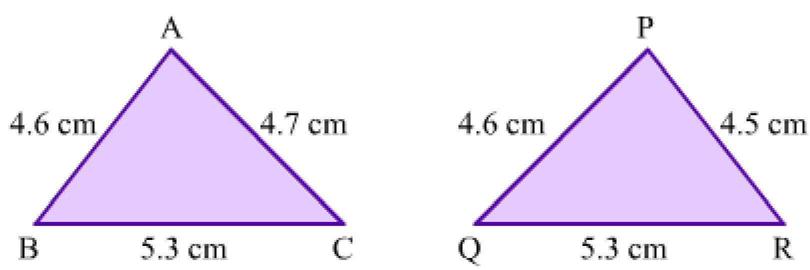
**Applying the SSS Congruence Rule**

**Solved Examples**

**Easy**

**Example 1:**

Are the following triangles congruent?




**Solution:**

In ΔABC and ΔPQR, we have:

AB = PQ = 4.6 cm

BC = QR = 5.3 cm

But AC ≠ PR

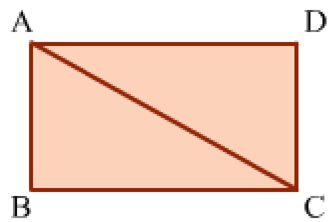
Therefore, ΔABC and ΔPQR are not congruent.

**Example 2**:

ABCD is a rectangle with AC as one of its diagonals. Prove that the triangles formed on the two sides of diagonal AC are congruent.

**Solution:**

The required rectangle ABCD with AC as its diagonal can be drawn as is shown.




In ΔABC and ΔCDA, we have:

AB = CD  (∵ Opposite sides of a rectangle are equal)

BC = DA  (∵ Opposite sides of a rectangle are equal)

CA = AC  (Common side)

Therefore, by the SSS congruence rule, we have:

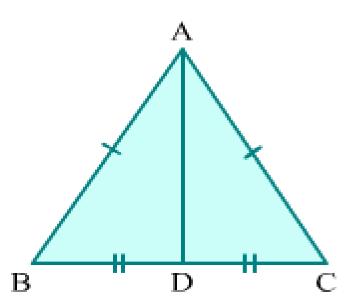
ΔABC ≅ ΔCDA

Thus, the triangles formed on the two sides of diagonal AC are congruent.

**Medium**

**Example 1**:

The given ΔABC is isosceles with AB = AC. AD is a median of the triangle. Prove that AD is perpendicular to BC.



**Solution:**

In ΔABD and ΔACD, we have:

AB = AC  (Given)

BD = DC  (∵ D is the midpoint of BC)

AD = AD  (Common side)

Therefore, by the SSS congruence rule, we obtain:

ΔABD ≅ ΔACD

⇒ ∠ADB = ∠ADC  (By CPCT)

Also, ∠ADB and ∠ADC form a linear pair.

So, ∠ADB + ∠ADC = 180°

⇒ ∠ADB + ∠ADB = 180°  (∵ ∠ADB = ∠ADC)

⇒ 2∠ADB = 180°

⇒ ∠ADB = 90°

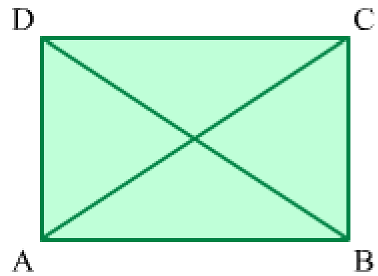
Thus, ∠ADB = ∠ADC = 90°, which means that AD is perpendicular to BC.

**Example 2**:

ABCD is a parallelogram. If the diagonals of ABCD are equal, then find the measure of ∠ABC.

**Solution**:

The given parallelogram ABCD with equal diagonals AC and BD is shown below.



In parallelogram ABCD, we have:

AB = CD and AD = BC (∵ Opposite sides of a parallelogram are equal)

In ΔADB and ΔBCA, we have:

AD = BC (Proved above)

BD = AC (Given)

BA = AB (Common side)

So, by the SSS congruence rule, we have:

ΔADB ≅ ΔBCA

⇒ ∠BAD **=** ∠ABC ... (1) [By CPCT]

Now, AD is parallel to BC and the transversal AB intersects them at A and B respectively.

We know that the sum of the interior angles on the same side of a transversal is supplementary.

∴ ∠BAD + ∠ABC = 180°

⇒ ∠ABC + ∠ABC = 180° (By equation 1)

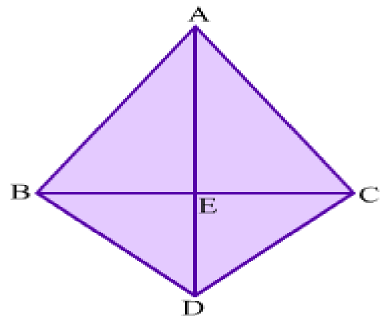
⇒ 2∠ABC = 180°

⇒ ∠ABC = 90°

**Hard**

**Example 1**:

In the given figure, ΔABC and ΔDBC are isosceles with AB = AC and DB = DC. Prove that AD is the perpendicular bisector of BC.




**Solution**:

In ΔABD and ΔACD, we have:

AB = AC  (Given)

DB = DC  (Given)

AD = AD  (Common side)

So, by the SSS congruence rule, we have:

ΔABD ≅ ΔACD

⇒ ∠BAE = ∠CAE  ... (1)    [By CPCT]

In ΔBAE and ΔCAE, we have:

AB = AC  (Given)

∠BAE = ∠CAE  (From equation 1)

AE = AE  (Common side)

So, by the SAS congruence rule, we have:

ΔBAE ≅ ΔCAE

⇒ BE = CE and ∠BEA = ∠CEA  (By CPCT)

We know that ∠BEA + ∠CEA = 180° as they form a linear pair.

So, 2∠BEA = 180°  (Proved above that ∠BEA = ∠CEA)

⇒ ∠BEA = 90°

Therefore, ∠BEA = ∠CEA = 90°

Since BE = CE and ∠BEA = ∠CEA = 90°, AD is the perpendicular bisector of BC.

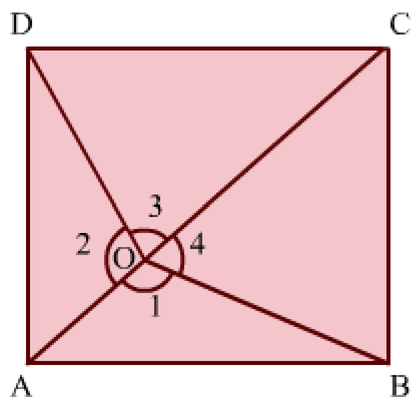
**Example 2**:

O is a point inside a square ABCD such that it is at an equal distance from points B and D. Prove that points A, O and C are collinear.

**Solution**:

The square with the given specifications is drawn as is shown.

