

Topics : Limits, Straight Line, Continuity & Derivability

Type of Questions

M.M., Min.

Comprehension (no negative marking) Q.1 to Q.2	(3 marks, 3 min.)	[6, 6]
Single choice Objective (no negative marking) Q.3,4	(3 marks, 3 min.)	[6, 6]
Multiple choice objective (no negative marking) Q.5	(5 marks, 4 min.)	[5, 4]
True or False (no negative marking) Q.6	(2 marks, 2 min.)	[2, 2]
Subjective Questions (no negative marking) Q.7,8	(4 marks, 5 min.)	[8, 10]

COMPREHENSION (FOR Q.NO. 1 TO 2)

If $f(x) = \text{maximum} \left(\cos x, \frac{1}{2}, \{\sin x\} \right)$, $0 \leq x \leq 2\pi$, where $\{ \cdot \}$ represents fractional part function, then

- Number of points of discontinuity of $f(x)$ is
(A) 1 (B) 2 (C) 3 (D) 4
- Number of points where $f(x)$ is not differentiable is
(A) 4 (B) 5 (C) 6 (D) 7
- Consider a function $f(x) : \mathbb{R} \rightarrow \mathbb{R}$ and if $\lim_{x \rightarrow a} [f(x)]$ does not exist, where $[\]$ denotes greatest integer function, then
(A) $\lim_{x \rightarrow a} f(x)$ will never exist (B) $f(x)$ may be continuous at $x = a$
(C) Function will not have a tangent at $x = a$ (D) None of these
- The angle between straight lines joining the origin and intersection points of the straight line $bx + ay = ab$ and circle $x^2 + y^2 = ax + by$ is
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$
- Two consecutive vertices of a rectangle of area 10 unit² are (1,3) and (-2, -1). Other two vertices are
(A) $\left(-\frac{3}{5}, \frac{21}{5} \right), \left(-\frac{18}{5}, \frac{1}{5} \right)$ (B) $\left(-\frac{3}{5}, \frac{21}{5} \right), \left(-\frac{2}{5}, -\frac{11}{5} \right)$
(C) $\left(-\frac{2}{5}, -\frac{11}{5} \right), \left(\frac{13}{5}, \frac{9}{5} \right)$ (D) $\left(\frac{13}{5}, \frac{9}{5} \right), \left(-\frac{18}{5}, \frac{1}{5} \right)$

6. True / False

(A) $\lim_{x \rightarrow \infty} \frac{\ell n x}{[x]} = \lim_{x \rightarrow \infty} \frac{\{x\}}{\ell n x}$

where $[.]$ is G.I.F. & $\{.\}$ denotes fractional part function

(B) If $\lim_{x \rightarrow \infty} \left(\sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 - cx^2 + 3x - d} \right) = 4,$

then absolute value of $a - c$ is 3.

(C) $\lim_{x \rightarrow 0} \left[\frac{\sin(\operatorname{sgn}(x))}{\operatorname{sgn}(x)} \right] = 1$ where $[.]$ is greatest integer function

(D) $\lim_{x \rightarrow \infty} \sec^{-1} \left(\frac{x}{\sin x} \right) = \lim_{x \rightarrow \infty} \sec^{-1} \left(\frac{\sin x}{x} \right)$

7. Consider the function $g(x) = \begin{cases} \frac{1 - a^x + xa^x \ell na}{a^x x^2} & ; x < 0 \\ \frac{2^x a^x - x \ell n 2 - x \ell na - 1}{x^2} & ; x > 0 \end{cases}$ where $a > 0$. Find the value of a and $g(0)$ so

that the function $g(x)$ is continuous at $x = 0$.

8. Consider the function $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{2x} \right| & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Find LHD and RHD at $x = \frac{1}{3}$

Answers Key

1. (B) 2. (C) 3. (B) 4. (D)

5. (A)(C)

6. (A) True (B) True (C) False (D) False

7. $a = \frac{1}{\sqrt{2}}$, $g(0) = \frac{1}{8}(\ln 2)^2$ 8. LHD = $-\frac{\pi}{2}$ and RHD = $\frac{\pi}{2}$