

## UNIT 10

# DIRECT AND INVERSE PROPORTIONS

### (A) Main Concepts and Results

- Two quantities  $x$  and  $y$  are said to be in direct proportion if they increase or decrease together in such a manner that the ratio of their corresponding values remains constant. That is,  $\frac{x}{y} = k$  where  $k$  is a positive number, if  $x$  and  $y$  are in direct proportion or vary directly. In case of direct proportion, if  $y_1, y_2$  are the values of  $y$  corresponding to the values  $x_1, x_2$  of  $x$  respectively, then  $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ .
- Two quantities  $x$  and  $y$  are said to be in inverse proportion if an increase in  $x$  causes a proportional decrease in  $y$  and vice-versa, in such a manner that the product of their corresponding values remains constant. That is,  $xy = k$  where  $k$  is a positive number, if  $x$  and  $y$  are in inverse proportion. In this case, if  $y_1, y_2$  are the values of  $y$  corresponding to the values  $x_1, x_2$  of  $x$  respectively, then  $x_1 y_1 = x_2 y_2$  or  $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ .
- Quantities increasing or decreasing together need not always be in direct proportion, same in the case of inverse proportion.
- When two quantities  $x$  and  $y$  are in direct proportion (or vary directly), they are written as  $x \propto y$ . Symbol “ $\propto$ ” stands for ‘is proportional to’.

- When two quantities  $x$  and  $y$  are in inverse proportion (or vary inversely) they are written as  $x \propto \frac{1}{y}$ .

## (B) Solved Examples

**In examples 1 to 3, there are four options out of which one is correct. Choose the correct answer,**

**Example 1** : If  $x$  and  $y$  are directly proportional and when  $x = 13$ ,  $y = 39$ , which of the following is not a possible pair of corresponding values of  $x$  and  $y$  ?

- (a) 1 and 3                      (b) 17 and 51  
(c) 30 and 10                  (d) 6 and 18

**Solution** : The correct answer is (c).

**Example 2** : A car covers a distance in 40 minutes with an average speed of 60 km per hour. The average speed to cover the same distance in 30 minutes is

- (a) 80 km/h                      (b)  $\frac{45}{2}$  km/h  
(c) 70 km/h                      (d) 45 km/h

**Solution** : The correct answer is (a).

**Example 3** : Which of the following is in direct proportion?

- One side of a cuboid and its volume.
- Speed of a vehicle and the distance travelled in a fixed time interval.
- Change in weight and height among individuals.
- Number of pipes to fill a tank and the time required to fill the same tank.

**Solution** : The correct answer is (b).

[Because, in a fixed time interval, as the speed of a vehicle increases, the distance travelled by it also increases in the same ratio.]

**In examples 4 to 6, fill in the blanks to make the statements true.**

**Example 4** : Amrita takes 18 hours to travel 720 kilometres. Time taken by her to travel 360 kilometres is \_\_\_\_\_.

**Solution** : 9 hours.

**Example 5** : If  $x$  and  $y$  are inversely proportional then \_\_\_\_\_ =  $k$  where  $k$  is positive constant.

**Solution** :  $xy$ .

**Example 6** : Side of a rhombus and its perimeter are in \_\_\_\_\_ proportion.

**Solution** : Direct.

**In examples 7 to 9, state whether the statements are true (T) or false (F):**

**Example 7** : When two quantities  $x$  and  $y$  are in inverse proportion, then  $\frac{x}{y}$  is a constant.

**Solution** : False.

**Example 8** : If the cost of 10 pencils is Rs 90, then the cost of 19 pencils is Rs 171.

**Solution** : True.

**Example 9** : If 5 persons can finish a job in 10 days then one person will finish it in 2 days.

**Solution** : False.

**Example 10** : In a scout camp, there is food provision for 300 cadets for 42 days. If 50 more persons join the camp, for how many days will the provision last?

**Solution** : More the persons, the sooner would be the provision exhausted. So, this is a case of inverse proportion.

Let the required number of days be  $x$ .

Hence,  $300 \times 42 = (300 + 50) \times x$

$300 \times 42 = 350 \times x$

$\frac{300 \times 42}{350} = x$

$x = 36$

**Example 11 :** If two cardboard boxes occupy 500 cubic centimetres space, then how much space is required to keep 200 such boxes?

**Solution :** As the number of boxes increases, the space required to keep them also increases.

So, this is a case of direct proportion.

Number of boxes	2	200
Space occupied (in cubic centimetres)	500	$x$

$$\text{So } \therefore \frac{2}{500} = \frac{200}{x}$$

$$2x = 500 \times 200$$

$$x = x = \frac{500 \times 200}{2}$$

$$x = 50,000$$

Thus, the required space is 50,000 cubic centimetres.

**Example 12 :** Under the condition that the temperature remains constant, the volume of gas is inversely proportional to its pressure. If the volume of gas is 630 cubic centimetres at a pressure of 360 mm of mercury, then what will be the pressure of the gas if its volume is 720 cubic centimetres at the same temperature?

**Solution :** Given that, at constant temperature pressure and volume of a gas are inversely proportional. Let the required pressure be  $x$ .

<b>Volume of gas (in cubic centimetres)</b>	<b>630</b>	<b>720</b>
<b>Pressure of gas (in mm)</b>	<b>360</b>	<b><math>x</math></b>

$$\text{Then, } 630 \times 360 = 720 \times x$$

$$\frac{630 \times 360}{720} = x$$

$$x = 315$$

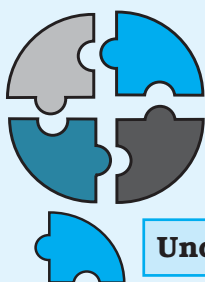
Therefore, the required pressure is 315 mm of mercury.

The time  $t$  required to download a 4-megabyte music file from an internet music seller is inversely proportional to the rate  $r$  at which data is transferred to the receiving computer.