***Construction***: Join point O to the vertices of the square.




In ΔAOD and ΔAOB, we have:

AD = AB   (Sides of a square)

AO = AO  (Common side)

OD = OB  (Given)

So, by the SSS congruence rule, we have:

ΔAOD ≅ ΔAOB

⇒ ∠1 = ∠2… (1)   [By CPCT]

Similarly, ΔDOC ≅ ΔBOC

⇒ ∠3 = ∠4 … (2)   [By CPCT]

We know that:

∠1 + ∠2 + ∠ 3 + ∠4 = 360°

⇒ 2∠2 + 2∠3 = 360°  (From equations 1 and 2)

⇒ ∠2 + ∠3 = 180°

Thus, ∠2 and ∠3 form a linear pair. Therefore, AOC is a line; in other words, points A, O and C are collinear.

**SAS Congruence Rule**

**Congruency of Triangles**

**Congruency** of triangles helps us find solutions to many problems in real life. For example, the distance travelled by a ball in a golf course is easy to measure when the ball is on land; however, when the land is separated by a water body like a pond or any other thing, the task of measurement becomes difficult. In such cases, we use the concept of congruency.

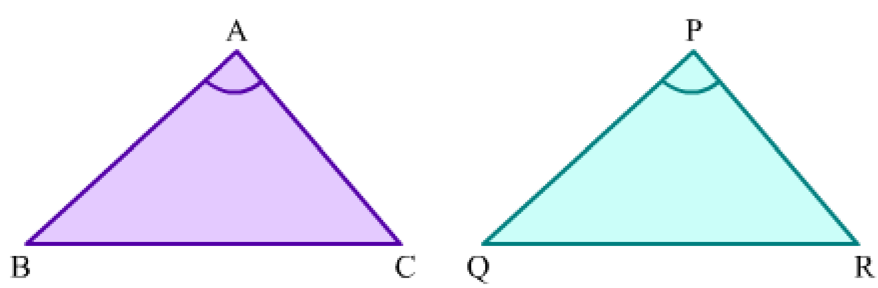
There are certain rules to check the congruency of triangles. One of these is the SAS (Side-Angle-Side) congruence rule. In this lesson, we will learn this rule and its applications.

**SAS Congruence Rule**

Consider a triangle two of whose sides and the included angle are known. We can check for the congruency of this triangle with respect to another triangle if we know the corresponding sides and angle of that triangle. Two triangles can, thus, be termed ‘congruent’ or ‘incongruent’ by using the SAS congruence rule. This rule states that:

***If two sides of a triangle and the angle between them are equal to the corresponding sides and angle of another triangle, then the two triangles are congruent.***

Look at the given ΔABC and ΔPQR.



Let us consider sides AB and AC and the included ∠BAC in ΔABC, and the corresponding sides and angle in ΔPQR, i.e., PQ, PR and ∠QPR.

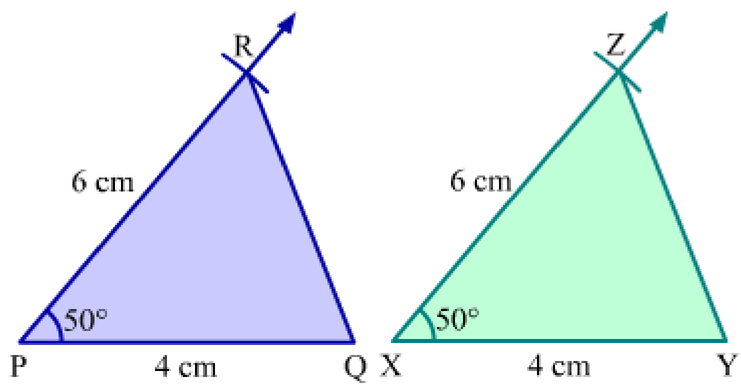
By the SAS congruence rule, the two triangles will be congruent if AB = PQ, AC = PR and ∠BAC = ∠QPR.

Similarly, we can check for congruency by taking other pairs of sides and included angles in these triangles.

**Verification of SAS Congruence Rule**

The SAS congruence rule for triangles is taken as a postulate, so there is no proof for the same but we we can verify it by doing an activity.

The steps of the activity are as follows:




i) Construct ΔPQR such that PQ = 4 cm, PR = 6 cm and ∠QPR = 50°.

ii) Construct ΔXYZ with the same measures such that XY = 4 cm, XZ = 6 cm and ∠YXZ = 50°.

iii) Cut both the triangles along their boundaries.

iv) Try to superpose one triangle by the other. One triangle can be placed on the other in six different ways such that vertex lie on vertex.

v) In one of the the trials, you will get P falling over X, Q falling over Y and R falling over Z. In this case, you will see that both the triangles cover each other exactly.

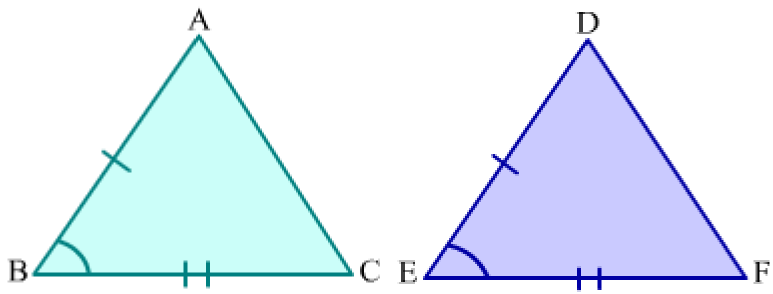
vi) Thus, under the correspondence PQR↔XYZ, the triangles are congruent.

This verifies the SAS congruence rule.

**CPCT**

CPCT stands for 'corresponding parts of congruent triangles'. 'Corresponding parts' means corresponding sides and angles of triangles. According to CPCT, if two or more triangles are congruent to one another, then all of their corresponding parts are equal.

For example, in the given ΔABC and ΔDEF, AB = DE, ∠B = ∠E and BC = EF. So, according to the SAS congruence criterion, we have ΔABC ≅ΔDEF.




Now, by CPCT, we can say that the remaining corresponding parts of the two congruent triangles are also equal. This means that AC = DF, ∠A = ∠D and ∠C = ∠F.

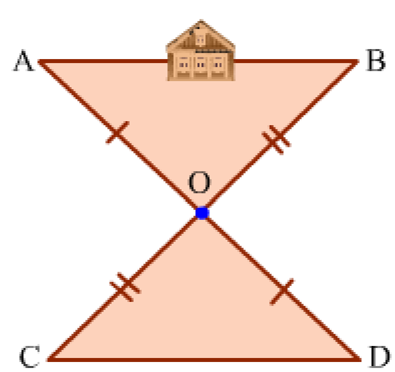
Similarly, we can apply CPCT in case of other congruent triangles.

