

Chapter 3. Solving Linear Equations

Ex. 3.9

Answer 2CU.

Define uniform motion problems as follows;

The problems that deal with the speed or rate at which an object moves are called uniform motion problems.

The attributes involved in the uniform motion problems are; the distance covered by the object, the speed or rate at which the object moves and the time taken by the object to cover the distance.

The formula used to solve the uniform motion problems is;

$$d = rt$$

Here;

The distance is represented by the variable d .

The rate is represented by the variable r and,

The time is represented by the variable t .

Answer 3CU.

Analyze the problem.

Ms. L owns \$2.55

The money consists of dimes and quarters.

The number of dimes is 8 more than the number of quarters.

The objective is to form a table that can be used to find the number of quarters that Ms. L has.

Suppose the number of quarters is x .

The number of dimes is 8 more than the number of quarters.

So, the number of dimes in mathematical form is $x + 8$.

A quarter is a 25 cents coin and its value in dollars is \$0.25

There are x quarters. Hence, the total value of the quarters with Ms. L is $\$0.25x$.

A dime is a 10 cents coin and its value in dollars is \$0.10

There are $x + 8$ dimes. Hence, the total value of dimes with her is $\$0.10(x + 8)$.

Write the processed information in the required table as follows;

Denomination	Number of coins	Value of each coin	Total value
Quarter	x	\$0.25	$\$0.25x$
Dime	$x + 8$	\$0.10	$\$0.10(x + 8)$

Answer 4CU.

Analyze the problem.

Mr. M creates 6 quarts of a 40% orange juice mixture by adding some quarts of pure orange juice to a 10% orange drink.

The number of quarts of pure orange juice added to the whole orange juice is p .

The total number of quarts in the 40% mixture of orange juice is 6.

Hence, the number of quarts of pure orange juice to be added to the 10% juice is $6 - p$.

Find the amount of orange juice in every percent of juice as follows;

The amount of orange juice in 10% juice is;

$$\frac{10}{100} \cdot (6 - p) = 0.10(6 - p)$$

The amount of orange juice in 100% juice is;

$$\frac{100}{100} \cdot p = p$$

The amount of orange juice in 40% juice is;

$$\frac{40}{100} \cdot 6 = 0.40(6)$$

Write the processed information in the required table as follows;

	Quarts	Amount of orange juice
10% juice	$6 - p$	$0.10(6 - p)$
100% juice	p	$1 \cdot p$
40% juice	6	$0.40(6)$

Answer 6CU.

Analyze the problem.

Mr. *M* creates 6 quarts of a 40% orange juice mixture by adding some quarts of pure orange juice to a 10% orange drink.

The number of quarts of pure orange juice added in the orange juice is p .

The objective is to find p .

The number of quarts of pure orange juice in the drink is p .

The total number of quarts in the 40% mixture of orange juice is 6.

Hence, the number of quarts of pure orange juice in 10% juice is $6 - p$.

Find the amount of orange juice in every percent of juice as follows;

The amount of orange juice in 10% juice is;

$$\frac{10}{100} \cdot (6 - p) = 0.10(6 - p)$$

The amount of orange juice in 100% juice is;

$$\frac{100}{100} \cdot p = p$$

The amount of orange juice in 40% juice is;

$$\frac{40}{100} \cdot 6 = 0.40(6)$$

Write the processed information in the required table as follows;

	Quarts	Amount of orange juice
10% juice	$6 - p$	$0.10(6 - p)$
100% juice	p	$1 \cdot p$
40% juice	6	$0.40(6)$

The amount of orange juice in the 10% juice plus the amount of juice in the 100% juice equals to the amount of juice in the 40% juice mixture.

So, write the equation that represents the situation in the problem, using the values from the table as follows;

$$0.10(6 - p) + 1 \cdot p = 0.40(6)$$

Solve the equation for p .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$0.10(6 - p) + 1 \cdot p = 0.40(6)$$

$$(0.10) \cdot 6 - (0.10) \cdot p + 1.00p = 2.40$$

$$0.60 - 0.10p + 1.00p = 2.40$$

$$0.60 + 0.90p = 2.40$$

Subtract 0.60 from both sides of the equation;

Combine the like terms

$$0.60 + 0.90p = 2.40$$

$$0.60 + 0.90p - 0.60 = 2.40 - 0.60$$

$$0.90p = 1.80$$

Divide both sides of the equation by 0.90;

Cancel the common factors

$$\frac{0.90p}{0.90} = \frac{1.80}{0.90}$$

$$p = 2$$

Therefore, Mr. M should use 2 quarts of pure orange juice.

Answer 7CU.

Analyze the problem.

Mr. M creates 6 quarts of a 40% orange juice mixture by adding some quarts of pure orange juice to a 10% orange drink.

The number of quarts of pure orange juice added in the orange juice is p .

The objective is to find the amount of 10% juice to be used.

The number of quarts of pure orange juice in the drink is p .

The total number of quarts in the 40% mixture of orange juice is 6.

Hence, the number of quarts of pure orange juice in 10% juice is $6 - p$.

Find the amount of orange juice in every percent of juice as follows;

The amount of orange juice in 10% juice is;

$$\frac{10}{100} \cdot (6 - p) = 0.10(6 - p)$$

The amount of orange juice in 100% juice is;

$$\frac{100}{100} \cdot p = p$$

The amount of orange juice in 40% juice is;

$$\frac{40}{100} \cdot 6 = 0.40(6)$$

Write the processed information in the required table as follows;

	Quarts	Amount of orange juice
10% juice	$6 - p$	$0.10(6 - p)$
100% juice	p	$1 \cdot p$
40% juice	6	$0.40(6)$

The amount of orange juice in the 10% juice plus the amount of juice in the 100% juice equals to the amount of juice in the 40% juice mixture.

So, write the equation that represents the situation in the problem, using the values from the table as follows;

$$0.10(6 - p) + 1 \cdot p = 0.40(6)$$

Solve the equation for p .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$0.10(6 - p) + 1 \cdot p = 0.40(6)$$

$$(0.10) \cdot 6 - (0.10) \cdot p + 1.00p = 2.40$$

$$0.60 - 0.10p + 1.00p = 2.40$$

$$0.60 + 0.90p = 2.40$$

Subtract 0.60 from both sides of the equation;

Combine the like terms

$$0.60 + 0.90p = 2.40$$

$$0.60 + 0.90p - 0.60 = 2.40 - 0.60$$

$$0.90p = 1.80$$

Divide both sides of the equation by 0.90;

Cancel the common factors

$$\frac{0.90p}{0.90} = \frac{1.80}{0.90}$$

$$p = 2$$

Thus, the number of quarts of pure orange juice in the drink is 2 quarts.

The number of quarts of pure orange juice in 10% juice is $6 - p$.

Substitute 2 for p and find the 10% juice;

$$\begin{aligned}6 - p &= 6 - 2 \\ &= 4\end{aligned}$$

Therefore, Mr. M should use 4 quarts of 10% juice.

Answer 8CU.

Analyze the problem.

The walnuts are sold by the Shoppe at a price of \$4.00 for a pound.

The cashews are sold at a price of \$7.00 per pound.

The mixture of 10 pounds of walnuts and some cashew nuts is sold at a price of \$5.50 per pound.

The objective is to find the number of cashews in the mixture.

Suppose the number of cashews in the mixture is x pounds

The price of the cashews is \$7.00 per pound.

Hence, the total sale price of the cashews is $\$7.00x$.

There are 10 pounds of walnuts in the mixture and each pound is for \$4.00

Hence, the total sale price of the walnuts is $(4.00) \cdot 10 = \$40$.

The mixture contains x pounds of cashews and 10 pounds of walnuts that is a total of $(x + 10)$ pounds, and the sale price of the mixture is \$5.50 per pound.

Hence, the total price of the mixture is $\$5.50(x + 10)$

Write the processed information in the following table;

Nut type	Number of pounds	Price per pound	Total price
Walnut	10	\$4.00	\$40
Cashew	x	\$7.00	$\$7.00x$
Mixture	$x + 10$	\$5.50	$\$5.50(x + 10)$

The total price of walnuts plus the total price of cashew equals the total price of the mixture.

So, write the equation that represents the situation in the problem, using the values from the table as follows;

$$40 + 7.00x = 5.50(x + 10)$$

Solve the equation for x .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$40 + 7.00x = 5.50(x + 10)$$

$$40 + 7.00x = 5.50x + 55$$

Subtract $5.50x$ from both sides of the equation;

$$40 + 7.00x = 5.50x + 55$$

$$40 + 7.00x - 5.50x = 5.50x + 55 - 5.50x$$

$$40 + 1.5x = 55$$

Subtract 40 from both sides of the equation;

Combine the like terms

$$40 + 1.5x = 55$$

$$40 + 1.5x - 40 = 55 - 40$$

$$1.5x = 15$$

Divide both sides of the equation by 1.5;

Cancel the common factors

$$\frac{1.5x}{1.5} = \frac{15}{1.5}$$

$$x = 10$$

Therefore, the Shoppe should mix **10 pounds** of cashew with 10 pounds of walnuts to sell the mixture at \$5.50 per pound.

Answer 9CU.

Draw the following table showing the grades of student B with respective credit ratings for a particular semester;

Class	Credit rating	Grade
A1	1	A
Sc	1	B
En	1	A
Sp	1	B
PE	$\frac{1}{2}$	A

The grade A equals 4 points and grade B equals 3 points.

So, redraw the table showing the grade points for each class as follows;

Class	Credit rating	Grade point
A1	1	4
Sc	1	3
En	1	4
Sp	1	3
PE	$\frac{1}{2}$	4

The grade point average is calculated using weighted average.

Weighted average for a set of data points is the ratio of the sum of the products of number of units and value per unit to the sum of the number of units.

So, write the grade point average of B as follows;

Calculate

$$\begin{aligned}
 \text{GPA} &= \frac{1(4) + 1(3) + 1(4) + 1(3) + \frac{1}{2}(4)}{1 + 1 + 1 + 1 + \frac{1}{2}} \\
 &= \frac{4 + 3 + 4 + 3 + 2}{4.5} \\
 &= \frac{24}{4.5} \\
 &\approx 3.56
 \end{aligned}$$

Therefore, the grade point average of B is about 3.56.

