Chapter 3

Progressions

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Leonardo Fibonacci

According to Boethius (510 A.D.) *arithmetic, Geometric and Harmonic sequences were known to early Greek writers. Among the Indian mathematician;* Aryabhatta (476 A.D.) *was the first to give the formula for the sum of squares and cubes of natural numbers in his famous work* Aryabhatiyam.

Another special type of sequence having important applications in mathematics, called Fibonacci *sequence, was discovered by Italian Mathematician* Leonardo Fibonacci (1170-1250 A.D.) *The general series was given by* Frenchman Francois-vieta (1540-1603 A.D.)

It was only through the rigorous developed of algebraic and set theoretic tools that the concepts related to sequence and series could be formulated suitably.

Progressions

3.1 Introduction

(1) **Sequence :** A sequence is a function whose domain is the set of natural numbers, N.

If $f : N \to C$ is a sequence, we usually denote it by $< f(n) > = < f(1), f(2), f(3), ... >$

It is not necessary that the terms of a sequence always follow a certain pattern or they are described by some explicit formula for the n^{th} term. Terms of a sequence are connected by commas. *Example* : 1, 1, 2, 3, 5, 8, ... is a sequence.

(2) **Series :** By adding or subtracting the terms of a sequence, we get a series.

If $t_1, t_2, t_3, ... t_n, ...$ is a sequence, then the expression $t_1 + t_2 + t_3 + ... + t_n ....$ is a series.

A series is finite or infinite as the number of terms in the corresponding sequence is finite or infinite.

Example : $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + ...$ is a series.

(3) **Progression :** A progression is a sequence whose terms follow a certain pattern *i.e.* the terms are arranged under a definite rule.

Example : 1, 3, 5, 7, 9, ... is a progression whose terms are obtained by the rule : $T_n = 2n - 1$, where T_n denotes the n^{th} term of the progression.

Progression is mainly of three types : Arithmetic progression, Geometric progression and Harmonic progression.

However, here we have classified the study of progression into five parts as :

- Arithmetic progression
- Geometric progression
- Arithmetico-geometric progression
- Harmonic progression
- Miscellaneous progressions

Arithmetic progression(A.P)

3.2 Definition

A sequence of numbers $< t_n >$ is said to be in arithmetic progression (A.P.) when the difference $t_n - t_{n-1}$ is a constant for all $n \in N$. This constant is called the common difference of the A.P., and is usually denoted by the letter d.

If 'a' is the first term and 'd' the common difference, then an A.P. can be represented as $a, a+d, a+2d, a+3d, ...$

Example : 2, 7, 12, 17, 22, ... is an A.P. whose first term is 2 and common difference 5.

Algorithm to determine whether a sequence is an A.P. or not.

Step I: Obtain a_n (the n^{th} term of the sequence).

Step II: Replace n by $n - 1$ in a_n to get a_{n-1}.

Step III: Calculate $a_n - a_{n-1}$.

If $a_n - a_{n-1}$ is independent of n, the given sequence is an A.P. otherwise it is not an A.P. An arithmetic progression is a linear function with domain as the set of natural numbers N.

$\therefore\ t_n = An + B$ represents the n^{th} term of an A.P. with common difference A.

3.3 General Term of an A.P.

(1) Let 'a' be the first term and 'd' be the common difference of an A.P. Then its n^{th} term is $a + (n-1)d$.

$$T_n = a + (n-1)d$$

(2) **p^{th} term of an A.P. from the end :** Let 'a' be the first term and 'd' be the common difference of an A.P. having n terms. Then p^{th} term from the end is $(n-p+1)^{th}$ term from the beginning.

$$p^{th} \text{ term from the end} = T_{(n-p+1)} = a + (n-p)d$$

Important Tips

- *General term (T_n) is also denoted by l (last term).*
- *Common difference can be zero, +ve or –ve.*
- *n (number of terms) always belongs to set of natural numbers.*
- *If T_k and T_p of any A.P. are given, then formula for obtaining T_n is $\frac{T_n - T_k}{n-k} = \frac{T_p - T_k}{p-k}$.*
- *If $pT_p = qT_q$ of an A.P., then $T_{p+q} = 0$.*
- *If p^{th} term of an A.P. is q and the q^{th} term is p, then $T_{p+q} = 0$ and $T_n = p + q - n$.*
- *If the p^{th} term of an A.P. is $\frac{1}{q}$ and the q^{th} term is $\frac{1}{p}$, then its pq^{th} term is 1.*
- *If $T_n = pn + q$, then it will form an A.P. of common difference p and first term p + q.*

Example: 1 Let T_r be rth term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n, m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals **[AIEEE 2004]**

(a) $\frac{1}{m} + \frac{1}{n}$ (b) 1 (c) $\frac{1}{mn}$ (d) 0

Solution: (d) $T_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$ .....(i)

and $T_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$ .....(ii)

Subtract (ii) from (i), we get $(m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow (m-n)d = \frac{(m-n)}{mn} \Rightarrow d = \frac{1}{mn}$, as $m - n \neq 0$

$a = \frac{1}{m} - (n-1)d = \frac{1}{m} - \frac{n-1}{mn} = \frac{1}{mn} = d$. Therefore $a - d = 0$

Example: 2 The 19^{th} term from the end of the series 2 + 6 + 10 + ... + 86 is

(a) 6 (b) 18 (c) 14 (d) 10

Solution: (c) $86 = 2 + (n-1)4 \Rightarrow n = 22$

19^{th} term from end $= t_{n-19+1} = t_{22-19+1} = t_4 = 2 + (4-1)4 = 14$

Example: 3 In a certain A.P., 5 times the 5^{th} term is equal to 8 times the 8^{th} term, then its 13^{th} term is **[AMU 1991]**

(a) 0 (b) – 1 (c) – 12 (d) – 13

Solution: (a) We have $5\,T_5 = 8\,T_8$

Let a and d be the first term and common difference respectively

$\therefore\ 5\{a + (5-1)d\} = 8\{a + (8-1)d\}$

$\Rightarrow 3a + 36d = 0 \Rightarrow a + 12d = 0$, *i.e.* $a + (13-1)d = 0$. Hence 13^{th} term = 0

Example: 4 If 7^{th} and 13^{th} term of an A.P. be 34 and 64 respectively, then its 18^{th} term is

(a) 87 (b) 88 (c) 89 (d) 90

Solution: (c) Let a be the first term and d be the common difference of the given A.P., then

$T_7 = 34 \Rightarrow a + 6d = 34$ .....(i)

$T_{13} = 64 \Rightarrow a + 12d = 64$ .....(ii)

From (i) and (ii), $d = 5$, $a = 4$

$\therefore\ T_{18} = a + 17d = 4 + 17 \times 5 = 89$

Trick: $\frac{T_n - T_k}{n-k} = \frac{T_p - T_k}{p-k} \Rightarrow \frac{T_{18} - T_7}{18-7} = \frac{T_{13} - T_7}{13-7} \Rightarrow \frac{T_{18} - 34}{11} = \frac{64-34}{6} \Rightarrow T_{18} = 89$

Example: 5 If $< a_n >$ is an arithmetic sequence, then $\Delta = \begin{vmatrix} a_m & a_n & a_p \\ m & n & p \\ 1 & 1 & 1 \end{vmatrix}$ equals

(a) 1 (b) –1 (c) 0 (d) None of these

Solution: (c) Let a be the first term and d the common difference. Then $a_r = a + (r-1)d$

$$\Delta = \begin{vmatrix} a+(m-1)d & a+(n-1)d & a+(p-1)d \\ m & n & p \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a & a & a \\ m & n & p \\ 1 & 1 & 1 \end{vmatrix} + d\begin{vmatrix} m-1 & n-1 & p-1 \\ m & n & p \\ 1 & 1 & 1 \end{vmatrix}$$

$$= a\begin{vmatrix} 1 & 1 & 1 \\ m & n & p \\ 1 & 1 & 1 \end{vmatrix} + d\begin{vmatrix} m & n & p \\ m & n & p \\ 1 & 1 & 1 \end{vmatrix} = a.0 + d.0 = 0$$

Example: 6 The n^{th} term of the series 3 + 10 + 17 + ... and 63 + 65 + 67 + ... are equal, then the value of n is **[Kerala (Engg.) 2002]**

(a) 11 (b) 12 (c) 13 (d) 15

Solution: (c) n^{th} term of 1^{st} series $= 3 + (n-1)7 = 7n - 4$

n^{th} term of 2^{nd} series $= 63 + (n-1) = 2n + 61$

$\therefore$ we have, $7n - 4 = 2n + 61 \Rightarrow n = 13$

3.4 Selection of Terms in an A.P.

When the sum is given, the following way is adopted in selecting certain number of terms :

Number of terms	Terms to be taken
3	$a - d, a, a + d$

4	$a - 3d, a - d, a + d, a + 3d$
5	$a - 2d, a - d, a, a + d, a + 2d$

In general, we take $a - rd, a - (r - 1)d, ..., a - d, a, a + d, ..., a + (r - 1)d, a + rd$, in case we have to take $(2r + 1)$ terms (*i.e.* odd number of terms) in an A.P.

And, $a-(2r-1)d, a-(2r-3)d, ..., a-d, a+d, ..., a+(2r-1)d$, in case we have to take $2r$ terms in an A.P.

When the sum is not given, then the following way is adopted in selection of terms.

Number of terms	Terms to be taken
3	$a, a+d, a+2d$
4	$a, a+d, a+2d, a+3d$
5	$a, a+d, a+2d, a+3d, a+4d$

Sum of *n* terms of an A.P. : The sum of n terms of the series $a+(a+d)+(a+2d)+.......+\{a+(n-1)d\}$ is given by

$$\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$$

Also, $S_n = \frac{n}{2}(a+l)$, where l = last term = $a+(n-1)d$

Important Tips

☞ *The common difference of an A.P is given by* $d = S_2 - 2S_1$ *where* S_2 *is the sum of first two terms and* S_1 *is the sum of first term or the first term.*

☞ *The sum of infinite terms* $= \begin{cases} \infty, & \text{when } d > 0 \\ -\infty, & \text{when } d < 0 \end{cases}$.

☞ *If sum of n terms* S_n *is given then general term* $T_n = S_n - S_{n-1}$, *where* S_{n-1} *is sum of (n – 1) terms of A.P.*

☞ *Sum of n terms of an A.P. is of the form* $An^2 + Bn$ *i.e. a quadratic expression in n, in such case, common difference is twice the coefficient of* n^2 *i.e. 2A.*

☞
- *If for the different A.P's* $\frac{S_n}{S'_n} = \frac{f_n}{\phi_n}$, *then* $\frac{T_n}{T'_n} = \frac{f(2n-1)}{\phi(2n-1)}$
- *If for two A.P.'s* $\frac{T_n}{T'_n} = \frac{An+B}{Cn+D}$ *then* $\frac{S_n}{S'_n} = \frac{A\left(\frac{n+1}{2}\right)+B}{C\left(\frac{n+1}{2}\right)+D}$

☞ *Some standard results*

- *Sum of first n natural numbers* $= 1+2+3+........+n = \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$
- *Sum of first n odd natural numbers* $= 1+3+5+.....+(2n-1) = \sum_{r=1}^{n}(2r-1) = n^2$
- *Sum of first n even natural numbers* $= 2+4+6+......+2n = \sum_{r=1}^{n} 2r = n(n+1)$

☞
- *If for an A.P. sum of p terms is q and sum of q terms is p, then sum of (p + q) terms is {–(p + q)}.*
- *If for an A.P., sum of p terms is equal to sum of q terms, then sum of (p + q) terms is zero.*

- *If the p^{th} term of an A.P. is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, then sum of pq terms is given by* $S_{pq}=\frac{1}{2}(pq+1)$

Example: 7 7^{th} term of an A.P. is 40, then the sum of first 13 terms is **[Karnataka CET 2003]**

(a) 53 (b) 520 (c) 1040 (d) 2080

Solution: (b) $S_{13}=\frac{13}{2}\{2a+12d\}=13\{a+6d\}=13\times T_7=13\times 40=520$

Example: 8 The first term of an A.P. is 2 and common difference is 4. The sum of its 40 terms will be **[MNR 1978; MP PET**

(a) 3200 (b) 1600 (c) 200 (d) 2800

Solution: (a) $S=\frac{n}{2}[2a+(n-1)d]=\frac{40}{2}[2\times 2+(40-1)4]=3200$

Example: 9 The sum of the first and third term of an A.P. is 12 and the product of first and second term is 24, the first term is

[MP PET 2003]

(a) 1 (b) 8 (c) 4 (d) 6

Solution: (c) Let $a-d, a, a+d, \ldots\ldots$ be an A.P.

$\therefore\ (a-d)+(a+d)=12 \Rightarrow a=6$. Also, $(a-d)a=24 \Rightarrow 6-d=\frac{24}{6}=4 \Rightarrow d=2$

$\therefore$ First term $=a-d=6-2=4$

Example: 10 If S_r denotes the sum of the first r terms of an A.P., then $\frac{S_{3r}-S_{r-1}}{S_{2r}-S_{2r-1}}$ is equal to

(a) $2r-1$ (b) $2r+1$ (c) $4r+1$ (d) $2r+3$

Solution: (b) $\frac{S_{3r}-S_{r-1}}{S_{2r}-S_{2r-1}}=\frac{\frac{3r}{2}\{2a+(3r-1)d\}-\frac{(r-1)}{2}\{2a+(r-1-1)d\}}{T_{2r}}=\frac{(2r+1)a+\frac{d}{2}\{3r(3r-1)-(r-1)(r-2)\}}{a+(2r-1)d}$

$=\frac{(2r+1)a+\frac{d}{2}\{8r^2-2\}}{a+(2r-1)d}=\frac{(2r+1)a+d(4r^2-1)}{a+(2r-1)d}=2r+1$

Example: 11 If the sum of the first $2n$ terms of 2, 5, 8.... is equal to the sum of the first n terms of 57, 59, 61...., then n is equal to

[IIT Screening 2001]

(a) 10 (b) 12 (c) 11 (d) 13

Solution: (c) We have, $\frac{2n}{2}\{2\times 2+(2n-1)3\}=\frac{n}{2}\{2\times 57+(n-1)2\} \Rightarrow 6n+1=n+56 \Rightarrow n=11$

Example: 12 If the sum of the 10 terms of an A.P. is 4 times to the sum of its 5 terms, then the ratio of first term and common difference is **[Rajasthan PET 1986]**

(a) 1 : 2 (b) 2 : 1 (c) 2 : 3 (d) 3 : 2

Solution: (a) Let a be the first term and d the common difference

Then, $\frac{10}{2}\{\{a+(10-1)d\}=4\times\frac{5}{2}\{2a+(5-1)d\} \Rightarrow 2a+9d=4a+8d \Rightarrow d=2a \Rightarrow \frac{a}{d}=\frac{1}{2}$, $\therefore$ $a : d = 1 : 2$

Example: 13 150 workers were engaged to finish a piece of work in a certain number of days. 4 workers dropped the second day, 4 more workers dropped the third day and so on. It takes eight more days to finish the work now. The number of days in which the work was completed is **[Kurukshetra CEE 1996]**

(a) 15 (b) 20 (c) 25 (d) 30

Solution: (c) Let the work was to be finished in x days. $\therefore$ Work of 1 worker in a day $=\frac{1}{150\,x}$

Now the work will be finished in $(x + 8)$ days. $\therefore$ Work done = Sum of the fraction of work done

$$1=\frac{1}{150x}\times 150+\frac{1}{150x}(150-4)+\frac{1}{150x}(150-8)+\ldots\ldots \text{ to } (x+8) \text{ terms}$$

$$\Rightarrow 1=\frac{x+8}{2}\left\{2\times\frac{150}{150x}+(x+8-1)\left(\frac{-4}{150x}\right)\right\} \Rightarrow 150x=(x+8)\{150-2(x+7)\} \Rightarrow (x+8)(x+7)-600=0$$

$$\Rightarrow (x+8)(x+7)=25\times 24\text{ , } \therefore\ x+8=25$$

Hence work completed in 25 days.

Example: 14 If the sum of first p terms, first q terms and first r terms of an A.P. be x, y and z respectively, then $\frac{x}{p}(q-r)+\frac{y}{q}(r-p)+\frac{z}{r}(p-q)$ is

(a) 0 (b) 2 (c) pqr (d) $\frac{8xyz}{pqr}$

Solution: (a) We have a, the first term and d, the common difference, $x=\{2a+(p-1)d\}\frac{p}{2} \Rightarrow \frac{x}{p}=a+(p-1)\frac{d}{2}$

Similarly, $\frac{y}{q}=a+(q-1)\frac{d}{2}$ and $\frac{z}{r}=a+(r-1)\frac{d}{2}$

$$\therefore\ \frac{x}{p}(q-r)+\frac{y}{q}(r-p)+\frac{z}{r}(p-q)=\left\{a+(p-1)\frac{d}{2}\right\}(q-r)+\left\{a+(q-1)\frac{d}{2}\right\}(r-p)+\left\{a+(r-1)\frac{d}{2}\right\}(p-q)$$

$$=a\{(q-r)+(r-p)+(p-q)\}+\frac{d}{2}\{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)\}$$

$$=a..0+\frac{d}{2}[\{pq-pr+rq-pq+pr-qr-\{(q-r)+(r-p)+(p-q)\}=0+\frac{d}{2}\{0-0\}=0$$

Example: 15 The sum of all odd numbers of two digits is **[Roorkee 1993]**

(a) 2475 (b) 2530 (c) 4905 (d) 5049

Solution: (a) Required sum, $S=11+13+15+\ldots\ldots+99$

Let the number of odd terms be n, then $99=11+(n-1)2 \Rightarrow n=45$

$\therefore\ S=\frac{45}{2}(11+99)=45\times 55=2475$ $\left[\because S=\frac{n}{2}(a+l)\right]$

Example: 16 If sum of n terms of an A.P. is $3n^2+5n$ and $T_m=164$, then m = **[Rajasthan PET 1991, 95; DCE 1999]**

(a) 26 (b) 27 (c) 28 (d) None of these

Solution: (b) $T_m=S_m-S_{m-1} \Rightarrow 164=(3m^2+5m)-\{3(m-1)^2+5(m-1)\} \Rightarrow 164=3(2m-1)+5 \Rightarrow m=27$

Example: 17 The sum of n terms of the series $\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ldots\ldots$ is **[UPSEAT 2002]**

(a) $\sqrt{2n+1}$ (b) $\frac{1}{2}\sqrt{2n+1}$ (c) $\sqrt{2n-1}$ (d) $\frac{1}{2}(\sqrt{2n+1}-1)$

Solution: (d) $S_n=\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ldots\ldots+\frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}$

$$=\frac{\sqrt{3}-1}{(\sqrt{3}-1)(\sqrt{3}+1)}+\frac{\sqrt{5}-\sqrt{3}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}+\ldots..+\frac{\sqrt{2n+1}-\sqrt{2n-1}}{2}$$

$$=\frac{1}{2}[\sqrt{3}-1+\sqrt{5}-\sqrt{3}+\sqrt{7}-\sqrt{5}+\ldots..+(\sqrt{2n+1}-\sqrt{2n-1})]=\frac{1}{2}[\sqrt{2n+1}-1]$$

Example: 18 If $a_1, a_2, \ldots\ldots, a_{n+1}$ are in A.P., then $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots..+\frac{1}{a_na_{n+1}}$ is **[AMU 2002]**

(a) $\frac{n-1}{a_1a_{n+1}}$ (b) $\frac{1}{a_1a_{n+1}}$ (c) $\frac{n+1}{a_1a_{n+1}}$ (d) $\frac{n}{a_1a_{n+1}}$

Solution: (d) $S=\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+....+\frac{1}{a_na_{n+1}}=\frac{\left(\frac{1}{a_1}-\frac{1}{a_2}\right)}{(a_2-a_1)}+\frac{\left(\frac{1}{a_2}-\frac{1}{a_3}\right)}{(a_3-a_2)}+......+\frac{\left(\frac{1}{a_n}-\frac{1}{a_{n+1}}\right)}{(a_{n+1}-a_n)}$

As $a_1, a_2, a_3, ..., a_n, a_{n+1}$ are in A.P., *i.e.* $a_2-a_1=a_3-a_2=........=a_{n+1}-a_n=d$ (say)

$\therefore\ S=\frac{1}{d}\left[\left(\frac{1}{a_1}-\frac{1}{a_2}\right)+\left(\frac{1}{a_2}-\frac{1}{a_3}\right)+......+\left(\frac{1}{a_n}-\frac{1}{a_{n+1}}\right)\right]=\frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_{n+1}}\right]=\frac{a_{n+1}-a_1}{d.a_1.a_{n+1}}=\frac{[a_1+(n+1-1)d]-a_1}{d.a_1.a_{n+1}}$

$S=\frac{nd}{d\,a_1\,a_{n+1}}=\frac{n}{a_1\,a_{n+1}}$

3.5 Arithmetic Mean

(1) **Definitions**

(i) If three quantities are in A.P. then the middle quantity is called Arithmetic mean (A.M.) between the other two.