**Solved Examples**

**Easy**

**Example 1**:

**Observe the following figure.**



Ajay wishes to determine the distance between two objects A and B, but there is a house in between. So, he devises an ingenious way to fix the problem. First, he fixes a pole at any point O so that both A and B are visible from O. He then fixes another pole at point D which is collinear to point O and object A, and is at the same distance from O as A, i.e., DO = AO. Similarly, he fixes a pole at point C which is collinear to point O and object B, and is at the same distance from O as B, i.e., CO = BO. Finally, he measures CD to find the distance between A and B. How can Ajay be sure that CD = AB?

**Solution**:

We have two triangles in the given figure, i.e., ΔAOB and ΔDOC.

In these two triangles, we have:

AO = DO (Given)

∠AOB = ∠DOC (Vertically opposite angles)

BO = CO (Given)

Therefore, by the SAS congruence rule, we can say that:

ΔAOB ≅ ΔDOC

⇒ AB = CD (By CPCT)

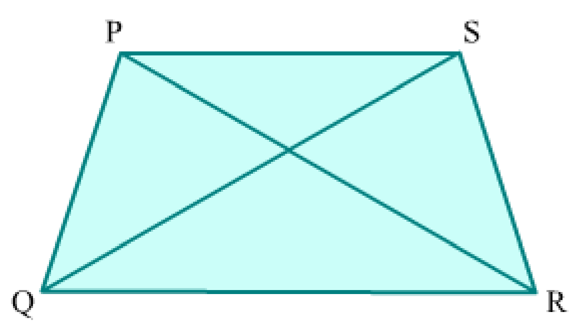
This is the reason why Ajay measures CD to find the distance between objects A and B.

**Example 2**:

In the given quadrilateral PQRS, PR bisects ∠QPS and PQ = PS. Prove that:

i) ΔPQR ≅ΔPSR

ii) QR = SR



**Solution**:

i) In ΔPQR and ΔPSR, we have:

PQ = PS(Given)

PR = PR(Common side)

∠QPR = ∠SPR(&because PR bisects ∠QPS)

So, by the SAS congruence rule, we obtain:

ΔPQR ≅ ΔPSR

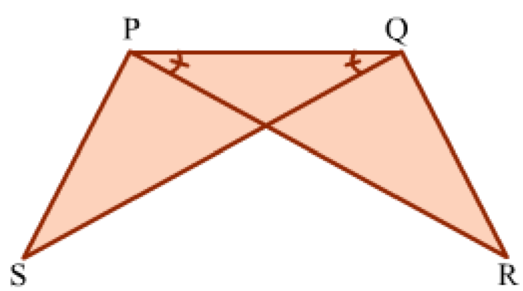
ii) We have proved that ΔPQR ≅ ΔPSR.

∴ QR = SR (&because Corresponding parts of congruent triangles are equal)

**Medium**

**Example 1**:

In the shown figure, PR = QS and ∠QPR = ∠PQS. Prove that ΔPQR ≅ ΔQPS. Also, show that PS = QR and ∠QPS = ∠PQR.




**Solution**:

In ΔPQR and ΔQPS, we have:

PR = QS(Given)

∠QPR = ∠PQS(Given)

PQ = PQ(Common side)

∴ ΔPQR ≅ ΔQPS(By the SAS congruence criterion)

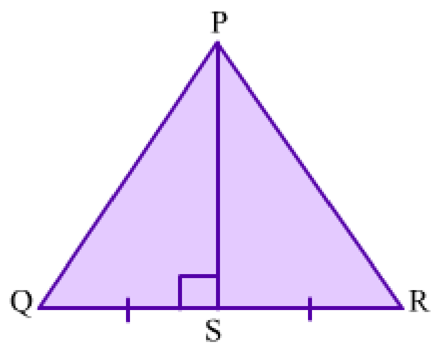
⇒ PS = QR and ∠QPS = ∠PQR (By CPCT)

**Example 2**:

Prove that ΔPQR is isosceles if the altitude drawn from a vertex bisects the opposite side.

**Solution**:

The given figure shows the ΔPQR having PS as an altitude that bisects the opposite side QR.




In ΔPSQ and ΔPSR, we have:

QS = SR (&because Altitude PS bisects QR)

PS = PS(Common side)

∠PSQ = ∠PSR = 90°(&because PS is the altitude to QR)

∴ ΔPSQ ≅ ΔPSR(By the SAS congruence rule)

⇒ PQ = PR(By CPCT)

Therefore, ΔPQR is isosceles.

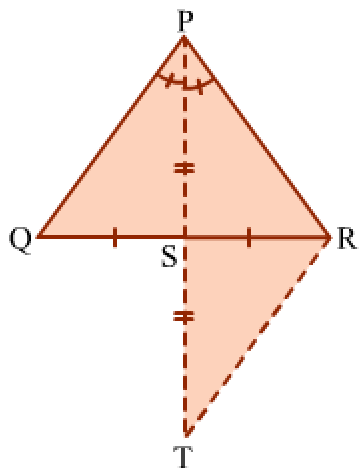
**Example 3**:

If the angle bisector of any angle of a triangle bisects the opposite side then show that the triangle is isosceles.

**Solution**:

Let ΔPQR be the given triangle and PS is the angle bisector of ∠QPR such that it bisects the side QR.

Let us extend the segment PS to point T such that PS = TS.




In ΔPQS and ΔTRS, we have

QS = RS (Given)

∠PSQ = ∠TSR (Vertically opposite angles)

PS = TS (By construction)

So, by the SAS congruence criterion, we have:

ΔPQS ≅ ΔTRS

By CPCT, we obtain

PQ = TR ...(1)

And ∠QPS = ∠RTS ...(2)

But ∠QPS = ∠RPS ...(3) (PS bisects ∠QPR)

∴ ∠RTS = ∠RPS [From (2) and (3)]

⇒ PR = TR (Sides opposite to equal angles)

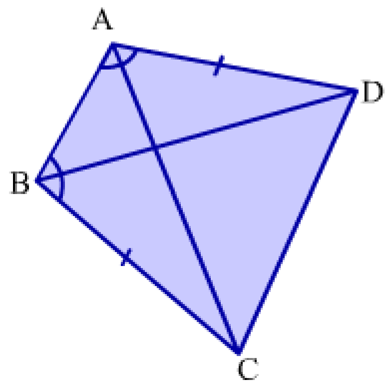
∴ PQ = PR [From (1)]

Thus, ΔPQR is an isosceles triangle.

**Hard**

**Example 1**:

ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA.




Prove that:

i) ΔABD ≅ ΔBAC

ii) BD = AC

iii) ∠ABD = ∠BAC

**Solution**:

i) In ΔABD and ΔBAC, we have:

AD = BC (Given)

∠DAB = ∠CBA (Given)

AB = BA (Common side)

So, by the SAS congruence criterion, we have:

ΔABD ≅ ΔBAC

ii) We have proved that ΔABD ≅ ΔBAC.