Answer 10CU.

Analyze the problem.

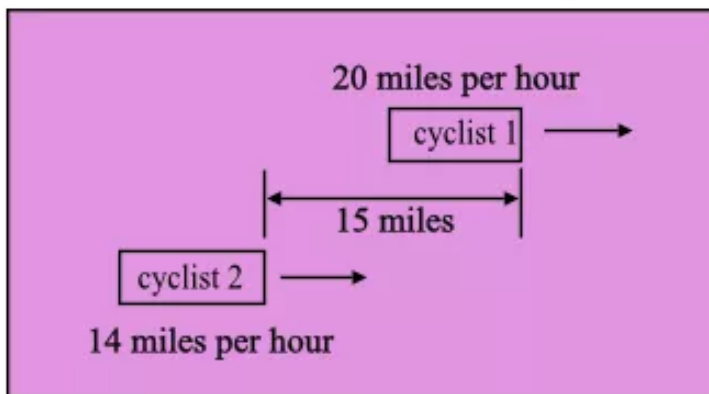
Two cyclists travel in the same direction on the same path.

The rate at which the cyclist 1 travels is 20 miles per hour.

The rate at which the cyclist 2 travels is 14 miles per hour.

Suppose the time since the cyclists start travelling is t .

Represent the information with the help of the following diagram;



If d is the distance, r is the rate at which a vehicle travels and t is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

Hence, the distance d_1 travelled by cyclist 1 at the rate of 20 miles per hour in time t is;

$$\begin{aligned} d_1 &= 20 \cdot t \\ &= 20t \end{aligned}$$

Similarly, the distance d_2 travelled by cyclist 2 at the rate of 14 miles per hour in time t is;

$$\begin{aligned} d_2 &= 14 \cdot t \\ &= 14t \end{aligned}$$

Write the processed information in the following table;

Cyclist	r	t	$d = rt$
1	20	t	$20t$
2	14	t	$14t$

The distance travelled by cyclist 1 minus the distance travelled by cyclist 2 equals the distance between the cyclists after time t .

From the table, the distance between the cyclists after time t is;

$$20t - 14t$$

To check when the cyclists will be 15 miles apart, equate the distance with 15.

So, write the equation as follows;

$$20t - 14t = 15$$

Thus, the equation that determines when the cyclists are 15 miles apart is $20t - 14t = 15$.

Solve the equation for t .

Combine the like terms and rewrite the equation as follows;

$$20t - 14t = 15$$

$$6t = 15$$

Divide both sides of the equation by 6;

Cancel the common factors

$$\frac{6t}{6} = \frac{15}{6}$$

$$t = 2.5$$

Therefore, the cyclists will be 15 miles apart after 2.5 or $2\frac{1}{2}$ hours.

Answer 11PA.

Analyze the problem.

The peanut butter cookies are sold by the shop at a price of \$6.50 per dozen.

The chocolate chip cookies are sold at a price of \$9.00 per dozen.

On a particular day the sale of peanut butter cookies was 85 dozen more than the chocolate chip cookies.

The total sales were \$4055.50

The objective is to form a table that can be used to find the number of peanut butter cookies sold.

Suppose the number of peanut butter cookies sold is p dozen.

The price of the peanut butter cookies is \$6.50 per dozen.

Hence, the total sale price of the peanut butter cookies is $\$6.50p$.

The sale of peanut butter cookies is 85 dozen more than the chocolate chip cookies.

So, the number of chocolate chip cookies sold is $p - 85$ dozen.

The price of chocolate chip cookies is \$9.00 per dozen.

Hence, the total sale price of the chocolate chip cookies is $\$9.00 \cdot (p - 85)$.

Write the processed information in the following table;

Cookie type	Number of dozens	Price per dozen	Total price
Peanut butter cookies	p	\$6.50	$\$6.50p$
Chocolate chip cookies	$p - 85$	\$7.00	$\$9.00 \cdot (p - 85)$

Answer 12PA.

Analyze the problem.

The peanut butter cookies are sold by the shop at a price of \$6.50 per dozen.

The chocolate chip cookies are sold at a price of \$9.00 per dozen.

On a particular day the sale of peanut butter cookies was 85 dozen more than the chocolate chip cookies.

The total sales were \$4055.50

The objective is to write the equation that represents the problem.

Suppose the number of peanut butter cookies sold is p dozen.

The price of the peanut butter cookies is \$6.50 per dozen.

Hence, the total sale price of the peanut butter cookies is $\$6.50p$.

The sale of peanut butter cookies is 85 dozen more than the chocolate chip cookies.

So, the number of chocolate chip cookies sold is $p - 85$ dozen.

The price of chocolate chip cookies is \$9.00 per dozen.

Hence, the total sale price of the chocolate chip cookies is $\$9.00 \cdot (p - 85)$.

Write the processed information in the following table;

Cookie type	Number of dozens	Price per dozen	Total price
Peanut butter cookies	p	\$6.50	$\$6.50p$
Chocolate chip cookies	$p - 85$	\$7.00	$\$9.00 \cdot (p - 85)$

The total sales of the cookies is \$4055.50

The total sales of the peanut butter cookies plus the total sales of the chocolate chip cookies equals the total sales of the cookies.

So, write the total sales of the cookies from the table and equate it with the actual sales as follows;

$$6.50p + 9.00(p - 85) = 4055.50$$

Therefore, the equation that represents the situation in the problem mathematically is;

$$6.50p + 9.00(p - 85) = 4055.50$$

Answer 13PA.

Analyze the problem.

The peanut butter cookies are sold by the shop at a price of \$6.50 per dozen.

The chocolate chip cookies are sold at a price of \$9.00 per dozen.

On a particular day the sale of peanut butter cookies was 85 dozen more than the chocolate chip cookies.

The total sales were \$4055.50

The objective is to find the number of peanut butter cookies sold.

Suppose the number of peanut butter cookies sold is p dozen.

The price of the peanut butter cookies is \$6.50 per dozen.

Hence, the total sale price of the peanut butter cookies is $\$6.50p$.

The sale of peanut butter cookies is 85 dozen more than the chocolate chip cookies.

So, the number of chocolate chip cookies sold is $p - 85$ dozen.

The price of chocolate chip cookies is \$9.00 per dozen.

Hence, the total sale price of the chocolate chip cookies is $\$9.00 \cdot (p - 85)$.

Write the processed information in the following table;

Cookie type	Number of dozens	Price per dozen	Total price
Peanut butter cookies	p	\$6.50	$\$6.50p$
Chocolate chip cookies	$p - 85$	\$7.00	$\$9.00 \cdot (p - 85)$

The total sales of the cookies is \$4055.50

The total sales of the peanut butter cookies plus the total sales of the chocolate chip cookies equals the total sales of the cookies.

So, write the total sales of the cookies from the table and equate it with the actual sales as follows;

$$6.50p + 9.00(p - 85) = 4055.50$$

The equation represents the situation in the problem mathematically.

Solve the equation for p .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$6.50p + 9.00(p - 85) = 4055.50$$

$$6.50p + (9.00) \cdot p - (9.00) \cdot 85 = 4055.50$$

$$6.50p + 9.00p - 765.00 = 4055.50$$

$$15.50p - 765.00 = 4055.50$$

Add 765.00 on both sides of the equation;

Combine the like terms

$$15.50p - 765.00 = 4055.50$$

$$15.50p - 765.00 + 765.00 = 4055.50 + 765.00$$

$$15.50p = 4820.50$$

Divide both sides of the equation by 15.50;

Cancel the common factors

$$\frac{15.50p}{15.50} = \frac{4820.50}{15.50}$$
$$p = 311$$

Therefore, the shop sold 311 dozen peanut butter cookies.

Answer 14PA.

Analyze the problem.

The peanut butter cookies are sold by the shop at a price of \$6.50 per dozen.

The chocolate chip cookies are sold at a price of \$9.00 per dozen.

On a particular day the sale of peanut butter cookies was 85 dozen more than the chocolate chip cookies.

The total sales were \$4055.50

The objective is to find the number of chocolate chip cookies sold.

Suppose the number of peanut butter cookies sold is p dozen.

The price of the peanut butter cookies is \$6.50 per dozen.

Hence, the total sale price of the peanut butter cookies is $\$6.50p$.

The sale of peanut butter cookies is 85 dozen more than the chocolate chip cookies.

So, the number of chocolate chip cookies sold is $p - 85$ dozen.

The price of chocolate chip cookies is \$9.00 per dozen.

Hence, the total sale price of the chocolate chip cookies is $\$9.00 \cdot (p - 85)$.

Write the processed information in the following table;

Cookie type	Number of dozens	Price per dozen	Total price
Peanut butter cookies	p	\$6.50	$\$6.50p$
Chocolate chip cookies	$p - 85$	\$9.00	$\$9.00 \cdot (p - 85)$

The total sales of the cookies is \$4055.50

The total sales of the peanut butter cookies plus the total sales of the chocolate chip cookies equals the total sales of the cookies.

So, write the total sales of the cookies from the table and equate it with the actual sales as follows;

$$6.50p + 9.00(p - 85) = 4055.50$$

The equation represents the situation in the problem mathematically.

Solve the equation for p .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$6.50p + 9.00(p - 85) = 4055.50$$

$$6.50p + (9.00) \cdot p - (9.00) \cdot 85 = 4055.50$$

$$6.50p + 9.00p - 765.00 = 4055.50$$

$$15.50p - 765.00 = 4055.50$$

Add 765.00 on both sides of the equation;

Combine the like terms

$$15.50p - 765.00 = 4055.50$$

$$15.50p - 765.00 + 765.00 = 4055.50 + 765.00$$

$$15.50p = 4820.50$$

Divide both sides of the equation by 15.50;

Cancel the common factors

$$\frac{15.50p}{15.50} = \frac{4820.50}{15.50}$$

$$p = 311$$

The number of chocolate chip cookies sold is $p - 85$.

Substitute 311 for p ;

$$\begin{aligned} p - 85 &= 311 - 85 \\ &= 226 \end{aligned}$$

Therefore, the shop sold 226 dozen chocolate chip cookies.