If *a*, *A*, *b* are in A.P., then *A* is called A.M. between *a* and *b*.

(ii) If $a, A_1, A_2, A_3, ..., A_n, b$ are in A.P., then $A_1, A_2, A_3, ..., A_n$ are called *n* A.M.'s between *a* and *b*.

(2) **Insertion of arithmetic means**

(i) **Single A.M. between *a* and *b* :** If *a* and *b* are two real numbers then single A.M. between *a* and *b* $=\frac{a+b}{2}$

(ii) ***n* A.M.'s between *a* and *b* :** If $A_1, A_2, A_3, ..., A_n$ are *n* A.M.'s between *a* and *b*, then

$A_1=a+d=a+\frac{b-a}{n+1}$, $A_2=a+2d=a+2\frac{b-a}{n+1}$, $A_3=a+3d=a+3\frac{b-a}{n+1}$, ..., $A_n=a+nd=a+n\frac{b-a}{n+1}$

Important Tips

☞ *Sum of n A.M.'s between a and b is equal to n times the single A.M. between a and b.*

i.e. $A_1+A_2+A_3+.........+A_n=n\left(\frac{a+b}{2}\right)$

☞ *If* A_1 *and* A_2 *are two A.M.'s between two numbers a and b, then* $A_1=\frac{1}{3}(2a+b), A_2=\frac{1}{3}(a+2b)$.

☞ *Between two numbers,* $\frac{\text{Sum of } m \text{ A.M.'s}}{\text{Sum of } n \text{ A.M.'s}}=\frac{m}{n}$.

☞ *If number of terms in any series is odd, then only one middle term exists which is* $\left(\frac{n+1}{2}\right)^{th}$ *term.*

☞ *If number of terms in any series is even then there are two middle terms, which are given by* $\left(\frac{n}{2}\right)^{th}$ *and* $\left\{\left(\frac{n}{2}\right)+1\right\}^{th}$ *term.*

Example: 19 After inserting *n* A.M.'s between 2 and 38, the sum of the resulting progression is 200. The value of *n* is **[MP PET 2001]**

(a) 10 (b) 8 (c) 9 (d) None of these

Solution: (b) There will be (*n* + 2) terms in the resulting A.P. $2, A_1, A_2, ..., A_n, 38$

Sum of the progression $=\frac{n+2}{2}(2+38) \Rightarrow 200=(n+2)\times 20 \Rightarrow n=8$

Example: 20 3 A.M.'s between 3 and 19 are

(a) 7, 11, 15 (b) 4, 6, 10 (c) 6, 10, 14 (d) None of these

Solution: (a) Let A_1, A_2, A_3 be three A.M.'s. Then $3, A_1, A_2, A_3, 19$ are in A.P.

$\Rightarrow$ common difference $d=\frac{19-3}{3+1}=4$ .Therefore $A_1=3+d=7$, $A_2=3+2d=11$, $A_3=3+3d=15$

Example: 21 If *a*, *b*, *c*, *d*, *e*, *f* are A.M.'s between 2 and 12, then $a+b+c+d+e+f$ is equal to

(a) 14 (b) 42 (c) 84 (d) None of these

Solution: (b) Since, *a*, *b*, *c*, *d*, *e*, *f* are six A.M.'s between 2 and 12

Therefore, $a+b+c+d+e+f=\frac{6}{2}(a+f)=\frac{6}{2}(2+12)=42$

3.6 Properties of A.P.

(1) If $a_1, a_2, a_3.....$ are in A.P. whose common difference is *d*, then for fixed non-zero number $K \in R$.

(i) $a_1 \pm K, a_2 \pm K, a_3 \pm K,.....$ will be in A.P., whose common difference will be *d*.

(ii) $Ka_1, Ka_2, Ka_3........$ will be in A.P. with common difference = *Kd*.

(iii) $\frac{a_1}{K}, \frac{a_2}{K}, \frac{a_3}{K}......$ will be in A.P. with common difference = *d*/*K*.

(2) The sum of terms of an A.P. equidistant from the beginning and the end is constant and is equal to sum of first and last term. *i.e.* $a_1+a_n=a_2+a_{n-1}=a_3+a_{n-2}=....$

(3) Any term (except the first term) of an A.P. is equal to half of the sum of terms equidistant from the term *i.e.* $a_n=\frac{1}{2}(a_{n-k}+a_{n+k})$, $k<n$.

(4) If number of terms of any A.P. is odd, then sum of the terms is equal to product of middle term and number of terms.

(5) If number of terms of any A.P. is even then A.M. of middle two terms is A.M. of first and last term.

(6) If the number of terms of an A.P. is odd then its middle term is A.M. of first and last term.

(7) If $a_1, a_2, ...a_n$ and $b_1, b_2, ...b_n$ are the two A.P.'s. Then $a_1 \pm b_1, a_2 \pm b_2, ...a_n \pm b_n$ are also A.P.'s with common difference $d_1 \neq d_2$, where d_1 and d_2 are the common difference of the given A.P.'s.

(8) Three numbers *a*, *b*, *c* are in A.P. iff $2b=a+c$.

(9) If T_n, T_{n+1} and T_{n+2} are three consecutive terms of an A.P., then $2T_{n+1}=T_n+T_{n+2}$.

(10) If the terms of an A.P. are chosen at regular intervals, then they form an A.P.

Example: 22 If $a_1, a_2, a_3, ..., a_{24}$ are in arithmetic progression and $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$, then $a_1+a_2+a_3+.....+a_{23}+a_{24}=$ **[MP PET 1999; AMU 1997]**

(a) 909 (b) 75 (c) 750 (d) 900

Solution: (d) $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225 \Rightarrow (a_1+a_{24})+(a_5+a_{20})+(a_{10}+a_{15})=225 \Rightarrow 3(a_1+a_{24})=225 \Rightarrow a_1+a_{24}=75$

($\because$ In an A.P. the sum of the terms equidistant from the beginning and the end is same and is equal to the sum of first and last term)

$a_1+a_2+.....+a_{24}=\frac{24}{2}(a_1+a_{24})=12\times 75=900$

Example: 23 If a, b, c are in A.P., then $\frac{1}{bc},\frac{1}{ca},\frac{1}{ab}$ will be in **[DCE 2002; MP PET 1985; Roorkee 1975]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Solution: (a) a, b, c are in A.P., $\Rightarrow \frac{1}{bc},\frac{1}{ca},\frac{1}{ab}$ will be in A.P. [Dividing each term by abc]

Example: 24 If log 2, $\log(2^n-1)$ and $\log(2^n+3)$ are in A.P., then n = **[MP PET 1998; Karnataka CET 2000]**

(a) 5/2 (b) $\log_2 5$ (c) $\log_3 5$ (d) $\frac{3}{2}$

Solution: (b) As, log 2, $\log(2^n-1)$ and $\log(2^n+3)$ are in A.P. Therefore

$2\log(2^n-1)=\log 2+\log(2^n+3) \Rightarrow (2^n-5)(2^n+1)=0$

As 2^n cannot be negative, hence $2^n-5=0 \Rightarrow 2^n=5$ or $n=\log_2 5$

Geometric progression(G.P.)

3.7 Definition

A progression is called a G.P. if the ratio of its each term to its previous term is always constant. This constant ratio is called its common ratio and it is generally denoted by r.

Example: The sequence 4, 12, 36, 108, ... is a G.P., because $\frac{12}{4}=\frac{36}{12}=\frac{108}{36}=.....=3$, which is constant.

Clearly, this sequence is a G.P. with first term 4 and common ratio 3.

The sequence $\frac{1}{3},-\frac{1}{2},\frac{3}{4},-\frac{9}{8},....$ is a G.P. with first term $\frac{1}{3}$ and common ratio $\left(-\frac{1}{2}\right)\Big/\left(\frac{1}{3}\right)=-\frac{3}{2}$

3.8 General Term of a G.P.

(1) We know that, $a,ar,ar^2,ar^3,.....ar^{n-1}$ is a sequence of G.P.

Here, the first term is 'a' and the common ratio is 'r'.

The general term or n^{th} term of a G.P. is $\boxed{T_n=ar^{n-1}}$

It should be noted that,

$$\boxed{r=\frac{T_2}{T_1}=\frac{T_3}{T_2}=......}$$

(2) **p^{th} term from the end of a finite G.P. :** If G.P. consists of 'n' terms, p^{th} term from the end $=(n-p+1)^{th}$ term from the beginning $=ar^{n-p}$.

Also, the p^{th} term from the end of a G.P. with last term l and common ratio r is $l\left(\frac{1}{r}\right)^{n-1}$

Important Tips

☞ *If a, b, c are in G.P.* $\Rightarrow \frac{b}{a}=\frac{c}{b}$ *or* $\boxed{b^2=ac}$

- *If T_k and T_p of any G.P. are given, then formula for obtaining T_n is*

$$\left(\frac{T_n}{T_k}\right)^{\frac{1}{n-k}}=\left(\frac{T_p}{T_k}\right)^{\frac{1}{p-k}}$$

- *If a, b, c are in G.P. then*

$\Rightarrow \frac{b}{a}=\frac{c}{b} \Rightarrow \frac{a+b}{a-b}=\frac{b+c}{b-c}$ *or* $\frac{a-b}{b-c}=\frac{a}{b}$ *or* $\frac{a+b}{b+c}=\frac{a}{b}$

- *Let the first term of a G.P be positive, then if r > 1, then it is an increasing G.P., but if r is positive and less than 1, i.e. 0< r < 1, then it is a decreasing G.P.*
- *Let the first term of a G.P. be negative, then if r > 1, then it is a decreasing G.P., but if 0< r < 1, then it is an increasing G.P.*
- *If a, b, c, d,... are in G.P., then they are also in continued proportion i.e.* $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=....=\frac{1}{r}$

Example: 25 The numbers $(\sqrt{2}+1), 1, (\sqrt{2}-1)$ will be in **[AMU 1983]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Solution: (b) Clearly $(1)^2=(\sqrt{2}+1).(\sqrt{2}-1)$

$\therefore\ \sqrt{2}+1, 1, \sqrt{2}-1$ are in G.P.

Example: 26 If the p^{th}, q^{th} and r^{th} term of a G.P. are a, b, c respectively, then $a^{q-r}\cdot b^{r-p}\cdot c^{p-q}$ is equal to **[Roorkee 1955, 63, 73; Pb. CET 1991, 95]**

(a) 0 (b) 1 (c) abc (d) pqr

Solution: (b) Let $x, xy, xy^2, xy^3,....$ be a G.P.

$\therefore\ a=xy^{p-1}, b=xy^{q-1}, c=xy^{r-1}$

Now, $a^{q-r}.b^{r-p}.c^{p-q}=(xy^{p-1})^{q-r}(xy^{q-1})^{r-p}(xy^{r-1})^{p-q}=x^{(q-r)+(r-p)+(p-q)}.y^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$

$=x^0.y^{p(q-r)+q(r-p)+r(p-q)-(q-r+r-p+p-q)}=x^0.y^{0-0}=(xy)^0=1$

Example: 27 If the third term of a G.P. is 4 then the product of its first 5 terms is **[IIT 1982; Rajasthan PET 1991]**

(a) 4^3 (b) 4^4 (c) 4^5 (d) None of these

Solution: (c) Given that $ar^2=4$

Then product of first 5 terms $=a(ar)(ar^2)(ar^3)(ar^4)=a^5r^{10}=[ar^2]^5=4^5$

Example: 28 If $x, 2x+2, 3x+3$ are in G.P., then the fourth term is **[MNR 1980, 81]**

(a) 27 (b) – 27 (c) 13.5 (d) – 13.5

Solution: (d) Given that $x, 2x+2, 3x+3$ are in G.P.

Therefore, $(2x+2)^2=x(3x+3) \Rightarrow x^2+5x+4=0 \Rightarrow (x+4)(x+1)=0 \Rightarrow x=-1,-4$

Now first term $a = x$, second term $ar=2(x+1)$

$\Rightarrow r=\frac{2(x+1)}{x}$, then 4^{th} term $=ar^3=x\left[\frac{2(x+1)}{x}\right]^3=\frac{8}{x^2}(x+1)^3$

Putting $x=-4$, we get

$T_4=\frac{8}{16}(-3)^3=-\frac{27}{2}=-13.5$

3.9 Sum of First '*n*' Terms of a G.P.

If a be the first term, r the common ratio, then sum S_n of first n terms of a G.P. is given by

$S_n=\frac{a(1-r^n)}{1-r}$, $|r|<1$

$$S_n = \frac{a(r^n - 1)}{r-1}, \qquad |r| > 1$$

$$S_n = na, \qquad r = 1$$

3.10 Selection of Terms in a G.P.

(1) When the product is given, the following way is adopted in selecting certain number of terms :

Number of terms	Terms to be taken
3	$\frac{a}{r}, a, ar$
4	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
5	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

(2) When the product is not given, then the following way is adopted in selection of terms

Number of terms	Terms to be taken
3	a, ar, ar^2
4	a, ar, ar^2, ar^3
5	a, ar, ar^2, ar^3, ar^4

Example: 29 Let a_n be the n^{th} term of the G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the common ratio is **[IIT 1992]**

(a) $\frac{\alpha}{\beta}$ (b) $\frac{\beta}{\alpha}$ (c) $\sqrt{\frac{\alpha}{\beta}}$ (d) $\sqrt{\frac{\beta}{\alpha}}$

Solution: (a) Let x be the first term and y, the common ratio of the G.P.

Then, $\alpha = \sum_{n=1}^{100} a_{2n} = a_2 + a_4 + a_6 + ... + a_{200}$ and $\beta = \sum_{n=1}^{100} a_{2n-1} = a_1 + a_3 + a_5 + ... + a_{199}$

$$\Rightarrow \alpha = xy + xy^3 + xy^5 + ... + xy^{199} = xy\frac{1-(y^2)^{100}}{1-y^2} = xy\left(\frac{1-y^{200}}{1-y^2}\right)$$

$$\beta = x + xy^2 + xy^4 + ... + xy^{198} = x \cdot \frac{1-(y^2)^{100}}{1-y^2} = x \cdot \left(\frac{1-y^{200}}{1-y^2}\right)$$

$\therefore \frac{\alpha}{\beta} = y$. Thus, common ratio $= \frac{\alpha}{\beta}$

Example: 30 The sum of first two terms of a G.P. is 1 and every term of this series is twice of its previous term, then the first term will be

[Rajasthan PET 1988]

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Solution: (b) We have, common ratio $r = 2$; $\left[\because \frac{a_n}{a_{n-1}} = 2\right]$

Let a be the first term, then $a + ar = 1 \Rightarrow a(1+r) = 1 \Rightarrow a = \frac{1}{1+r} = \frac{1}{1+2} = \frac{1}{3}$

3.11 Sum of Infinite Terms of a G.P.

(1) When $|r| < 1$, $(\text{or } -1 < r < 1)$

$$S_\infty = \frac{a}{1-r}$$

(2) If $r \geq 1$, then S_∞ doesn't exist

Example: 31 The first term of an infinite geometric progression is x and its sum is 5. Then **[IIT Screening 2004]**

(a) $0 \leq x \leq 10$ (b) $0 < x < 10$ (c) $-10 < x < 0$ (d) $x > 10$

Solution: (b) According to the given conditions, $5 = \frac{x}{1-r}$, r being the common ratio $\Rightarrow r = 1 - \frac{x}{5}$

Now, $|r| < 1$ *i.e.* $-1 < r < 1 \Rightarrow$ $-1 < 1 - \frac{x}{5} < 1 \Rightarrow -2 < -\frac{x}{5} < 0 \Rightarrow 2 > \frac{x}{5} > 0$ *i.e.* $0 < \frac{x}{5} < 2$, $\therefore 0 < x < 10$

Example: 32 $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is **[AIEEE 2004]**

(a) $e + 1$ (b) $e - 1$ (c) $1 - e$ (d) e

Solution: (b) $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{r/n} = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} e^{r/n} = \lim_{n\to\infty} \frac{1}{n} \cdot (e^{1/n} + e^{2/n} + e^{3/n} + ... + e^{n/n}) = \lim_{n\to\infty} \frac{1}{n} \cdot [e^{1/n} + (e^{1/n})^2 + (e^{1/n})^3 + ... + (e^{1/n})^n]$

$$= \lim_{n\to\infty} \frac{1}{n} e^{1/n} \frac{1-(e^{1/n})^n}{1-e^{1/n}} = \lim_{n\to\infty} \frac{1}{n} e^{1/n} \frac{1-e}{1-e^{1/n}} = \lim_{n\to\infty} \frac{(1-e)(e^{1/n}-1+1)}{n(1-e^{1/n})} = \lim_{n\to\infty} \frac{(e-1)}{n} + \lim_{n\to\infty} \frac{(e-1)\cdot\frac{1}{n}}{e^{1/n}-1}$$

Put $\frac{1}{n} = h$, we get $h \to 0$

$$= 0 + (e-1) \lim_{h\to 0} \frac{h}{e^h - 1} \qquad \left[\frac{0}{0} \text{ form}\right]$$

$$= (e-1) \lim_{h\to 0} \frac{1}{e^h} = (e-1).1 = e - 1.$$

Example: 33 The value of $.2\dot{3}\dot{4}$ .234 is **[MNR 1986; UPSEAT 2000]**

(a) $\frac{232}{990}$ (b) $\frac{232}{9990}$ (c) $\frac{0.232}{990}$ (d) $\frac{232}{9909}$

Solution: (a) $.2\dot{3}\dot{4} = .234343434 .... = \frac{2}{10} + \frac{34}{1000} + \frac{34}{100000} + \frac{34}{10^7} + ... = \frac{2}{10} + \frac{34}{1000}\left(1 + \frac{1}{100} + \frac{1}{(100)^2} + ...\right)$

$$= \frac{1}{5} + \frac{17}{500}\left(\frac{1}{1-\frac{1}{100}}\right) = \frac{1}{5} + \frac{17}{500} \times \frac{100}{99} = \frac{1}{5}\left\{1 + \frac{17}{99}\right\} = \frac{116}{495} = \frac{232}{990}$$

Example: 34 If a, b, c are in A.P. and $|a|, |b|, |c| < 1$, and

$x = 1 + a + a^2 + ...\infty$

$y = 1 + b + b^2 + ...\infty$

$z = 1 + c + c^2 + ...\infty$

Then x, y, z shall be in **[Karnataka CET 1995]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Solution: (c) $x = 1 + a + a^2 + ...\infty = \frac{1}{1-a}$

$y = 1 + b + b^2 + ...\infty = \frac{1}{1-b}$

$z = 1 + c + c^2 + ...\infty = \frac{1}{1-c}$

Now, a, b, c are in A.P.

$\Rightarrow$ $1 - a, 1 - b, 1 - c$ are in A.P. $\Rightarrow \frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in H.P. Therefore x, y, z are in H.P.

3.12 Geometric Mean

(1) **Definition :** (i) If three quantities are in G.P., then the middle quantity is called geometric mean (G.M.) between the other two. If a, G, b are in G.P., then G is called G.M. between a and b.

(ii) If $a, G_1, G_2, G_3, ... G_n, b$ are in G.P. then $G_1, G_2, G_3, ... G_n$ are called n G.M.'s between a and b.

(2) **Insertion of geometric means :** (i) **Single G.M. between *a* and *b* :** If a and b are two real numbers then single G.M. between a and $b = \sqrt{ab}$

(ii) ***n* G.M.'s between *a* and *b* :** If $G_1, G_2, G_3, ..., G_n$ are n G.M.'s between a and b, then

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, \ G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, \ G_3 = ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}, \ ...., \ G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Important Tips

☞ *Product of n G.M.'s between a and b is equal to nth power of single geometric mean between a and b.*
i.e. $G_1 \, G_2 \, G_3 ...... \, G_n = (\sqrt{ab})^n$

☞ *G.M. of* $a_1 \, a_2 \, a_3 ...... \, a_n$ *is* $(a_1 \, a_2 \, a_3 ..... a_n)^{1/n}$

☞ *If* G_1 *and* G_2 *are two G.M.'s between two numbers a and b is* $G_1 = (a^2 b)^{1/3}, G_2 = (ab^2)^{1/3}$.

☞ *The product of n geometric means between a and* $\frac{1}{a}$ *is 1.*

☞ *If n G.M.'s inserted between a and b then* $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

3.13 Properties of G.P.

(1) If all the terms of a G.P. be multiplied or divided by the same non-zero constant, then it remains a G.P., with the same common ratio.

(2) The reciprocal of the terms of a given G.P. form a G.P. with common ratio as reciprocal of the common ratio of the original G.P.