- How long will it take to download a 4-megabyte file if the transmission occurs at a rate of 2.5 megabytes per minute? How long if the transmission rate is 0.8 megabytes per minute?
- How can the relationship of  $t$  and  $r$  be expressed in symbolic form?
- How does the value of  $t$  change as the value of  $r$  increases steadily? How is this pattern of change related to the constant of proportionality?

### Application on Problem Solving Strategy



#### Example 13:

Lemons were bought at Rs 60 a dozen and sold at the rate of Rs 40 per 10. Find the gain or loss per cent.

#### Understand and Explore the problem

- Rewrite given equation in your own words.  
30 men can reap a field in 17 days. If the field is to be reaped in 10 days then how many men will be required? How many extra men are to be employed?
- What do you know?  
30 men can reap a field in 17 days.

#### Plan a Strategy

- Think that 30 men are reaping the field in 17 days, so to reap the field in 10 days, i.e. in less number of days, men required will be more or less?
- No. of days has reduced so men to be employed will increase. Therefore we will use indirect variation.
- Find difference between no. of men required and the number 30.

#### Solve

- Let number of men required to finish the job in 10 days be  $x$

No. of days	17 decrease 10
No. of men	30 increase $x$

- No. of days has decreased so number of men will increase. Hence we will use inverse variation.

$$\text{Hence } 30 \times 17 = x \times 10$$

$$x = \frac{30 \times 17}{10} = 51$$

51 men will be required.

$$\text{Extra number of men required} = 51 - 30 = 21 \text{ men.}$$



### Revise

- Verify your answer by adopting some other method e.g. Here instead of using variation we can use unitary method.

e.g.	No. of days	No. of men
	17	30
	1	?
	10	$x$

No. of men required to complete the job in 17 days = 30

No. of men required to complete the job in 1 day =  $30 \times 17$

No. of men required to complete the job in 10 days =

$$\frac{30 \times 17}{10} = 51$$

No. of extra men required =  $51 - 30 = 21$  men.

Hence verified.

### Think and Discuss

- What will happen if question is:

If 30 men can reap a field in 17 days, then 10 men reap the field in how many days?

- In the questions of men and work we always use indirect variation. Now think of some situation related to men where direct variation will be used, e.g. If maximum 15 men can travel by three cars, then find minimum maximum number of cars required for (a) 25 men (b) 38 men.

**(C) Exercises**

**In questions 1 to 16, there are four options out of which one is correct.**

**Write the correct answer.**

- Both  $u$  and  $v$  vary directly with each other. When  $u$  is 10,  $v$  is 15, which of the following is not a possible pair of corresponding values of  $u$  and  $v$ ?  
(a) 2 and 3      (b) 8 and 12      (c) 15 and 20      (d) 25 and 37.5
- Both  $x$  and  $y$  vary inversely with each other. When  $x$  is 10,  $y$  is 6, which of the following is not a possible pair of corresponding values of  $x$  and  $y$ ?  
(a) 12 and 5      (b) 15 and 4      (c) 25 and 2.4      (d) 45 and 1.3
- Assuming land to be uniformly fertile, the area of land and the yield on it vary  
(a) directly with each other.  
(b) inversely with each other.  
(c) neither directly nor inversely with each other.  
(d) sometimes directly and sometimes inversely with each other.
- The number of teeth and the age of a person vary  
(a) directly with each other.  
(b) inversely with each other.  
(c) neither directly nor inversely with each other.  
(d) sometimes directly and sometimes inversely with each other.

**Direct Variation:** If the relationship of variables  $y$  and  $x$  can be expressed in the form:

$$y = kx \text{ for some constant } k,$$

then we say that  $y$  varies directly with  $x$  or that  $y$  is directly proportional to  $x$ . The number  $k$  is called the constant of proportionality for the relationship.

The close connection between multiplication and division of numbers

implies that if  $y$  is directly proportional to  $x$ , then  $\frac{y}{x} = k$ . The symbolic

form  $\frac{y}{x} = k$  shows that the ratio of  $y$  to  $x$  is constant, for any corresponding values of  $y$  and  $x$ .

5. A truck needs 54 litres of diesel for covering a distance of 297 km. The diesel required by the truck to cover a distance of 550 km is  
 (a) 100 litres (b) 50 litres (c) 25.16 litres (d) 25 litres
6. By travelling at a speed of 48 kilometres per hour, a car can finish a certain journey in 10 hours. To cover the same distance in 8 hours, the speed of the car should be  
 (a) 60 km/h (b) 80 km/h (c) 30 km/h (d) 40 km/h
7. In which of the following case, do the quantities vary directly with each other?

(a) 

$x$	0.5	2	8	32
$y$	2	8	32	128

(b) 

$p$	$1^2$	$2^2$	$3^2$	$4^2$
$q$	$1^3$	$2^3$	$3^3$	$4^3$

(c) 

$r$	2	5	10	25	50
$s$	25	10	5	2	0.5

(d) 

$u$	2	4	6	9	12
$v$	18	9	6	4	3

8. Which quantities in the previous question vary inversely with each other?  
 (a)  $x$  and  $y$  (b)  $p$  and  $q$  (c)  $r$  and  $s$  (d)  $u$  and  $v$
9. Which of the following vary inversely with each other?  
 (a) speed and distance covered.  
 (b) distance covered and taxi fare.  
 (c) distance travelled and time taken.  
 (d) speed and time taken.
10. Both  $x$  and  $y$  are in direct proportion, then  $\frac{1}{x}$  and  $\frac{1}{y}$  are  
 (a) in indirect proportion.  
 (b) in inverse proportion.  
 (c) neither in direct nor in inverse proportion.  
 (d) sometimes in direct and sometimes in inverse proportion.