Answer 15PA.

Analyze the problem.

The price of gold is \$270 per ounce.

The price of silver is \$5.00 per ounce.

Gold and silver are mixed to form an alloy weighing 15 ounces and its price is \$164 per ounce.

The objective is to form a table that will help to find the amount of gold used in the alloy.

Suppose the amount of gold used in the alloy is g ounce.

The price of gold is \$270 per ounce.

Hence, the total price of the gold in the alloy is $\$270g$.

The total weight of the alloy is 15 ounce.

So, the amount of silver in the alloy is $15 - g$ ounce.

The price of silver is \$5.00 per ounce.

Hence, the total price of the silver in the alloy is $\$5.00 \cdot (15 - g)$.

Write the processed information in the required table as follows;

Metal	Number of ounces	Price per ounce	Total price
Gold	g	\$270	$\$270g$
Silver	$15 - g$	\$5.00	$\$5.00 \cdot (15 - g)$
Alloy	15	\$164	$\$15 \cdot 164 = \2460

Answer 16PA.

Analyze the problem.

The price of gold is \$270 per ounce.

The price of silver is \$5.00 per ounce.

Gold and silver are mixed to form an alloy weighing 15 ounces and its price is \$164 per ounce.

The objective is to form an equation that represents the situation in the problem.

Suppose the amount of gold used in the alloy is g ounce.

The price of gold is \$270 per ounce.

Hence, the total price of the gold in the alloy is $\$270g$.

The total weight of the alloy is 15 ounce.

So, the amount of silver in the alloy is $15 - g$ ounce.

The price of silver is \$5.00 per ounce.

Hence, the total price of the silver in the alloy is $\$5.00 \cdot (15 - g)$.

Write the processed information in the required table as follows;

Metal	Number of ounces	Price per ounce	Total price
Gold	g	\$270	$\$270g$
Silver	$15 - g$	\$5.00	$\$5.00 \cdot (15 - g)$
Alloy	15	\$164	$\$15 \cdot 164 = \2460

The total price of the gold used in the alloy plus the total price of the silver used in the alloy equals the total price of the alloy.

So, write the equation from the table as follows;

$$270g + 5.00(15 - g) = 2460$$

Therefore, the equation that represents the situation in the problem mathematically is;

$$270g + 5.00(15 - g) = 2460$$

Answer 17PA.

Analyze the problem.

The price of gold is \$270 per ounce.

The price of silver is \$5.00 per ounce.

Gold and silver are mixed to form an alloy weighing 15 ounces and its price is \$164 per ounce.

The objective is to find the amount of gold used in the alloy.

Suppose the amount of gold used in the alloy is g ounce.

The price of gold is \$270 per ounce.

Hence, the total price of the gold in the alloy is $\$270g$.

The total weight of the alloy is 15 ounce.

So, the amount of silver in the alloy is $15 - g$ ounce.

The price of silver is \$5.00 per ounce.

Hence, the total price of the silver in the alloy is $\$5.00 \cdot (15 - g)$.

Write the processed information in the required table as follows;

Metal	Number of ounces	Price per ounce	Total price
Gold	g	\$270	$\$270g$
Silver	$15 - g$	\$5.00	$\$5.00 \cdot (15 - g)$
Alloy	15	\$164	$\$15 \cdot 164 = \2460

The total price of the gold used in the alloy plus the total price of the silver used in the alloy equals the total price of the alloy.

So, write the equation from the table as follows;

$$270g + 5.00(15 - g) = 2460$$

The equation represents the situation in the problem mathematically.

Solve the equation for g .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$270g + 5.00(15 - g) = 2460$$

$$270g + (5.00) \cdot 15 - (5.00) \cdot g = 2460$$

$$270g + 75.00 - 5.00g = 2460$$

$$265g + 75 = 2460$$

Subtract 75 from both sides of the equation;

Combine the like terms

$$265g + 75 = 2460$$

$$265g + 75 - 75 = 2460 - 75$$

$$265g = 2385$$

Divide both sides of the equation by 265;

Cancel the common factors

$$\frac{265g}{265} = \frac{2385}{265}$$

$$g = 9$$

Therefore, the amount of gold used in the alloy is **9 ounce**.

Answer 18PA.

Analyze the problem.

The price of gold is \$270 per ounce.

The price of silver is \$5.00 per ounce.

Gold and silver are mixed to form an alloy weighing 15 ounces and its price is \$164 per ounce.

The objective is to find the amount of silver used in the alloy.

Suppose the amount of gold used in the alloy is g ounce.

The price of gold is \$270 per ounce.

Hence, the total price of the gold in the alloy is $\$270g$.

The total weight of the alloy is 15 ounce.

So, the amount of silver in the alloy is $15 - g$ ounce.

The price of silver is \$5.00 per ounce.

Hence, the total price of the silver in the alloy is $\$5.00 \cdot (15 - g)$.

Write the processed information in the required table as follows;

Metal	Number of ounces	Price per ounce	Total price
Gold	g	\$270	$\$270g$
Silver	$15 - g$	\$5.00	$\$5.00 \cdot (15 - g)$
Alloy	15	\$164	$\$15 \cdot 164 = \2460

The total price of the gold used in the alloy plus the total price of the silver used in the alloy equals the total price of the alloy.

So, write the equation from the table as follows;

$$270g + 5.00(15 - g) = 2460$$

The equation represents the situation in the problem mathematically.

Solve the equation for g .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$270g + 5.00(15 - g) = 2460$$

$$270g + (5.00) \cdot 15 - (5.00) \cdot g = 2460$$

$$270g + 75.00 - 5.00g = 2460$$

$$265g + 75 = 2460$$

Subtract 75 from both sides of the equation;

Combine the like terms

$$265g + 75 = 2460$$

$$265g + 75 - 75 = 2460 - 75$$

$$265g = 2385$$

Divide both sides of the equation by 265;

Cancel the common factors

$$\frac{265g}{265} = \frac{2385}{265}$$

$$g = 9$$

The amount of silver used in the alloy is $15 - g$.

Substitute 9 for g ;

$$15 - g = 15 - 9$$

$$= 6$$

Therefore, the amount of silver used in the alloy is **6 ounce**.

Answer 19PA.

Analyze the problem.

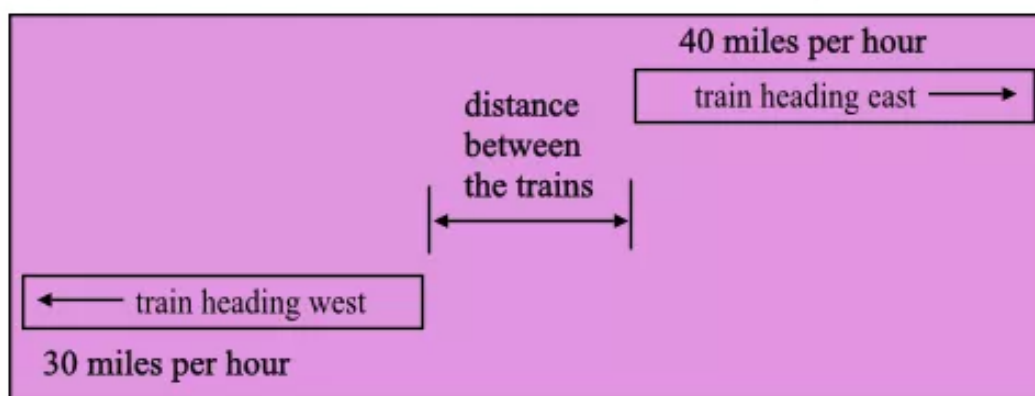
Two trains leave the station at the same time, one heading east and the other heading west.

The rate at which the train heading towards east travels is 40 miles per hour.

The rate at which the train heading towards west travels is 30 miles per hour.

The time since the departure of the trains is t .

Represent the information with the help of the following diagram;



If d is the distance, r is the rate at which a vehicle travels and t is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

Hence, the distance d_e travelled by the train heading east at the rate of 40 miles per hour in time t is;

$$\begin{aligned} d_e &= 40 \cdot t \\ &= 40t \end{aligned}$$

Similarly, the distance d_w travelled by the train heading west at the rate of 30 miles per hour in time t is;

$$\begin{aligned} d_w &= 30 \cdot t \\ &= 30t \end{aligned}$$

Write the processed information in the required table as follows;

Train	r	t	$d = rt$
Train heading east	40	t	$40t$
Train heading west	30	t	$30t$

Answer 20PA.

Analyze the problem.

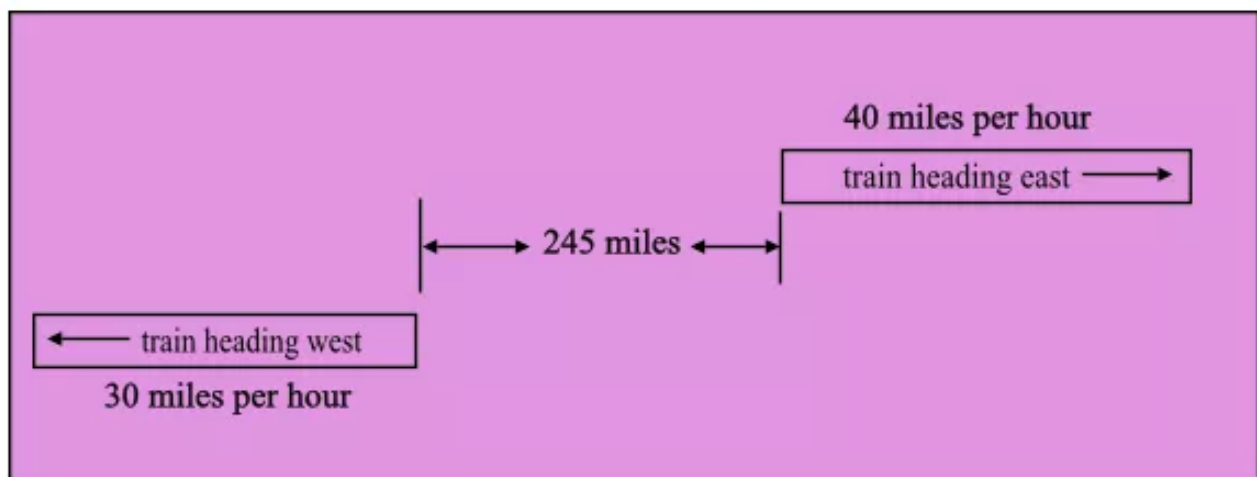
Two trains leave the station at the same time, one heading east and the other heading west.