(3) If each term of a G.P. with common ratio r be raised to the same power k, the resulting sequence also forms a G.P. with common ratio r^k.

(4) In a finite G.P., the product of terms equidistant from the beginning and the end is always the same and is equal to the product of the first and last term.

i.e., if $a_1, a_2, a_3, ... a_n$ be in G.P. Then $a_1\, a_n = a_2\, a_{n-1} = a_3\, a_{n-2} = a_n\, a_{n-3} = ... = a_r \,.\, a_{n-r+1}$

(5) If the terms of a given G.P. are chosen at regular intervals, then the new sequence so formed also forms a G.P.

(6) If $a_1, a_2, a_3, ..., a_n ......$ is a G.P. of non-zero, non-negative terms, then $\log a_1, \log a_2, \log a_3, ... \log a_n, ...$ is an A.P. and vice-versa.

(7) Three non-zero numbers *a*, *b*, *c* are in G.P. iff $b^2 = ac$.

(8) Every term (except first term) of a G.P. is the square root of terms equidistant from it.

i.e. $T_r = \sqrt{T_{r-p} \cdot T_{r+p}}$; [*r* > *p*]

(9) If first term of a G.P. of *n* terms is *a* and last term is *l*, then the product of all terms of the G.P. is $(al)^{n/2}$.

(10) If there be *n* quantities in G.P. whose common ratio is *r* and S_m denotes the sum of the first *m* terms, then the sum of their product taken two by two is $\frac{r}{r+1} S_n\, S_{n-1}$.

Example: 35 The two geometric mean between the number 1 and 64 are **[Kerala (Engg.) 2002]**

(a) 1 and 64 (b) 4 and 16 (c) 2 and 16 (d) 8 and 16

Solution: (b) Let G_1 and G_2 are two G.M.'s between the number $a = 1$ and $b = 64$

$G_1 = (a^2 b)^{\frac{1}{3}} = (1.64)^{\frac{1}{3}} = 4$, $G_2 = (ab^2)^{\frac{1}{3}} = (1.64^2)^{\frac{1}{3}} = 16$

Example: 36 The G.M. of the numbers $3, 3^2, 3^3 ...... 3^n$ is **[DCE 2002]**

(a) $3^{\frac{2}{n}}$ (b) $3^{\frac{n+1}{2}}$ (c) $3^{\frac{n}{2}}$ (d) $3^{\frac{n-1}{2}}$

Solution: (b) G.M. of $(3.\,3^2.\,3^3 ...... 3^n) = (3.\,3^2.\,3^3 ...... 3^n)^{1/n} = (3)^{\frac{1+2+3+....+n}{n}} = 3^{\frac{n(n+1)}{2n}} = 3^{\frac{n+1}{2}}$

Example: 37 If *a*, *b*, *c* are in A.P. *b* – *a*, *c* – *b* and *a* are in G.P., then *a* : *b* : *c* is

(a) 1 : 2 : 3 (b) 1 : 3 : 5 (c) 2 : 3 : 4 (d) 1 : 2 : 4

Solution: (a) Given, *a*, *b*, *c* are in A.P. ⇒ 2*b* = *a* + *c*

b – a, c – *b*, *a* are in G.P. So $(c-b)^2 = a(b-a)$

$\Rightarrow (b-a)^2 = (b-a)a$ $\quad \begin{bmatrix} \because\ 2b = a+c \\ \Rightarrow b+b = a+c \\ \Rightarrow b-a = c-b \end{bmatrix}$

$\Rightarrow b = 2a$ [$\because b \neq a$]

Put in 2*b* = *a* + *c*, we get c = 3a. Therefore *a* : b : c = 1 : 2 : 3

Harmonic progression(H.P.)

3.14 Definition

A progression is called a harmonic progression (H.P.) if the reciprocals of its terms are in A.P.

Standard form : $\frac{1}{a}+\frac{1}{a+d}+\frac{1}{a+2d}+....$

Example: The sequence $1,\frac{1}{3},\frac{1}{5},\frac{1}{7},\frac{1}{9},...$ is a H.P., because the sequence 1, 3, 5, 7, 9, ... is an A.P.

3.15 General Term of an H.P.

If the H.P. be as $\frac{1}{a},\frac{1}{a+d},\frac{1}{a+2d},....$ then corresponding A.P. is $a, a+d, a+2d, ...$

T_n of A.P. is $a+(n-1)d$

$\therefore T_n$ of H.P. is $\frac{1}{a+(n-1)d}$

In order to solve the question on H.P., we should form the corresponding A.P.

Thus, General term : $\boxed{T_n=\frac{1}{a+(n-1)d}}$ or $\boxed{T_n \text{ of H.P.}=\frac{1}{T_n \text{ of A.P.}}}$

Example: 38 The 4^{th} term of a H.P. is $\frac{3}{5}$ and 8^{th} term is $\frac{1}{3}$ then its 6^{th} term is **[MP PET 2003]**

(a) $\frac{1}{6}$ (b) $\frac{3}{7}$ (c) $\frac{1}{7}$ (d) $\frac{3}{5}$

Solution: (b) Let $\frac{1}{a},\frac{1}{a+d},\frac{1}{a+2d},.......$ be an H.P.

$\therefore 4^{th}$ term $=\frac{1}{a+3d} \Rightarrow \frac{3}{5}=\frac{1}{a+3d}$

$\Rightarrow \frac{5}{3}=a+3d$ .....(i)

Similarly, $3=a+7d$ .....(ii)

From (i) and (ii), $d=\frac{1}{3}$, $a=\frac{2}{3}$

$\therefore 6^{th}$ term $=\frac{1}{a+5d}=\frac{1}{\frac{2}{3}+\frac{5}{3}}=\frac{3}{7}$

Example: 39 If the roots of $a(b-c)x^2+b(c-a)x+c(a-b)=0$ be equal, then *a*, *b*, *c* are in **[Rajasthan PET 1997]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Solution: (c) As the roots are equal, discriminate = 0

$\Rightarrow \{b(c-a)\}^2-4a(b-c)c(a-b)=0 \Rightarrow b^2c^2+a^2b^2-2ab^2c-4a^2bc+4a^2c^2+4ab^2c-4abc^2=0$

$\Rightarrow (b^2c^2+2ab^2c+a^2b^2)=4ac\{ab+bc-ac\} \Rightarrow (ab+bc)^2=4ac(ab+bc-ac) \Rightarrow \{b(a+c)\}^2=4abc(a+c)-4a^2c^2$

$\Rightarrow b^2(a+c)^2-2b(a+c)\cdot 2ac+(2ac)^2=0 \Rightarrow [b(a+c)-2ac]^2=0$

$\therefore b=\frac{2ac}{a+c}$

Thus, *a*, *b*, *c* are in H.P.

Example: 40 If the first two terms of an H.P. be $\frac{2}{5}$ and $\frac{12}{23}$ then the largest positive term of the progression is the

(a) 6th term (b) 7th term (c) 5th term (d) 8th term

Solution: (c) For the corresponding A.P., the first two terms are $\frac{5}{2}$ and $\frac{23}{12}$ *i.e.* $\frac{30}{12}$ and $\frac{23}{12}$

Common difference $= -\frac{7}{12}$

$\therefore$ The A.P. will be $\frac{30}{12}, \frac{23}{12}, \frac{16}{12}, \frac{9}{12}, \frac{2}{12}, -\frac{5}{12}, ...$

The smallest positive term is $\frac{2}{12}$, which is the 5^{th} term. $\therefore$ The largest positive term of the H.P. will be the 5^{th} term.

3.16 Harmonic Mean

(1) **Definition :** If three or more numbers are in H.P., then the numbers lying between the first and last are called harmonic means (H.M.'s) between them. For example 1, 1/3, 1/5, 1/7, 1/9 are in H.P. So 1/3, 1/5 and 1/7 are three H.M.'s between 1 and 1/9.

Also, if *a*, *H*, *b* are in H.P., then *H* is called harmonic mean between *a* and *b*.

(2) **Insertion of harmonic means :**

(i) Single H.M. between *a* and *b* $= \frac{2ab}{a+b}$

(ii) *H*, H.M. of *n* non-zero numbers $a_1, a_2, a_3, ..., a_n$ is given by $\frac{1}{H} = \frac{\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}}{n}$.

(iii) Let *a*, *b* be two given numbers. If *n* numbers $H_1, H_2, ... H_n$ are inserted between *a* and *b* such that the sequence $a, H_1, H_2, H_3 ...... H_n, b$ is an H.P., then $H_1, H_2, ... H_n$ are called *n* harmonic means between *a* and *b*.

Now, $a, H_1, H_2, ... H_n, b$ are in H.P. $\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, ... \frac{1}{H_n}, \frac{1}{b}$ are in A.P.

Let *D* be the common difference of this A.P. Then,

$$\frac{1}{b} = (n+2)^{th} \text{ term} = T_{n+2}$$

$$\frac{1}{b} = \frac{1}{a} + (n+1)D \Rightarrow D = \frac{a-b}{(n+1)ab}$$

Thus, if *n* harmonic means are inserted between two given numbers *a* and *b*, then the common difference of the corresponding A.P. is given by $D = \frac{a-b}{(n+1)ab}$

Also, $\frac{1}{H_1} = \frac{1}{a} + D$, $\frac{1}{H_2} = \frac{1}{a} + 2D, ..., \frac{1}{H_n} = \frac{1}{a} + nD$ where $D = \frac{a-b}{(n+1)ab}$

Important Tips

☞ *If a, b, c are in H.P. then* $b = \frac{2ac}{a+c}$.

☞ *If* H_1 *and* H_2 *are two H.M.'s between a and b, then* $H_1 = \frac{3ab}{a+2b}$ *and* $H_2 = \frac{3ab}{2a+b}$

3.17 Properties of H.P.

(1) No term of H.P. can be zero.

(2) If a, b, c are in H.P., then $\frac{a-b}{b-c}=\frac{a}{c}$.

(3) If H is the H.M. between a and b, then

(i) $\frac{1}{H-a}+\frac{1}{H-b}=\frac{1}{a}+\frac{1}{b}$ (ii) $(H-2a)(H-2b)=H^2$ (iii) $\frac{H+a}{H-a}+\frac{H+b}{H-b}=2$

Example: 41 The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2-(4+\sqrt{3})x+8+2\sqrt{3}=0$ is **[IIT 1999]**

(a) 2 (b) 4 (c) 6 (d) 8

Solution: (b) Let α and β be the roots of the given equation

$\therefore \quad \alpha+\beta=\frac{4+\sqrt{3}}{5+\sqrt{2}}$, $\alpha\beta=\frac{8+2\sqrt{3}}{5+\sqrt{2}}$

Hence, required harmonic mean $=\frac{2\alpha\beta}{\alpha+\beta}=\frac{2\left(\frac{8+2\sqrt{3}}{5+\sqrt{2}}\right)}{\frac{4+\sqrt{3}}{5+\sqrt{2}}}=\frac{2(8+2\sqrt{3})}{4+\sqrt{3}}=\frac{4(4+\sqrt{3})}{4+\sqrt{3}}=4$

Example: 42 If a, b, c are in H.P., then the value of $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$ is **[MP PET 1998; Pb. CET 2000]**

(a) $\frac{2}{bc}+\frac{1}{b^2}$ (b) $\frac{3}{c^2}+\frac{2}{ca}$ (c) $\frac{3}{b^2}-\frac{2}{ab}$ (d) None of these

Solution: (c) a, b, c are in H.P. $\Rightarrow \frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

$\therefore \frac{1}{a}+\frac{1}{c}=\frac{2}{b}$

Now, $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)=\left\{\frac{1}{b}+\left(\frac{1}{a}+\frac{1}{c}\right)-\frac{2}{a}\right\}\left(\frac{2}{b}-\frac{1}{b}\right)=\left(\frac{1}{b}+\frac{2}{b}-\frac{2}{a}\right)\left(\frac{1}{b}\right)=\frac{1}{b}\left(\frac{3}{b}-\frac{2}{a}\right)=\frac{3}{b^2}-\frac{2}{ab}$

Example: 43 If a, b, c are in H.P., then which one of the following is true **[MNR 1985]**

(a) $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{b}$ (b) $\frac{ac}{a+c}=b$ (c) $\frac{b+a}{b-a}+\frac{b+c}{b-c}=1$ (d) None of these

Solution: (d) a, b, c are in H.P. $\Rightarrow b=\frac{2ac}{a+c}$, $\therefore$ option (b) is false

$b-a=\frac{2ac}{a+c}-a=\frac{a(c-a)}{c+a} \Rightarrow b-c=\frac{c(a-c)}{a+c}$

$\therefore \frac{1}{b-a}+\frac{1}{b-c}=\frac{a+c}{a-c}\left\{-\frac{1}{a}+\frac{1}{c}\right\}=\frac{a+c}{a-c}\cdot\frac{a-c}{ac}=\frac{a+c}{ac}=\frac{a+c}{2ac}\cdot 2=\frac{2}{b}$, $\therefore$ option (a) is false

$\frac{b+a}{b-a}+\frac{b+c}{b-c}=\frac{(c+a)(b+a)}{a(c-a)}+\frac{(b+c)(a+c)}{c(a-c)}=\frac{a+c}{a-c}\left\{-\left(\frac{b+a}{a}\right)+\frac{b+c}{c}\right\}=\frac{a+c}{a-c}\left(\frac{b}{c}-\frac{b}{a}\right)=\frac{a+c}{a-c}\cdot\frac{(a-c)b}{ac}$

$=\frac{a+c}{ac}\cdot b=\frac{a+c}{2ac}\cdot 2b=\frac{1}{b}\cdot 2b=2$

$\therefore$ option (c) is false.

Arithmetico-geometric progression(A.G.P.)

3.18 n^{th} Term of A.G.P.

If $a_1, a_2, a_3, ..., a_n, ...$ is an A.P. and $b_1, b_2, ..., b_n, ...$ is a G.P., then the sequence $a_1b_1, a_2b_2, a_3b_3, \; ......, a_nb_n, ...$ is said to be an arithmetico-geometric sequence.

Thus, the general form of an arithmetico geometric sequence is $a, (a+d)r, (a+2d)r^2, (a+3d)r^3, ...$

From the symmetry we obtain that the nth term of this sequence is $[a+(n-1)d]r^{n-1}$

Also, let $a, (a+d)r, (a+2d)r^2, (a+3d)r^3, ...$ be an arithmetico-geometric sequence. Then, $a+(a+d)r \;+(a+2d)r^2+(a+3d)r^3+...$ is an arithmetico-geometric series.

3.19 Sum of A.G.P.

(1) **Sum of *n* terms :** The sum of n terms of an arithmetico-geometric sequence $a, (a+d)r, (a+2d)r^2, \; (a+3d)r^3, ...$ is given by

$$S_n = \begin{cases} \dfrac{a}{1-r} + dr\dfrac{(1-r^{n-1})}{(1-r)^2} - \dfrac{\{a+(n-1)d\}r^n}{1-r}, \text{ when } r \neq 1 \\ \dfrac{n}{2}[2a+(n-1)d], \text{ when } r = 1 \end{cases}$$

(2) **Sum of infinite sequence :** Let $|r| < 1$. Then $r^n, r^{n-1} \to 0$ as $n \to \infty$ and it can also be shown that $n.r^n \to 0$ as $n \to \infty$. So, we obtain that $S_n \to \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$, as $n \to \infty$.

In other words, when $|r| < 1$ the sum to infinity of an arithmetico-geometric series is

$$\boxed{S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}}$$

3.20 Method for Finding Sum

This method is applicable for both sum of n terms and sum of infinite number of terms.

First suppose that sum of the series is S, then multiply it by common ratio of the G.P. and subtract. In this way, we shall get a G.P., whose sum can be easily obtained.

3.21 Method of Difference

If the differences of the successive terms of a series are in A.P. or G.P., we can find n^{th} term of the series by the following steps :

Step I: Denote the n^{th} term by T_n and the sum of the series upto n terms by S_n.

Step II: Rewrite the given series with each term shifted by one place to the right.

Step III: By subtracting the later series from the former, find T_n.

Step IV: From T_n, S_n can be found by appropriate summation.

Example: 44 $1+\frac{3}{2}+\frac{5}{2^2}+\frac{7}{2^3}+......\infty$ is equal to **[DCE 1999]**

(a) 3 (b) 6 (c) 9 (d) 12

Solution: (b)

$$S = 1+\frac{3}{2}+\frac{5}{2^2}+\frac{7}{2^3}+.......\infty$$

$$\frac{1}{2}S = \frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+.......\infty$$

$$\frac{1}{2}S = 1+\frac{2}{2}+\frac{2}{2^2}+\frac{2}{2^3}+........ \quad \text{(on subtracting)}$$

$$\Rightarrow \frac{S}{2} = 1+2\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....\infty\right) \Rightarrow \frac{S}{2} = 1+2\times\left(\frac{1/2}{1-1/2}\right) = 3$$. Hence $S = 6$

Example: 45 Sum of the series $1+2.2+3.2^2+4.2^3+.....+100.2^{99}$ is **[IIIT (Hydrabad) 2000; Kerala (Engg.) 2001]**

(a) $100.2^{100}+1$ (b) $99.2^{100}+1$ (c) $99.2^{100}-1$ (d) $100.2^{100}-1$

Solution: (b) Let $S = 1+2.2+3.2^2+4.2^3+....+100.2^{99}$ .....(i)

$2S = 1.2+2.2^2+3.2^3+.....+99.2^{99}+100.2^{100}$ .....(ii)

Equation (i) – Equation (ii) gives,

$$-S = 1+(1.2+1.2^2+1.2^3+..... \text{ upto 99 terms}) - 100.2^{100} = 1+\frac{2(2^{99}-1)}{2-1}-100.2^{100}$$

$$\Rightarrow S = -1-2^{100}+2+100.2^{100} = 1+99.2^{100}$$

Example: 46 The sum of the series 3 + 33 + 333 + ... + n terms is **[Rajasthan PET 2000]**

(a) $\frac{1}{27}(10^{n+1}+9n-28)$ (b) $\frac{1}{27}(10^{n+1}-9n-10)$ (c) $\frac{1}{27}(10^{n+1}+10n-9)$ (d) None of these

Solution: (b)

$$S = 3+33+333+...... \text{ to } n \text{ terms}$$

$$S = 3+33+.......$$

$$0 = 3+30+300+......... \text{ to } n \text{ terms} - T_n \quad \text{(on subtracting)}$$

$$\therefore T_n = 3(1+10+100+....... \text{ to } n \text{ terms}) = 3\times 1\cdot\frac{10^n-1}{10-1} = \frac{1}{3}(10^n-1)$$

$$S_n = \sum_{n=1}^{n}\frac{1}{3}(10^n-1) = \frac{1}{3}\sum_{n=1}^{n}10^n - \frac{1}{3}\sum_{n=1}^{n}1 = \frac{1}{3}\left(10\cdot\frac{10^n-1}{10-1}\right)-\frac{1}{3}n$$

$$S = \frac{1}{27}(10^{n+1}-9n-10)$$

Example: 47 The sum of n terms of the following series $1+(1+x)+(1+x+x^2)+....$ will be **[IIT 1962]**

(a) $\frac{1-x^n}{1-x}$ (b) $\frac{x(1-x^n)}{1-x}$ (c) $\frac{n(1-x)-x(1-x^n)}{(1-x)^2}$ (d) None of these

Solution: (c)

$$S = 1+(1+x)+(1+x+x^2)+......$$

$$S = 1+(1+x)+.......$$

$$0 = (1+x+x^2+..... \text{ to } n \text{ terms}) - T_n \quad \text{(on subtracting)}$$

$$\therefore T_n = \frac{1-x^n}{1-x}$$

$$S_n = \sum_{n=1}^{n}T_n = \sum_{n=1}^{n}\frac{1-x^n}{1-x} = \frac{1}{1-x}\sum_{n=1}^{n}1-\frac{1}{1-x}\sum_{n=1}^{n}x^n = \frac{1}{1-x}\cdot n-\frac{1}{1-x}\cdot x\cdot\left(\frac{1-x^n}{1-x}\right)$$

$$= \frac{n}{1-x} - \frac{x(1-x^n)}{(1-x)^2} = \frac{n(1-x) - x(1-x^n)}{(1-x)^2}$$

Example: 48 The sum to n terms of the series $1+3+7+15+31+.......$ is **[IIT 1963]**

(a) $2^{n+1} - n$ (b) $2^{n+1} - n - 2$ (c) $2^n - n - 2$ (d) None of these

Solution: (b)

$$S = 1+3+7+15+31+......$$
$$S = \quad 1+3+7+15+.......$$
$$0 = (1+2+4+8+16+..... \text{ to } n \text{ terms}) - T_n \quad \text{(on subtracting)}$$

$$\therefore \quad T_n = 1+2+4+8+.......... \text{ to } n \text{ terms} = 1 \cdot \frac{2^n - 1}{2-1} = 2^n - 1$$

$$S_n = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n} (2^n - 1) = \sum_{n=1}^{n} 2^n - \sum_{n=1}^{n} 1 = 2 \cdot \left(\frac{2^n - 1}{2-1} \right) - n = 2^{n+1} - n - 2$$

Miscellaneous series

3.22 Special Series

There are some series in which n^{th} term can be predicted easily just by looking at the series.