11. Meenakshee cycles to her school at an average speed of 12 km/h and takes 20 minutes to reach her school. If she wants to reach her school in 12 minutes, her average speed should be
- (a)  $\frac{20}{3}$  km/h (b) 16 km/h  
(c) 20 km/h (d) 15 km/h
12. 100 persons had food provision for 24 days. If 20 persons left the place, the provision will last for
- (a) 30 days (b)  $\frac{96}{5}$  days  
(c) 120 days (d) 40 days
13. If two quantities  $x$  and  $y$  vary directly with each other, then
- (a)  $\frac{x}{y}$  remains constant. (b)  $x - y$  remains constant.  
(c)  $x + y$  remains constant. (d)  $x \times y$  remains constant.
14. If two quantities  $p$  and  $q$  vary inversely with each other, then
- (a)  $\frac{p}{q}$  remains constant. (b)  $p + q$  remains constant.  
(c)  $p \times q$  remains constant. (d)  $p - q$  remains constant.
15. If the distance travelled by a rickshaw in one hour is 10 km, then the distance travelled by the same rickshaw with the same speed in one minute is
- (a)  $\frac{250}{9}$  m (b)  $\frac{500}{9}$  m (c) 1000 m (d)  $\frac{500}{3}$  m

**Inverse Variation:** If the relationship of variables  $y$  and  $x$  can be expressed in the form.

$$y = \frac{k}{x} \text{ for some constant } k,$$

then we say that  $y$  varies directly with  $x$  or that  $y$  is inversely proportional to  $x$ . The number  $k$  is called the constant of proportionality for the relationship.

Once again, the close connection between multiplication and division of numbers implies that if  $y$  is inversely proportional to  $x$ , then  $xy = k$ . The symbolic form  $xy = k$  shows that the product of  $y$  and  $x$  is constant, for any corresponding values of  $x$  and  $y$ .

- 16.** Both  $x$  and  $y$  vary directly with each other and when  $x$  is 10,  $y$  is 14, which of the following is not a possible pair of corresponding values of  $x$  and  $y$ ?
- (a) 25 and 35    (b) 35 and 25    (c) 35 and 49    (d) 15 and 21

**In questions 17 to 42, fill in the blanks to make the statements true:**

- 17.** If  $x = 5y$ , then  $x$  and  $y$  vary \_\_\_\_\_ with each other.
- 18.** If  $xy = 10$ , then  $x$  and  $y$  vary \_\_\_\_\_ with each other.
- 19.** When two quantities  $x$  and  $y$  are in \_\_\_\_\_ proportion or vary \_\_\_\_\_ they are written as  $x \propto y$ .
- 20.** When two quantities  $x$  and  $y$  are in \_\_\_\_\_ proportion or vary \_\_\_\_\_ they are written as  $x \propto \frac{1}{y}$ .
- 21.** Both  $x$  and  $y$  are said to vary \_\_\_\_\_ with each other if for some positive number  $k$ ,  $xy = k$ .
- 22.**  $x$  and  $y$  are said to vary directly with each other if for some positive number  $k$ , \_\_\_\_\_  $= k$ .
- 23.** Two quantities are said to vary \_\_\_\_\_ with each other if they increase (decrease) together in such a manner that the ratio of their corresponding values remains constant.
- 24.** Two quantities are said to vary \_\_\_\_\_ with each other if an increase in one causes a decrease in the other in such a manner that the product of their corresponding values remains constant.
- 25.** If 12 pumps can empty a reservoir in 20 hours, then time required by 45 such pumps to empty the same reservoir is \_\_\_\_\_ hours.



#### Understand the problem

- If you write a problem in your own words, you may understand it better. Before writing a problem in your own words, you may need to read it over several times – perhaps aloud, so that you can hear yourself say the words.
- Once you have written the problem in your own words, you may want to make sure you have included all of the necessary information to solve the problem.

26. If  $x$  varies inversely as  $y$ , then

$x$	—	60
$y$	2	10

27. If  $x$  varies directly as  $y$ , then

$x$	12	6
$y$	48	—

28. When the speed remains constant, the distance travelled is \_\_\_\_\_ proportional to the time.
29. On increasing  $a$ ,  $b$  increases in such a manner that  $\frac{a}{b}$  remains \_\_\_\_\_ and positive, then  $a$  and  $b$  are said to vary directly with each other.
30. If on increasing  $a$ ,  $b$  decreases in such a manner that \_\_\_\_\_ remains \_\_\_\_\_ and positive, then  $a$  and  $b$  are said to vary inversely with each other.
31. If two quantities  $x$  and  $y$  vary directly with each other, then \_\_\_\_\_ of their corresponding values remains constant.
32. If two quantities  $p$  and  $q$  vary inversely with each other then \_\_\_\_\_ of their corresponding values remains constant.
33. The perimeter of a circle and its diameter vary \_\_\_\_\_ with each other.
34. A car is travelling 48 km in one hour. The distance travelled by the car in 12 minutes is \_\_\_\_\_.
35. An auto rickshaw takes 3 hours to cover a distance of 36 km. If its speed is increased by 4 km/h, the time taken by it to cover the same distance is \_\_\_\_\_.
36. If the thickness of a pile of 12 cardboard sheets is 45 mm, then the thickness of a pile of 240 sheets is \_\_\_\_\_ cm.
37. If  $x$  varies inversely as  $y$  and  $x = 4$  when  $y = 6$ , then when  $x = 3$  the value of  $y$  is \_\_\_\_\_.
38. In direct proportion,  $\frac{a_1}{b_1}$  \_\_\_\_\_  $\frac{a_2}{b_2}$

39. In case of inverse proportion,  $\frac{a_2}{a_1} = \frac{b_1}{b_2}$
40. If the area occupied by 15 postal stamps is 60 cm<sup>2</sup>, then the area occupied by 120 such postal stamps will be \_\_\_\_\_.
41. If 45 persons can complete a work in 20 days, then the time taken by 75 persons will be \_\_\_\_\_ hours.
42. Devangi travels 50 m distance in 75 steps, then the distance travelled in 375 steps is \_\_\_\_\_ km.