The rate at which the train heading towards east travels is 40 miles per hour.

The rate at which the train heading towards west travels is 30 miles per hour.

The time since the departure of the trains is t .

Represent the information with the help of the following diagram;



If d is the distance, r is the rate at which a vehicle travels and t is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

Hence, the distance d_e travelled by the train heading east at the rate of 40 miles per hour in time t is;

$$\begin{aligned} d_e &= 40 \cdot t \\ &= 40t \end{aligned}$$

Similarly, the distance d_w travelled by the train heading west at the rate of 30 miles per hour in time t is;

$$\begin{aligned} d_w &= 30 \cdot t \\ &= 30t \end{aligned}$$

Write the processed information in the following table;

Train	r	t	$d = rt$
Train heading east	40	t	$40t$
Train heading west	30	t	$30t$

The distance travelled by the train heading east plus the distance travelled by the train heading west equals the distance between the trains after time t .

From the table, the distance between the trains since departure is;

$$40t + 30t$$

After time t the trains are 245 miles apart.

So, write the equation as follows;

$$40t + 30t = 245$$

Therefore, the equation that determines when the trains are 245 miles apart is

$$40t + 30t = 245$$

Answer 21PA.

Analyze the problem.

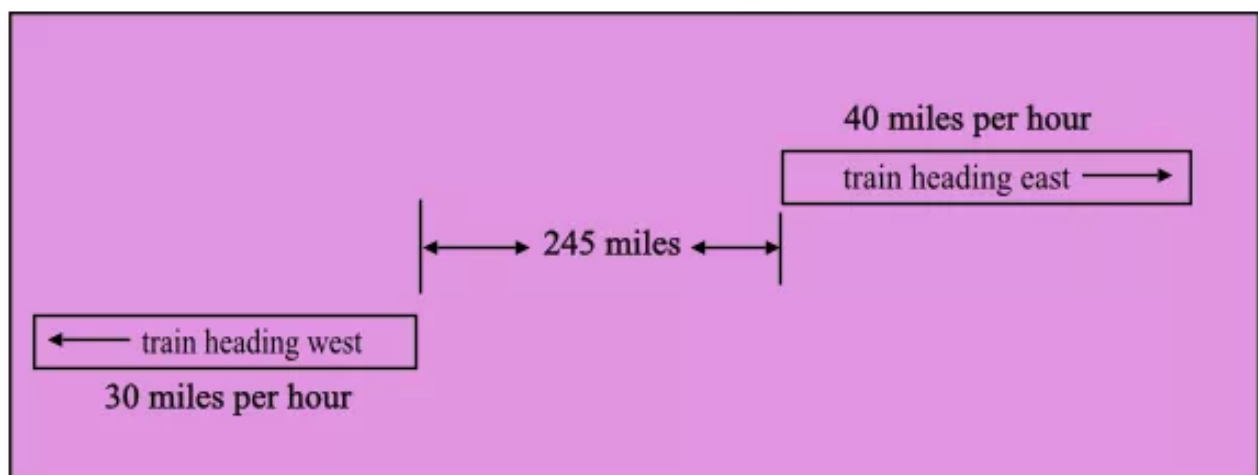
Two trains leave the station at the same time, one heading east and the other heading west.

The rate at which the train heading towards east travels is 40 miles per hour.

The rate at which the train heading towards west travels is 30 miles per hour.

The time since the departure of the trains is t .

Represent the information with the help of the following diagram;



If d is the distance, r is the rate at which a vehicle travels and t is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

Hence, the distance d_e travelled by the train heading east at the rate of 40 miles per hour in time t is;

$$\begin{aligned} d_e &= 40 \cdot t \\ &= 40t \end{aligned}$$

Similarly, the distance d_w travelled by the train heading west at the rate of 30 miles per hour in time t is;

$$\begin{aligned} d_w &= 30 \cdot t \\ &= 30t \end{aligned}$$

Write the processed information in the following table;

Train	r	t	$d = rt$
Train heading east	40	t	$40t$
Train heading west	30	t	$30t$

The distance travelled by the train heading east plus the distance travelled by the train heading west equals the distance between the trains after time t .

From the table, the distance between the trains since departure is;

$$40t + 30t$$

After time t the trains are 245 miles apart.

So, write the equation as follows;

$$40t + 30t = 245$$

Thus, the equation that determines when the trains are 245 miles apart is $40t + 30t = 245$.

Solve the equation for t .

Combine the like terms and rewrite the equation as follows;

$$\begin{aligned} 40t + 30t &= 245 \\ 70t &= 245 \end{aligned}$$

Divide both sides of the equation by 70;

Cancel the common factors

$$\begin{aligned} \frac{70t}{70} &= \frac{245}{70} \\ t &= 3.5 \end{aligned}$$

Therefore, the trains will be 245 miles apart after 3.5 or $3\frac{1}{2}$ hours.

Answer 22PA.

Analyze the problem.

The gift wrap in solid colors is sold by the band at a price of \$4.00 per roll.

The print gift wrap is sold at a price of \$6.00 per roll.

A total of 480 rolls are sold.

The total sales are \$2340.

The objective is to find the number of rolls of each kind sold.

Suppose the number of rolls of solid colors gift wrap sold is x .

The price of the solid color gift wrap is \$4.00 per roll.

Hence, the total sale price of the solid color gift wrap rolls is $\$4.00x$.

The total number of rolls sold is 480.

So, the number of print gift wrap rolls sold is $480 - x$.

The price of print gift wrap rolls is \$6.00 per roll.

Hence, the total sale price of the print gift wrap rolls is $\$6.00 \cdot (480 - x)$.

Write the processed information in the following table;

Gift wrap type	Number of rolls sold	Price per roll	Total price
Solid color gift wrap	x	\$4.00	$\$4.00x$
Print gift wrap	$480 - x$	\$6.00	$\$6.00 \cdot (480 - x)$

The total sales of the rolls is \$2340.00

The total sales of the solid color gift wrap rolls plus the total sales of the print gift wrap rolls equals the total sales of the rolls.

So, write the total sales of the gift wrap rolls from the table and equate it with the actual sales as follows;

$$4.00x + 6.00(480 - x) = 2340.00$$

The equation represents the situation in the problem mathematically.

Solve the equation for x .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Cancel the common factors

$$4.00x + 6.00(480 - x) = 2340.00$$

$$4.00x + (6.00) \cdot 480 - (6.00) \cdot x = 2340.00$$

$$4.00x + 2880.00 - 6.00x = 2340.00$$

$$-2.00x + 2880.00 = 2340.00$$

Subtract 2880.00 from both sides of the equation;

Combine the like terms

$$-2.00x + 2880.00 = 2340.00$$

$$-2.00x + 2880.00 - 2880.00 = 2340.00 - 2880.00$$

$$-2x = -540$$

Divide both sides of the equation by -2 ;

Cancel the common factors

$$\frac{-2x}{-2} = \frac{-540}{-2}$$

$$x = 270$$

Thus, the number of rolls of solid color gift wraps sold is 270 .

The number of rolls of print gift wraps sold is $480 - x$.

Substitute 270 for x ;

$$480 - x = 480 - 270$$

$$= 210$$

Thus, the number of rolls of print gift wraps sold is 210.

Therefore, the band sold 270 solid color and 210 print gift wraps.

Answer 23PA.

Analyze the problem.

The shop sells type I coffee at a price of \$6.40 per pound.

The type II coffee is sold at a price of \$7.28 per pound.

Type II coffee is mixed with 9 pounds of type I coffee to form a mixture that is sold at a price of \$6.95 per pound.

The objective is to find the amount of type II coffee in the mixture.

Suppose the amount of type II coffee in the mixture is x pounds.

The price of the type II coffee is \$7.28 per pound.

Hence, the total sale price of the type II coffee is $\$7.28x$.

The amount of type I coffee in the mixture is 9 pounds.

The price of the type I coffee is \$6.40 per pound.

Hence, the total sale price of the type I coffee is $6.40 \cdot (9) = \$57.60$.

The amount of coffee in the mixture is $(x + 9)$ pounds.

The mixture is sold at \$6.95 per pound.

Hence, the total sale price of the mixture is $\$6.95 \cdot (x + 9)$

Write the processed information in the following table;

Coffee	Weight in pounds	Price per pound	Total price
Type I	9	\$6.40	\$57.60
Type II	x	\$7.28	$\$7.28x$
Mixture	$x + 9$	\$6.95	$\$6.95 \cdot (x + 9)$

The total sales of type I coffee plus the total sales of type II coffee equals the total sales of the mixture.

So, write the following equation using the values from the table;

$$57.60 + 7.28x = 6.95(x + 9)$$

The equation represents the situation in the problem mathematically.

Solve the equation for x .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

$$57.60 + 7.28x = 6.95(x + 9)$$

$$57.60 + 7.28x = (6.95) \cdot x + (6.95) \cdot 9$$

$$57.60 + 7.28x = 6.95x + 62.55$$

Subtract 57.60 from both sides of the equation;

Cancel the common factors

$$57.60 + 7.28x = 6.95x + 62.55$$

$$57.60 + 7.28x - 57.60 = 6.95x + 62.55 - 57.60$$

$$7.28x = 6.95x + 4.95$$

Subtract $6.95x$ from both sides of the equation;

Combine the like terms

$$7.28x = 6.95x + 4.95$$

$$7.28x - 6.95x = 6.95x + 4.95 - 6.95x$$

$$0.33x = 4.95$$

Divide both sides of the equation by 0.33 ;

Cancel the common factors

$$\frac{0.33x}{0.33} = \frac{4.95}{0.33}$$

$$x = 15$$

Therefore, the amount of \$7.28 coffee in the mixture is 15 pounds.