If $T_n = \alpha n^3 + \beta n^2 + \gamma n + \delta$

Then $S_n = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n} (\alpha n^3 + \beta n^2 + \gamma n + \delta) = \alpha \sum_{n=1}^{n} n^3 + \beta \sum_{n=1}^{n} n^2 + \gamma \sum_{n=1}^{n} n + \delta \sum_{n=1}^{n} 1$

$$= \alpha \left(\frac{n(n+1)}{2} \right)^2 + \beta \left(\frac{n(n+1)(2n+1)}{6} \right) + \gamma \left(\frac{n(n+1)}{2} \right) + \delta n$$

Note : ❑ Sum of squares of first n natural numbers $= 1^2 + 2^2 + 3^2 + ... + n^2 = \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

❑ Sum of cubes of first n natural numbers $= 1^3 + 2^3 + 3^3 + 4^3 + ... + n^3 = \sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2} \right]^2$

3.23 V_n Method

(1) To find the sum of the series $\frac{1}{a_1 a_2 a_3 .....a_r} + \frac{1}{a_2 a_3 .....a_{r+1}} + ... + \frac{1}{a_n a_{n+1} .....a_{n+r-1}}$

Let d be the common difference of A.P. Then $a_n = a_1 + (n-1)d$.

Let S_n and T_n denote the sum to n terms of the series and n^{th} term respectively.

$$S_n = \frac{1}{a_1 a_2 .....a_r} + \frac{1}{a_2 a_3 .....a_{r+1}} + ... + \frac{1}{a_n a_{n+1} .....a_{n+r-1}}$$

$$\therefore \ T_n = \frac{1}{a_n a_{n+1} .....a_{n+r-1}}$$

Let $V_n = \dfrac{1}{a_{n+1}a_{n+2}.....a_{n+r-1}}$; $\quad V_{n-1} = \dfrac{1}{a_n a_{n+1}.....a_{n+r-2}}$

$\Rightarrow V_n - V_{n-1} = \dfrac{1}{a_{n+1}a_{n+2}.....a_{n+r-1}} - \dfrac{1}{a_n a_{n+1}.....a_{n+r-2}} = \dfrac{a_n - a_{n+r-1}}{a_n a_{n+1}.....a_{n+r-1}}$

$= \dfrac{[a_1 + (n-1)d] - [a_1 + \{(n+r-1)-1\}d]}{a_n a_{n+1}.....a_{n+r-1}} = d(1-r)T_n$

$\therefore T_n = \dfrac{1}{d(r-1)}\{V_{n-1} - V_n\}$, $\quad \therefore S_n = \sum_{n=1}^{n} T_n = \dfrac{1}{d(r-1)}(V_0 - V_n)$

$S_n = \dfrac{1}{(r-1)(a_2 - a_1)}\left\{\dfrac{1}{a_1 a_2....a_{r-1}} - \dfrac{1}{a_{n+1}a_{n+2}......a_{n+r-1}}\right\}$

Example: If $a_1, a_2, ...a_n$ are in A.P., then

$$\frac{1}{a_1a_2a_3} + \frac{1}{a_2a_3a_4} + ... + \frac{1}{a_n a_{n+1} a_{n+2}} = \frac{1}{2(a_2 - a_1)}\left\{\frac{1}{a_1a_2} - \frac{1}{a_{n+1}a_{n+2}}\right\}$$

(2) If $S_n = a_1a_2.....a_r + a_2a_3.....a_{r+1}.... + a_n a_{n+1}...a_{n+r-1}$

$T_n = a_n a_{n+1}.....a_{n+r-1}$

Let $V_n = a_n a_{n+1}....a_{n+r-1}a_{n+r}$, $\therefore V_{n-1} = a_{n-1}a_{n+1}......a_{n+r-1}$

$\Rightarrow V_n - V_{n-1} = a_n a_{n+1} a_{n+2}.....a_{n+r-1}(a_{n+r} - a_{n-1}) = T_n\{[a_1 + (n+r-1)d] - [a_1 + (n-2)d]\} = T_n(r+1)d$

$\therefore T_n = \dfrac{V_n - V_{n-1}}{(r+1)d}$

$S_n = \sum_{n=1}^{n} T_n = \dfrac{1}{(r+1)d}\sum_{n=1}^{n}(V_n - V_{n-1}) = \dfrac{1}{(r+1)d}(V_n - V_0) = \dfrac{1}{(r+1)d}\{(a_n a_{n+1}....a_{n+r}) - (a_0 a_1....a_r)\}$

$= \dfrac{1}{(r+1)(a_2 - a_1)}\{a_n a_{n+1}....a_{n+r} - a_0 a_1.....a_r\}$

Example: $1.2.3.4 + 2.3.4.5 + ... + n(n+1)(n+2)(n+3) = \dfrac{1}{5.1}\{n(n+1)(n+2)(n+3) - 0.1.2.3\}$

$= \dfrac{1}{5}\{n(n+1)(n+2)(n+3)\}$

Example: 49 The sum of $1^3 + 2^3 + 3^3 + 4^3 + ... + 15^3$ is **[MP PET 2003]**

(a) 22000 (b) 10000 (c) 14400 (d) 15000

Solution: (c) $S = 1^3 + 2^3 + 3^3 + ... + 15^3$; For $n = 15$, the value of $\left(\dfrac{n(n+1)}{2}\right)^2 = \left(\dfrac{15 \times 16}{2}\right)^2 = 14400$

Example: 50 A series whose n^{th} term is $\left(\dfrac{n}{x}\right) + y$, the sum of r terms will be **[UPSEAT 1999]**

(a) $\left\{\dfrac{r(r+1)}{2x}\right\} + ry$ (b) $\left\{\dfrac{r(r-1)}{2x}\right\}$ (c) $\left\{\dfrac{r(r-1)}{2x}\right\} - ry$ (d) $\left\{\dfrac{r(r+1)}{2x}\right\} - rx$

Solution: (a) $S_r = \sum_{n=1}^{r} t_n = \sum_{n=1}^{r}\left(\dfrac{n}{x} + y\right) = \dfrac{1}{x}\sum_{n=1}^{r} n + y\sum_{n=1}^{r} 1 = \dfrac{1}{x}\dfrac{r(r+1)}{2} + yr = \dfrac{r(r+1)}{2x} + ry$

Example: 51 If $(1^2 - t_1) + (2^2 - t_2) + ... + (n^2 - t_n) = \dfrac{1}{3}n(n^2 - 1)$, then t_n is

(a) $\frac{n}{2}$ (b) $n-1$ (c) $n+1$ (d) n

Solution: (d) $\frac{1}{3}n(n^2-1)=(1^2+2^2+....+n^2)-(t_1+t_2+.....+t_n)$

$\Rightarrow t_1+t_2+.....+t_n=1^2+2^2+3^2+.......+n^2-\frac{1}{3}n(n^2-1)=\frac{n(n+1)(2n+1)}{6}-\frac{1}{3}n(n^2-1)=\frac{n(n+1)}{6}[2n+1-(2n-2)]$

$\therefore\ t_1+t_2+t_3+.....+t_n=\frac{n(n+1)}{2}\Rightarrow S_n=\frac{n(n+1)}{2}$

$t_n=S_n-S_{n-1}=\frac{n(n+1)}{2}-\frac{(n-1)n}{2}=n$

Example: 52 The sum of the series $\frac{1}{3\times 7}+\frac{1}{7\times 11}+\frac{1}{11\times 15}+....$ is **[MNR 1984; UPSEAT 2000]**

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{12}$

Solution: (d) $S=\left(\frac{1}{3\times 7}+\frac{1}{7\times 11}+\frac{1}{11\times 15}+....\right)=\frac{1}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{15}\right)+.......\frac{1}{\infty}\right]=\frac{1}{4}\left[\frac{1}{3}-\frac{1}{\infty}\right]=\frac{1}{4}\left[\frac{1}{3}-0\right]=\frac{1}{12}$

Example: 53 The sum of the series 1.2.3 + 2.3.4 + 3.4.5 + ... to n terms is **[Kurukshetra CEE 1998]**

(a) $n(n+1)(n+2)$ (b) $(n+1)(n+2)(n+3)$

(c) $\frac{1}{4}n(n+1)(n+2)(n+3)$ (d) $\frac{1}{4}(n+1)(n+2)(n+3)$

Solution: (c) $T_n=n(n+1)(n+2)=n^3+3n^2+2n$

$\therefore\ S=1.2.3+2.3.4+3.4.5+......\text{ to } n \text{ terms}=\sum_{n=1}^{n}(n^3+3n^2+2n)=\sum_{n=1}^{n}n^3+3\sum_{n=1}^{n}n^2+2\sum_{n=1}^{n}n$

$S=\left(\frac{n(n+1)}{2}\right)^2+3\frac{n(n+1)(2n+1)}{6}+2\frac{n(n+1)}{2}=\frac{1}{4}n(n+1)[n(n+1)+2(2n+1)+4]$

$=\frac{1}{4}n(n+1)[n^2+5n+6]=\frac{1}{4}n(n+1)(n+2)(n+3)$

3.24 Properties of Arithmetic, Geometric and Harmonic means between Two given Numbers

Let A, G and H be arithmetic, geometric and harmonic means of two numbers a and b. Then, $A=\frac{a+b}{2}, G=\sqrt{ab}$ and $H=\frac{2ab}{a+b}$

These three means possess the following properties :

(1) $A\geq G\geq H$

$A=\frac{a+b}{2}, G=\sqrt{ab}$ and $H=\frac{2ab}{a+b}$

$\therefore\ A-G=\frac{a+b}{2}-\sqrt{ab}=\frac{(\sqrt{a}-\sqrt{b})^2}{2}\geq 0$

$\Rightarrow A\geq G$ .....(i)

$G-H=\sqrt{ab}-\frac{2ab}{a+b}=\sqrt{ab}\left(\frac{a+b-2\sqrt{ab}}{a+b}\right)=\frac{\sqrt{ab}}{a+b}(\sqrt{a}-\sqrt{b})^2\geq 0$

$\Rightarrow G \geq H$ .....(ii)

From (i) and (ii), we get $A \geq G \geq H$

Note that the equality holds only when *a* = *b*

(2) *A*, *G*, *H* from a G.P., *i.e.* $G^2 = AH$

$$AH = \frac{a+b}{2} \times \frac{2ab}{a+b} = ab = (\sqrt{ab})^2 = G^2$$

Hence, $G^2 = AH$

(3) The equation having *a* and *b* as its roots is $x^2 - 2Ax + G^2 = 0$

The equation having *a* and *b* its roots is $x^2 - (a+b)x + ab = 0$

$\Rightarrow x^2 - 2Ax + G^2 = 0$ $\left[\because A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}\right]$

The roots *a*, *b* are given by $A \pm \sqrt{A^2 - G^2}$

(4) If *A*, *G, H* are arithmetic, geometric and harmonic means between three given numbers *a*, *b* and *c*, then the equation having *a*, *b*, *c* as its roots is $x^3 - 3Ax^2 + \frac{3G^3}{H}x - G^3 = 0$

$$A = \frac{a+b+c}{3}, G = (abc)^{1/3} \text{ and } \frac{1}{H} = \frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3}$$

$$\Rightarrow a+b+c = 3A, abc = G^3 \text{ and } \frac{3G^3}{H} = ab+bc+ca$$

The equation having *a*, *b*, *c* as its roots is $x^3 - (a+b+c)x^2 + (ab+bc+ca)x - abc = 0$

$$\Rightarrow x^3 - 3Ax^2 + \frac{3G^3}{H}x - G^3 = 0$$

3.25 Relation between A.P., G.P. and H.P.

(1) If *A*, *G*, *H* be A.M., G.M., H.M. between *a* and *b*, then $\boxed{\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \begin{cases} A \text{ when } n = 0 \\ G \text{ when } n = -1/2 \\ H \text{ when } n = -1 \end{cases}}$

(2) If A_1, A_2 be two A.M.'s; G_1, G_2 be two G.M.'s and H_1, H_2 be two H.M.'s between two numbers *a* and *b* then $\boxed{\frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2}}$

(3) **Recognization of A.P., G.P., H.P. :** If *a*, *b*, *c* are three successive terms of a sequence.

Then if, $\frac{a-b}{b-c} = \frac{a}{a}$, then *a*, *b*, *c* are in A.P.

If, $\frac{a-b}{b-c} = \frac{a}{b}$, then *a*, *b*, *c* are in G.P.

If, $\frac{a-b}{b-c}=\frac{a}{c}$, then a, b, c are in H.P.

(4) If number of terms of any A.P./G.P./H.P. is odd, then A.M./G.M./H.M. of first and last terms is middle term of series.

(5) If number of terms of any A.P./G.P./H.P. is even, then A.M./G.M./H.M. of middle two terms is A.M./G.M./H.M. of first and last terms respectively.

(6) If p^{th}, q^{th} and r^{th} terms of a G.P. are in G.P. Then p, q, r are in A.P.

(7) If a, b, c are in A.P. as well as in G.P. then $a=b=c$.

(8) If a, b, c are in A.P., then x^a, x^b, x^c will be in G.P. $(x \neq \pm 1)$

Example: 54 If the A.M., G.M. and H.M. between two positive numbers a and b are equal, then **[Rajasthan PET 2003]**

(a) $a = b$ (b) $ab = 1$ (c) $a > b$ (d) $a < b$

Solution: (a) $\because$ A.M. = G.M.

$\Rightarrow \frac{a+b}{2}=\sqrt{ab} \Rightarrow \frac{(\sqrt{a})^2-2\sqrt{a}\sqrt{b}+(\sqrt{b})^2}{2}=0 \Rightarrow \frac{(\sqrt{a}-\sqrt{b})^2}{2}=0 \Rightarrow a=b$

$\because$ G.M. = H.M.

$\Rightarrow \sqrt{ab}=\frac{2ab}{a+b} \Rightarrow a+b-2\sqrt{ab}=0 \Rightarrow (\sqrt{a}-\sqrt{b})^2=0 \Rightarrow \sqrt{a}=\sqrt{b} \quad \therefore \quad a=b$

Thus A.M. =(G.M.) (H.M.) So $a=b$

Example: 55 Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation **[AIEEE 2004]**

(a) $x^2-18x-16=0$ (b) $x^2-18x+16=0$ (c) $x^2+18x-16=0$ (d) $x^2+18x+16=0$

Solution: (b) A = 9, G = 4 are respectively the A.M. and G.M. between two numbers, then the quadratic equation having its roots as the two numbers, is given by $x^2-2Ax+G^2=0$ *i.e.* $x^2-18x+16=0$

Example: 56 If $\frac{a}{b},\frac{b}{c},\frac{c}{a}$ are in H.P., then **[UPSEAT 2002]**

(a) a^2b, c^2a, b^2c are in A.P. (b) a^2b, b^2c, c^2a are in H.P.

(c) a^2b, b^2c, c^2a are in G.P. (d) None of these

Solution: (a) $\frac{a}{b},\frac{b}{c},\frac{c}{a}$ are in H.P.

$\Rightarrow \frac{b}{a},\frac{c}{b},\frac{a}{c}$ are in A.P. $\Rightarrow abc\times\frac{b}{a}, abc\times\frac{c}{b}, abc\times\frac{a}{c}$ are in A.P. $\Rightarrow b^2c, ac^2, a^2b$ are in A.P.

$\therefore a^2b, c^2a, b^2c$ are in A.P.

Example: 57 If a, b, c are in G.P., then $\log_a x, \log_b x, \log_c x$ are in **[Rajasthan PET 2002]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Solution: (c) a, b, c are in G.P.

$\Rightarrow \log_x a, \log_x b, \log_x c$ are in A.P. $\Rightarrow \frac{1}{\log_a x},\frac{1}{\log_b x},\frac{1}{\log_c x}$ are in A.P.

$\therefore \log_a x, \log_b x, \log_c x$ are in H.P.

Example: 58 If A_1, A_2 ; G_1, G_2 and H_1, H_2 be two A.M.'s, G.M.'s and H.M.'s between two quantities, then the value of $\frac{G_1G_2}{H_1H_2}$ is

[Roorkee 1983; AMU 2000]

(a) $\frac{A_1+A_2}{H_1+H_2}$ (b) $\frac{A_1-A_2}{H_1+H_2}$ (c) $\frac{A_1+A_2}{H_1-H_2}$ (d) $\frac{A_1-A_2}{H_1-H_2}$

Solution: (a) Let a and b be the two numbers

$\therefore\ A_1 = a+\left(\frac{b-a}{3}\right)=\frac{2a+b}{3}$, $A_2 = a+2\left(\frac{b-a}{3}\right)=\frac{a+2b}{3}$

$G_1 = a\left(\frac{b}{a}\right)^{1/3} = a^{2/3}\,b^{1/3}$, $G_2 = a\left(\left(\frac{b}{a}\right)^{1/3}\right)^2 = a^{1/3}b^{2/3}$

$H_1 = \frac{1}{\frac{1}{a}+\left(\frac{1}{b}-\frac{1}{a}\right)\frac{1}{3}} = \frac{3}{\frac{2}{a}+\frac{1}{b}} = \frac{3ab}{a+2b}$, $H_2 = \frac{3ab}{2a+b}$

$\therefore\ \frac{G_1G_2}{H_1H_2} = \frac{(a^{2/3}b^{1/3})(a^{1/3}b^{2/3})}{\frac{3ab}{a+2b}\cdot\frac{3ab}{2a+b}} = \frac{(a+2b)(2a+b)}{9ab}$

$A_1+A_2 = \frac{2a+b}{3}+\frac{a+2b}{3} = a+b$

$H_1+H_2 = \frac{3ab}{a+2b}+\frac{3ab}{2a+b} = 3ab\left(\frac{2a+b+a+2b}{(a+2b)(2a+b)}\right) = \frac{9ab\,(a+b)}{(a+2b)(2a+b)}$

$\therefore\ \frac{A_1+A_2}{H_1+H_2} = \frac{(a+2b)(2a+b)}{9ab} = \frac{G_1G_2}{H_1H_2}$

Example: 59 If the ratio of H.M. and G.M. of two quantities is 12 : 13, then the ratio of the numbers is **[Rajasthan PET 1990]**

(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) None of these

Solution: (d) Let x and y be the numbers

$\therefore$ H.M. $= \frac{2xy}{x+y}$, G.M. $= \sqrt{xy}$

$\therefore\ \frac{\text{H.M.}}{\text{G.M.}} = \frac{2\sqrt{xy}}{x+y} = \frac{2\sqrt{x/y}}{\frac{x}{y}+1} \Rightarrow \frac{12}{13} = \frac{2r}{r^2+1}$, $(\because\ r = \sqrt{\frac{x}{y}}) \Rightarrow 12r^2-26r+12=0 \Rightarrow 6r^2-13r+6=0$

$\therefore\ r = \frac{13\pm\sqrt{13^2-4.6.6}}{2\times 6} = \frac{13\pm 5}{12} = \frac{18}{12},\frac{8}{12} = \frac{3}{2},\frac{2}{3}$

$\therefore$ Ratio of numbers $= \frac{x}{y} = r^2 : 1 = \frac{9}{4}:1$ or $\frac{4}{9}:1 = 9:4$ or 4 : 9

Example: 60 If the A.M. of two numbers is greater than G.M. of the numbers by 2 and the ratio of the numbers is 4 : 1, then the numbers are **[Rajasthan PET 1988]**

(a) 4, 1 (b) 12, 3 (c) 16, 4 (d) None of these

Solution: (c) Let x and y be the numbers $\therefore$ A.M. = G.M. + 2 $\Rightarrow \frac{x+y}{2} = \sqrt{xy}+2$

Also, $\frac{x}{y} = 4:1 \Rightarrow x = 4y$

$\therefore\ \frac{4y+y}{2} = \sqrt{4y.y}+2 \Rightarrow \frac{5y}{2} = 2y+2 \Rightarrow y = 4 \Rightarrow x = 4\times 4 = 16$

$\therefore$ The numbers are 16, 4.