**In questions from 43 to 59, state whether the statements are true (T) or false (F).**

43. Two quantities  $x$  and  $y$  are said to vary directly with each other if for some rational number  $k$ ,  $xy = k$ .
44. When the speed is kept fixed, time and distance vary inversely with each other.
45. When the distance is kept fixed, speed and time vary directly with each other.
46. Length of a side of a square and its area vary directly with each other.
47. Length of a side of an equilateral triangle and its perimeter vary inversely with each other.



### Plan a strategy

- Concept maps are visual tools for organising information. A concept map shows how key concepts are related and can help you summarise and analyse information in lessons or chapters.

### Create a Concept Map

- Give your concept map a title;
- Identify the main idea of your concept map;
- List the key concepts;
- Link the concepts to show the relationships between the concepts and the main idea.

48. If  $d$  varies directly as  $t^2$ , then we can write  $dt^2 = k$ , where  $k$  is some constant.
49. If a tree 24 m high casts a shadow of 15 m, then the height of a pole that casts a shadow of 6 m under similar conditions is 9.6 m.
50. If  $x$  and  $y$  are in direct proportion, then  $(x - 1)$  and  $(y - 1)$  are also in direct proportion.
51. If  $x$  and  $y$  are in inverse proportion, then  $(x + 1)$  and  $(y + 1)$  are also in inverse proportion.
52. If  $p$  and  $q$  are in inverse variation then  $(p + 2)$  and  $(q - 2)$  are also in inverse proportion.
53. If one angle of a triangle is kept fixed then the measure of the remaining two angles vary inversely with each other.
54. When two quantities are related in such a manner that, if one increases, the other also increases, then they always vary directly.
55. When two quantities are related in such a manner that if one increases and the other decreases, then they always vary inversely.
56. If  $x$  varies inversely as  $y$  and when  $x = 6$ ,  $y = 8$ , then for  $x = 8$  the value of  $y$  is 10.
57. The number of workers and the time to complete a job is a case of direct proportion.
58. For fixed time period and rate of interest, the simple interest is directly proportional to the principal.
59. The area of cultivated land and the crop harvested is a case of direct proportion.

**In questions 60 to 62, which of the following vary directly and which vary inversely with each other and which are neither of the two?**

60. (i) The time taken by a train to cover a fixed distance and the speed of the train.
- (ii) The distance travelled by CNG bus and the amount of CNG used.
- (iii) The number of people working and the time to complete a given work.

- (iv) Income tax and the income.
- (v) Distance travelled by an auto-rickshaw and time taken.
- 61.** (i) Number of students in a hostel and consumption of food.
- (ii) Area of the walls of a room and the cost of white washing the walls.
- (iii) The number of people working and the quantity of work.
- (iv) Simple interest on a given sum and the rate of interest.
- (v) Compound interest on a given sum and the sum invested.
- 62.** (i) The quantity of rice and its cost.
- (ii) The height of a tree and the number of years.
- (iii) Increase in cost and number of shirts that can be purchased if the budget remains the same.
- (iv) Area of land and its cost.
- (v) Sales Tax and the amount of the bill.

**Solve the following :**

- 63.** If  $x$  varies inversely as  $y$  and  $x = 20$  when  $y = 600$ , find  $y$  when  $x = 400$ .
- 64.** The variable  $x$  varies directly as  $y$  and  $x = 80$  when  $y$  is 160. What is  $y$  when  $x$  is 64?
- 65.**  $l$  varies directly as  $m$  and  $l$  is equal to 5, when  $m = \frac{2}{3}$ . Find  $l$  when  $m = \frac{16}{3}$ .
- 66.** If  $x$  varies inversely as  $y$  and  $y = 60$  when  $x = 1.5$ . Find  $x$  when  $y = 4.5$ .
- 67.** In a camp, there is enough flour for 300 persons for 42 days. How long will the flour last if 20 more persons join the camp?
- 68.** A contractor undertook a contract to complete a part of a stadium in 9 months with a team of 560 persons. Later on, it was required to complete the job in 5 months. How many extra persons should he employ to complete the work?
- 69.** Sobi types 108 words in 6 minutes. How many words would she type in half an hour?

- 70.** A car covers a distance in 40 minutes with an average speed of 60 km/h. What should be the average speed to cover the same distance in 25 minutes?
- 71.** It is given that  $l$  varies directly as  $m$ .
- Write an equation which relates  $l$  and  $m$ .
  - Find the constant of proportion ( $k$ ), when  $l$  is 6 then  $m$  is 18.
  - Find  $l$ , when  $m$  is 33.
  - Find  $m$  when  $l$  is 8.
- 72.** If a deposit of Rs 2,000 earns an interest of Rs 500 in 3 years, how much interest would a deposit of Rs 36,000 earn in 3 years with the same rate of simple interest?
- 73.** The mass of an aluminium rod varies directly with its length. If a 16 cm long rod has a mass of 192 g, find the length of the rod whose mass is 105 g.
- 74.** Find the values of  $x$  and  $y$  if  $a$  and  $b$  are in inverse proportion:
- $12 \times 8$
  - $30 \times y$
- 75.** If Naresh walks 250 steps to cover a distance of 200 metres, find the distance travelled in 350 steps.
- 76.** A car travels a distance of 225 km in 25 litres of petrol. How many litres of petrol will be required to cover a distance of 540 kilometres by this car?
- 77.** From the following table, determine if  $x$  and  $y$  are in direct proportion or not.

(i)

$x$	3	6	15	20	30
$y$	12	24	45	60	120

(ii)

$x$	4	7	10	16
$y$	24	42	60	96

(iii)

$x$	1	4	9	20
$y$	1.5	6	13.5	30

- 78.** If  $a$  and  $b$  vary inversely to each other, then find the values of  $p, q, r$ ;  $x, y, z$  and  $l, m, n$ .

(i)

$a$	6	8	$q$	25
$b$	18	$p$	39	$r$

(ii)

$a$	2	$y$	6	10
$b$	$x$	12.5	15	$z$

(iii)

$a$	$l$	9	$n$	6
$b$	5	$m$	25	10

- 79.** If 25 metres of cloth costs Rs 337.50, then

- What will be the cost of 40 metres of the same type of cloth?
- What will be the length of the cloth bought for Rs 810?