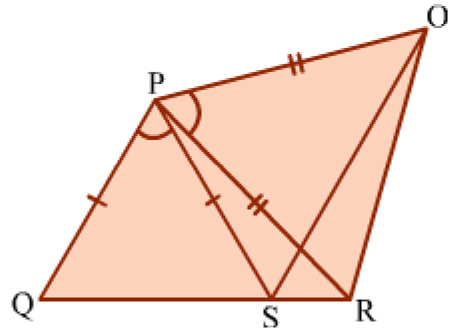
∴ BD = AC (By CPCT)

iii) Since ΔABD ≅ ΔBAC, we have:

∠ABD = ∠BAC(By CPCT)

**Example 2**:

In the given figure, PR = PO, PQ = PS and ∠QPS = ∠OPR. Show that QR = SO.




**Solution**:

It is given that ∠QPS = ∠OPR.

∴ ∠QPS + ∠SPR = ∠OPR + ∠SPR

⇒ ∠QPR = ∠SPO… (1)

In ΔQPR and ΔSPO, we have:

PQ = PS (Given)

∠QPR = ∠SPO (From equation 1)

PR = PO (Given)

So, by the SAS congruence rule, we have:

ΔQPR ≅ ΔSPO

⇒ QR = SO (By CPCT)

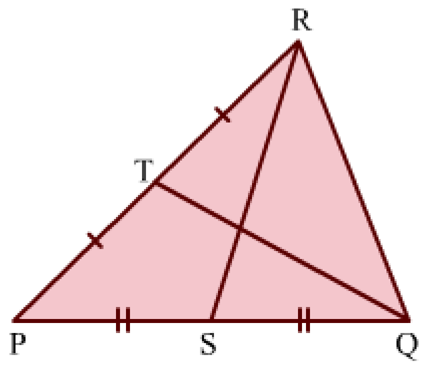
**Example 3**:

In an isosceles triangle, prove that the medians on the equal sides are equal.

**Solution**:

Let ΔPQR be an isosceles triangle such that PQ = PR.

Also, let RS and QT be the medians to the sides PQ and PR respectively.




In ΔPQR, we have

PS = SQ = $\frac{1}{2}PQ$ (RS is the median)

And PT = TR = $\frac{1}{2}PR$ (QT is the median)

But PQ = PR

∴ PS = SQ = PT = TR ...(1)

In ΔPRS and ΔPQT, we have

PQ = PR (Given)

∠RPS = ∠QPT (Common angle)

PS = PT [From (1)]

So, by the SAS congruence rule, we have:

ΔPRS ≅ ΔPQT

∴ RS = QT (By CPCT)

Thus, the medians on the equal sides of an isosceles triangle are equal.

Proving Theorem of Right Angled Triangle

There is a theorem of right angled triangles which states that:

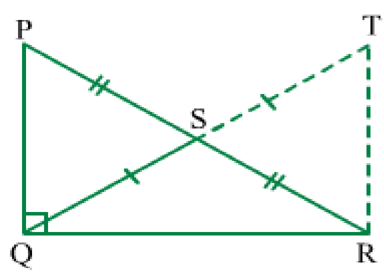
**In a right angled triangle, median drawn to the hypotenuse from the opposite vertex is equal to the half of the hypotenuse.**

Let us prove the theorem.

**Given:** Right angled ΔPQR, ∠PQR = 90° and median QS to hypotenuse PR.

**To prove:** QS = $\frac{1}{2}PR$

**Construction:** Extend QS to T such that QS = ST and join T to R.




**Proof:**

In ΔPSQ and ΔRST, we have

QS = TS (By construction)

∠PSQ = ∠RST (Vertically opposite angles)

PS = RS (Given)

So, by SAS congruence criterion, we have

ΔPSQ ≅ ΔRST

∴ PQ = RT ...(1) (By CPCT)

And ∠QPS = ∠TRS (By CPCT)

Thus, PQ || RT (∠QPS and ∠TRS alternate interior angles formed by transversal PR)

Now, QR is also a transversal to parallel line segments PQ and RT.

∠PQR + ∠TRQ = 180° (Sum of interior angles on the same side of transversal)

But ∠PQR = 90°

∴ ∠TRQ = 90°  (2)

In ΔPQR and ΔTRQ, we have

PQ = RT  [From (1)]

∠PQR = ∠TRQ = 90°

QR = QR  (Common side)

So, by SAS congruence criterion, we have

ΔPQR ≅ ΔTRQ

∴ PR = QT  (By CPCT)

$\Rightarrow \frac{1}{2}PR = \frac{1}{2}QT$

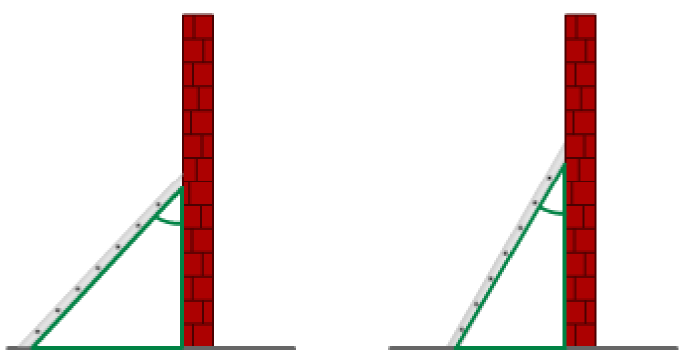
But $\frac{1}{2}QT = QS$

$\therefore QS = \frac{1}{2}PR$

Hence, proved.

**ASA Congruence Rule**

Look at the given figure.



Observe how the ladder, the wall and the horizontal together make triangles in the figure. It can be seen that the angle marked between the ladder and the wall on the left is greater than the same angle marked on the right. Clearly, the triangles are not congruent although it is the same ladder on both sides.

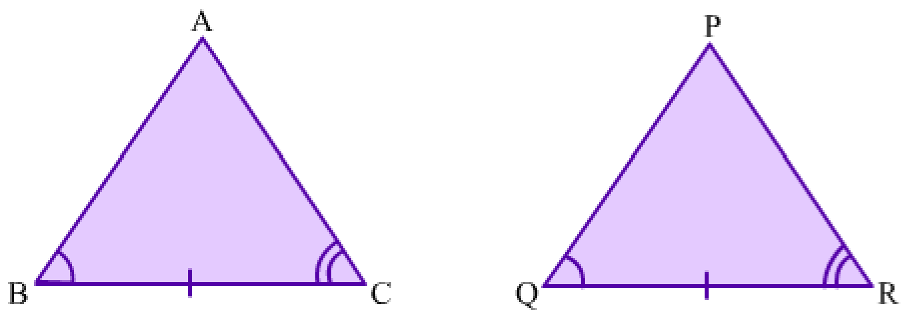
Both angles and sides play an important role in determining the congruency of triangles. In this lesson, we will discuss the ASA (Angle-Side-Angle) congruence rule and solve some problems based on it.

**ASA Congruence Rule**

The ASA congruence rule for triangles states that:

***If two angles of a triangle and the side between them are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.***

Consider the given ΔABC and ΔPQR.




Observe how corresponding components of the two triangles are marked.