Answer 24PA.

Analyze the problem.

The whipping cream consists of 9% butterfat.

The 2% milk consists of 2% butterfat.

A mixture of 35 gallons of milk is made with 4% butterfat.

The objective is to find the amount of whipping cream and 2% milk in the mixture.

Suppose the amount of whipping cream in the mixture is x gallons.

The whipping cream has 9% butterfat.

Hence, the amount of butterfat in x gallons of whipping cream is $0.09x$.

The total quantity of milk is 35 gallons.

So, the amount of 2% milk in the mixture is $35 - x$.

The 2% milk has 2% of butterfat.

Hence, the amount of butterfat in $(35 - x)$ gallons is $0.02 \cdot (35 - x)$.

Write the processed information in the following table;

	Quantity in gallons	Amount of butterfat
Whipping Cream	x	$0.09x$
2% milk	$35 - x$	$0.02 \cdot (35 - x)$
Total milk	35	$(0.04) \cdot 35$

The amount of butterfat in the whipping cream plus the amount of butterfat in the 2% milk equals the amount of butterfat in 35 gallons of milk.

So, write the following equation using the values from the table;

$$0.09x + 0.02(35 - x) = (0.04) \cdot 35$$

The equation represents the situation in the problem mathematically.

Solve the equation for x .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$0.09x + 0.02(35 - x) = (0.04) \cdot 35$$

$$0.09x + (0.02) \cdot 35 - (0.02) \cdot x = 1.40$$

$$0.09x + 0.70 - 0.02x = 1.40$$

$$0.07x + 0.70 = 1.40$$

Subtract 0.70 from both sides of the equation;

Combine the like terms

$$0.07x + 0.70 = 1.40$$

$$0.07x + 0.70 - 0.70 = 1.40 - 0.70$$

$$0.07x = 0.70$$

Divide both sides of the equation by 0.07 ;

Cancel the common factors

$$\frac{0.07x}{0.07} = \frac{0.70}{0.07}$$
$$x = 10$$

Thus, the amount of whipping cream in the mixture is 10 gallons.

The amount of 2% milk in the mixture is $35 - x$.

Substitute 10 for x ;

$$\begin{aligned} 35 - x &= 35 - 10 \\ &= 25 \end{aligned}$$

Thus, the amount of 2% milk in the mixture is 25 gallons.

Therefore, **10 gallons of whipping cream** and **25 gallons of 2% milk** should be mixed to get 35 gallons of milk with 4% butterfat in it.

Answer 25PA.

Analyze the problem.

There are two types of alloys.

The type I alloy consists of 25% copper.

The type II alloy consists of 50% copper.

A mixture of 1000 grams of alloy is made with 45% of copper.

The objective is to find the amount of each alloy in the mixture.

Suppose the amount of type I alloy in the mixture is x grams.

The type I alloy has 25% copper.

Hence, the amount of copper in x grams of type I alloy is $0.25x$ grams.

The total quantity of alloy is 1000 grams.

So, the amount of type II alloy in the mixture is $1000 - x$ grams.

The type II alloy has 50% of copper.

Hence, the amount of copper in $(1000 - x)$ grams is $0.50 \cdot (1000 - x)$.

Write the processed information in the following table;

Alloy	Quantity in grams	Amount of copper
Type I	x	$0.25x$
Type II	$1000 - x$	$0.50 \cdot (1000 - x)$
Mixture alloy	1000	$(0.45) \cdot 1000$

The amount of copper in type I alloy plus the amount of copper in type II alloy equals the amount of copper in 1000 grams of alloy.

So, write the following equation using the values from the table;

$$0.25x + 0.50(1000 - x) = (0.45) \cdot 1000$$

The equation represents the situation in the problem mathematically.

Solve the equation for x .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$0.25x + 0.50(1000 - x) = (0.45) \cdot 1000$$

$$0.25x + (0.50) \cdot 1000 - (0.50) \cdot x = 450$$

$$0.25x + 500 - 0.50x = 450$$

$$-0.25x + 500 = 450$$

Subtract 500 from both sides of the equation;

Combine the like terms

$$-0.25x + 500 = 450$$

$$-0.25x + 500 - 500 = 450 - 500$$

$$-0.25x = -50$$

Divide both sides of the equation by -0.25 ;

Cancel the common factors

$$\frac{-0.25x}{-0.25} = \frac{-50}{-0.25}$$

$$x = 200$$

Thus, the amount of type I alloy in the mixture is 200 grams.

The amount of type II alloy in the mixture is $1000 - x$.

Substitute 200 for x ;

$$1000 - x = 1000 - 200$$

$$= 800$$

Thus, the amount of type II alloy in the mixture is 800 grams.

Therefore, 200 grams of 25% copper alloy and 800 grams of 50% copper alloy should be mixed to get 1000 grams of alloy with 45% copper in it.

Answer 26PA.

The time taken by the airplane to travel a distance of 1000 miles due east is 2 hours.

It takes 3 hours for the plane to travel the same distance due south.

The objective is to find the average speed of the airplane.

Write the relationship between speed (rate), distance and time as follows;

$$d = rt$$

$$r = \frac{d}{t}$$

Here, the speed or rate at which the vehicle travels is r .

The distance travelled by the vehicle is d and,

The time taken to travel the distance d at rate r is t .

Use the formula to find the speed of the airplane in both directions.

Substitute 1000 for d and 2 for t to find the speed of the airplane while moving east.

So, the speed of the airplane due east is;

$$\begin{aligned}r_{east} &= \frac{1000}{2} \\ &= 500\end{aligned}$$

Thus, the speed of the airplane while moving east is 500 miles per hour.

Substitute 1000 for d and 3 for t to find the speed of the airplane while moving south.

So, the speed of the airplane due east is;

$$r_{south} = \frac{1000}{3}$$

Thus, the speed of the airplane while moving south is $\frac{1000}{3}$ miles per hour.

The time taken by the airplane to travel in each direction is different. Hence calculate the average speed using weighted average.

Weighted average for a set of data points is the ratio of the sum of the products of number of units and value per unit to the sum of the number of units.

So, calculate the average speed as follows;

$$r_{average} = \frac{r_{east}(2) + r_{south}(3)}{2 + 3}$$

Substitute 500 for r_{east} and $\frac{1000}{3}$ for r_{south} and calculate the average speed as follows;

Calculate

$$\begin{aligned}r_{average} &= \frac{500(2) + \frac{1000}{3}(3)}{2 + 3} \\ &= \frac{1000 + 1000}{5} \\ &= \frac{2000}{5} \\ &= 400\end{aligned}$$

Therefore, the average speed of the airplane is 400 miles per hour.

Answer 27PA.

Analyze the problem.

There are two types of solutions; 25% copper sulfate solution and a 60% copper sulfate solution.

A quantity of 140 milliliters of 30% copper sulfate solution is required.

The objective is to find the amount of each solution in the mixture.

Suppose the amount of 25% copper sulfate solution in the mixture is x milliliters.

Hence, the amount of copper sulfate in x milliliters of 25% solution is $0.25x$ milliliters.

The total quantity of mixture is 140 milliliters.

So, the amount of 60% copper sulfate solution in the mixture is $140 - x$ milliliters.

Hence, the amount of copper sulfate in $(140 - x)$ milliliters of 60% solution is $0.60 \cdot (140 - x)$.

Write the processed information in the following table;

Solution	Quantity in milliliters	Amount of copper sulfate
25% solution	x	$0.25x$
60% solution	$140 - x$	$0.60 \cdot (140 - x)$
30% solution	30	$(0.30) \cdot 140$

The amount of copper sulfate in 25% solution plus the amount of copper sulfate in 60% solution equals the amount of copper sulfate in 140 milliliters of 30% solution.

So, write the following equation using the values from the table;

$$0.25x + 0.60(140 - x) = (0.30) \cdot 140$$

The equation represents the situation in the problem mathematically.

Solve the equation for x .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$0.25x + 0.60(140 - x) = (0.30) \cdot 140$$

$$0.25x + (0.60) \cdot 140 - (0.60) \cdot x = 42$$

$$0.25x + 84 - 0.60x = 42$$

$$-0.35x + 84 = 42$$

Subtract 84 from both sides of the equation;

Combine the like terms

$$-0.35x + 84 = 42$$

$$-0.35x + 84 - 84 = 42 - 84$$

$$-0.35x = -42$$

Divide both sides of the equation by -0.35 ;

Cancel the common factors

$$\frac{-0.35x}{-0.35} = \frac{-42}{-0.35}$$

$$x = 120$$

Thus, the amount of 25% copper sulfate solution in the mixture is 120 milliliters.

The amount of 60% copper sulfate solution in the mixture is $140 - x$.

Substitute 120 for x ;

$$140 - x = 140 - 120$$

$$= 20$$

Thus, the amount of 60% copper sulfate solution in the mixture is 20 milliliters.

Therefore, **120 ml of 25% solution** and **20 ml of 60% solution** should be mixed to get 140 milliliters of 30% copper sulfate solution.

Answer 28PA.

Analyze the problem.

There are two types of antifreeze; 40% glycol and a 60% glycol.

A quantity of 100 gallons of 48% glycol antifreeze is required.

The objective is to find the amount of each type of antifreeze in the mixture.

Suppose the amount of 40% glycol antifreeze in the mixture is x gallons.

Hence, the amount of glycol in x gallons of 40% glycol antifreeze is $0.40x$ gallons.

The total quantity of mixture is 100 gallons.

So, the amount of 60% glycol antifreeze in the mixture is $100 - x$ gallons.

Hence, the amount of glycol in $(100 - x)$ gallons of 60% glycol antifreeze is $0.60 \cdot (100 - x)$.

Write the processed information in the following table;

Antifreeze	Quantity in gallons	Amount of glycol
40% glycol	x	$0.40x$
60% glycol	$100 - x$	$0.60 \cdot (100 - x)$
48% glycol	48	$(0.48) \cdot 100$

The amount of glycol in 40% glycol antifreeze plus the amount of glycol in 60% glycol antifreeze equals the amount of glycol in 48% glycol antifreeze.