- 80.** A swimming pool can be filled in 4 hours by 8 pumps of the same type. How many such pumps are required if the pool is to be filled in  $2\frac{2}{3}$  hours?

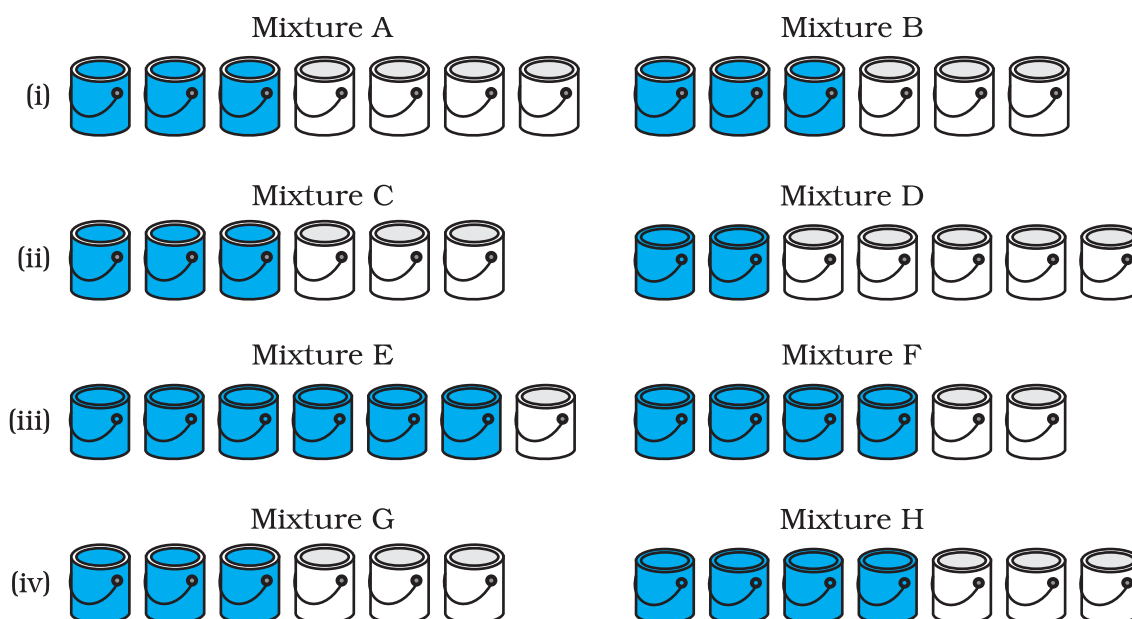
- 81.** The cost of 27 kg of iron is Rs 1,080, what will be the cost of 120 kg of iron of the same quality?

- 82.** At a particular time, the length of the shadow of Qutub Minar whose height is 72 m is 80 m. What will be the height of an electric pole, the length of whose shadow at the same time is 1000 cm?

- 83.** In a hostel of 50 girls, there are food provisions for 40 days. If 30 more girls join the hostel, how long will these provisions last?

- 84.** Campus and Welfare Committee of school is planning to develop a blue shade for painting the entire school building. For this purpose various shades are tried by mixing containers of blue paint and white paint. In each of the following mixtures, decide which is a lighter shade of blue and also find the lightest blue shade among all of them.





If one container has one litre paint and the building requires 105 litres for painting, how many container of each type is required to paint the building by darkest blue shade?

### 85. Posing a question

Work with a partner to write at least five ratio statements about this quilt, which has white, blue, and purple squares.



How many squares of each colour will be there in 12 such quilts?

86. A packet of sweets was distributed among 10 children and each of them received 4 sweets. If it is distributed among 8 children, how many sweets will each child get?

87. 44 cows can graze a field in 9 days. How many less/more cows will graze the same field in 12 days?
88. 30 persons can reap a field in 17 days. How many more persons should be engaged to reap the same field in 10 days?
89. Shabnam takes 20 minutes to reach her school if she goes at a speed of 6 km/h. If she wants to reach school in 24 minutes, what should be her speed?
90. Ravi starts for his school at 8:20 a.m. on his bicycle. If he travels at a speed of 10km/h, then he reaches his school late by 8 minutes but on travelling at 16 km/h he reaches the school 10 minutes early. At what time does the school start?

**91. Match each of the entries in Column I with the appropriate entry in Column II**

**Column I**

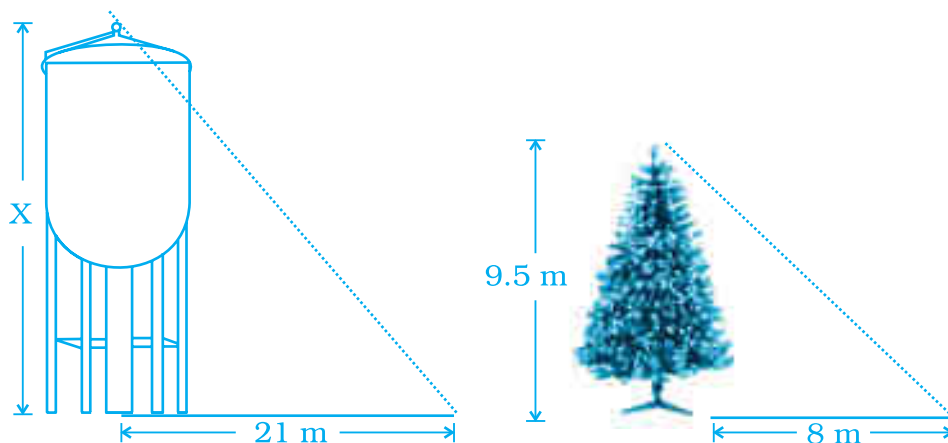
1.  $x$  and  $y$  vary inversely to each other
2. Mathematical representation of inverse variation of quantities  $p$  and  $q$
3. Mathematical representation of direct variation of quantities  $m$  and  $n$
4. When  $x = 5$ ,  $y = 2.5$  and when  $y = 5$ ,  $x = 10$
5. When  $x = 10$ ,  $y = 5$  and when  $x = 20$ ,  $y = 2.5$
6.  $x$  and  $y$  vary directly with each other
7. If  $x$  and  $y$  vary inversely then on decreasing  $x$
8. If  $x$  and  $y$  vary directly then on decreasing  $x$