Now, by the ASA congruence rule, the two triangles will be congruent if these corresponding components are equal, i.e., if ∠ABC = ∠PQR, BC = QR and ∠ACB = ∠PRQ, then ΔABC ≅ ΔPQR.

Note that, under the above condition of congruence, we cannot write ΔABC ≅ ΔQRP. The order of the vertices matters in any congruency.

**Did You Know?**

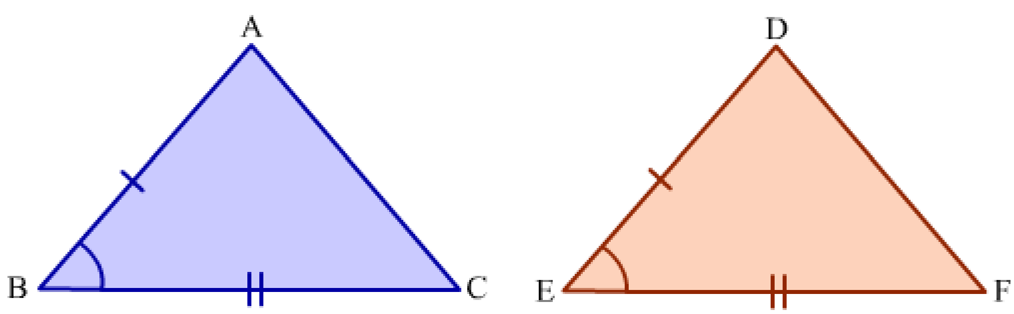
A bright meteor was seen in the sky above Greenland on December 9, 1997. In an attempt to find the fragments of the meteorite, scientists collected data from eyewitnesses who observed the meteor passing through the sky. As is shown in the figure below, the scientists considered sightlines of observers in different towns.


One such sightline was of observers in the town called Paamiut (Point P). Another was of observers in the town called Narsarsuaq (Point N). Using the ASA congruence rule, the scientists were able to gather enough information to successfully locate the fragments of the meteorite (Point M).

**Proof of the ASA Congruence Rule: Case 1**

Let us consider ΔABC and ΔDEF such that ∠ABC = ∠DEF, ∠ACB = ∠DFE and BC = EF. By the ASA congruence rule, ΔABC and ΔDEF are congruent. By CPCT, we have AB = DE.

**Case 1**: Let us prove ΔABC ≅ ΔDEF by taking AB = DE.




In this case, we have

AB = DE (Given)

∠ABC = ∠DEF (Given)

BC = EF  (Given)

So, by the SAS congruence rule, we have:

ΔABC ≅ ΔDEF

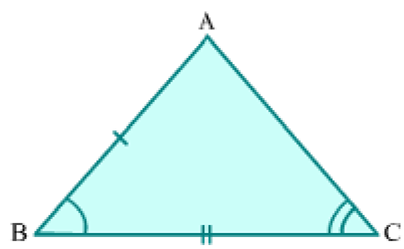
**Proof of the ASA Congruence Rule: Case 2**

Let us consider ΔABC and ΔDEF such that ∠ABC = ∠DEF, ∠ACB = ∠DFE and BC = EF. By the ASA congruence rule, ΔABC and ΔDEF are congruent. By CPCT, we have AB = DE. Let us assume AB ≠ DE.

**Case 2**: Let us prove ΔABC ≅ ΔDEF by taking AB < DE.

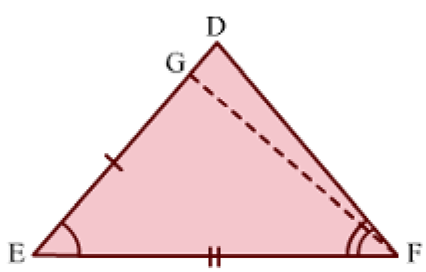
***Construction*****:** Mark a point G on DE such that GE = AB. Join G to F.

Now, in ΔABC and ΔGEF, we have:




AB = GE (By construction)

∠ABC = ∠GEF (∵∠ABC = ∠DEF and ∠DEF = ∠GEF)




BC = EF (Given)

So, by the SAS congruence rule, we obtain:

ΔABC ≅ ΔGEF

⇒ ∠ACB = ∠GFE (By CPCT)

But ∠ACB = ∠DFE  (Given)

∴ ∠GFE = ∠DFE

This can be possible only when line segment GF coincides with line segment DF or point G coincides with point D. Therefore, AB must be equal to DE and ΔGEF must be ΔDEF.

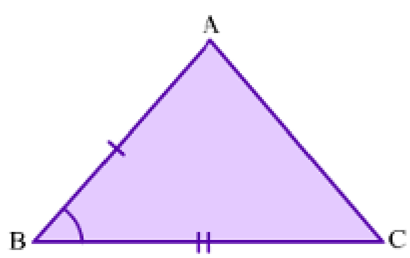
∴ ΔABC ≅ ΔDEF

**Proof of the ASA Congruence Rule: Case 3**

Let us consider ΔABC and ΔDEF such that ∠ABC = ∠DEF, ∠ACB = ∠DFE and BC = EF. By the ASA congruence rule, ΔABC and ΔDEF are congruent. By CPCT, we have AB = DE. Let us assume AB ≠ DE.

**Case 3**: Let us prove ΔABC ≅ ΔDEF by taking AB > DE.

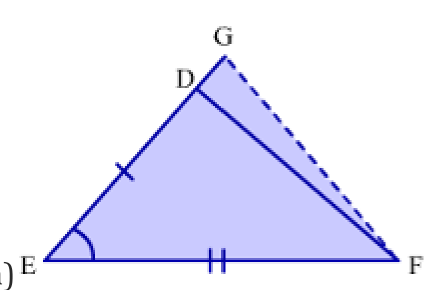
***Construction***: Extend ED to point G such that GE = AB. Join G to F.




Now, in ΔABC and ΔGEF, we have:

AB = GE (By construction)

∠ABC = ∠DEF (∵∠ABC = ∠DEF and ∠DEF = ∠GEF)




BC = EF (Given)

So, by the SAS congruence rule, we obtain:

ΔABC ≅ ΔGEF

⇒ ∠ACB = ∠GFE (By CPCT)

But ∠ACB = ∠DFE (Given)

∴ ∠GFE = ∠DFE

This can be possible only when line segment GF coincides with line segment DF or point G coincides with point D. Therefore, AB must be equal to DE and ΔGEF must be ΔDEF.

∴ ΔABC ≅ ΔDEF

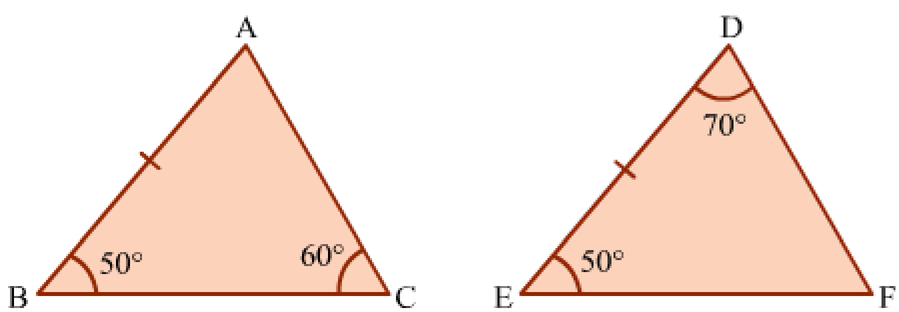
**Applying the ASA Congruence Rule**

**Solved Examples**

**Easy**

**Example 1**:

Check whether the given triangles are congruent or not.