So, write the following equation using the values from the table;

$$0.40x + 0.60(100 - x) = (0.48) \cdot 100$$

The equation represents the situation in the problem mathematically.

Solve the equation for x .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$0.40x + 0.60(100 - x) = (0.48) \cdot 100$$

$$0.40x + (0.60) \cdot 100 - (0.60) \cdot x = 48$$

$$0.40x + 60 - 0.60x = 48$$

$$-0.20x + 60 = 48$$

Subtract 60 from both sides of the equation;

Combine the like terms

$$-0.20x + 60 = 48$$

$$-0.20x + 60 - 60 = 48 - 60$$

$$-0.20x = -12$$

Divide both sides of the equation by -0.20 ;

Cancel the common factors

$$\frac{-0.20x}{-0.20} = \frac{-12}{-0.20}$$

$$x = 60$$

Thus, the amount of 40% glycol antifreeze in the mixture is 60 gallons.

The amount of 60% glycol antifreeze in the mixture is $100 - x$.

Substitute 60 for x ;

$$100 - x = 100 - 60$$

$$= 40$$

Thus, the amount of 60% glycol antifreeze in the mixture is 40 gallons.

Hence, 60 gallons of 40% glycol antifreeze and 40 gallons of 60% glycol antifreeze should be mixed to get 100 gallons of 48% glycol antifreeze.

Answer 29PA.

Analyze the problem.

The weight of a test score is three times as much as that of a quiz score.

So, consider the grade point for a quiz as 1.

Hence, the grade point of a test is 3.

A student scores 85 and 92 grades in her tests.

She scores grades of 82, 75 and 95 in her quizzes.

The objective is to find the average grade of the student.

Write the processed information in the following table;

Class	Score	Grade point
Test	85	3
Test	92	3
Quiz	82	1
Quiz	75	1
Quiz	95	1

The average grade is calculated using weighted average.

Weighted average for a set of data points is the ratio of the sum of the products of number of units and value per unit to the sum of the number of units.

So, write the average grade of the student as follows;

Calculate

$$\begin{aligned}\text{Average grade} &= \frac{85(3) + 92(3) + 82(1) + 75(1) + 95(1)}{3 + 3 + 1 + 1 + 1} \\ &= \frac{255 + 276 + 82 + 75 + 95}{9} \\ &= \frac{783}{9} \\ &= 87\end{aligned}$$

Therefore, the average grade of the student is 87.

Answer 30PA.

Analyze the problem.

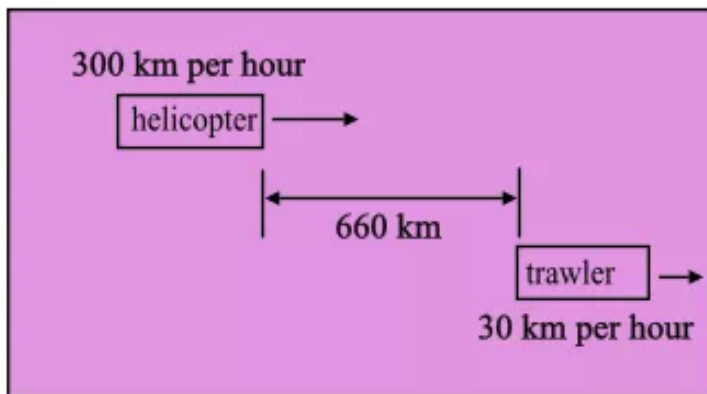
The difference between the trawler and the helicopter is 660 km.

The rate at which the trawler travels is 30 km per hour.

The rate at which the helicopter travels is 300 km per hour.

Suppose the time since the helicopter starts heading towards the trawler is t .

Represent the information with the help of the following diagram;



If d is the distance, r is the rate at which a vehicle travels and t is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

Hence, the distance d_t travelled by the trawler at the rate of 30 km per hour in time t is;

$$\begin{aligned} d_t &= 30 \cdot t \\ &= 30t \end{aligned}$$

Similarly, the distance d_h travelled by the helicopter at the rate of 300 km per hour in time t is;

$$\begin{aligned} d_h &= 300 \cdot t \\ &= 300t \end{aligned}$$

Write the processed information in the following table;

	r	t	$d = rt$
Trawler	30	t	$30t$
Helicopter	300	t	$300t$

The helicopter will reach the trawler when it covers the distance travelled by the trawler in time t plus the distance between itself and the trawler.

Thus, the helicopter will reach the trawler when the distance covered by it in time t equals the distance covered by the trawler in time t plus the distance between the helicopter and the trawler.

So, write the equation using the values from the table as follows;

$$300t = 30t + 660$$

Subtract $30t$ from both sides of the equation;

Combine the like terms

$$300t = 30t + 660$$

$$300t - 30t = 30t + 660 - 30t$$

$$270t = 660$$

Divide both sides of the equation by 270;

Cancel the common factors

$$\frac{270t}{270} = \frac{660}{270}$$

$$t \approx 2.4$$

Write the value of t as follows;

$$t = 2.4 \text{ hours}$$

$$= 2 \text{ hours} + 0.4 \text{ hours}$$

Convert 0.4 hours to minutes using the following relation;

$$1 \text{ hour} = 60 \text{ minutes}$$

So, 0.4 hours is;

$$0.4 \times 60 = 24 \text{ minutes}$$

Therefore, the helicopter will reach the trawler after approximately **2 hours 24 minutes**.

Answer 31PA.

Analyze the problem.

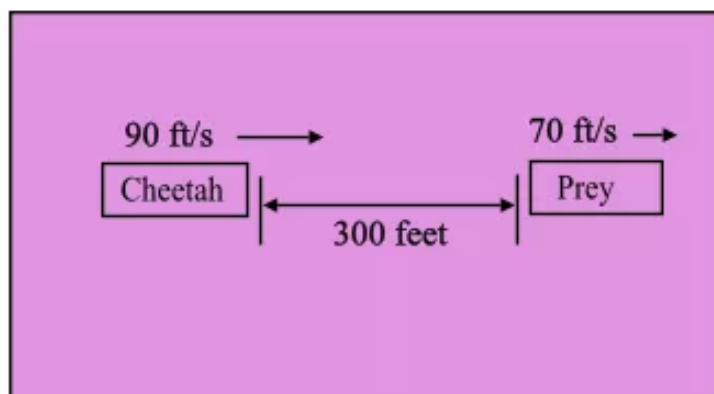
The difference between the cheetah and its prey is 300 ft.

The rate at which the cheetah runs is 90 ft per second.

The rate at which the prey runs is 70 ft per second.

Suppose the time since the cheetah starts heading towards the prey is t .

Represent the information with the help of the following diagram;



If d is the distance, r is the rate at which a vehicle travels and t is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

Hence, the distance d_c travelled by the cheetah at the rate of 90 ft per second in time t is;

$$\begin{aligned} d_c &= 90 \cdot t \\ &= 90t \end{aligned}$$

Similarly, the distance d_p travelled by the prey at the rate of 70 ft per second in time t is;

$$\begin{aligned} d_p &= 70 \cdot t \\ &= 70t \end{aligned}$$

Write the processed information in the following table;

	r	t	$d = rt$
Cheetah	90	t	$90t$
Prey	70	t	$70t$

The cheetah will catch the prey when it covers the distance travelled by the prey in time t plus the distance between itself and the prey.

Thus, the cheetah will catch the prey when the distance covered by it in time t equals the distance covered by the prey in time t plus the distance between the cheetah and the prey.

So, write the equation using the values from the table as follows;

$$90t = 70t + 300$$

Subtract $70t$ from both sides of the equation;

Combine the like terms

$$90t = 70t + 300$$

$$90t - 70t = 70t + 300 - 70t$$

$$20t = 300$$

Divide both sides of the equation by 20;

Cancel the common factors

$$\frac{20t}{20} = \frac{300}{20}$$

$$t = 15$$

Therefore, the cheetah will catch its prey after **15 seconds**.

Answer 32PA.

Analyze the problem.

The sprinter runs at a rate of 8.2 meters per second.

His opponent runs at a rate of 8 meters per second.

The sprinter starts the 200 meter race 1 second late than his opponent.

Suppose the time since the race starts is t .

If d is the distance, r is the rate at which a vehicle or an object travels and t is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

$$t = \frac{d}{r}$$

Hence, the time taken by the opponent to complete the 200 meter race is;

$$\begin{aligned} t_o &= \frac{200}{8} \\ &= 25 \end{aligned}$$

Thus, the opponent completes the race in 25 seconds.

The sprinter will catch his opponent before the end if he completes the race before 25 seconds.

That is, if he covers the 200 meter distance plus the distance covered by his opponent in the initial 1 second, before 25 seconds.

The opponent runs at a rate of 8 meters per second. Hence, he covers a distance of 8 meters in 1 second.

So, find the time taken by the sprinter to cover a distance of $200 + 8 = 208$ meters at a rate of 8.2 meters per second.

So, substitute 208 for d and 8.2 for r in the formula for t ,

$$\begin{aligned} t_s &= \frac{208}{8.2} \\ &\approx 25.37 \end{aligned}$$

Thus, completes the race in 25.37 seconds which is 37 seconds late than that of the opponent.

Therefore, the sprinter will not catch the opponent before the end of the race.

Answer 33PA.

Analyze the problem.

The radiator of the car is filled with 25% antifreeze solution.

The total capacity of the radiator is 16 quarts.

Some of the antifreeze solution is drained out and replaced with pure antifreeze to get a 40% antifreeze solution.

The objective is to find the amount of 25% antifreeze solution drained.

Suppose the amount of the amount of 25% antifreeze solution drained is x quarts.

Thus, the amount of solution replaced with pure antifreeze is x quarts.

So, the amount of pure antifreeze that is 100% antifreeze is $1.00x$.

The total capacity of the radiator is 16 quarts.

So, the amount of 25% antifreeze solution in the 40% antifreeze solution is $16 - x$ quarts.

Hence, the amount of antifreeze in $(16 - x)$ quarts of 25% antifreeze solution is $0.25 \cdot (16 - x)$.

