Probability

Exercise-16

Question 1:

15 defective ballpens are accidentally mixed with 135 good ones. It is not possible to just look at a ballpen and say whether it is defective or not. One ballpen is picked up at random from it. Find the probability that the ballpen selected is a good one.

Solution :

Total number of ballpens = No. of defective ballpens + No. good ballpens

$\therefore$ Total ballpens = 15 + 135 = 150.

Let A be the event that the ballpen picked is a good one.

The number of favourable outcomes for A is 135.

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to event A}}{\text{Number of all possible outcomes}}$$

$$= \frac{135}{150}$$

$$= 0.9$$

$\therefore$ Probability that the selected ballpen is a good one is 0.9.

Question 2:

A box contains 5 green, 8 yellow and 7 brown balls. One ball is taken out from a box at random. What is the probability that the ball taken out is (i) yellow ? (ii) brown ? (iii) neither green nor brown ? (iv) not brown ?

Solution :

Total number of balls in the box = 5 + 8 + 7 = 20

i. Let A be the event that the ball taken out is yellow.

Number of yellow balls in the box = 8

$$\therefore P(A) = \frac{8}{20} = \frac{2}{5}$$

ii. Let B be the event that the ball taken out is brown.

Number of brown balls in the box = 7

$$\therefore P(B) = \frac{7}{20}$$

iii. Let C be the event that the ball taken out is neither green nor brown, i.e., it is yellow.

Number of yellow balls in the box = 8

$$\therefore P(C) = \frac{8}{20} = \frac{2}{5}$$

iv. Let D be the event that the ball taken out is not brown.

Number of balls in the box which are not brown = 20 – 7 = 13.

$$\therefore P(D) = \frac{13}{20}$$

Question 3:

A bag contains orange flavoured candies only. Rahi takes out one candy without looking into the bag. What is the probability that she takes out (i) the orange flavoured candy ? (ii) a lemon flavoured candy ?

Solution :

Let us assume that the bag contains n candies.
It is given that the bag contains only orange candies in the box.
Number of orange flavoured candies in the bag = n
∴ Number of lemon flavoured candies in the bag = 0.

i. Let A be the event that the orange flavoured candy is taken out.
As the number of orange flavoured candies in the bag is n,

$$\therefore P(A) = \frac{n}{n} = 1$$

ii. Let B be the event that the lemon flavoured candy is taken out.
Number of lemon flavoured candies in the bag is 0.

$$\therefore P(B) = \frac{0}{n} = 0$$

Question 4:

A box contain 100 cards marked with numbers 1 to 100. If one card is drawn from the box, find the probability that it bears (i) single digit number, (ii) two-digit numbers (iii) three-digit number (iv) a number divisible by 8 (v) a multiple of 9, (vi) a multiple of 5.

Solution :

There are 100 cards in a box marked with numbers 1 to 100
A card is drawn randomly from it.
∴ Number of possible outcomes = 100

i. Let A be the event that the card bears a single digit number.
There are 9 single digit numbers (1 to 9) among the numbers from 1 to 100.
∴ The number of outcomes favourable to A is 9.

$$\therefore P(A) = \frac{9}{100} = 0.09$$

ii. Let B be the event that the card bears a two digit number.
There are 90 two digit numbers (10 to 99) among the numbers from 1 to 100.
∴ The number of outcomes favourable to B is 90.

$$\therefore P(B) = \frac{90}{100} = 0.9$$

iii. Let C be the event that the card bears a three digit number.
There is only 1 three digit number,i.e. 100, among the number from 1 to 100.
∴ The number of outcomes favourable to event C is 1.

$$\therefore P(C) = \frac{1}{100} = 0.01$$

iv. Let D be the event that the card bears a number divisible by 8.
Numbers divisible by 8: 8, 16, 24, ..., 96.
There are 12 numbers in 1 to 100 that are divisible by 8.
∴ Number of outcomes favourable to D is 12.

$$\therefore P(D) = \frac{12}{100} = 0.12$$

v. Let E be the event that the card bears a multiple of 9.
Mulitples of 9: 9, 18, 27, ……..,99.
There are 11 multiples of 9 from the numbers 1 to 100.
∴ Number of outcomes favourable to E is 11.

$\therefore P(E) = \frac{11}{100} = 0.11$

vi. Let F be the event that the card bears a multiple of 5.
Multiples of 5: 5, 10, 15, ……. 100
There are 20 multiples of 5 among the numbers 1 to 100.
∴ The number of outcomes favourable to F is 20.

$\therefore P(F) = \frac{20}{100} = 0.2$

Question 5:

A carton consist of 100 trousers of which 73 are good, 12 have minor defects and 15 have major defects. Kanu, a trader, will only accept the trousers which are good, but Radha, another trader, will only reject the trousers which have major defects. One trouser is drawn at random from the carton. What is the probability that,
(i) it is acceptable to Kanu ? (ii) it is acceptable to Radha ?