**Solution**:

In ΔABC, we have:

∠ABC + ∠BCA + ∠BAC = 180° (By the angle sum property)

⇒ 50° + 60° + ∠BAC = 180°

⇒ 110° + ∠BAC = 180°

⇒ ∠BAC = 180° − 110°

⇒ ∠BAC = 70°

In ΔABC and ΔDEF, we have:

∠BAC = ∠EDF = 70°

AB = DE (Given)

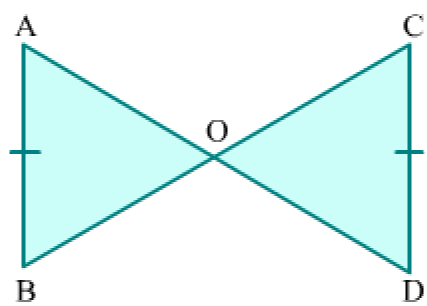
∠ABC = ∠DEF = 50°

Therefore, by the ASA congruence rule, we have:

ΔABC ≅ ΔDEF

**Example 2**:

In the given figure, AB and CD are two equal and parallel lines. Prove that ΔABO ≅ ΔCDO.




**Solution**:

It is given that AB||CD. AD and BC are transversals lying on lines AB and CD.

So, by the alternate angles axiom, we obtain:

∠OAB = ∠ODC… (1)

∠OBA = ∠OCD… (2)

In ΔABO and ΔCDO, we have:

∠OAB = ∠ODC   (By equation 1)

AB = CD(Given)

∠OBA = ∠OCD   (By equation 2)

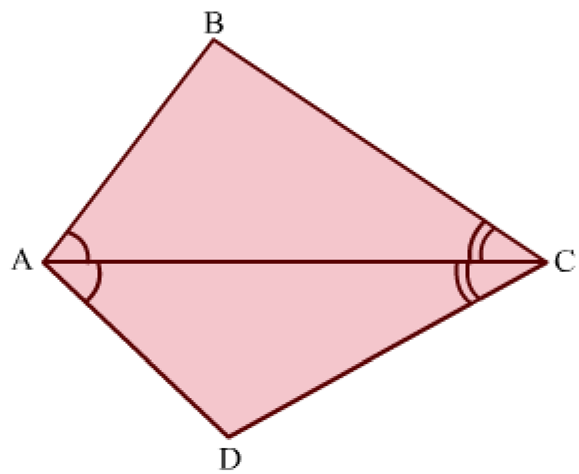
Thus, by the ASA congruence rule, we obtain:

ΔABO ≅ ΔCDO

**Medium**

**Example 1**:

In the given quadrilateral ABCD, diagonal AC bisects ∠BAD and ∠BCD. Prove that AB = AD and CB = CD.




**Solution**:

Since diagonal AC bisects ∠BAD and ∠CAD, we have:

∠BAC = ∠DAC and ∠BCA = ∠DCA

In ΔACB and ΔACD, we have:

∠BAC **=** ∠DAC (Given)

∠BCA = ∠DCA (Given)

AC = AC (Common side)

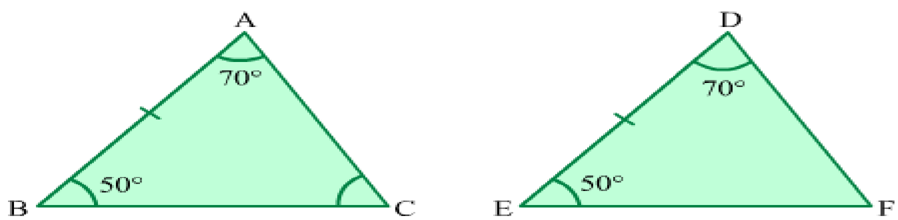
So, by the ASA congruence rule, we have:

ΔACB ≅ ΔACD

⇒ AB = AD and CB = CD (By CPCT)

**Example 2**:

Consider the two triangular parks ABC and DEF shown below.




Tina jogs around park ABC and Aliya jogs around park DEF daily. Paths AB and DE are equal in length. If both girls jog an equal number of rounds daily, then check whether or not they cover the same distance while jogging?

**Solution**:

In ΔABC and ΔDEF, we have:

∠BAC = ∠EDF = 70°(Given)

AB = DE(Given)

∠ABC = ∠DEF = 50°(Given)

Therefore, by the ASA congruency rule, we obtain:

ΔABC ≅ ΔDEF

⇒ AC = DF and BC = EF(By CPCT)

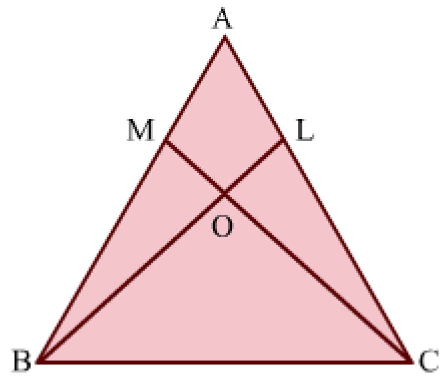
∴ AB + BC + CA = DE + EF + FD

Hence, both Tina and Aliya cover the same distance daily while jogging.

**Hard**

**Example 1**:

The given ΔABC is isosceles with AB = AC. ∠LOC = 2∠OBC and ∠MOB = 2∠OCB. Prove that ΔBCM ≅ ΔCBL.




**Solution**:

It is given that:

∠LOC = 2∠OBC… (1)

∠MOB = 2∠OCB… (2)

Now, ∠LOC = ∠MOB   (Vertically opposite angles)

Using equations (1) and (2), we obtain:

∠OCB = ∠OBC

⇒ ∠MCB = ∠LBC… (3)

In ΔBCM and ΔCBL, we have:

∠MBC = ∠LCB   (∵ ΔABC is isosceles with AB = AC)

BC = CB (Common side)

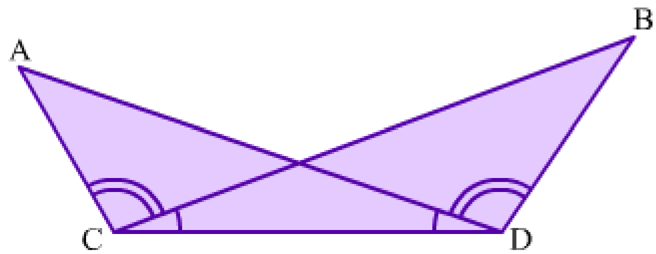
∠MCB = ∠LBC  (Using equation 3)

Thus, by the ASA congruence rule, we obtain:

ΔBCM ≅ ΔCBL

**Example 2**:

In the given figure, ∠BCD = ∠ADC and ∠ACB = ∠BDA. Prove that AD = BC and ∠CAD = ∠DBC.