Write the processed information in the following table;

Antifreeze	Quantity in quarts	Amount of antifreeze
25% solution	$16 - x$	$0.25 \cdot (16 - x)$
100% solution	x	$1.00 \cdot x$
40% solution	16	$(0.40) \cdot 16$

The amount of antifreeze in 25% solution plus the amount of antifreeze in 100% solution equals the amount of antifreeze in 40% solution.

So, write the following equation using the values from the table;

$$0.25(16 - x) + 1.00x = (0.40) \cdot 16$$

The equation represents the situation in the problem mathematically.

Solve the equation for x .

Use the distributive property to open the parentheses.

The distributive states that for any real numbers a , b and c ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

Combine the like terms

$$0.25(16 - x) + 1.00x = (0.40) \cdot 16$$

$$(0.25) \cdot 16 - (0.25) \cdot x + 1.00x = 6.40$$

$$4.00 - 0.25x + 1.00x = 6.40$$

$$0.75x + 4.00 = 6.40$$

Subtract 4.00 from both sides of the equation;

Combine the like terms

$$0.75x + 4.00 = 6.40$$

$$0.75x + 4.00 - 4.00 = 6.40 - 4.00$$

$$0.75x = 2.40$$

Divide both sides of the equation by **0.75**;

Cancel the common factors

$$\frac{0.75x}{0.75} = \frac{2.40}{0.75}$$

$$x = 3.2$$

Therefore, the amount of 25% antifreeze that should be drained and replaced with pure antifreeze is **3.2 quarts**.

Answer 34PA.

Analyze the problem.

Two trains; an express train and a local train travel between two same stations.

The rate at which the express train travels is 80 km per hour.

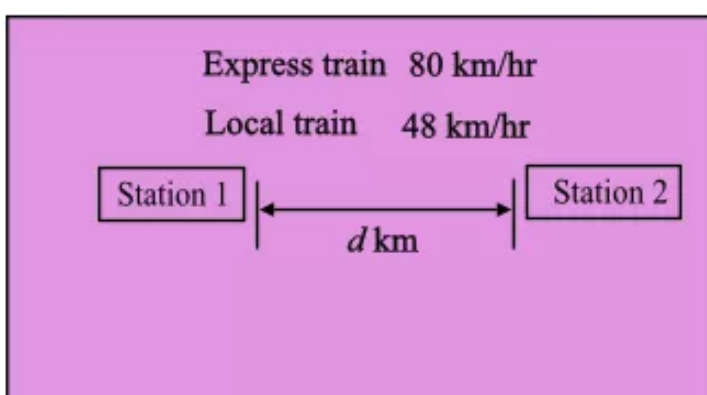
The rate at which the local train travels is 48 km per hour.

The local train takes two hours more than the express train.

The objective is to find the distance between the two stations.

Suppose the distance between the stations is d .

Represent the information with the help of the following diagram;



If d is the distance, r is the rate at which a vehicle travels and t is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

$$t = \frac{d}{r}$$

Hence, the time taken by the express train to travel the distance d at the rate of 80 km per hour is;

$$t_e = \frac{d}{80}$$

Similarly, the time taken by the local train to travel the distance d at the rate of 48 km per hour is;

$$t_l = \frac{d}{48}$$

Write the processed information in the following table;

	r	d	$t = \frac{d}{r}$
Express train	80	d	$\frac{d}{80}$
Local train	48	d	$\frac{d}{48}$

The local train takes 2 hours longer than the express train to cover the same distance.

Hence, the time taken by the express train equals the time taken by the local train to cover the distance between the stations plus 2.

So, write the equation using the values from the table as follows;

$$\frac{d}{80} = \frac{d}{48} + 2$$

Solve the equation for d .

Subtract $\frac{d}{48}$ from both sides of the equation;

Cancel the common factors

Combine the like terms

$$\frac{d}{80} - \frac{d}{48} = 2$$

$$\frac{5 \cdot d - 3 \cdot d}{240} = 2$$

$$\frac{2d}{240} = 2$$

$$\frac{d}{120} = 2$$

Multiply both sides of the equation by 120;

$$\frac{d}{120} = 2$$

$$120 \cdot \frac{d}{120} = 120 \cdot 2$$

$$d = 240$$

Therefore, the distance between the stations is 240 kilometers.

Answer 35PA.

Write the following formula that gives the rating of quarterbacks of football teams.

$$R = [50 + 2000(C \div A) + 8000(T \div A) - 10000(I \div A) + 100(Y \div A)] \div 24$$

Here;

The rating of the quarterback is R .

The number of completions is C .

The number of passing attempts is A .

The number of touchdown passes is T .

The number of interceptions is I and,

The number of yards gained by passing is Y .

For a player D ;

The number of completions is 297.

The number of passing attempts is 474.

The number of touchdown passes is 33.

The number of interceptions is 16 and,

The number of yards gained by passing is 3937

The objective is to find the rating of player D .

So, substitute 297 for C , 474 for A , 33 for T , 16 for I and 3937 for Y in the formula;

$$\begin{aligned} R &= [50 + 2000(C \div A) + 8000(T \div A) - 10000(I \div A) + 100(Y \div A)] \div 24 \\ &= [50 + 2000(297 \div 474) + 8000(33 \div 474) - 10000(16 \div 474) + 100(3937 \div 474)] \div 24 \end{aligned}$$

Perform the operations in the parentheses first;

$$\begin{aligned} R &\approx [50 + 2000(0.6266) + 8000(0.0696) - 10000(0.0338) + 100(8.3059)] \div 24 \\ &\approx [50 + 1253.2 + 556.8 - 338 + 830.59] \div 24 \\ &\approx (2352.59) \div 24 \\ &\approx 98.02 \end{aligned}$$

Thus, the rating is approximately 98.

Therefore, the rating of player D according to the mentioned weighted average formula is about **98.0**.

Answer 36PA.

Write the following equation;

$$1.00x + 0.28(40) = 0.40(x + 40)$$

The objective is to form a question that describes the equation.

Suppose the problem consists of a mixture of alloy formed using two metal alloys containing copper.

The first term in the equation relates to the amount of copper in the first alloy, the second term relates to the amount of copper in the second alloy and the term to the right side of the equality relates to the amount of copper in the mixture alloy.

The first term of the equation is $1.00x$.

Hence, the amount of copper in the first alloy is 100% and there are x units of first alloy in the mixture.

The second term of the equation is $0.28(40)$.

Hence, the amount of copper in the second alloy is 28% and there are 40 units of second alloy in the mixture.

The term on the right side of the equality is $0.40(x + 40)$.

Hence, the amount of copper in the mixture alloy is 40% and the total mixture formed is $x + 40$ units.

So, form the problem as follows;

How many units of a 100% copper alloy should be added to 40 units of 28% copper alloy to form a mixture of a 40% copper alloy?

Answer 38PA.

Analyze the problem.

Ms. *E* invests \$6000 in two accounts.

The amount invested in an account paying 4.5% is \$*d*.

She invests the remaining amount in the account paying 6%.

Hence, the amount invested in the account paying 6% is $\$(6000 - d)$.

The objective is to find the interest earned by the account paying 6% in one year.

The interest *I* on an investment *P* at the rate of *r*% for *n* years is given by the following formula;

$$I = \frac{P \cdot R \cdot N}{100}$$

So, substitute $(6000 - d)$ for *P*, 6 for *r* and 1 for *n* in the formula;

$$\begin{aligned} I &= \frac{P \cdot R \cdot N}{100} \\ &= \frac{(6000 - d) \cdot 6 \cdot 1}{100} \\ &= \frac{6}{100}(6000 - d) \\ &= 0.06(6000 - d) \end{aligned}$$

Thus, the amount of interest obtained on the investment made at the account paying 6% after 1 year is $\$0.06(6000 - d)$.

This answer is shown in option D.

Therefore, choose option D.

Answer 39PA.

Analyze the problem.

The distance of the trip travelled by Mr. *T* is 616 miles.

The time taken to complete the trip is 16 hours.

He spent 2 hours for petrol, food and breaks.

Hence, the time taken to drive the total distance is $16 - 2 = 14$ hours.

The objective is to find the average speed of Mr. *T*.

If d is the distance, r is the rate at which a vehicle travels and t is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

$$r = \frac{d}{t}$$

The total distance covered by Mr. T is 616 miles.

The total time taken to cover this distance is 14 hours.

So, substitute 616 for d and 14 for t in the formula for rate;

$$\begin{aligned} r &= \frac{d}{t} \\ &= \frac{616}{14} \\ &= 44 \end{aligned}$$

Thus, the average speed of Mr. T is **44 miles per hour**.

This answer is shown in option C.

Therefore, choose option **C**.

Answer 40MYS.

Write the following original equation;

$$3t - 4 = 6t - s$$

The objective is to solve the equation for the variable t .

So, collect the terms containing the variable t on one side of the equation.

Add s on both sides of the equation;

Collect the like terms together and combine them

$$3t - 4 = 6t - s$$

$$3t - 4 + s = 6t - s + s$$

$$3t + s - 4 = 6t$$

Subtract $3t$ from both sides of the equation and rewrite the equation as follows;

Collect the like terms together and combine them

$$3t + s - 4 = 6t$$

$$3t + s - 4 - 3t = 6t - 3t$$

$$s - 4 = 3t$$

$$3t = s - 4$$

Divide both sides of the equation by 3;

Cancel the common factors

$$\frac{3t}{3} = \frac{s-4}{3}$$
$$t = \frac{s-4}{3}$$

Thus, the solution of the equation in terms of the variable t is $t = \frac{s-4}{3}$.

Answer 41MYS.

Write the following original equation;

$$a+6 = \frac{b-1}{4}$$

The objective is to solve the equation for the variable b .

So, collect the terms containing the variable b on one side of the equation.

Multiply both sides of the equation by 4;

$$a+6 = \frac{b-1}{4}$$
$$4 \cdot (a+6) = 4 \cdot \frac{b-1}{4}$$

Use the distributive property to open the parenthesis.