Example: 61 If the ratio of A.M. between two positive real numbers a and b to their H.M. is $m : n$, then $a : b$ is

(a) $\dfrac{\sqrt{m-n}+\sqrt{n}}{\sqrt{m-n}-\sqrt{n}}$ (b) $\dfrac{\sqrt{n}+\sqrt{m-n}}{\sqrt{n}-\sqrt{m-n}}$ (c) $\dfrac{\sqrt{m}+\sqrt{m-n}}{\sqrt{m}-\sqrt{m-n}}$ (d) None of these

Solution: (c) We have, $\dfrac{m}{n}=\dfrac{(a+b)/2}{2ab/(a+b)} \Rightarrow \dfrac{m}{n}=\dfrac{(a+b)^2}{4ab}=\dfrac{\left(\dfrac{a}{b}+1\right)^2}{4\dfrac{a}{b}} \Rightarrow 4\dfrac{m}{n}\left(\dfrac{a}{b}\right)=\left(\dfrac{a}{b}+1\right)^2 \Rightarrow 2\dfrac{\sqrt{m}}{\sqrt{n}}\sqrt{\dfrac{a}{b}}=\left(1+\dfrac{a}{b}\right)$

Let $\dfrac{a}{b}=r^2$, $\therefore \dfrac{2\sqrt{m}}{\sqrt{n}}r=(1+r^2) \Rightarrow 2\sqrt{m}\,r=\sqrt{n}+\sqrt{n}\,r^2 \Rightarrow \sqrt{n}\,r^2-2\sqrt{m}\,r+\sqrt{n}=0$

$\therefore r=\dfrac{2\sqrt{m}\pm\sqrt{4m-4n}}{2\sqrt{n}}=\dfrac{\sqrt{m}\pm\sqrt{m-n}}{\sqrt{n}}$

Considering +*ve* sign, $r=\dfrac{\sqrt{m}+\sqrt{m-n}}{\sqrt{n}}=\dfrac{(\sqrt{m}+\sqrt{m-n})(\sqrt{m}-\sqrt{m-n})}{\sqrt{n}(\sqrt{m}-\sqrt{m-n})}=\dfrac{m-(m-n)}{\sqrt{n}(\sqrt{m}-\sqrt{m-n})}=\dfrac{\sqrt{n}}{\sqrt{m}-\sqrt{m-n}}$

$\therefore r^2=\dfrac{\sqrt{m}+\sqrt{m-n}}{\sqrt{n}}\cdot\dfrac{\sqrt{n}}{\sqrt{m}-\sqrt{m-n}}$. Hence, $\dfrac{a}{b}=\dfrac{\sqrt{m}+\sqrt{m-n}}{\sqrt{m}-\sqrt{m-n}}$.

3.26 Applications of Progressions

There are many applications of progressions is applied in science and engineering. Properties of progressions are applied to solve problems of inequality and maximum or minimum values of some expression can be found by the relation among A.M., G.M. and H.M.

Example: 62 If $x=\log_5 3+\log_7 5+\log_9 7$ then

(a) $x\ge\dfrac{3}{2}$ (b) $x\ge\dfrac{1}{\sqrt[3]{2}}$ (c) $x\ge\dfrac{3}{\sqrt[3]{2}}$ (d) None of these

Solution: (c) $x=\log_5 3+\log_7 5+\log_9 7$

$\dfrac{\log_5 3+\log_7 5+\log_9 7}{3}\ge(\log_5 3.\log_7 5.\log_9 7)^{1/3}$ [A.M. ≥ G.M.]

$\Rightarrow \dfrac{x}{3}\ge(\log_9 3)^{1/3} \Rightarrow x\ge 3(\log_9 9^{1/2})^{1/3} \Rightarrow x\ge 3\left(\dfrac{1}{2}\right)^{1/3}$. Hence $x\ge\dfrac{3}{\sqrt[3]{2}}$

Example: 63 If *a*, *b*, *c*, *d* are four positive numbers then

(a) $\left(\dfrac{a}{b}+\dfrac{b}{c}\right)\left(\dfrac{c}{d}+\dfrac{d}{e}\right)\ge 4\cdot\sqrt{\dfrac{a}{e}}$ (b) $\left(\dfrac{a}{b}+\dfrac{c}{d}\right)\left(\dfrac{b}{c}+\dfrac{d}{e}\right)\ge 4\cdot\sqrt{\dfrac{a}{e}}$

(c) $\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{d}+\dfrac{d}{e}+\dfrac{e}{a}\ge 5$ (d) $\dfrac{b}{a}+\dfrac{c}{b}+\dfrac{d}{c}+\dfrac{e}{d}+\dfrac{a}{e}\ge\dfrac{1}{5}$

Solution: (a,b,c) We have $\dfrac{\dfrac{a}{b}+\dfrac{b}{c}}{2}\ge\left(\dfrac{a}{b}\cdot\dfrac{b}{c}\right)^{1/2}$; ($\because$ A.M. ≥ G.M.)

$\Rightarrow \dfrac{a}{b}+\dfrac{b}{c}\ge 2\sqrt{\dfrac{a}{c}}$ .....(i)

Similarly, $\dfrac{c}{d}+\dfrac{d}{e}\ge 2\sqrt{\dfrac{c}{e}}$ .....(ii)

Multiplying (i) by (ii),

$\left(\dfrac{a}{b}+\dfrac{b}{c}\right)\left(\dfrac{c}{d}+\dfrac{d}{e}\right)\ge 4\sqrt{\dfrac{a}{c}}\sqrt{\dfrac{c}{e}} \Rightarrow \left(\dfrac{a}{b}+\dfrac{b}{c}\right)\left(\dfrac{c}{d}+\dfrac{d}{e}\right)\ge 4\sqrt{\dfrac{a}{e}}$, $\therefore$ (a) is true

Next, $\left(\dfrac{a}{b}+\dfrac{c}{d}\right)\left(\dfrac{b}{c}+\dfrac{d}{e}\right)\ge 2\left(\dfrac{a}{b}\cdot\dfrac{c}{d}\right)^{1/2}\cdot 2\left(\dfrac{b}{c}\cdot\dfrac{d}{e}\right)^{1/2} \Rightarrow \left(\dfrac{a}{b}+\dfrac{c}{d}\right)\left(\dfrac{b}{c}+\dfrac{d}{e}\right)\ge 4\sqrt{\dfrac{a}{e}}$, $\therefore$ (b) is true

$$\frac{\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{e}+\frac{e}{a}}{5} \ge \left(\frac{a}{b}\cdot\frac{b}{c}\cdot\frac{c}{d}\cdot\frac{d}{e}\cdot\frac{e}{a}\right)^{1/5} \Rightarrow \frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{e}+\frac{e}{a} \ge 5, \quad \therefore \text{(c) is true}$$

Now, $\frac{b}{a}+\frac{c}{b}+\frac{d}{c}+\frac{e}{d}+\frac{a}{e} \ge 5\left(\frac{b}{a}\cdot\frac{c}{b}\cdot\frac{d}{c}\cdot\frac{e}{d}\cdot\frac{a}{e}\right)^{1/5} \Rightarrow \frac{b}{a}+\frac{c}{b}+\frac{d}{c}+\frac{e}{d}+\frac{a}{e} \ge 5$, $\therefore$ (d) is false

Example: 64 Let $a_n =$ product of first n natural numbers. Then for all $n \in N$

(a) $n^n \ge a_n$ (b) $\left(\frac{n+1}{2}\right)^n \ge n!$ (c) $n^n \ge a_n + 1$ (d) None of these

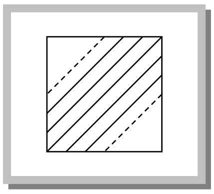
Solution: (a,b) We have $a_n = 1.2.3.........\,...n = n!$, $n^n = n.n.n........$ to n times

$\Rightarrow n^n \ge n.(n-1)(n-2)....\{n-(n-1)\} \Rightarrow n^n \ge n.(n-1)(n-2).........2.1 \Rightarrow n^n \ge n! \;\therefore\; n^n \ge a_n$. So (a) is true

$n^n \ngeq (n+1)! \Rightarrow n^n \ngeq a_n + 1$. So (c) is false

$\frac{1+2+3+.....+n}{n} \ge (1.2.3......n)^{1/n} \Rightarrow \frac{n(n+1)}{2n} \ge (n!)^{1/n} \Rightarrow \frac{n+1}{2} \ge (n!)^{1/n}$. $\therefore \left(\frac{n+1}{2}\right)^n \ge n!$. So (b) is true.

Example: 65 In the given square, a diagonal is drawn and parallel line segments joining points on the adjacent sides are drawn on both sides of the diagonal. The length of the diagonal is $n\sqrt{2}$ *cm*. If the distance between consecutive line segments be $\frac{1}{\sqrt{2}}$ *cm* then the sum of the lengths of all possible line segments and the diagonal is



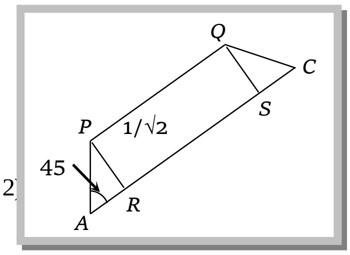
(a) $n(n+1)\sqrt{2}\,cm$ (b) $n^2\,cm$ (c) $n(n+2)\,cm$ (d) $n^2\sqrt{2}\,cm$

Solution: (d) Let us consider the diagonal and an adjacent parallel line

Length of the line $PQ = RS = AC - (AR + SC) = AC - 2AR$

$= AC - 2.PR$ $\quad (\because AR = PR)$

$= n\sqrt{2} - 2\cdot\frac{1}{\sqrt{2}} = n\sqrt{2} - \sqrt{2} = (n-1)\sqrt{2}\,cm$



Length of line adjacent to PQ, other than AC, will be $((n-1)-1)\sqrt{2} = (n-2$

$\therefore$ Sum of the lengths of all possible line segments and the diagonal

$= 2 \times [n\sqrt{2} + (n-1)\sqrt{2} + (n-2)\sqrt{2} + ...] - n\sqrt{2}$, $\quad n \in N$

$= 2 \times \sqrt{2}[n + (n-1) + (n-2) + ... + 1] - n\sqrt{2} = 2\sqrt{2} \times \frac{n(n+1)}{2} - n\sqrt{2} = n\sqrt{2}\{n+1-1\} = n^2\sqrt{2}\,cm$

Example: 66 Let $f(x) = \frac{1-x^{n+1}}{1-x}$ and $g(x) = 1 - \frac{2}{x} + \frac{3}{x^2} - .... + (-1)^n \frac{n+1}{x^n}$. Then the constant term in $f'(x) \times g(x)$ is equal to

(a) $\frac{n(n^2-1)}{6}$ when n is even (b) $\frac{n(n+1)}{2}$ when n is odd (c) $-\frac{n}{2}(n+1)$ when n is even

(d) $-\frac{n(n-1)}{2}$ when n is odd

Solution: (b,c) $f(x) = \frac{1-x^{n+1}}{1-x} = \frac{(1-x)(1+x+x^2+.....+x^n)}{(1-x)} = 1 + x + x^2 + ... + x^n$; $\quad f'(x) = 1 + 2x + 3x^2 + ... + n\,x^{n-1}$

$f'(x).g(x) = (1 + 2x + 3x^2 + ... + n\,x^{n-1}) \times \left(1 - \frac{2}{x} + \frac{3}{x^2} - ...... + (-1)^n \frac{n+1}{x^n}\right)$

$\therefore$ constant term in $f'(x)\times g(x)$ is

$$c=1^2-2^2+3^2-4^2+.......+n^2(-1)^{n-1}=[1^2+2^2+3^2+4^2+......+n^2]-2[2^2+4^2+6^2+....]$$

when n is odd,

$$c=[1^2+2^2+.....+n^2]-2[2^2+4^2+6^2+......+(n-1)^2]=\left[\frac{n(n+1)(2n+1)}{6}-2.2^2[1^2+2^2+3^2+......+\left(\frac{n-1}{2}\right)^2\right]$$

$$=\frac{n(n+1)(2n+1)}{6}-8\,\frac{\left(\frac{n-1}{2}\right)\cdot\left(\frac{n-1}{2}+1\right)\left(2\,\frac{n-1}{2}+1\right)}{6}=\frac{n(n+1)(2n+1)}{6}-\frac{n(n-1)(n+1)}{3}$$

$$=\frac{n(n+1)}{6}(2n+1-2(n-1))=\frac{n(n+1)}{6}\times 3=\frac{n(n+1)}{2}$$

when n is even, $c=[1^2+2^2+.....+n^2]-2[2^2+4^2+.....+n^2]=\frac{n(n+1)(2n+1)}{6}-2.2^2\left[1^2+2^2+....+\left(\frac{n}{2}\right)^2\right]$

$$=\frac{n(n+1)(2n+1)}{6}-8\,\frac{\left(\frac{n}{2}\right)\cdot\left(\frac{n}{2}+1\right)\left(2\cdot\frac{n}{2}+1\right)}{6}=\frac{n(n+1)(2n+1)}{6}-\frac{1}{3}n(n+1)(n+2)$$

$$=\frac{1}{6}n(n+1)(2n+1-2(n+2))=-\frac{1}{2}n(n+1)$$

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Assignment

General term of an Arithmetic progression

Basic Level

1. The sequence $\frac{5}{\sqrt{7}}, \frac{6}{\sqrt{7}}, \sqrt{7}........$ is

(a) H.P. (b) G.P. (c) A.P. (d) None of these

2. p^{th} term of the series $\left(3-\frac{1}{n}\right)+\left(3-\frac{2}{n}\right)+\left(3-\frac{3}{n}\right)+.....$ will be

(a) $\left(3+\frac{p}{n}\right)$ (b) $\left(3-\frac{p}{n}\right)$ (c) $\left(3+\frac{n}{p}\right)$ (d) $\left(3-\frac{n}{p}\right)$

3. If the 9^{th} term of an A.P. be zero, then the ratio of its 29^{th} and 19^{th} term is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

4. Which of the following sequence is an arithmetic sequence

(a) $f(n)=an+b; n \in N$ (b) $f(n)=kr^n; n \in N$ (c) $f(n)=(an+b)kr^n; n \in N$ (d) $f(n)=\frac{1}{a\left(n+\frac{b}{n}\right)}; n \in N$

5. If the p^{th} term of an A.P. be q and q^{th} term be p, then its r^{th} term will be **[Rajasthan PET 1999]**

(a) $p+q+r$ (b) $p+q-r$ (c) $p+r-q$ (d) $p-q-r$

6. If the 9^{th} term of an A.P. is 35 and 19^{th} is 75, then its 20^{th} term will be **[Rajasthan PET 1989]**

(a) 78 (b) 79 (c) 80 (d) 81

7. If $(a+1), 3a, (4a+2)$ are in A.P. then 7^{th} term of the series is

(a) $10a+4$ (b) – 33 (c) 33 (d) 10 a – 4

8. It x, y, z are in A.P., then its common difference is

(a) $\sqrt{x^2-yz}$ (b) $\sqrt{y^2-xz}$ (c) $\sqrt{z^2-xy}$ (d) None of these

9. The 10^{th} term of the sequence $\sqrt{3}, \sqrt{12}, \sqrt{27}, ...$is

(a) $\sqrt{243}$ (b) $\sqrt{300}$ (c) $\sqrt{363}$ (d) $\sqrt{432}$

10. Which term of the sequence $(-8+18i), (-6+15i), (-4+12i)$, ...is purely imaginary

(a) 5^{th} (b) 7^{th} (c) 8^{th} (d) 6^{th}

11. If $(m+2)^{th}$ term of an A.P. is $(m+2)^2-m^2$, then its common difference is

(a) 4 (b) – 4 (c) 2 (d) – 2

12. For an A.P., $T_2+T_5-T_3=10,\ T_2+T_9=17,$ then common difference is

(a) 0 (b) 1 (c) – 1 (d) 13

Advance Level

13. If tan $n\theta=\tan m\theta,$ then the different values of θ will be in **[Karnataka CET 1998]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

14. If the p^{th},q^{th} and r^{th} term of an arithmetic sequence are a, b and c respectively, then the value of [a $(q-r)$+$b(r-p)$+ c $(p-q)$]=

[MP PET 1985]

(a) 1 (b) – 1 (c) 0 (d) $\frac{1}{2}$

15. If n^{th} terms of two A.P.'s are $3n + 8$ and $7n + 15$, then the ratio of their 12th terms will be **[MP PET 1986]**

(a) $\frac{4}{9}$ (b) $\frac{7}{16}$ (c) $\frac{3}{7}$ (d) $\frac{8}{15}$

16. The 6th term of an A.P. is equal to 2, the value of the common difference of the A.P. which makes the product $a_1a_4a_5$ least is given by

(a) $\frac{8}{5}$ (b) $\frac{5}{4}$ (c) $\frac{2}{3}$ (d) None of these

17. If p times the p^{th} term of an A.P. is equal to q times the q^{th} term of an A.P., then $(p+q)^{th}$ term is

[MP PET 1997; Karnataka CET 2002]

(a) 0 (b) 1 (c) 2 (d) 3

18. The numbers $t(t^2+1)$, $-\frac{1}{2}t^2$ and 6 are three consecutive terms of an A.P. If t be real, then the next two terms of A.P. are

(a) –2, –10 (b) 14, 6 (c) 14, 22 (d) None of these

19. If the p^{th} term of the series 25, $22\frac{3}{5}$, $20\frac{1}{2}$, $18\frac{1}{4}$,...... is numerically the smallest, then p=

(a) 11 (b) 12 (c) 13 (d) 14

20. The second term of an A.P. is $(x - y)$ and the 5th term is $(x + y)$, then its first term is **[AMU 1989]**

(a) $x-\frac{1}{3}y$ (b) $x-\frac{2}{3}y$ (c) $x-\frac{4}{3}y$ (d) $x-\frac{5}{3}y$

21. The number of common terms to the two sequences 17, 21, 25, ...417 and 16, 21, 26, ... 466 is

(a) 21 (b) 19 (c) 20 (d) 91

22. In an A.P. first term is 1. If $T_1\,T_3+T_2T_3$ is minimum, then common difference is

(a) –5/4 (b) –4/5 (c) 5/4 (d) 4/5

23. Let the sets A={2, 4, 6, 8,......} and B= {3, 6, 9, 12, ...}, and $n(A)$ = 200, $n(B)$ = 250. Then

(a) $n(A\cap B)$ = 67 (b) $n(A\cup B)$ = 450 (c) $n(A\cap B)$ = 66 (d) $n(A\cup B)$ = 384

Sum to n terms of an Arithmetic progression

Basic Level

24. The sum of first n natural numbers is **[MP PET 1984; Rajasthan PET 1995]**

(a) $n(n-1)$ (b) $\frac{n(n-1)}{2}$ (c) $n(n+1)$ (d) $\frac{n(n+1)}{2}$

25. The sum of the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+......$ to 9 terms is **[MNR 1985]**

(a) $-\frac{5}{6}$ (b) $-\frac{1}{2}$ (c) 1 (d) $-\frac{3}{2}$

26. The sum of all natural numbers between 1 and 100 which are multiples of 3 is **[MP PET 1984]**

(a) 1680 (b) 1683 (c) 1681 (d) 1682

27. The sum of 1+3+5+7+..... upto n terms is **[MP PET 1984]**

(a) $(n+1)^2$ (b) $(2n)^2$ (c) n^2 (d) $(n-1)^2$

28. If the sum of the series 2+ 5+ 8+11 .... is 60100, then the number of terms are **[MNR 1991; DCE 2001]**

(a) 100 (b) 200 (c) 150 (d) 250

29. If the first term of an A.P. be 10, last term is 50 and the sum of all the terms is 300, then the number of terms are **[Rajasthan PET 1987]**

(a) 5 (b) 8 (c) 10 (d) 15

30. The sum of the numbers between 100 and 1000 which is divisible by 9 will be **[MP PET 1982]**

(a) 55350 (b) 57228 (c) 97015 (d) 62140

31. If the sum of three numbers of a arithmetic sequence is 15 and the sum of their squares is 83, then the numbers are **[MP PET 1985]**

(a) 4, 5, 6 (b) 3, 5, 7 (c) 1, 5, 9 (d) 2, 5, 8

32. If the sum of three consecutive terms of an A.P. is 51 and the product of last and first term is 273, then the numbers are

[MP PET 1986]

(a) 21, 17, 13 (b) 20, 16, 12 (c) 22, 18, 14 (d) 24, 20, 16

33. There are 15 terms in an arithmetic progression. Its first term is 5 and their sum is 390. The middle term is **[MP PET 19**

(a) 23 (b) 26 (c) 29 (d) 32

34. If $S_n = nP + \frac{1}{2}n(n-1)Q$, where S_n denotes the sum of the first n terms of an A.P. then the common difference is

[JEE West Bengal 1994]

(a) $P+Q$ (b) $2P+3Q$ (c) $2Q$ (d) Q

35. The sum of numbers from 250 to 1000 which are divisible by 3 is **[Rajasthan PET 1997]**

(a) 135657 (b) 136557 (c) 161575 (d) 156375

36. Four numbers are in arithmetic progression. The sum of first and last term is 8 and the product of both middle terms is 15. The least number of the series is **[MP PET 2001]**