**Solution**:

It is given that:

∠BCD = ∠ADC … (1)

∠ACB = ∠BDA … (2)

On adding equations (1) and (2), we get:

∠BCD + ∠ACB = ∠ADC + ∠BDA

⇒ ∠ACD = ∠BDC… (3)

In ΔACD and ΔBDC, we have:

∠ADC = ∠BCD(Given)

CD = DC  (Common side)

∠ACD = ∠BDC  (By equation 3)

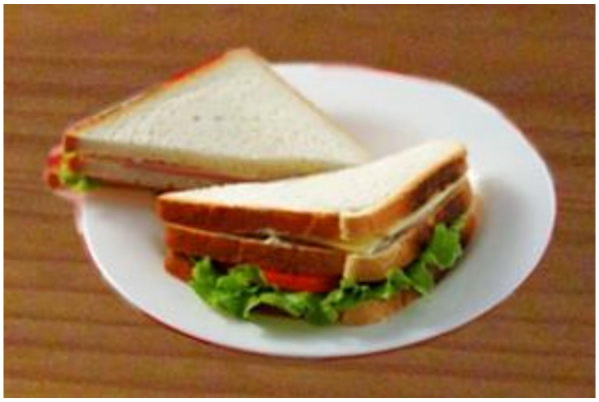
So, by the ASA congruence rule, we have:

ΔACD ≅ ΔBDC

⇒ AD = BC and ∠CAD = ∠DBC  (By CPCT)

**RHS Congruence Rule**

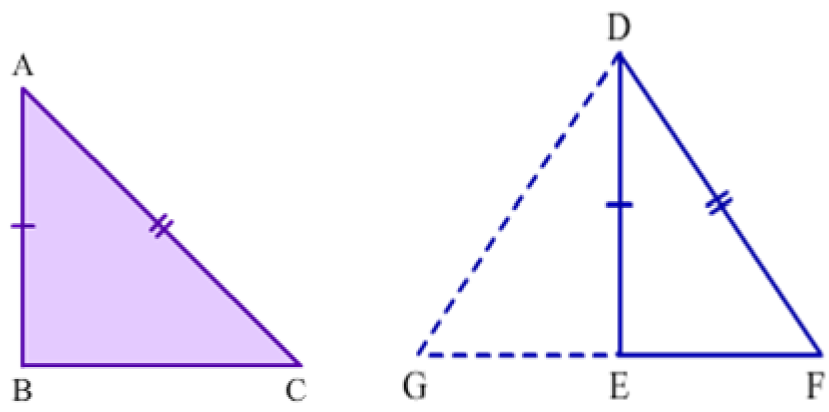
Right angles are all around us. For example, while building houses, the walls are kept at a right angle to the horizontal. Different square and rectangular figures surround us and all of them consist of right angles. The following figure also shows right angles.



In the figure, the pieces of bread resemble right-angled triangles. They also seem to be congruent. Right-angled triangles are special and their congruency is checked by a special congruence rule known as the RHS (Right-Hypotenuse-Side) rule.

We will study the RHS congruence rule in this lesson and solve some examples to familiarize ourselves with the concept.

**RHS congruence theorem**: **Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are equal to the hypotenuse and the corresponding side of the other triangle.**




**Given:** Two right-angled triangle ABC and DEF such that ∠B = ∠E = 90°; Hypotenuse AC = Hypotenuse DF and AB = DE.

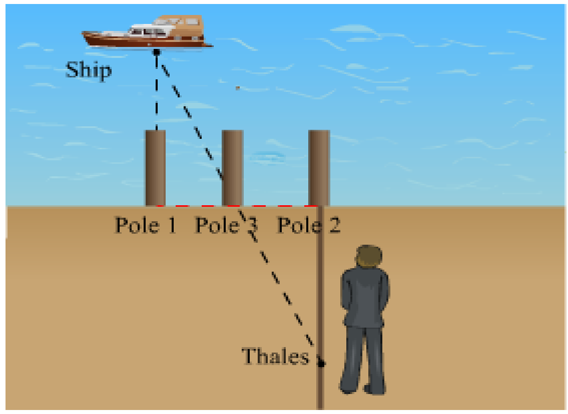
**To prove:** ΔABC ≅ ΔDEF.

**Construction:** Produce FE to G so that EG = BC and join DG.

**Proof:**

In triangles ABC and DEF,
AB = DE (Given)
BC = EG (By construction)
∠ABC= ∠DEF (Each equal to 90°)
Thus, by SAS congruence criterion,
ΔABC ≅ ΔDEG
⇒ ∠ACB = ∠ DGE and AC = DG (CPCT)
Given, AC = DF
∴ DG = AC = DF
In ΔDGF, we have
DG = DF
∠G = ∠F (Angles opposite to equal sides are equal)
In ΔDEF and ΔDEG,
∠G = ∠F (Proved)
∠DEG = ∠DEF (Both equal to 90°)
Thus, ∠GDE = 180° – (∠G + ∠DEG) = 180° – (∠F +∠DEF) = ∠FDE
In ΔDEG and ΔDEF,
DG = DF (Proved)
DE = DE (Common)
∠GDE = ∠FDE (Proved)
Thus, by SAS congruence criterion
ΔDEG ≅ ΔDEF
But, we have ΔABC ≅ ΔDEG
Thus, ΔABC ≅ ΔDEF
**Whiz Kid**

RHS congruence rule was used by the famous ancient Greek mathematician Thales to calculate the distance of a ship anchored at sea from the shore. For this, he stuck three poles on the shore such that the first one was directly in front of the ship, the second was at some distance from the first pole and the third was exactly between the other two poles. He then walked backward along a line from the second pole perpendicular to the shore until the middle pole and the ship were in the same line of sight. Then, he marked his position. This is shown in the following figure.




It can be seen that the triangle formed on the sea is congruent to the triangle formed on the shore by the RHS rule. So, the distance between the ship and the shore is equal to the distance between the second pole and the spot where Thales stands.

**Applying the RHS Rule**

**Solved Examples**

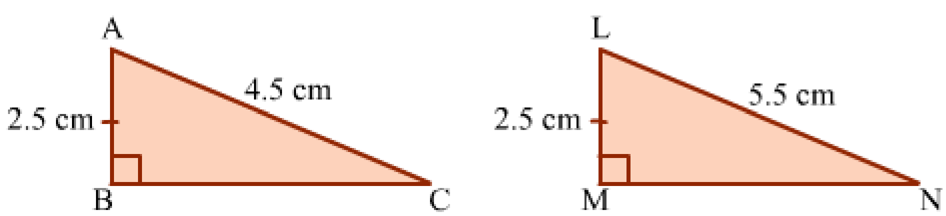
**Easy**

**Example 1**:

ΔABC and ΔLMN are right-angled at ∠ABC and ∠LMN respectively. In ΔABC, AB = 2.5 cm and AC = 4.5 cm. In ΔLMN, LN = 5.5 cm and LM = 2.5 cm. Examine whether the two triangles are congruent.