**Column II**

- A.  $\frac{x}{y} = \text{Constant}$
- B.  $y$  will increase in proportion
- C.  $xy = \text{Constant}$
- D.  $p \propto \frac{1}{q}$
- E.  $y$  will decrease in proportion
- F.  $x$  and  $y$  are directly proportional
- G.  $m \propto n$
- H.  $x$  and  $y$  vary inversely
- I.  $p \propto q$
- J.  $m \propto \frac{1}{n}$

92. There are 20 grams of protein in 75 grams of sauted fish. How many grams of protein is in 225 gm of that fish?
93. Ms. Anita has to drive from Jhareda to Ganwari. She measures a distance of 3.5 cm between these villages on the map. What is the actual distance between the villages if the map scale is 1 cm = 10 km?

- 94.** A water tank casts a shadow 21 m long. A tree of height 9.5 m casts a shadow 8 m long at the same time. The lengths of the shadows are directly proportional to their heights. Find the height of the tank.

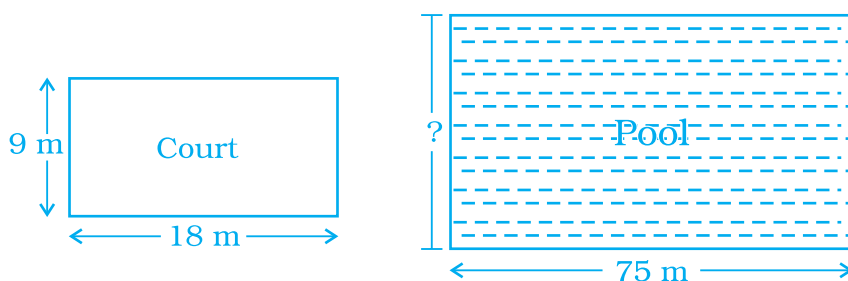


- 95.** The table shows the time four elevators take to travel various distances. Find which elevator is fastest and which is slowest.

	Distance (m)	Time (sec.)
Elevator- A	435	29
Elevator- B	448	28
Elevator- C	130	10
Elevator- D	85	5

How much distance will be travelled by elevators B and C separately in 140 sec? Who travelled more and by how much?

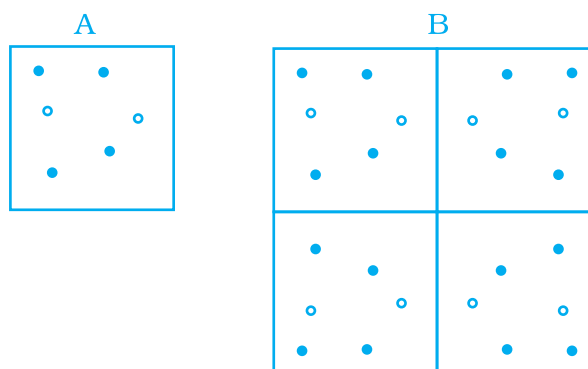
- 96.** A volleyball court is in a rectangular shape and its dimensions are directly proportional to the dimensions of the swimming pool given below. Find the width of the pool.



- 97.** A recipe for a particular type of muffins requires 1 cup of milk and 1.5 cups of chocolates. Riya has 7.5 cups of chocolates. If she is using the recipe as a guide, how many cups of milk will she need to prepare muffins?



- 98.** Pattern B consists of four tiles like pattern A. Write a proportion involving red dots and blue dots in pattern A and B. Are they in direct proportion? If yes, write the constant of proportion.



- 99.** A bowler throws a cricket ball at a speed of 120 km/h. How long does this ball take to travel a distance of 20 metres to reach the batsman?
- 100.** The variable  $x$  is inversely proportional to  $y$ . If  $x$  increases by  $p\%$ , then by what per cent will  $y$  decrease?
- 101.** Here is a key board of a harmonium:
- (a) Find the ratio of white keys to black keys on the keyboard.



- (b) What is the ratio of black keys to all keys on the given keyboard.
- (c) This pattern of keys is repeated on larger keyboard. How many black keys would you expect to find on a keyboard with 14 such patterns.

- 102.** The following table shows the distance travelled by one of the new eco-friendly energy-efficient cars travelled on gas.

Litres of gas	1	0.5	2	2.5	3	5
Distance (km)	15	7.5	30	37.5	45	75

Which type of properties are indicated by the table? How much distance will be covered by the car in 8 litres of gas?

- 103.** Kritika is following this recipe for bread. She realises her sister used most of sugar syrup for her breakfast. Kritika has only  $\frac{1}{6}$  cup of syrup, so she decides to make a small size of bread. How much of each ingredient shall she use?

**Bread recipe**

1 cup quick cooking oats

2 cups bread flour

$\frac{1}{3}$  cup sugar syrup

1 tablespoon cooking oil

$1\frac{1}{3}$  cups water

3 tablespoons yeast

1 teaspoon salt.

- 104.** Many schools have a recommended students-teacher ratio as 35:1. Next year, school expects an increase in enrolment by 280 students. How many new teachers will they have to appoint to maintain the students-teacher ratio?
- 105.** Kusum always forgets how to convert miles to kilometres and back again. However she remembers that her car's speedometer shows both miles and kilometres. She knows that travelling 50 miles per hour is same as travelling 80 kilometres per hour. To cover a distance of 200 km, how many miles Kusum would have to go?

- 106.** The students of Anju's class sold posters to raise money. Anju wanted to create a ratio for finding the amount of money her class would make for different numbers of posters sold. She knew they could raise Rs 250 for every 60 posters sold.
- How much money would Anju's class make for selling 102 posters?
  - Could Anju's class raise exactly Rs 2,000? If so, how many posters would they need to sell? If not, why?