Solution :

The carton contains 100 trousers.
∴ Total number of possible outcomes is 100.

i. Let A be the event that the trouser drawn is acceptable to Kanu,
i.e., the trouser is good.
There are 73 good trousers in the carton.
∴ The number of outcomes favourable to A is 73.

$\therefore P(A) = \frac{73}{100} = 0.73$

ii. Let B be the event that the trouser drawn is rejected by Radha,
i.e., the trouser has major defects.
There are 15 trousers having major defects.
∴ The number of outcomes favourable to B is 15.

$\therefore P(B) = \frac{15}{100} = 0.15$

Question 6:

Marks obtained by 50 students from 100 are as follows

Marks	0-34	35-50	51-70	71-90	91-100
Number of student	8	9	14	11	8

Find the probability that a student get marks :

1. below 34,
2. between 71-90
3. more than 70
4. less than or equal to 50

Solution :

Total number of students = 8 + 9 + 14 + 11 + 8 = 50

Total number of possible outcomes = 50.

i. Let A be the event that the selected student gets less than or equal to 34 marks.

Number of students who got marks less than or equal to 34 = 8

∴ The number of outcomes favourable to A is 8.

$$\therefore P(A) = \frac{8}{50} = \frac{4}{25}$$

ii. Let B be the event that the selected student gets marks between 71 and 90.

Number students who got marks between 71 and 90 = 11

∴ The number of outcomes favourable to B is 11.

$$\therefore P(B) = \frac{11}{50}$$

iii. Let C be the event that the selected student gets more than 70 marks.

The number of students who got more than 70 marks = 11 + 8 = 19

∴ The number of outcomes favourable to event C is 19.

$$\therefore P(C) = \frac{19}{50}$$

iv. Let D be the event that the selected student gets less than or equal to 50 marks.

Number of students who got marks less than or equal to 50 = 8 + 9 = 17

∴ The number of outcomes favourable to event D is 17.

$$\therefore P(D) = \frac{17}{50}$$

v. Let E be the event that the selected student gets marks above 90.

Number of students getting marks above 90 = 8.

∴ The number of outcomes favourable to event E is 8.

$$\therefore P(D) = \frac{8}{50} = \frac{4}{25}$$

Question 7:

Two fair dice are rolled simultaneously. Find the probability of the following events :

1. A : getting the same number on both dice.
2. B : the sum of the integers on two dice is more than 4 but less than 8.
3. C : the product of numbers on two dice is divisible by 2.
4. D : the sum of numbers on two dice is greater than 12.

Solution :

In the experiment of throwing two dice we get
36 elementary outcomes as given below:
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6),

1. Here A is the event that the same number appears on both the dice.
   The favourable outcomes for event A are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), and (6, 6).
   ∴ Number of outcomes favourable to event A is 6.
   ∴ $P(A) = \frac{6}{36} = \frac{1}{6}$
2. Here B is the event that the sum of integers on the two dice is more than 4 but less than 8, i.e., the sum is 5, 6, or 7.
   Favourable outcomes giving sum 5, 6 or 7 are (1, 4), (2, 3), (3, 2), (4, 1), (1, 5), (2, 4), (3, 3), (4, 2) and (5, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1).
   ∴ Total number of favourable outcomes giving sum 5, 6 or 7 = 4 + 5 + 6 = 15
   ∴ Number of outcomes favourable to B is 15.
   ∴ $P(B) = \frac{15}{36} = \frac{5}{12}$
3. Here C is the event that the product of the numbers on two dice is divisible by 2, i.e., the product is even.
   For the product to be even, at least one of the two numbers should be even.
   Similarly, for the product to be odd both the numbers should be odd.
   So the favourable outcomes giving the product an odd number are (1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3) and (5, 5).
   ∴ Number of outcomes favourable giving product an odd number = 9.
   ∴ Number of outcomes favourable giving product an even number =36 – 9 = 27.
   ∴ Number of outcomes favourable to C is 27.
   ∴ $P(C) = \frac{27}{36} = \frac{3}{4}$
4. Here D is the event that the sum of numbers on two dice is greater than 12.
   But it is not possible to get sum greater than 12 since the maximum sum of numbers on two dice is 6 + 6 = 12.
   ∴ Event D is an impossible event.
   ∴ $P(D) = 0$

Question 8:

A coin is tossed three times. Find the probability of the following events :

1. A : getting at least two heads,
2. B : getting exactly two heads,
3. C : getting at most one head,
4. D : getting more heads than tails.

Solution :

The possible outcomes of the event of tossing a coin three times are (HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT).
So, the number of outcomes is 8.