(a) 4 (b) 3 (c) 2 (d) 1

37. The number of terms of the A.P. 3, 7, 11, 15 ...... to be taken so that the sum is 406 is **[Kerala (Engg.) 2002]**

(a) 5 (b) 10 (c) 12 (d) 14

38. The consecutive odd integers whose sum is $45^2 - 21^2$ are

(a) 43, 45, ..., 75 (b) 43, 45,...... 79 (c) 43, 45, ..., 85 (d) 43, 45, ..., 89

39. If common difference of m A.P.'s are respectively 1, 2,...... m and first term of each series is 1, then sum of their m^{th} terms is

(a) $\frac{1}{2}m(m+1)$ (b) $\frac{1}{2}m(m^2+1)$ (c) $\frac{1}{2}m(m^2-1)$ (d) None of these

40. The sum of all those numbers of three digits which leave remainder 5 after division by 7 is

(a) 551 × 129 (b) 550 × 130 (c) 552 × 128 (d) None of these

41. If $S_n = n^2 p$ and $S_m = m^2 p, m \neq n,$ in A.P., then S_p is

(a) p^2 (b) p^3 (c) p^4 (d) None of these

42. An A.P. consists of n (odd terms) and its middle term is m. Then the sum of the A.P. is

(a) $2mn$ (b) $\frac{1}{2}mn$ (c) mn (d) mn^2

43. The minimum number of terms of $1+3+5+7+.....$ that add up to a number exceeding 1357 is

(a) 15 (b) 37 (c) 35 (d) 17

Advance Level

44. If the ratio of the sum of n terms of two A.P.'s be $(7n+1) : (4n+27)$, then the ratio of their 11^{th} terms will be **[AMU 1996]**

(a) 2 : 3 (b) 3 : 4 (c) 4 : 3 (d) 5 : 6

45. The interior angles of a polygon are in A.P. If the smallest angle be 120° and the common difference be 5, then the number of sides is **[IIT 1980]**

(a) 8 (b) 10 (c) 9 (d) 6

46. The sum of integers from 1 to 100 that are divisible by 2 or 5 is **[IIT 1984]**

(a) 3000 (b) 3050 (c) 4050 (d) None of these

47. If the sum of first n terms of an A.P. be equal to the sum of its first m terms, $(m \neq n)$, then the sum of its first $(m+n)$ terms will be **[MP PET 1984]**

(a) 0 (b) n (c) m (d) $m+n$

48. If $a_1, a_2, ..., a_n$ are in A.P. with common difference d, then the sum of the following series is

$\sin d(\text{cosec } a_1 . \text{cosec } a_2 + \text{cosec } a_2 . \text{cosec } a_3 + ... + \text{cosec } a_{n-1} \text{cosec } a_n)$ **[Rajasthan PET 2000]**

(a) $\sec a_1 - \sec a_n$ (b) $\cot a_1 - \cot a_n$ (c) $\tan a_1 - \tan a_n$ (d) $\text{cosec } a_1 - \text{cosec } a_n$

49. The odd numbers are divided as follows

1 3
5 7 9 11
13 15 17 19 21 23
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Then the sum of n^{th} row is

(a) $2^{n-2}[2^n + 2^{n-1} - 1]$ (b) $\frac{1}{2}(2n+1)$ (c) $2n$ (d) $4n^3$

50. If the sum of n terms of an A.P. is $2n^2+5n$, then the n^{th} term will be **[Rajasthan PET 1992]**

(a) $4n+3$ (b) $4n+5$ (c) $4n+6$ (d) $4n+7$

51. The nth term of an A.P. is $3n-1$. Choose from the following the sum of its first five terms **[MP PET 1983]**

(a) 14 (b) 35 (c) 80 (d) 40

52. If the sum of two extreme numbers of an A.P. with four terms is 8 and product of remaining two middle term is 15, then greatest number of the series will be **[Roorkee 1965]**

(a) 5 (b) 7 (c) 9 (d) 11

53. The ratio of sum of m and n terms of an A.P. is $m^2:n^2$, then the ratio of m^{th} and n^{th} term will be**[Roorkee 1963; MP PET 19**

(a) $\frac{m-1}{n-1}$ (b) $\frac{n-1}{m-1}$ (c) $\frac{2m-1}{2n-1}$ (d) $\frac{2n-1}{2m-1}$

54. The value of x satisfying $\log_a x+\log_{\sqrt{a}} x+\log_{\sqrt[3]{a}} x+.....+\log_{\sqrt[n]{a}} x=\frac{a+1}{2}$ will be

(a) $x=a$ (b) $x=a^a$ (c) $x=a^{-1/a}$ (d) $x=a^{1/a}$

55. Sum of first n terms in the following series $\cot^{-1}3+\cot^{-1}7+\cot^{-1}13+\cot^{-1}21+.....$ is given by

(a) $\tan^{-1}\left(\frac{n}{n+2}\right)$ (b) $\cot^{-1}\left(\frac{n+2}{n}\right)$ (c) $\tan^{-1}(n+1)-\tan^{-1}1$ (d) All of these

56. Let S_n denotes the sum of n terms of an A.P. If $S_{2n}=3S_n$, then ratio $\frac{S_{3n}}{S_n}=$ **[MNR 1993; UPSEAT 2001]**

(a) 4 (b) 6 (c) 8 (d) 10

57. If the sum of the first n terms of a series be $5n^2+2n$, then its second term is **[MP PET 1996]**

(a) 7 (b) 17 (c) 24 (d) 42

58. All the terms of an A.P. are natural numbers. The sum of its first nine terms lies between 200 and 220. If the second term is 12, then the common difference is

(a) 2 (b) 3 (c) 4 (d) None of these

59. If $S_1=a_2+a_4+a_6+.....$up to 100 terms and $S_2=a_1+a_3+a_5+......$ up to 100 terms of a certain A.P. then its common difference d is

(a) S_1-S_2 (b) S_2-S_1 (c) $\frac{S_1-S_2}{2}$ (d) None of these

60. In the arithmetic progression whose common difference is non-zero, the sum of first $3n$ terms is equal to the sum of the next n terms. Then the ratio of the sum of the first $2n$ terms to the next $2n$ terms is

(a) $\frac{1}{5}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) None of these

61. If the sum of n terms of an A.P. is $nA+n^2B$, where A, B are constants, then its common difference will be**[MNR 1977]**

(a) $A-B$ (b) $A+B$ (c) $2A$ (d) $2B$

Arithmetic mean

Basic Level

62. A number is the reciprocal of the other. If the arithmetic mean of the two numbers be $\frac{13}{12}$, then the numbers are

(a) $\frac{1}{4},\frac{4}{1}$ (b) $\frac{3}{4},\frac{4}{3}$ (c) $\frac{2}{5},\frac{5}{2}$ (d) $\frac{3}{2},\frac{2}{3}$

63. The arithmetic mean of first n natural number **[Rajasthan PET 1986]**

(a) $\frac{n-1}{2}$ (b) $\frac{n+1}{2}$ (c) $\frac{n}{2}$ (d) n

64. The four arithmetic means between 3 and 23 are **[MP PET 1985]**

(a) 5, 9, 11, 13 (b) 7, 11, 15, 19 (c) 5, 11, 15, 22 (d) 7, 15, 19, 21

65. The mean of the series a, $a + nd$, $a + 2nd$ is **[DCE 2002]**

(a) $a+(n-1)d$ (b) $a+nd$ (c) $a+(n+1)d$ (d) None of these

66. If n A.M. s are introduced between 3 and 17 such that the ratio of the last mean to the first mean is 3 : 1, then the value of n is

(a) 6 (b) 8 (c) 4 (d) None of these

Advance Level

67. The sum of n arithmetic means between a and b, is **[Rajasthan PET 1986]**

(a) $\frac{n(a+b)}{2}$ (b) $n(a+b)$ (c) $\frac{(n+1)(a+b)}{2}$ (d) $(n+1)(a+b)$

68. Given that n A.M.'s are inserted between two sets of numbers a, $2b$ and $2a$, b, where $a, b \in R$. Suppose further that m^{th} mean between these sets of numbers is same, then the ratio $a : b$ equals

(a) $n - m + 1 : m$ (b) $n - m + 1 : n$ (c) $n : n - m + 1$ (d) $m : n - m + 1$

69. Given two number a and b. Let A denote the single A.M. and S denote the sum of n A.M.'s between a and b, then S/A depends on

[Pb. CET 1992]

(a) n, a, b (b) n, b (c) n, a (d) n

70. The A.M. of series $a+(a+d)+(a+2d)+.....+(a+2nd)$ is **[Pb. CET 1998]**

(a) $a+(n-1)d$ (b) $a+nd$ (c) $a+(n-1)d$ (d) None of these

71. If 11 AM's are inserted between 28 and 10, then three mid terms of the series are **[MNR 1997]**

(a) $\frac{41}{2},19,\frac{35}{2}$ (b) $20,\frac{41}{2},\frac{43}{2}$ (c) $20,\frac{61}{2},\frac{62}{3}$ (d) $20, 22, 24$

72. If $f(x+y,x-y)=xy$, then the arithmetic mean of $f(x,y)$ and $f(y,x)$ is **[AMU 2002]**

(a) x (b) y (c) 0 (d) 1

73. If A.M. of the roots of a quadratic equation is $\frac{8}{5}$ and the A.M. of their reciprocals is $\frac{8}{7}$, then the quadratic equation is

(a) $7x^2+16x+5=0$ (b) $7x^2-16x+5=0$ (c) $5x^2-16x+7=0$ (d) $5x^2-8x+7=0$

74. If $a_1=0$ and $a_1, a_2, a_3,.....a_n$ are real numbers such that $|a_i|=|a_{i-1}+1|$ for all i, then A.M. of the numbers $a_1, a_2, ...a_n$ has the value x where

(a) $x<1$ (b) $x<-\frac{1}{2}$ (c) $x\geq-\frac{1}{2}$ (d) $x=\frac{1}{2}$

75. If A.M. of the numbers 5^{1+x} and 5^{1-x} is 13 then the set of possible real values of x is

(a) $\{5, \frac{1}{5}\}$ (b) $\{1, -1\}$ (c) $\{x \mid x^2 - 1 \mid = 0, x \in R\}$ (d) None of these

Properties of A.P.

Basic Level

76. If $2x$, $x + 8$, $3x + 1$ are in A.P., then the value of x will be **[MP PET 1984]**

(a) 3 (b) 7 (c) 5 (d) – 2

77. If $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in A.P., then x is equal to **[IIT 1990]**

(a) $1, \frac{1}{2}$ (b) $1, \frac{1}{3}$ (c) $1, \frac{3}{2}$ (d) None of these

78. If a_m denotes the m^{th} term of an A.P., then $a_m =$

(a) $\frac{a_{m+k} + a_{m-k}}{2}$ (b) $\frac{a_{m+k} - a_{m-k}}{2}$ (c) $\frac{2}{a_{m+k} + a_{m-k}}$ (d) None of these

79. If 1, $\log_y x$, $\log_z y$, $-15 \log_x z$ are in A.P., then

(a) $z^3 = x$ (b) $x = y^{-1}$ (c) $z^{-3} = y$ (d) $x = y^{-1} = z^3$

(e) All of these

80. If $\frac{1}{p+q}, \frac{1}{r+p}, \frac{1}{q+r}$ are in A.P., then **[Rajasthan PET 1995]**

(a) p, q, r are in A.P. (b) p^2, q^2, r^2 are in A.P. (c) $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in A.P. (d) None of these

81. If a, b, c, are in A.P., then $b^2 - ac$ is equal to **[Roorkee 1975]**

(a) $\frac{1}{4}(a+c)^2$ (b) $\frac{1}{4}(a-c)^2$ (c) $\frac{1}{2}(a+c)^2$ (d) $\frac{1}{2}(a-c)^2$

82. If $a_1, a_2, a_3, ...$ are in A.P. then a_p, a_q, a_r are in A.P. if p, q, r are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Advance Level

83. If the sum of the roots of the equation $ax^2 + bx + c = 0$ be equal to the sum of the reciprocals of their squares, then bc^2, ca^2, ab^2 will be in **[IIT 1976]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

84. If $\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ be consecutive terms of an A.P., then $(b - c)^2$, $(c - a)^2$, $(a - b)^2$ will be in

(a) G.P. (b) A.P. (c) H.P. (d) None of these

85. If a^2, b^2, c^2 are in A.P., then $(b + c)^{-1}$, $(c + a)^{-1}$ and $(a + b)^{-1}$ will be in **[Roorkee 1968; Rajasthan PET 1996]**

(a) H.P. (b) G.P. (c) A.P. (d) None of these

86. If the sides of a right angled triangle are in A.P., then the sides are proportional to **[Roorkee 1974]**

(a) 1, 2, 3 (b) 2, 3, 4 (c) 3, 4, 5 (d) 4, 5, 6

87. If a, b, c are in A.P., then the straight line $ax + by + c = 0$ will always pass through the point **[IIT 1984]**

(a) $(-1, -2)$ (b) $(1, -2)$ (c) $(-1, 2)$ (d) $(1, 2)$

88. If a, b, c are in A.P. then $\frac{(a-c)^2}{(b^2-ac)} =$ **[Roorkee 1975]**

(a) 1 (b) 2 (c) 3 (d) 4

89. If a, b, c, d, e, f are in A.P., then the value of $e - c$ will be **[Pb. CET 1989, 91]**

(a) $2(c-a)$ (b) $2(f-d)$ (c) $2(d-c)$ (d) $d-c$

90. If p, q, r are in A.P. and are positive, the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for **[IIT 1995]**

(a) $\left|\frac{r}{p}-7\right| \ge 4\sqrt{3}$ (b) $\left|\frac{p}{r}-7\right| < 4\sqrt{3}$ (c) All p and r (d) No p and r

91. If $a_1, a_2, a_3, \ldots\ldots\, a_n$ are in A.P., where $a_i > 0$ for all i, then the value of $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots\ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} =$ **[IIT 1982**

(a) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ (b) $\frac{n+1}{\sqrt{a_1}+\sqrt{a_n}}$ (c) $\frac{n-1}{\sqrt{a_1}-\sqrt{a_n}}$ (d) $\frac{n+1}{\sqrt{a_1}-\sqrt{a_n}}$

92. Given $a+d > b+c$ where a, b, c, d are real numbers, then **[Kurukshetra CEE 1998]**

(a) a, b, c, d are in A.P. (b) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ are in A.P.

(c) $(a+b), (b+c), (c+d), (a+d)$ are in A.P. (d) $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+d}, \frac{1}{a+d}$ are in A.P.

93. If a, b, c are in A.P., then $(a + 2b - c)\ (2b + c - a)\ (c + a - b)$ equals **[Pb. CET 1999]**

(a) $\frac{1}{2}abc$ (b) abc (c) $2\,abc$ (d) $4\,abc$

94. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference will be
[UPSEAT 1994, 99, 2001; Rajasthan PET 2001]

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

95. If 1, $\log_9(3^{1-x}+2)$, $\log_3(4\,.\,3^x - 1)$ are in A.P., then x equals **[AIEEE 2002]**

(a) $\log_3 4$ (b) $1 - \log_3 4$ (c) $1 - \log_4 3$ (d) $\log_4 3$

96. If a, b, c, d, e are in A.P. then the value of $a+b+4c-4d+e$ in terms of a, if possible is **[Rajasthan PET 2002]**

(a) $4a$ (b) $2a$ (c) 3 (d) None of these

97. If $a_1, a_2, a_3, \ldots\ldots. a_{2n+1}$ are in A.P. then $\frac{a_{2n+1}-a_1}{a_{2n+1}+a_1} + \frac{a_{2n}-a_2}{a_{2n}+a_2} + \ldots\ldots. + \frac{a_{n+2}-a_n}{a_{n+2}+a_n}$ is equal to

(a) $\frac{n(n+1)}{2} \cdot \frac{a_2 - a_1}{a_{n+1}}$ (b) $\frac{n(n+1)}{2}$ (c) $(n+1)(a_2 - a_1)$ (d) None of these

98. If the non-zero numbers x, y, z are in A.P. and $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are also in A.P., then

(a) $x = y = z$ (b) $xy = yz$ (c) $x^2 = yz$ (d) $z^2 = xy$

99. If three positive real numbers a, b, c are in A.P. such that $abc = 4$, then the minimum value of b is

(a) $2^{1/3}$ (b) $2^{2/3}$ (c) $2^{1/2}$ (d) $2^{3/2}$

100. If $\sin\alpha, \sin^2\alpha, 1, \sin^4\alpha$ and $\sin^5\alpha$ are in A.P., where $-\pi < \alpha < \pi$, then α lies in the interval

(a) $(-\pi/2, \pi/2)$ (b) $(-\pi/3, \pi/3)$ (c) $(-\pi/6, \pi/6)$ (d) None of these

101. If the sides of a triangle are in A.P. and the greatest angle of the triangle is double the smallest angle, the ratio of the sides of the triangle is

(a) 3 : 4 : 5 (b) 4 : 5 : 6 (c) 5 : 6 : 7 (d) 7 : 8 : 9

102. If *a, b, c* of a ΔABC are in A.P., then $\cot\frac{c}{2} =$ **[T.S. Rajendra 1990]**

(a) $3\tan\frac{A}{2}$ (b) $3\tan\frac{B}{2}$ (c) $3\cot\frac{A}{2}$ (d) $3\cot\frac{B}{2}$

103. If *a, b, c* are in A.P. then the equation $(a-b)x^2 + (c-a)x + (b-c) = 0$ has two roots which are

(a) Rational and equal (b) Rational and distinct (c) Irrational conjugates (d) Complex conjugates

104. The least value of 'a' for which $5^{1+x} + 5^{1-x}, \frac{a}{2}, 25^x + 25^{-x}$ are three consecutive terms of an A.P. is

(a) 10 (b) 5 (c) 12 (d) None of these

105. $\alpha, \beta, \gamma, \delta$ are in A.P. and $\int_0^2 f(x)dx = -4$, where $f(x) = \begin{vmatrix} x+\alpha & x+\beta & x+\alpha-\gamma \\ x+\beta & x+\gamma & x-1 \\ x+\gamma & x+\delta & x-\beta+\delta \end{vmatrix}$, then the common difference *d* is

(a) 1 (b) –1 (c) 2 (d) – 2

106. If the sides of a right angled triangle form an A.P. then the sines of the acute angles are

(a) $\frac{3}{5}, \frac{4}{5}$ (b) $\sqrt{3}, \frac{1}{3}$ (c) $\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}+1}{2}}$ (d) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

107. If x, y, z are positive numbers in A.P., then

(a) $y^2 \geq xz$ (b) $y \geq 2\sqrt{xz}$

(c) $\frac{x+y}{2y-x} + \frac{y+z}{2y-z}$ has the minimum value 2 (d) $\frac{x+y}{2y-x} + \frac{y+z}{2y-z} \geq 4$

General term of Geometric progression

Basic Level

108. If the $4^{th}, 7^{th}$ and 10^{th} terms of a G.P. be *a, b, c* respectively, then the relation between *a, b, c* is **[MNR 1995; Karnataka CET 1999]**

(a) $b = \frac{a+c}{2}$ (b) $a^2 = bc$ (c) $b^2 = ac$ (d) $c^2 = ab$

109. 7^{th} term of the sequence $\sqrt{2}, \sqrt{10}, 5\sqrt{2}, ...$ is

(a) $125\sqrt{10}$ (b) $25\sqrt{2}$ (c) 125 (d) $125\sqrt{2}$

110. If the 5^{th} term of a G.P. is $\frac{1}{3}$ and 9^{th} term is $\frac{16}{243}$, then the 4^{th} term will be **[MP PET 1982]**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{5}$

111. If the 10^{th} term of a geometric progression is 9 and 4^{th} term is 4, then its 7^{th} term is **[MP PET 1996]**

(a) 6 (b) 36 (c) $\frac{4}{9}$ (d) $\frac{9}{4}$

112. The third term of a G.P. is the square of first term. If the second term is 8, then the 6^{th} term is **[MP PET 1997]**

(a) 120 (b) 124 (c) 128 (d) 132

113. The 6^{th} term of a G.P. is 32 and its 8^{th} term is 128, then the common ratio of the G.P. is **[Pb. CET 1999]**

(a) – 1 (b) 2 (c) 4 (d) – 4

114. The first and last terms of a G.P. are a and l respectively, r being its common ratio; then the number of term in this G.P. is

(a) $\frac{\log l - \log a}{\log r}$ (b) $1 - \frac{\log l - \log a}{\log r}$ (c) $\frac{\log a - \log l}{\log r}$ (d) $1 + \frac{\log l - \log a}{\log r}$

