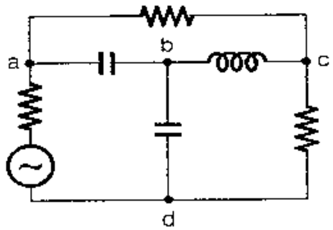
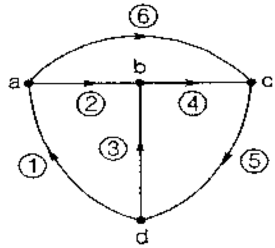
3

Graph Theory



Circuit diagram of linear graph



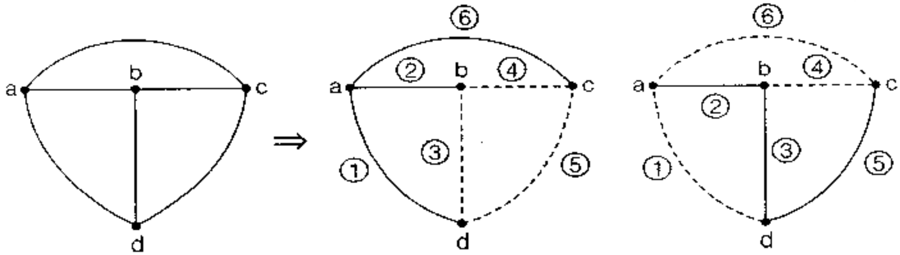
Oriented graph

For the graph shown in the above figure

Node or vertices : [a b c d]

Branch or edge: [1, 2, 3, 4, 5, 6]

1. In fully connected graph, each node is connected to all other nodes of the graph.
2. A closed loop or a closed circuit may not contain all the nodes of the graph.
3. Degree of any node represents the number of branches which are connected to it.
4. For a fully connected graph, the degree of each node is equal to the rank of the graph.
5. Tree of the graph
   (a) It contain all the nodes of the graph.
   (b) If graph contains N nodes, its tree will contain n-1 branches.
   (c) There is no closed path and hence, a tree is circuitless.
   (d) The tree of a graph is not unique.
6. The branches of the ***tree*** are represented by the tree branches or the ***twigs***, whereas the branches of the ***co-tree*** are represented by ***link*** or the ***chords***.



(a) (b)

For figure (a)

Tree: $[\underbrace{1,\ 2,\ 6}_{\text{twig}}]$ Co-tree $[\underbrace{3, 4, 5}_{\text{link or chords}}]$

For figure (b)

Tree: $[\underbrace{2,\ 3,\ 5}_{\text{twig}}]$ Co-tree $[\underbrace{1,\ 4,\ 7}_{\text{link or chords}}]$

7. (a) Number of trees =
   (i) $n^{(n-2)}$ (for fully connected graph)
   (ii) det $\{[A][A]^t\}$ (general expression)

   where, n = Number of nodes
   A = Reduced incident matrix (RIM)
   $[A]^t$ = Transpose of RIM
   det = Determinant

   (b) Number of branches in a fully connected graph = $\frac{n(n-2)}{2}$

   (c) Number of tree branches
   = n − 1
   = Number of KCL equations
   = Degree of each node of fully connected graph
   = Rank of graph
   = Number of fundamental cut-sets.

   (d) Number of link/chords
   = b − (n − 1)
   = Number of KVL equations
   = Number of tie-sets

8. A tree can be used to solve the electrical network using:
   (a) tie-set schedule
   (b) cut-set schedule

Matrixes

1. Incidence Matrix

(i) This matrix translates all the geometrical feature of the graph into an algebric expression.

(ii) Every graph has incidence matrix and vice-versa.

(iii) Each row of matrix contains +1, −1, 0 depending upon the orientation of the branches with the node.

- •⟶ = +1 (if branch is pointing away from node)
- ⟶• = −1 (if branch is pointing towards a node)
- • = 0 (if branch is not connected to the node)

(iv) Each column of matrix contain only one entry of +1 and only one entry of −1 so that the sum of the element of each column is always equal to zero.

(v) The determinant of the incident matrix of a closed loop is equal to zero.

(vi) Order of matrix is [n × b]
where n = node and b = branch.

(vii) Two graphs having same incidence matrix are called isomorphic graph.

Reduced incidence matrix

(i) A particular node is taken as a reference node and the row corresponding to that node is deleted, resulting in the reduced incidence matrix.

(ii) The order of matrix is $[(n-1)\times b]$.

(iii) This matrix can be utilised to find number of trees in a graph whether that graph is fully connected or not.

2. Cut set Matrix

It is a group of branches containing only one twig and a number of links.

Fundamental cut set matrix

(i) Fundamental cut set is a group of branch containing only one twig and the minimum number of links.

(ii) Fundamental cut set matrix can be used to write KCL equations for the given network.

Entry in the matrix

+1 ⇒ If orientation of branch is same as the orientation of cut sets related to it

−1 ⇒ If orientation of branch is opposite to orientation of cut set related to it

0 ⇒ If a cut set is not related to the branch.

Remember:

- Number of fundamental cut sets of a graph

  = Number of twigs

  = Number of KCL equations

  = $(n - 1)$

  = Number of node pair voltage
- Number of rows of the matrix = Number of fundamental cut sets
- Number of columns = Number of branches.
- Number of KCL equation = Number of fundamental cut set.

3. Tie set Matrix

It is a group of branches containing only one link and a number of twigs

Fundamental tie set matrix

(i) It is a mathematical representation of fundamental tie sets of a graph in form of matrix.

(ii) Fundamental Tie set is a group of branches containing only one link and minimum number of twigs.

Entry in the matrix

+1 ⇒ If orientation of the branch is same as the orientation of loop current .

−1 ⇒ If orientation of branch is opposite to the orientation of the loop current.

0 ⇒ If branch is not related to the loop.

Remember:

Number of fundamental tie sets for a graph.

= Number of links

= Number of KVL(mesh equation)

= $b - n + 1$