The distributive property states that for any real numbers a , b and c ;

$$a \cdot (b \pm c) = a \cdot b \pm a \cdot c$$

So, write the equation as follows;

$$4 \cdot (a+6) = 4 \cdot \frac{b-1}{4}$$
$$4 \cdot a + 4 \cdot 6 = b-1$$
$$4a + 24 = b-1$$

Add 1 on both sides of the equation;

Collect the like terms together and combine them

$$4a + 24 = b-1$$
$$4a + 24 + 1 = b-1+1$$
$$4a + 25 = b$$
$$b = 4a + 25$$

Thus, the solution of the equation in terms of the variable b is $b = 4a + 25$.

Answer 42MYS.

The original value of the product is 25.

The new value of the product is 14.

The objective is to find the percent of change.

If the new value is greater than the old value then this means that there is an increase in the old value of the quantity. Hence the change in percent is called as the percent of increase.

If the new value is less than the old value then this means that there is a decrease in the original value. Hence the change in percent is called the percent of decrease.

Observe that the new value of the product, 14, is less than the original value, 25.

Hence, the percent of change is a percent decrease.

Use the following proportion to find the percent of change;

$$\frac{\text{change in value}}{\text{original value}} = \frac{r}{100}$$

Here r is the percent of change.

The new value is less than the original value. Hence, find the change in value as follows;

Change in value = Original value – new value.

So, calculate;

$$\begin{aligned}\text{change in value} &= 25 - 14 \\ &= 11\end{aligned}$$

Substitute 11 for change in value and 25 for original value in the proportion mentioned above and rewrite the proportion as follows;

$$\begin{aligned}\frac{\text{change in value}}{\text{original value}} &= \frac{r}{100} \\ \frac{11}{25} &= \frac{r}{100}\end{aligned}$$

Cross multiply the terms and form the following equation;

$$\begin{aligned}\frac{11}{25} &= \frac{r}{100} \\ 11 \cdot 100 &= r \cdot 25 \\ 1100 &= 25r \\ 25r &= 1100\end{aligned}$$

Divide both sides of the equation by 25;

$$\begin{aligned}\frac{25r}{25} &= \frac{1100}{25} \\ r &= 44\end{aligned}$$

The percent of change is 44%.

Therefore, the percent of decrease in the original value is 44%.

Answer 43MYS.

The original value of the product is 35.

The new value of the product is 42.

The objective is to find the percent of change.

If the new value is greater than the old value then this means that there is an increase in the old value of the quantity. Hence the change in percent is called as the percent of increase.

If the new value is less than the old value then this means that there is a decrease in the original value. Hence the change in percent is called the percent of decrease.

Observe that the new value of the product, 42, is greater than the original value, 35.

Hence, the percent of change is a percent increase.

Use the following proportion to find the percent of change;

$$\frac{\text{change in value}}{\text{original value}} = \frac{r}{100}$$

Here r is the percent of change.

The new value is greater than the original value. Hence, find the change in value as follows;

Change in value = new value – original value.

So, calculate;

$$\begin{aligned}\text{change in value} &= 42 - 35 \\ &= 7\end{aligned}$$

Substitute 7 for change in value and 35 for original value in the proportion mentioned above and rewrite the proportion as follows;

$$\begin{aligned}\frac{\text{change in value}}{\text{original value}} &= \frac{r}{100} \\ \frac{7}{35} &= \frac{r}{100}\end{aligned}$$

Cross multiply the terms and form the following equation;

$$\begin{aligned}\frac{7}{35} &= \frac{r}{100} \\ 7 \cdot 100 &= r \cdot 35 \\ 700 &= 35r \\ 35r &= 700\end{aligned}$$

Divide both sides of the equation by 35;

$$\begin{aligned}\frac{35r}{35} &= \frac{700}{35} \\ r &= 20\end{aligned}$$

The percent of change is 20%.

Therefore, the percent of increase in the original value is 20%.

Answer 44MYS.

The original value of the product is 244.

The new value of the product is 300.

The objective is to find the percent of change.

If the new value is greater than the old value then this means that there is an increase in the old value of the quantity. Hence the change in percent is called as the percent of increase.

If the new value is less than the old value then this means that there is a decrease in the original value. Hence the change in percent is called the percent of decrease.

Observe that the new value of the product, 300, is greater than the original value, 244.

Hence, the percent of change is a percent increase.

Use the following proportion to find the percent of change;

$$\frac{\text{change in value}}{\text{original value}} = \frac{r}{100}$$

Here r is the percent of change.

The new value is greater than the original value. Hence, find the change in value as follows;

Change in value = new value – original value.

So, calculate;

$$\begin{aligned}\text{change in value} &= 300 - 244 \\ &= 56\end{aligned}$$

Substitute 56 for change in value and 244 for original value in the proportion mentioned above and rewrite the proportion as follows;

$$\begin{aligned}\frac{\text{change in value}}{\text{original value}} &= \frac{r}{100} \\ \frac{56}{244} &= \frac{r}{100}\end{aligned}$$

Cross multiply the terms and form the following equation;

$$\begin{aligned}\frac{56}{244} &= \frac{r}{100} \\ 56 \cdot 100 &= r \cdot 244 \\ 5600 &= 244r \\ 244r &= 5600\end{aligned}$$

Divide both sides of the equation by 244;

$$\begin{aligned}\frac{244r}{244} &= \frac{5600}{244} \\ r &\approx 22.95\end{aligned}$$

Round up the number to the nearest whole number.

So, the percent of change is 23%.

Therefore, the percent of increase in the original value is about 23%.

Answer 45MYS.

The probability of the occurrence of the event is $\frac{2}{3}$.

The objective is to find the odds that the event will occur.

The probability of an event is defined as the ratio of the favorable outcomes for the event to the total number of outcomes.

In mathematical form, the probability that an event occurs is given by the following formula;

$$P(\text{event}) = \frac{\text{favorable outcomes}}{\text{total outcomes}}$$

The odds of an event are the ratio of the favorable outcomes to the unfavorable outcomes for the event.

In mathematical form, the odds of an event are given by the following formula;

$$\text{odds of an event} = \frac{\text{favorable outcomes}}{\text{unfavorable outcomes}}$$

The probability that the event occurs is $\frac{2}{3}$.

Thus, the number of favorable outcomes is 2 and the number of total outcomes is 3.

Hence, the number of unfavorable outcomes is $3 - 2 = 1$.

Find the number of odds.

Substitute 2 for the favorable outcomes and 1 for the unfavorable outcome in the formula to calculate the odds;

$$\begin{aligned} \text{odds of the event} &= \frac{\text{favorable outcomes}}{\text{unfavorable outcomes}} \\ &= \frac{2}{1} \end{aligned}$$

Therefore, the odds that the event will occur are $\boxed{2:1}$.

Answer 46MYS.

Write the following expression;

$$(2b)(-3a)$$

The objective is to simplify the expression.

Multiply

Collect the constant terms and the variables in one bracket and rewrite the expression as follows;

$$\begin{aligned} (2b)(-3a) &= (2 \cdot (-3))(b \cdot a) \\ &= -6(ba) \\ &= -6ab \end{aligned}$$

Therefore, the value of the expression $(2b)(-3a)$ is $\boxed{-6ab}$.

Answer 47MYS.

Write the following expression;

$$3x(-3y) + (-6x)(-2y)$$

The objective is to simplify the expression.

Use the following facts for multiplication;

The product of a two negative numbers is positive.

The product of a negative and a positive number is negative.

Multiply

Collect the constant terms and the variables in one bracket and rewrite the expression as follows;

$$\begin{aligned} 3x(-3y) + (-6x)(-2y) &= (3 \cdot (-3))(x \cdot y) + ((-6) \cdot (-2))(x \cdot y) \\ &= -9xy + 12xy \end{aligned}$$

Add the coefficients of the like terms;

$$\begin{aligned} 3x(-3y) + (-6x)(-2y) &= -9xy + 12xy \\ &= (-9 + 12)xy \\ &= 3xy \end{aligned}$$

Therefore, the value of the expression $3x(-3y) + (-6x)(-2y)$ is $\boxed{3xy}$.

Answer 48MYS.

Write the following expression;

$$5s(-6t) + 2s(-8t)$$

The objective is to simplify the expression.

Use the following facts for multiplication;

The product of a two negative numbers is positive.

The product of a negative and a positive number is negative.

Collect the constant terms and the variables in one bracket and rewrite the expression as follows;

Multiply

$$\begin{aligned} 5s(-6t) + 2s(-8t) &= (5 \cdot (-6))(s \cdot t) + (2 \cdot (-8))(s \cdot t) \\ &= -30st + (-16)st \end{aligned}$$

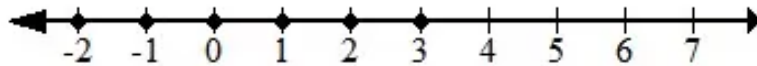
Add the coefficients of the like terms;

$$\begin{aligned} 5s(-6t) + 2s(-8t) &= -30st + (-16)st \\ &= (-30 + (-16))st \\ &= (-30 - 16)st \\ &= -46st \end{aligned}$$

Therefore, the value of the expression $5s(-6t) + 2s(-8t)$ is $\boxed{-46st}$.

Answer 49MYS.

Draw the following graph;



The objective is to write the set of numbers graphed on the number line.

Observe that the points marked on the graph are all the points that lie to the left side of the number 3 including 3.

Thus, the numbers are;

$3, 2, 1, 0, -1, -2, \dots$

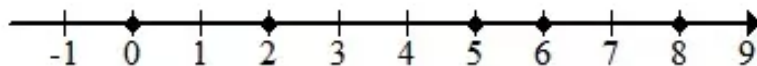
Write these numbers in the increasing order as set of numbers as follows;

$\{\dots - 2, -1, 0, 1, 2, 3\}$

Therefore, the numbers graphed on the number line are $\boxed{\{\dots - 2, -1, 0, 1, 2, 3\}}$.

Answer 50MYS.

Draw the following graph;



The objective is to write the set of numbers graphed on the number line.

Observe that the points marked on the graph are the points with coordinates 0, 2, 5, 6 and 8 on the number line.

Thus, the numbers are;

$0, 2, 5, 6, 8$

Write these numbers in the increasing order as set of numbers as follows;

$\{0, 2, 5, 6, 8\}$

Therefore, the numbers graphed on the number line are $\boxed{\{0, 2, 5, 6, 8\}}$.