115. If first term and common ratio of a G.P. are both $\frac{\sqrt{3}+i}{2}$. The absolute value of n^{th} term will be

(a) 2^n (b) 4^n (c) 1 (d) 4

116. In any G.P. the last term is 512 and common ratio is 2, then its 5^{th} term from last term is

(a) 8 (b) 16 (c) 32 (d) 64

117. Given the geometric progression 3, 6, 12, 24, ... the term 12288 would occur as the **[SCRA 1999]**

(a) 11^{th} term (b) 12^{th} term (c) 13^{th} term (d) 14^{th} term

118. Let $\{t_n\}$ be a sequence of integers in GP in which $t_4 : t_6 = 1 : 4$ and $t_2 + t_5 = 216$. Then t_1 is

(a) 12 (b) 14 (c) 16 (d) None of these

Advance Level

119. α, β are the roots of the equation $x^2 - 3x + a = 0$ and γ, δ are the roots of the equation $x^2 - 12x + b = 0$. If $\alpha, \beta, \gamma, \delta$ form an increasing G.P., then $(a, b) =$ **[DCE 2000]**

(a) (3, 12) (b) (12, 3) (c) (2, 32) (d) (4, 16)

120. If $(p+q)^{th}$ term a G.P. be m and $(p-q)^{th}$ term be n, then the p^{th} term will be **[Rajasthan PET 1997; MP PET 1985, 99]**

(a) m / n (b) $\sqrt{mn}$ (c) mn (d) 0

121. If the third term of a G.P. is 4 then the product of its first 5 terms is **[IIT 1982; Rajasthan PET 1991]**

(a) 4^3 (b) 4^4 (c) 4^5 (d) None of these

122. If the first term of a G.P. $a_1, a_2, a_3, ...$ is unity such that $4a_2 + 5a_3$ is least, then the common ratio of G.P. is

(a) $-\frac{2}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{2}{5}$ (d) None of these

123. Fifth term of a G.P. is 2, then the product of its 9 terms is **[Pb. CET 1990, 94; AIEEE 2002]**

(a) 256 (b) 512 (c) 1024 (d) None of these

124. If the nth term of geometric progression $5, -\frac{5}{2}, \frac{5}{4}, -\frac{5}{8}, ...$ is $\frac{5}{1024}$, then the value of n is **[Kerala (Engg.) 2002]**

(a) 11 (b) 10 (c) 9 (d) 4

Sum to n terms of Geometric progression

Basic Level

125. The sum of 100 terms of the series .9+ .09 + .009...... will be

(a) $1-\left(\frac{1}{10}\right)^{100}$ (b) $1+\left(\frac{1}{10}\right)^{106}$ (c) $1-\left(\frac{1}{10}\right)^{106}$ (d) $1+\left(\frac{1}{10}\right)^{100}$

126. If the sum of three terms of G.P. is 19 and product is 216, then the common ratio of the series is **[Roorkee 1972]**

(a) $-\frac{3}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) 3

127. If the sum of first 6 terms is 9 times to the sum of first 3 terms of the same G.P., then the common ratio of the series will be

[Rajasthan PET 1985]

(a) – 2 (b) 2 (c) 1 (d) $\frac{1}{2}$

128. If the sum of n terms of a G.P. is 255 and n^{th} term is 128 and common ratio is 2, then first term will be**[Rajasthan PET 1**

(a) 1 (b) 3 (c) 7 (d) None of these

129. The sum of 3 numbers in geometric progression is 38 and their product is 1728. The middle number is**[MP PET 1994]**

(a) 12 (b) 8 (c) 18 (d) 6

130. The sum of few terms of any ratio series is 728, if common ratio is 3 and last term is 486, then first term of series will be

[UPSEAT 1999]

(a) 2 (b) 1 (c) 3 (d) 4

131. The sum of n terms of a G.P. is $3-\frac{3^{n+1}}{4^{2n}}$, then the common ratio is equal to

(a) $\frac{3}{16}$ (b) $\frac{3}{256}$ (c) $\frac{39}{256}$ (d) None of these

132. The value of n for which the equation $1+r+r^2.....+r^n=(1+r)(1+r^2)(1+r^4)(1+r^8)$ holds is

(a) 13 (b) 12 (c) 15 (d) 16

133. The value of the sum $\sum_{n=1}^{13}(i^n+i^{n+1})$, where $i=\sqrt{-1}$, equals **[IIT 1998]**

(a) i (b) i – 1 (c) – i (d) 0

134. For a sequence $a_1, a_2.........a_n$ given $a_1=2$ and $\frac{a_{n+1}}{a_n}=\frac{1}{3}$. Then $\sum_{r=1}^{20} a_r$ is

(a) $\frac{20}{2}[4+19\times 3]$ (b) $3\left(1-\frac{1}{3^{20}}\right)$ (c) $2(1-3^{-20})$ (d) None of these

135. The sum of $(x+2)^{n-1}+(x+2)^{n-2}(x+1)+(x+2)^{n-3}(x+1)^2+.....(x+1)^{n-1}$ is equal to **[IIT 1990]**

(a) $(x+2)^{n-2}-(x+1)^n$
(b) $(x+2)^{n-1}-(x+1)^{n-1}$
(c) $(x+2)^n-(x+1)^n$
(d) None of these

Advance Level

136. The sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+........$ is

[IIT 1988; MP PET 1996; Rajasthan PET 1996, 2000; Pb. CET 1994; DCE 1995, 96]

(a) 2^n-n-1
(b) $1-2^{-n}$
(c) $n+2^{-n}-1$
(d) 2^n-1

137. If the product of three consecutive terms of G.P. is 216 and the sum of product of pair – wise is 156, then the numbers will be

[MNR 1978]

(a) 1, 3, 9
(b) 2, 6, 18
(c) 3, 9, 27
(d) 2, 4, 8

138. If $f(x)$ is a function satisfying $f(x+y)=f(x)f(y)$ for all $x,y\in N$ such that $f(1)=3$ and $\sum_{x=1}^{n} f(x)=120$. Then the value of n is

[IIT 1992]

(a) 4
(b) 5
(c) 6
(d) None of these

139. The first term of a G.P. is 7, the last term is 448 and sum of all terms is 889, then the common ratio is **[MP PET 2003]**

(a) 5
(b) 4
(c) 3
(d) 2

140. The sum of a G.P. with common ratio 3 is 364, and last term is 243, then the number of terms is **[MP PET 2003]**

(a) 6
(b) 5
(c) 4
(d) 10

141. A G.P. consists of $2n$ terms. If the sum of the terms occupying the odd places is S_1, and that of the terms in the even places is S_2, then S_2/S_1 is

(a) Independent of a
(b) Independent of r
(c) Independent of a and r
(d) Dependent on r

142. Sum of the series $\frac{2}{3}+\frac{8}{9}+\frac{26}{27}+\frac{80}{81}+.....$ to n terms is **[Karnataka CET 2001]**

(a) $n-\frac{1}{2}(3^n-1)$
(b) $n+\frac{1}{2}(3^n-1)$
(c) $n+\frac{1}{2}(1-3^{-n})$
(d) $n+\frac{1}{2}(3^{-n}-1)$

143. If the sum of the n terms of G.P. is S product is P and sum of their inverse is R, then P^2 is equal to**[IIT 1966; Roorkee 19**

(a) $\frac{R}{S}$
(b) $\frac{S}{R}$
(c) $\left(\frac{R}{S}\right)^n$
(d) $\left(\frac{S}{R}\right)^n$

144. The minimum value of n such that $1+3+3^2+.....+3^n>1000$ is

(a) 7
(b) 8
(c) 9
(d) None of these

145. If every term of a G.P. with positive terms is the sum of its two previous terms, then the common ratio of the series is

[Rajasthan PET 1986]

(a) 1
(b) $\frac{2}{\sqrt{5}}$
(c) $\frac{\sqrt{5}-1}{2}$
(d) $\frac{\sqrt{5}+1}{2}$

146. If $(1.05)^{50} = 11.658$, then $\sum_{n=1}^{49} (1.05)^n$ equals **[Roorkee 1991]**

(a) 208.34 (b) 212.12 (c) 212.16 (d) 213.16

147. If $a_1, a_2, a_3.....a_n$ are in G.P. with first term 'a' and common ratio 'r' then $\frac{a_1 a_2}{a_1^2 - a_2^2} + \frac{a_2 a_3}{a_2^2 - a_3^2} + \frac{a_3 a_4}{a_3^2 - a_n^2} + ... + \frac{a_{n-1} a_n}{a_{n-1}^2 - a_n^2}$ is equal to

(a) $\frac{nr}{1-r^2}$ (b) $\frac{(n-1)^r}{1-r^2}$ (c) $\frac{nr}{1-r}$ (d) $\frac{(n-1)r}{1-r}$

148. The sum of the squares of three distinct real numbers which are in G.P. is S^2. If their sum is αS, then

(a) $1 < \alpha^2 < 3$ (b) $\frac{1}{3} < \alpha^2 < 1$ (c) $1 < \alpha < 3$ (d) $\frac{1}{3} < \alpha < 1$

Sum to infinite terms

Basic Level

149. If the sum of the series $1 + \frac{2}{x} + \frac{4}{x^2} + \frac{8}{x^3} + ...\infty$ is a finite number, then **[UPSEAT 2002]**

(a) $x > 2$ (b) $x > -2$ (c) $x > \frac{1}{2}$ (d) None of these

150. If $y = x - x^2 + x^3 - x^4 + ...\infty$, then value of x will be **[MNR 1975; Rajasthan PET 1988; MP PET 2002]**

(a) $y + \frac{1}{y}$ (b) $\frac{y}{1+y}$ (c) $y - \frac{1}{y}$ (d) $\frac{y}{1-y}$

151. If the sum of an infinite G.P. be 9 and the sum of first two terms be 5, then the common ratio is

(a) $\frac{1}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

152. $2.\dot{3}\dot{5}\dot{7} =$ **[IIT 1983; Rajasthan PET 1995]**

(a) $\frac{2355}{1001}$ (b) $\frac{2370}{997}$ (c) $\frac{2355}{999}$ (d) None of these

153. The first term of a G.P. whose second term is 2 and sum to infinity is 8, will be **[MNR 1979; Rajasthan PET 1992, 95]**

(a) 6 (b) 3 (c) 4 (d) 1

154. The sum of infinite terms of a G.P. is x and on squaring the each term of it, the sum will be y, then the common ratio of this series is

[Rajasthan PET 1988]

(a) $\frac{x^2 - y^2}{x^2 + y^2}$ (b) $\frac{x^2 + y^2}{x^2 - y^2}$ (c) $\frac{x^2 - y}{x^2 + y}$ (d) $\frac{x^2 + y}{x^2 - y}$

155. If $3 + 3\alpha + 3\alpha^2 + ...\infty = \frac{45}{8}$, then the value of α will be **[Pb. CET 1989]**

(a) $\frac{15}{23}$ (b) $\frac{7}{15}$ (c) $\frac{7}{8}$ (d) $\frac{15}{7}$

156. The sum can be found of a infinite G.P. whose common ratio is r **[AMU 1982]**

(a) For all values of r (b) For only positive value of r (c) Only for $0 < r < 1$ (d) Only for $-1 < r < 1 (r \neq 0)$

157. The sum of infinity of a geometric progression is $\frac{4}{3}$ and the first term is $\frac{3}{4}$. The common ratio is **[MP PET 1994]**

(a) $\frac{7}{16}$ (b) $\frac{9}{16}$ (c) $\frac{1}{9}$ (d) $\frac{7}{9}$

158. The value of $4^{1/3}.4^{1/9}.4^{1/27}.....\infty$ is **[Rajasthan PET 2003]**

(a) 2 (b) 3 (c) 4 (d) 9

159. 0.14189189189.... can be expressed as a rational number **[AMU 2000]**

(a) $\frac{7}{3700}$ (b) $\frac{7}{50}$ (c) $\frac{525}{111}$ (d) $\frac{21}{148}$

160. The sum of the series $5.05+1.212+0.29088+...\infty$ is **[AMU 2000]**

(a) 6.93378 (b) 6.87342 (c) 6.74384 (d) 6.64474

161. Sum of infinite number of terms in G.P. is 20 and sum of their square is 100. The common ratio of G.P. is **[AIEEE 2002]**

(a) 5 (b) 3/5 (c) 8/5 (d) 1/5

162. If in an infinite G.P. first term is equal to the twice of the sum of the remaining terms, then its common ratio is **[Rajas**

(a) 1 (b) 2 (c) 1/3 (d) – 1/3

163. The sum of infinite terms of the geometric progression $\frac{\sqrt{2}+1}{\sqrt{2}-1}, \frac{1}{2-\sqrt{2}}, \frac{1}{2}.....$ is **[Kerala (Engg.) 2002]**

(a) $\sqrt{2}(\sqrt{2}+1)^2$ (b) $(\sqrt{2}+1)^2$ (c) $5\sqrt{2}$ (d) $3\sqrt{2}+\sqrt{5}$

164. If $x > 0$, then the sum of the series $e^{-x}-e^{-2x}+e^{-3x}......\infty$ is **[AMU 1989]**

(a) $\frac{1}{1-e^{-x}}$ (b) $\frac{1}{e^{x}-1}$ (c) $\frac{1}{1+e^{-x}}$ (d) $\frac{1}{1+e^{x}}$

165. The sum of the series $0.4+0.004+0.00004+.......\infty$ is **[AMU 1989]**

(a) $\frac{11}{25}$ (b) $\frac{41}{100}$ (c) $\frac{40}{99}$ (d) $\frac{2}{5}$

166. A ball is dropped from a height of 120 *m* rebounds $(4/5)^{th}$ of the height from which it has fallen. If it continues to fall and rebound in this way. How far will it travel before coming to rest ?

(a) 240 *m* (b) 140 *m* (c) 1080 *m* (d) ∞

167. The series $C+\frac{C^2}{1+C}+\frac{C^3}{(1+C)^2}+\frac{C^4}{(1+C)^3}+.....$ has a finite sum if C is greater than

(a) – 1/2 (b) – 1 (c) – 2/3 (d) None of these

Advance Level

168. If $A=1+r^z+r^{2z}+r^{3z}+.....\infty$, then the value of r will be

(a) $A(1-A)^z$ (b) $\left(\frac{A-1}{A}\right)^{1/z}$ (c) $\left(\frac{1}{A}-1\right)^{1/z}$ (d) $A(1-A)^{1/z}$

169. The sum to infinity of the following series $2+\frac{1}{2}+\frac{1}{3}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^3}+.....,$ will be **[AMU 1984]**

(a) 3 (b) 4 (c) $\frac{7}{2}$ (d) $\frac{9}{2}$

170. $x=1+a+a^2+.......\infty(a<1)$, $y=1+b+b^2+.......\infty(b<1)$. Then the value of $1+ab+a^2b^2+......\infty$ is **[MNR 1980; MP PET 1985]**

(a) $\frac{xy}{x+y-1}$ (b) $\frac{xy}{x+y+1}$ (c) $\frac{xy}{x-y-1}$ (d) $\frac{xy}{x-y+1}$

171. The value of $a^{\log_b x}$, where $a=0.2, b=\sqrt{5}$, $x=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+......$ to ∞ is

(a) 1 (b) 2 (c) 1/2 (d) 4

172. The sum of an infinite geometric series is 3. A series, which is formed by squares of its terms have the sum also 3. First series will be \

[Rajasthan PET 1999; Roorkee 1972; UPSEAT 1999]

(a) $\frac{3}{2},\frac{3}{4},\frac{3}{8},\frac{3}{16},.....$ (b) $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},.....$ (c) $\frac{1}{3},\frac{1}{9},\frac{1}{27},\frac{1}{81},.....$ (d) $1,-\frac{1}{3},\frac{1}{3^2},-\frac{1}{3^3},.....$

173. If $1+\cos\alpha+\cos^2\alpha+.......\infty=2-\sqrt{2}$, then α, $(0<\alpha<\pi)$ is **[Roorkee 2000]**

(a) $\pi/8$ (b) $\pi/6$ (c) $\pi/4$ (d) $3\pi/4$

174. Consider an infinite G.P. with first term *a* and common ratio *r*, its sum is 4 and the second term is 3/4 , then

[IIT Screening 2000; DCE 2001]

(a) $a=\frac{7}{4}, r=\frac{3}{7}$ (b) $a=\frac{3}{2}, r=\frac{1}{2}$ (c) $a=2, r=\frac{3}{8}$ (d) $a=3, r=\frac{1}{4}$

175. Let $n(>1)$ be a positive integer, then the largest integer *m* such that (n^m+1) divides $(1+n+n^2+.....+n^{127})$, is **[IIT 1995]**

(a) 32 (b) 63 (c) 64 (d) 127

176. If |*a*|<1 and |*b*|<1, then the sum of the series $a(a+b)+a^2(a^2+b^2)+a^3(a^3+b^3)+.....$ upto ∞ is

(a) $\frac{a}{1-a}+\frac{ab}{1-ab}$ (b) $\frac{a^2}{1-a^2}+\frac{ab}{1-ab}$ (c) $\frac{b}{a-b}+\frac{a}{1-a}$ (d) $\frac{b^2}{1-b^2}+\frac{ab}{1-ab}$

177. If *S* is the sum to infinity of a G.P., whose first term is *a*, then the sum of the first *n* terms is **[UPSEAT 2002]**

(a) $S\left(1-\frac{a}{S}\right)^n$ (b) $S\left[1-\left(1-\frac{a}{S}\right)^n\right]$ (c) $a\left[1-\left(1-\frac{a}{S}\right)^n\right]$ (d) None of these

178. If S denotes the sum to infinity and S_n the sum of *n* terms of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....$, such that $S-S_n<\frac{1}{1000}$, then the least value of *n* is

(a) 8 (b) 9 (c) 10 (d) 11

179. If exp. $\{(\sin^2 x+\sin^4 x+\sin^4 x+....+\infty)\ \log_e 2\}$ satisfies the equation $x^2-9x+8=0$, then the value of $\frac{\cos x}{\cos x+\sin x}, 0<x<\frac{\pi}{2}$ is

(a) $\frac{1}{2}(\sqrt{3}+1)$ (b) $\frac{1}{2}(\sqrt{3}-1)$ (c) 0 (d) None of these

Geometric mean

Basic Level

180. If G be the geometric mean of *x* and *y*, then $\frac{1}{G^2-x^2}+\frac{1}{G^2-y^2}=$

(a) G^2 (b) $\frac{1}{G^2}$ (c) $\frac{2}{G^2}$ (d) $3G^2$

181. If *n* geometric means be inserted between *a* and *b*, then the n^{th} geometric mean will be

(a) $a\left(\frac{b}{a}\right)^{\frac{n}{n-1}}$ (b) $a\left(\frac{b}{a}\right)^{\frac{n-1}{n}}$ (c) $a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$ (d) $a\left(\frac{b}{a}\right)^{\frac{1}{n}}$

182. If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ be the geometric mean of a and b, then n=

(a) 0 (b) 1 (c) 1/2 (d) None of these

183. The G.M. of roots of the equation $x^2-18x+9=0$ is **[Rajasthan PET 1997]**

(a) 3 (b) 4 (c) 2 (d) 1

184. If five G.M.'s are inserted between 486 and 2/3 then fourth G.M. will be **[Rajasthan PET 1999]**

(a) 4 (b) 6 (c) 12 (d) – 6

185. If 4 G.M's be inserted between 160 and 5 them third G.M. will be

(a) 8 (b) 118 (c) 20 (d) 40

186. The product of three geometric means between 4 and $\frac{1}{4}$ will be

(a) 4 (b) 2 (c) – 1 (d) 1

187. The geometric mean between –9 and –16 is

(a) 12 (b) – 12 (c) – 13 (d) None of these

Advance Level

188. If n geometric means between a and b be $G_1, G_2, \ldots\ldots G_n$ and a geometric mean be G, then the true relation is

(a) $G_1.G_2\ldots\ldots G_n = G$ (b) $G_1.G_2\ldots\ldots G_n = G^{1/n}$ (c) $G_1.G_2\ldots\ldots G_n = G^n$ (d) $G_1.G_2\ldots\ldots G_n = G^{2/n}$

189. If x and y be two real numbers and n geometric means are inserted between x and y. now x is multiplied by k and y is multiplied $\frac{1}{k}$ and then n G.M's. are inserted. The ratio of the n^{th} G.M's. in the two cases is

(a) $k^{\frac{n-1}{n+1}}:1$ (b) $1:k^{\frac{1}{n+1}}$ (c) $1:1$ (d) None of these

Properties of G.P.