1. Here A is the event of getting atleast two heads, i.e., getting two or three heads.
   Favourable outcomes are (HHH), (HHT), (HTH) and (THH).
   ∴ Number of outcomes favourable to A is 4.
   $\therefore P(A) = \frac{4}{8} = \frac{1}{2}$
2. Here B is the event of getting exactly two heads.
   Favourable outcomes are (HHT), (HTH) and (THH).
   ∴ Number of outcomes favourable to B is 3.
   $\therefore P(B) = \frac{3}{8}$
3. Here C is the event of getting at most one head i.e., getting one or zero head.
   Favourable outcomes are (HTT), (THT), (TTH) and (TTT).
   ∴ Number of outcomes favourable to C is 4
   $\therefore P(C) = \frac{4}{8} = \frac{1}{2}$
4. Here D is the event of getting more heads than tails.
   Favourable outcomes are (THH), (HTH), (HHT) and (HHH).
   ∴ Number of outcomes favourable to D is 4.
   $\therefore P(D) = \frac{4}{8} = \frac{1}{2}$

Question 9:

A game of chance consists of spinning an arrow which comes to rest pointing numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 (see figure 16.1) and there are outcomes. What is the probability that it will point at

1. 7?
2. a number greater than 9 ?
3. an odd number ?
4. an even number ?
5. a number less than 5 ?

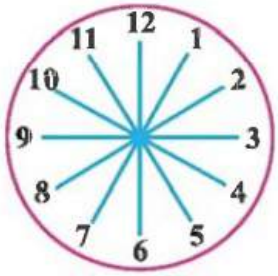

Figure 16.1

Solution :

The arrow can point at any number from 1 to 12.

∴ Total number of possible outcomes is 12.

1. Let A be the event that the arrow points at 7.
   ∴ Number of outcomes favourable to A is only 1.
   ∴ $P(A) = \frac{1}{12}$
2. Let B be the event that the arrow points at a number greater than 9, i.e., at 10, 11 or 12.
   ∴ Number of outcomes favourable to B is 3.
   ∴ $P(B) = \frac{3}{12} = \frac{1}{4}$
3. Let C be the event that the arrow points at an odd number, i.e., at 1, 3, 5, 7, 9 or 11.
   ∴ Number of outcomes favourable to C is 6.
   ∴ $P(C) = \frac{6}{12} = \frac{1}{2}$
4. Let D be the event that the arrow points at an even number, i.e., at 2, 4, 6, 8, 10 or 12.
   ∴ Number of outcomes favourable to D is 6.
   ∴ $P(D) = \frac{6}{12} = \frac{1}{2}$
5. Let E be the event that the arrow points at a number less than 5, i.e., at 1, 2, 3 or 4.
   ∴ Number of outcomes favourable to E is 4.
   ∴ $P(E) = \frac{4}{12} = \frac{1}{3}$

Question 10:

Select a proper option (a), (b), (c) or (d) from given options and write in the box given on the right so that the statement becomes correct :

Question 10(1):

Solution :

b. 1

For any event A, there are two possibilities - either event A occurs or event A does not occur. Hence, we get

$P(A) + P(\overline{A}) = 1$.

Question 10(2):

Solution :

d. 1

An event which always occurs is known as a certain event.

∴ The probability of a certain event is 1.

Question 10(3):

Solution :

a. 0

An event which never occurs is known as an impossible event.

∴ The probability of an impossible event is 0.

Question 10(4):

Solution :

d. 0

For any event A, $0 \leq P(A) \leq 1$.
∴ The probability of any event is greater than or equal to 0.

Question 10(5):

Solution :

d. 1

Solution: For any event A, $0 \leq P(A) \leq 1$.
∴ The probability of any event is less than or equal to 1.

Question 10(6):

Solution :

c. 0.65

For any event A,
$P(A) + P(\bar{A}) = 1$
$\therefore P(\bar{A}) = 1 - P(A)$
$\therefore P(\bar{A}) = 1 - 0.35$
$\therefore P(\bar{A}) = 0.65$

Question 10(7):

Solution :

d. 0.53

For any event E,
$P(E) + P(\bar{E}) = 1$
$\therefore P(E) = 1 - P(\bar{E})$
$\therefore P(E) = 1 - 0.47$
$\therefore P(E) = 0.53$

Question 10(8):

Solution :

c. 0

The paper in my hand is of 100 marks. So, to get 101 marks out of 100 in this paper is an impossible event. The probability of an impossible event is 0.

Question 10(9):

Solution :

c. 0

The event ‘The Sun rises in the West’ is an impossible event and thus its probability is 0.

Question 10(10):

Solution :

c. 1

If A_1, A_2, A_3, ……., A_k are all the elementary events of an experiment, then $P(A_1) + P(A_2) + P(A_3)$ ……. + $P(A_k) = 1$.