**(D) Application, Games and Puzzles**

1. 
$$\text{Speed} = \frac{\text{Distance Travelled}}{\text{Time Taken}}$$

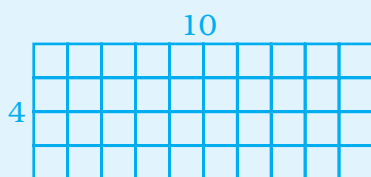
Calculate the speed for at least 10 students of your class by giving them a certain distance to walk. Measure the distance each student has walked and record the time taken by each to cover the distance. Then, complete the table given below:

Name of the student	Distance walked (in metres)	Time taken (in minutes)	Rate of speed (in m/min)
1			
2			

Which student ran the fastest?

- 2.** Figures that have the same shape but not necessarily the same size are called similar figures. We can make rectangles with similar figures by increasing or decreasing its dimensions in the same ratio. Let us make similar figures using square tiles.

What is the length of a similar rectangle where width is made up of 12 tiles? Let us consider a rectangle having 10 square tiles along the length and 4 along the breadth as shown in the figure.



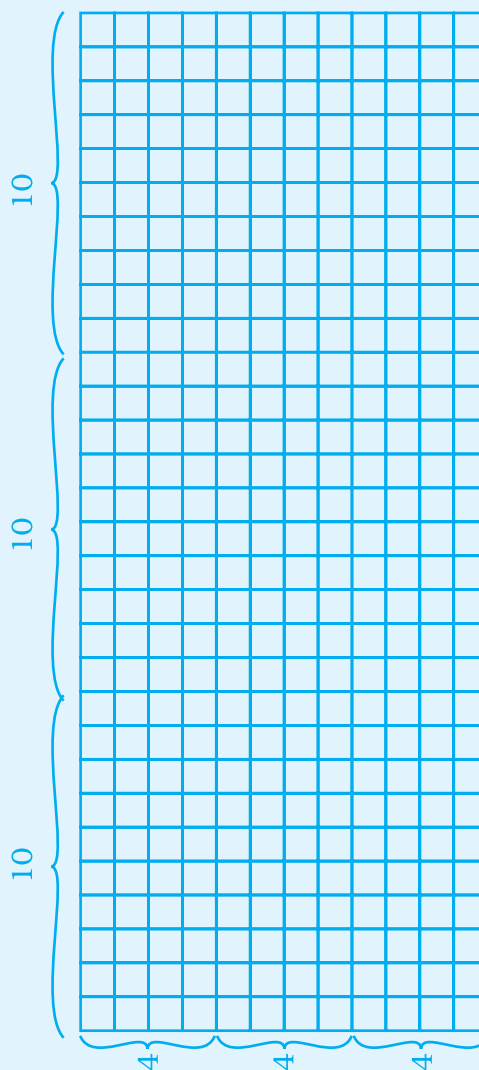
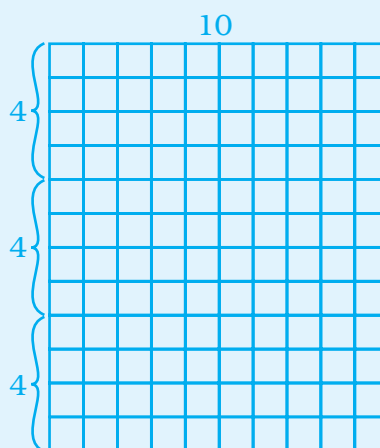
Use tiles to make a  $10 \times 4$  rectangles.

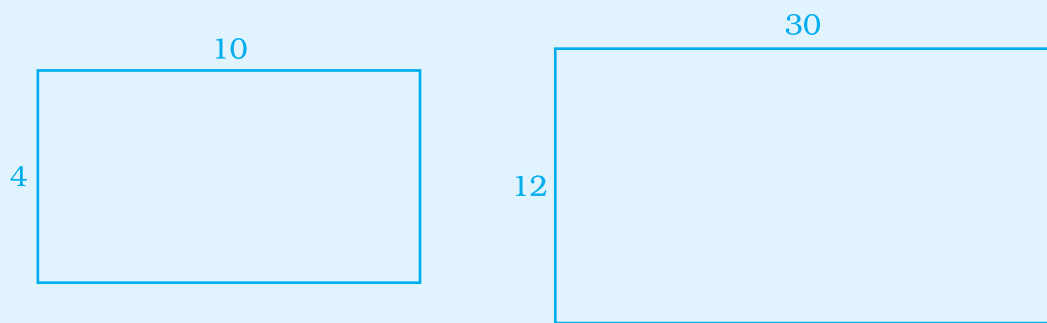
Add tiles to increase width of the rectangle to 12 tiles.

The width of the new rectangle is three times the width of the original rectangle. To keep the ratios of the lengths of two rectangles proportional, the length of this new rectangle must also be three times the length of the original rectangle.

Add tiles to increase the length of the rectangle to 30 tiles.

To check our answers, we can use the idea of direct proportion.





$$\frac{4}{10} = \frac{12}{30}$$

$$\text{or, } \frac{2}{5} = \frac{2}{5}$$

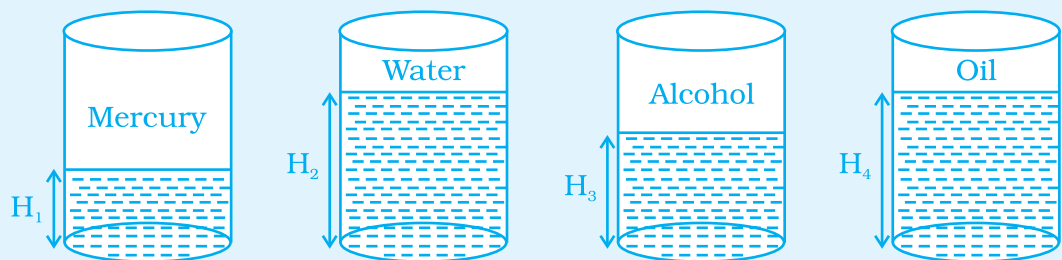
### Do yourself

Use square tiles to make similar rectangles with given dimensions and find  $x$ .