**Solution**:

On the basis of the given information, the two triangles can be drawn as is shown.




In ΔABC and ΔLMN, we have:

∠ABC = ∠LMN  (Right angles)

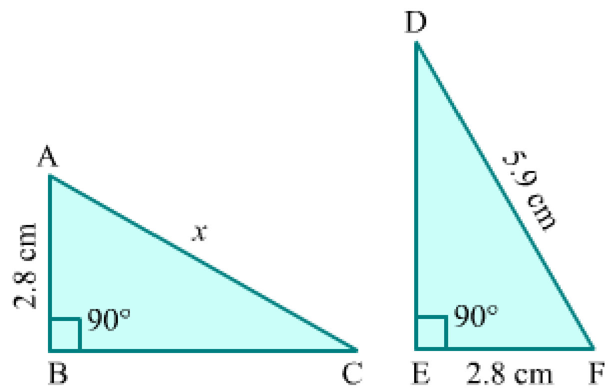
AB = LM = 2.5 cm  (Given)

But AC ≠ LN

Hence, ΔABC and ΔLMN are not congruent.

**Example 2**:

Find the value of *x* if the shown triangles ABC and DEF are congruent.




**Solution**:

It is given that ΔABC ≅ ΔDEF.

When two triangles are congruent, their corresponding sides are equal.

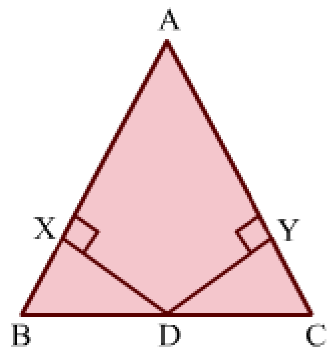
∴ AC = DF = 5.9 cm

Thus, the value of *x* is 5.9 cm.

**Medium**

**Example 1**:

In the given ΔABC, D is the midpoint of side BC. The perpendiculars DX and DY drawn from point D to sides AB and BC respectively are of the same length. Prove that DX and DY make the same angle with BC.




**Solution**:

On comparing ΔDXB and ΔDYC, we get:

DX = DY  (Given)

∠DXB = ∠DYC = 90°  (∵ DX and DY are perpendiculars)

BD = CD  (∵ D is the midpoint of BC)

Thus, by the RHS congruence rule, we have:

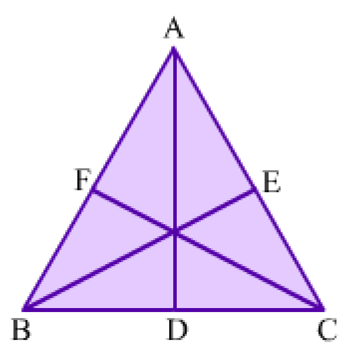
ΔDXB ≅ ΔDYC

⇒ ∠BDX = ∠CDY  (By CPCT)

Thus, the perpendiculars DX and DY make the same angle with side BC.

**Example 2**:

In the given ΔABC, AD, BE and CF are the altitudes. If the three altitudes are equal, then prove that the triangle is equilateral.




**Solution**:

In ΔBEC and ΔCFB, we have:

BC = CB (Common side)

BE = CF (Given)

∠BEC = ∠CFB = 90° (∵ BE and CF are altitudes)

So, by the RHS congruence rule, we obtain:

ΔBEC ≅ ΔCFB

⇒ ∠BCE = ∠CBF (By CPCT)

⇒ ∠CBA = ∠BCA

⇒ AC = AB … (1) [∵ Sides opposite equal angles are equal]

Similarly, we can prove that ΔADB ≅ ΔBEA.

⇒ ∠DBA = ∠BAE (By CPCT)

⇒ ∠CBA = ∠BAC

⇒ AC = BC … (2) [∵ Sides opposite equal angles are equal]

From equations (1) and (2), we get:

AB = BC = AC

Hence, ΔABC is equilateral.

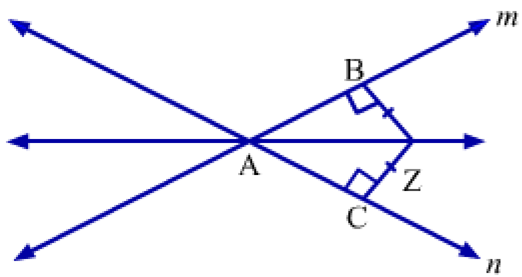
**Hard**

**Example 1**:

If Z is a point equidistant from two lines *m* and *n* intersecting at point A, then prove that AZ bisects the angle between *m* and *n*.

**Solution**:

The following figure can be drawn as per the given information.




***Construction***: ZB and ZC are perpendiculars drawn from point Z to lines *m* and *n* respectively.

Since Z is equidistant from *m* and *n*, we have:

ZB = ZC

In ΔZBA and ΔZCA, we have:

ZB = ZC (Proved above)

∠ZBA = ∠ZCA = 90° (∵ ZB and ZC are perpendiculars)

ZA = ZA (Common side)

So, by the RHS congruence rule, we have:

ΔZBA ≅ ΔZCA

⇒ ∠ZAB = ∠ZAC (By CPCT)

Now, ∠ZAB + ∠ZAC = ∠BAC

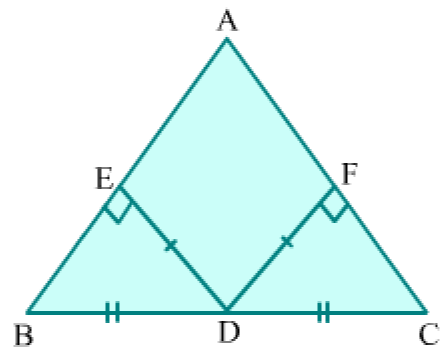
Therefore, AZ bisects the angle between lines *m* and *n*.

**Example 2**:

In a ΔABC, BD = DC. If the perpendiculars from point D to sides AB and AC are equal, then prove that AB = AC.

**Solution**:

The triangle with the given specifications is drawn below.




In ΔABC, D is the midpoint of BC. Also, DE and DF are the perpendiculars from D to AB and AC respectively.

In ΔDEB and ΔDFC, we have:

∠DEB = ∠DFC = 90° (∵ DE and DF are perpendiculars)

DB = DC (Given)

DE = DF (Given)

So, by the RHS congruence rule, we obtain:

ΔDEB ≅ ΔDFC

⇒ ∠DBE = ∠DCF (By CPCT)

⇒ ∠CBA = ∠BCA

⇒ AC = AB (As sides opposite to equal angles are equal)