- The original rectangle is 8 tiles wide and 6 tiles long. The similar rectangle is 16 tiles wide and  $x$  tiles long.
- The original rectangle is 3 tiles wide and 7 tiles long. The similar rectangle is 9 tiles wide and  $x$  tiles long.

### 3 Inverse Variation

Take four cylindrical containers of the same size each of radius 5 cm. Fill the containers with different types of liquids with same mass (different density) like Mercury, Water, Alcohol, Oil.



Note the height in each case at which the level of liquid stands. Tabulate this information in the following table and show that this is case of inverse proportion.



Density	Mercury	Water	Alcohol	Oil
(g/cm <sup>3</sup> )	13.6 (D <sub>1</sub> )	.99 (D <sub>2</sub> )	.78 (D <sub>3</sub> )	.96 (D <sub>4</sub> )
Height (cm)	H <sub>1</sub>	H <sub>2</sub>	H <sub>3</sub>	H <sub>4</sub>

Density  $\times$  Height = Constant.

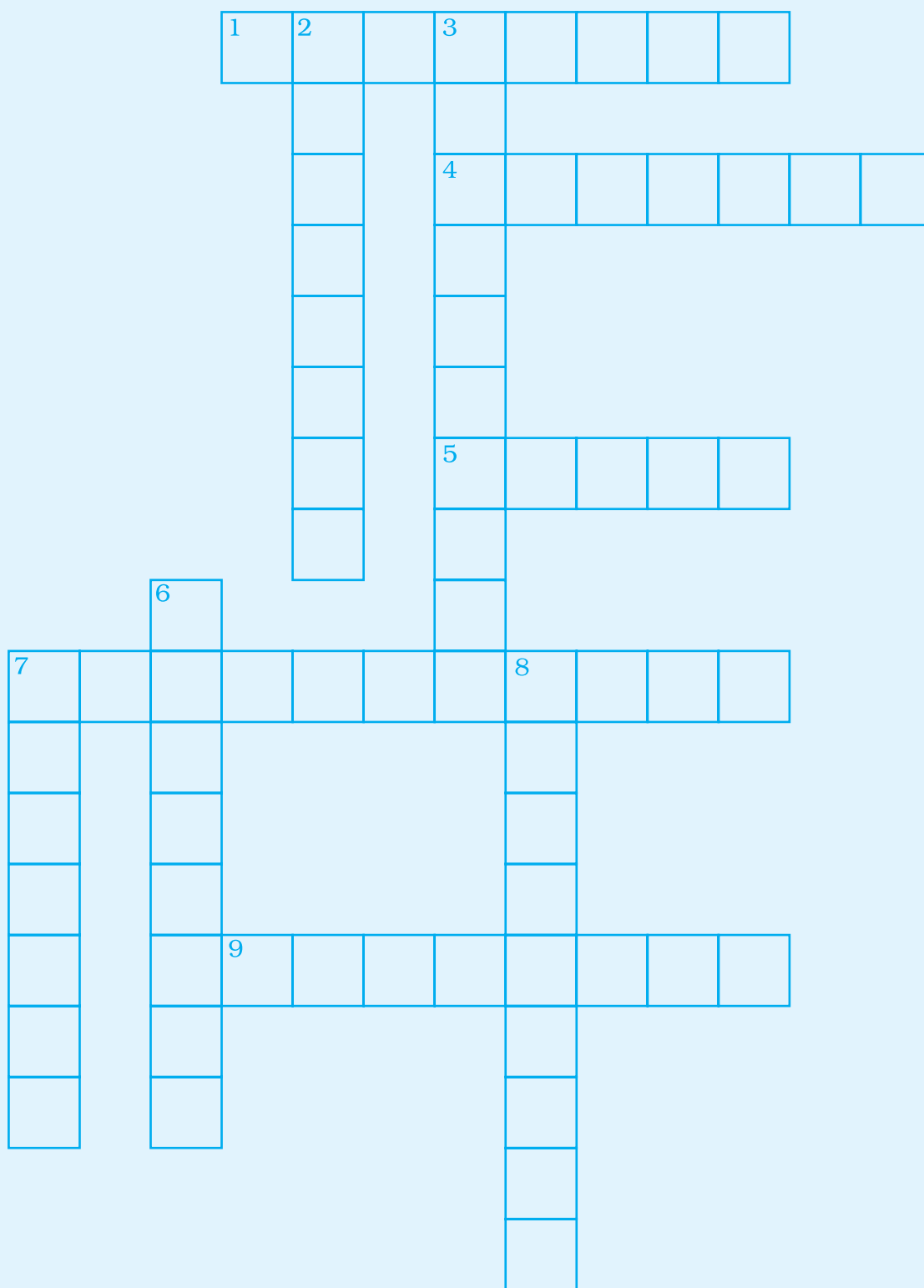
#### 4. Crossword

##### Across

- Two things are said to be varying \_\_\_\_\_ if they increase (decrease) together such that ratio of their corresponding values remains constant.
- Problems based on direct proportion can be solved using \_\_\_\_\_ method.
- More the number of workers, \_\_\_\_\_ the number of days to finish a job.
- Indirect \_\_\_\_\_.
- Two quantities are said to be in inverse proportion if an increase in one quantity causes a proportional \_\_\_\_\_ in other.

##### Down

- Speed and time are in \_\_\_\_\_ proportion to each other if distance remains the same.
- It is used to compare two ratios or make \_\_\_\_\_ fractions.
- 'k' is called \_\_\_\_\_ of variation.
- In inverse proportion, \_\_\_\_\_ of corresponding values remains constant.
- With an increase in quantity of milk, cost of milk also \_\_\_\_\_.



## Rough Work

## Rough Work