Basic Level

190. If a, b, c are in G.P., then

(a) $a(b^2+a^2)=c(b^2+c^2)$ (b) $a(b^2+c^2)=c(a^2+b^2)$ (c) $a^2(b+c)=c^2(a-b)$ (d) None of these

191. If x is added to each of numbers 3, 9, 21 so that the resulting numbers may be in G.P., then the value of x will be**[MP PE**

(a) 3 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{3}$

192. If $\log_x a, a^{x/2}$ and $\log_b x$ are in G.P., then x =

(a) $-\log_a(\log_b a)$ (b) $-\log_a(\log_a b)$ (c) $\log_a(\log_e a)-\log_a(\log_e b)$ (d) $\log_a(\log_e b)-\log_a(\log_e a)$

193. If $\sum_{n=1}^{n} n, \frac{\sqrt{10}}{3}.\sum_{n=1}^{n} n^2, \sum_{n=1}^{n} n^3$ are in G.P. then the value of n is

(a) 2 (b) 3 (c) 4 (d) Nonexistent

194. If *p*, *q*, *r* are in A.P., then p^{th}, q^{th} and r^{th} terms of any G.P. are in

(a) AP (b) G.P.

(c) Reciprocals of these terms are in A.P. (d) None of these

195. If *a*, *b*, *c* are in G.P., then **[Rajasthan PET 1995]**

(a) a^2, b^2, c^2 are in G.P. (b) $a^2(b+c), c^2(a+b), b^2(a+c)$ are in G.P.

(c) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in G.P. (d) None of these

196. Let *a* and *b* be roots of $x^2 - 3x + p = 0$ and let *c* and *d* be the roots of $x^2 - 12x + q = 0$, where *a, b, c, d* form an increasing G.P. Then the ratio of $(q + p) : (q - p)$ is equal to

(a) 8 : 7 (b) 11 : 10 (c) 17 : 15 (d) None of these

197. If the roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$ are in G.P., then

(a) $c^3 a = b^3 d$ (b) $ca^3 = bd^3$ (c) $a^3 b = c^3 d$ (d) $ab^3 = cd^3$

198. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) **[IIT 199**

(a) Lie on a straight line (b) Lie on an ellipse (c) Lie on a circle (d) Are vertices of a triangle

199. Let $f(x) = 2x + 1$. Then the number of real values of *x* for which the three unequal numbers $f(x), f(2x), f(4x)$ are in GP is

(a) 1 (b) 2 (c) 0 (d) None of these

200. S_r denotes the sum of the first *r* terms of a G.P. Then $S_n, S_{2n} - S_n, S_{3n} - S_{2n}$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

201. If $a^{1/x} = b^{1/y} = c^{1/z}$ and a, b, c are in G.P., then *x, y, z* will be in **[IIT 1969; UPSEAT 2001]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

202. If *x, y, z* are in G.P. and $a^x = b^y = c^z$, then **[IIT 1966, 1968]**

(a) $\log_a c = \log_b a$ (b) $\log_b a = \log_c b$ (c) $\log_c b = \log_a c$ (d) None of these

General term of Harmonic progression

Basic Level

203. Three consecutive terms of a progression are 30, 24, 20. The next term of the progression is

(a) 18 (b) $17\frac{1}{7}$ (c) 16 (d) None of these

204. The 5^{th} term of the H.P., $2, 2\frac{1}{2}, 3\frac{1}{3}, ...$ will be **[MP PET 1984]**

(a) $5\frac{1}{5}$ (b) $3\frac{1}{5}$ (c) $1/10$ (d) 10

205. If 5^{th} term of a H.P. is $\frac{1}{45}$ and 11^{th} term is $\frac{1}{69}$, then its 16^{th} term will be **[Rajasthan PET 1987, 97]**

(a) $\frac{1}{89}$ (b) $\frac{1}{85}$ (c) $\frac{1}{80}$ (d) $\frac{1}{79}$

206. If the 7^{th} term of a H.P. is $\frac{1}{10}$ and the 12^{th} term is $\frac{1}{25}$, then the 20^{th} term is **[MP PET 1997]**

(a) $\frac{1}{37}$ (b) $\frac{1}{41}$ (c) $\frac{1}{45}$ (d) $\frac{1}{49}$

207. If 6^{th} term of a H.P. is $\frac{1}{61}$ and its tenth term is $\frac{1}{105}$, then first term of that H.P. is **[Karnataka CET 2001]**

(a) $\frac{1}{28}$ (b) $\frac{1}{39}$ (c) $\frac{1}{6}$ (d) $\frac{1}{17}$

Advance Level

208. The 9^{th} term of the series 27+ 9 + $5\frac{2}{5}+3\frac{6}{7}+.....$ will be **[MP PET 1983]**

(a) $1\frac{10}{17}$ (b) $\frac{10}{17}$ (c) $\frac{16}{27}$ (d) $\frac{17}{27}$

209. In a H.P., p^{th} term is q and the q^{th} term is p. Then pq^{th} term is **[Karnataka CET 2002]**

(a) 0 (b) 1 (c) pq (d) $pq(p+q)$

210. If *a, b, c* be respectively the p^{th}, q^{th} and r^{th} terms of a H.P., then $\Delta = \begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

(a) 1 (b) 0 (c) – 1 (d) None of these

Harmonic mean

Basic Level

211. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ be the harmonic mean between a and b, then the value of n is **[Assam PET 1986]**

(a) 1 (b) – 1 (c) 0 (d) 2

212. If the harmonic mean between a and b be H, then $\frac{H+a}{H-a}+\frac{H+b}{H-b}$ **[AMU 1998]**

(a) 4 (b) 2 (c) 1 (d) $a + b$

213. If H is the harmonic mean between p and q, then the value of $\frac{H}{p}+\frac{H}{q}$ is **[MNR 1990; UPSEAT 2000; 2001]**

(a) 2 (b) $\frac{pq}{p+q}$ (c) $\frac{p+q}{pq}$ (d) None of these

214. H. M. between the roots of the equation $x^2-10x+11=0$ is **[MP PET 1995]**

(a) $\frac{1}{5}$ (b) $\frac{5}{21}$ (c) $\frac{21}{20}$ (d) $\frac{11}{5}$

215. The harmonic mean of $\frac{a}{1-ab}$ and $\frac{a}{1+ab}$ is **[MP PET 1996]**

(a) $\frac{a}{\sqrt{1-a^2b^2}}$ (b) $\frac{a}{1-a^2b^2}$ (c) a (d) $\frac{1}{a-a^2b^2}$

216. The sixth H.M. between 3 and $\frac{6}{13}$ is **[Rajasthan PET 1996]**

(a) $\frac{63}{120}$ (b) $\frac{63}{12}$ (c) $\frac{126}{105}$ (d) $\frac{120}{63}$

Advance Level

217. If there are n harmonic means between 1 and $\frac{1}{31}$ and the ratio of 7^{th} and $(n-1)^{th}$ harmonic means is 9 : 5, then the value of n will be

[Rajasthan PET 1986]

(a) 12 (b) 13 (c) 14 (d) 15

218. If m is a root of the given equation $(1-ab)x^2-(a^2+b^2)x-(1+ab)=0$ and m harmonic means are inserted between a and b, then the difference between last and the first of the means equals

(a) $b-a$ (b) $ab(b-a)$ (c) $a(b-a)$ (d) $ab(a-b)$

Properties of Harmonic progression

Basic Level

219. If $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$, then a, b, c are in **[MNR 1984; MP PET 1997; UPSEAT 2000]**

(a) A.P. (b) G.P. (c) H.P. (d) In G.P. and H.P. both

220. If a, b, c are in H.P., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in **[Roorkee 1980]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

221. If a, b, c, d are any four consecutive coefficients of any expanded binomial, then $\frac{a+b}{a}, \frac{b+c}{b}, \frac{c+d}{c}$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

222. $\log_3 2, \log_6 2, \log_{12} 2$ are in **[Rajasthan PET 1993, 2001]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

223. If a, b, c are in H.P., then for all $n \in N$ the true statement is **[Rajasthan PET 1995]**

(a) $a^n+c^n<2b^n$ (b) $a^n+c^n>2b^n$ (c) $a^n+c^n=2b^n$ (d) None of these

224. Which number should be added to the numbers 13, 15, 19 so that the resulting numbers be the consecutive term of a H.P.

(a) 7 (b) 6 (c) – 6 (d) – 7

Advance Level

225. If b^2, a^2, c^2 are in A.P., then $a+c, b+c, c+a$ will be in **[AMU 1974]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

226. If a, b, c, d be in H.P., then

(a) $a^2+c^2>b^2+d^2$ (b) $a^2+d^2>b^2+c^2$ (c) $ac+bd>b^2+c^2$ (d) $ac+bd>b^2+d^2$

227. If $a_1, a_2, a_3, ..., a_n$ are in H.P., then $a_1a_2+a_2a_3+........+a_{n-1}a_n$ will be equal to **[IIT 1975]**

(a) a_1a_n (b) na_1a_n (c) $(n-1)a_1a_n$ (d) None of these

228. If x, y, z are in H.P., then the value of expression $\log(x+z)+\log(x-2y+z)$ will be **[Rajasthan PET 1985, 2000]**

(a) $\log(x-z)$ (b) $2\log(x-z)$ (c) $3\log(x-z)$ (d) $4\log(x-z)$

229. If $\frac{x+y}{2}, y, \frac{y+z}{2}$ are in H.P., then x, y, z are in **[Rajasthan PET 1989; MP PET 2003]**

(a) A.P.(b) G.P. (c) H.P. (d)

230. If a, b, c, d are in H.P., then **[Rajasthan PET 1991]**

(a) $a + d > b + c$ (b) $ad > bc$ (c) Both (a) and (b) (d) None of these

Arithmetio-geometric progression

Basic Level

231. If $|x| < 1$, then the sum of the series $1+2x+3x^2+4x^3+........\infty$ will be

(a) $\frac{1}{1-x}$ (b) $\frac{1}{1+x}$ (c) $\frac{1}{(1+x)^2}$ (d) $\frac{1}{(1-x)^2}$

232. The sum of 0.2+0.004 + 0.00006 + 0.0000008+...... to ∞ is

(a) $\frac{200}{891}$ (b) $\frac{2000}{9801}$ (c) $\frac{1000}{9801}$ (d) None of these

233. The n^{th} term of the sequence 1.1, 2.3, 4.5, 8.7,...... will be

(a) $2^n(2n-1)$ (b) $2^{n-1}(2n+1)$ (c) $2^{n-1}(2n-1)$ (d) $2^n(2n+1)$

Advance Level

234. The sum of infinite terms of the following series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+$.....will be

[MP PET 1981; Rajasthan PET 1997; Roorkee 1992; DCE 1996, 2000]

(a) $\frac{3}{16}$ (b) $\frac{35}{8}$ (c) $\frac{35}{4}$ (d) $\frac{35}{16}$

235. The sum of the series $1+3x+6x^2+10x^3+.......\infty$ will be

(a) $\frac{1}{(1-x)^2}$ (b) $\frac{1}{1-x}$ (c) $\frac{1}{(1+x)^2}$ (d) $\frac{1}{(1-x)^3}$

236. $2^{1/4}.4^{1/8}.8^{1/16}.16^{1/32}.......$ is equal to **[MNR 1984; MP PET 1998; AIEEE 2002]**

(a) 1 (b) 2 (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

237. The sum of $1+\frac{2}{5}+\frac{3}{5^2}+\frac{4}{5^3}+........$ upto n terms is **[MP PET 1982]**

(a) $\frac{25}{16}-\frac{4n+5}{16\times5^{n-1}}$ (b) $\frac{3}{4}-\frac{2n+5}{16\times5^{n+1}}$ (c) $\frac{3}{7}-\frac{3n+5}{16\times5^{n-1}}$ (d) $\frac{1}{2}-\frac{5n+1}{3\times5^{n+2}}$

238. The sum of $i - 2 - 3i + 4 +$ .... upto 100 terms, where $i=\sqrt{-1}$ is

(a) $50(1-i)$ (b) $25\,i$ (c) $25(1+i)$ (d) $100(1-i)$

Method of Difference

Basic Level

239. n^{th} term of the series $2+4+7+11+......$ will be **[Roorkee 1977]**

(a) $\frac{n^2+n+1}{2}$ (b) n^2+n+2 (c) $\frac{n^2+n+2}{2}$ (d) $\frac{n^2+2n+2}{2}$

240. If t_n denotes the n^{th} term of the series $2+3+6+11+18+....$ then t_{50} is

(a) 49^2-1 (b) 49^2 (c) 50^2+1 (d) 49^3+2

241. First term of the 11$^{\text{th}}$ group in the following groups (1), (2, 3, 4), (5, 6, 7, 8, 9), ... is

(a) 89 (b) 97 (c) 101 (d) 123

242. The sum of the series $6+66+666+......$ upto n terms is **[IIT 1974]**

(a) $(10^{n-1}-9n+10)/81$ (b) $2(10^{n+1}-9n-10)/27$ (c) $2(10^n-9n-10)/27$ (d) None of these

243. Sum of n terms of series $12+16+24+40+.....$ will be **[UPSEAT 1999]**

(a) $2(2^n-1)+8n$ (b) $2(2^n-1)+6n$ (c) $3(2^n-1)+8n$ (d) $4(2^n-1)+8n$

244. If $|a|<1$ and $|b|<1$, then the sum of the series $1+(1+a)b+(1+a+a^2)b^2+(1+a+a^2+a^3)b^3+......$ is

(a) $\frac{1}{(1-a)(1-b)}$ (b) $\frac{1}{(1-a)(1-ab)}$ (c) $\frac{1}{(1-b)(1-ab)}$ (d) $\frac{1}{(1-a)(1-b)(1-ab)}$

n^{th} Term of Special series

Basic Level

245. n^{th} term of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+$ ....will be **[Pb. CET 2000]**

(a) n^2+2n+1 (b) $\frac{n^2+2n+1}{8}$ (c) $\frac{n^2+2n+1}{4}$ (d) $\frac{n^2-2n+1}{4}$

246. The n^{th} term of series $\frac{1}{1}+\frac{1+2}{2}+\frac{1+2+3}{3}+.........$ will be **[AMU 1982]**

(a) $\frac{n+1}{2}$ (b) $\frac{n-1}{2}$ (c) $\frac{n^2+1}{2}$ (d) $\frac{n^2-1}{2}$

247. If $a_1=a_2=2, a_n=a_{n-1}-1(n>2)$, then a_5 is

(a) 1 (b) – 1 (c) 0 (d) – 2

Advance Level

248. The number 111.......1 (91 times) is a

(a) Even number (b) Prime number (c) Not prime (d) None of these

249. The difference between an integer and its cube is divisible by **[MP PET 1999]**

(a) 4 (b) 6 (c) 9 (d) None of these

250. In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8,......, where n consecutive terms have the value n, the 1025$^{\text{th}}$ term is

(a) 2^9 (b) 2^{10} (c) 2^{11} (d) 2^8

251. Observe that $1^3=1, 2^3=3+5, 3^3=7+9+11, 4^3=13+15+17+19$. Then n^3 as a similar series is

(a) $\left[2\left\{\frac{n(n-1)}{2}+1\right\}-1\right]+\left[2\left\{\frac{(n+1)n}{2}+1\right\}+1\right]+......+\left[2\left\{\frac{(n+1)n}{2}+1\right\}+2n-3\right]$

(b) $(n^2+n+1)+(n^2+n+3)+(n^2+n+5)+.....+(n^2+3n-1)$

(c) $(n^2-n+1)+(n^2-n+3)+(n^2-n+5)+.....+(n^2+n-1)$

(d) None of these

Sum to n terms and infinite number of terms

Basic Level

252. The sum of the series 3 . 6 + 4 . 7 + 5 . 8 +....... upto (n – 2) terms **[EAMCET 1980]**

(a) n^3+n^2+n+2 (b) $\frac{1}{6}(2n^3+12n^2+10n-84)$ (c) n^3+n^2+n (d) None of these

253. The sum of the series $1+(1+2)+(1+2+3)+......$ upto n terms, will be **[MP PET 1986]**

(a) n^2-2n+6 (b) $\frac{n(n+1)(2n-1)}{6}$ (c) n^2+2n+6 (d) $\frac{n(n+1)(n+2)}{6}$

254. The sum to n terms of the series $2^2+4^2+6^2+.......$ is **[MP PET 1994]**

(a) $\frac{n(n+1)(2n+1)}{3}$ (b) $\frac{2n(n+1)(2n+1)}{3}$ (c) $\frac{n(n+1)(2n+1)}{6}$ (d) $\frac{n(n+1)(2n+1)}{9}$

255. $11^2+12^2+13^2+.......+20^2=$ **[MP PET 1995]**

(a) 2481 (b) 2483 (c) 2485 (d) 2487

256. The sum to n terms of $(2n-1)+2(2n-3)+3(2n-5)+.....$ is **[AMU 2001]**

(a) $(n+1)(n+2)(n+3)/6$ (b) $n(n+1)(n+2)/6$ (c) $n(n+1)(2n+3)$ (d) $n(n+1)(2n+1)/6$

257. $\frac{1^3+2^3+3^3+4^3+.....+12^3}{1^2+2^2+3^3+4^2+.....+12^2}=$ **[MP PET 1998]**

(a) $\frac{234}{25}$ (b) $\frac{243}{35}$ (c) $\frac{263}{27}$ (d) None of these

258. Sum of the squares of first n natural numbers exceeds their sum by 330, then n= **[Karnataka CET 1998]**

(a) 8 (b) 10 (c) 15 (d) 20

259. $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{n.(n+1)}$ equals **[AMU 1995; Rajasthan PET 1996; UPSEAT 1999, 2001]**

(a) $\frac{1}{n(n+1)}$ (b) $\frac{n}{n+1}$ (c) $\frac{2n}{n+1}$ (d) $\frac{2}{n(n+1)}$

260. The sum to n terms of the infinite series $1.3^2+2.5^2+3.7^2+.....\infty$ is **[AMU 1982]**

(a) $\frac{n}{6}(n+1)(6n^2+14n+7)$ (b) $\frac{n}{6}(n+1)(2n+1)(3n+1)$ (c) $4n^3+4n^2+n$ (d) None of these

Advance Level

261. The sum of all the products of the first n natural numbers taken two at a time is

(a) $\frac{1}{24}n(n-1)(n+1)(3n+2)$ (b) $\frac{n^2}{48}(n-1)(n-2)$ (c) $\frac{1}{6}n(n+1)(n+2)(n+5)$ (d) None of these

262. The sum of the series 1. 3. 5 + 2. 5. 8 +3. 7. 11+.....up to 'n' terms is **[Dhanbad Engg. 1972]**

(a) $\frac{n(n-1)(9n^2+23n+13)}{6}$ (b) $\frac{n(n-1)(9n^2+23n+12)}{6}$ (c) $\frac{(n+1)(9n^2+23n+13)}{6}$ (d) $\frac{n(9n^2+23n+13)}{6}$

263. The sum of first n terms of the given series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+.......$ is $\frac{n(n+1)^2}{2}$, when n is even. When n is odd, the sum will be **[IIT 1988; AIEEE 2004]**

(a) $\frac{n(n+1)^2}{2}$ (b) $\frac{1}{2}n^2(n+1)$ (c) $n(n+1)^2$ (d) None of these

264. The value of $\sum_{r=1}^{n} \log\left(\frac{a^r}{b^{r-1}}\right)$ is

(a) $\frac{n}{2}\log\left(\frac{a^n}{b^n}\right)$ (b) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^2}\right)$ (c) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^{n-1}}\right)$ (d) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^{n+1}}\right)$

265. The sum of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}$......... to n terms is

(a) $\frac{n(n^2+1)}{n^2+n+1}$ (b) $\frac{n(n+1)}{2(n^2+n+1)}$ (c) $\frac{n(n^2-1)}{2(n^2+n+1)}$ (d) None of these

266. For any odd integer $n \ge 1$, $n^3-(n-1)^3+......+(-1)^{n-1}1^3 =$ **[IIT 1996]**

(a) $\frac{1}{2}(n-1)^2(2n-1)$ (b) $\frac{1}{4}(n-1)^2(2n-1)$ (c) $\frac{1}{2}(n+1)^2(2n-1)$ (d) $\frac{1}{4}(n+1)^2(2n-1)$

267. The sum of the infinite terms of the sequence $\frac{5}{3^2.7^2}+\frac{9}{7^2.11^2}+\frac{13}{11^2.15^2}+.....$ is

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{54}$ (d) $\frac{1}{72}$

268. The sum of the infinite series $1^2+2^2x+3^2x^2+.....$ is

(a) $(1+x)/(1-x)^3$ (b) $(1+x)/(1-x)$ (c) $x/(1-x)^3$ (d) $1/(1-x)^3$

269. If in a series $t_n = \frac{n}{(n+1)!}$, then $\sum_{n=1}^{20} t_n$ is equal to

(a) $\frac{20!-1}{20!}$ (b) $\frac{21!-1}{21!}$ (c) $\frac{1}{2(n-1)!}$ (d) None of these

270. $\sum_{r=1}^{n} r^2 - \sum_{m=1}^{n}\sum_{r=1}^{m} r$ is equal to

(a) 0 (b) $\frac{1}{2}\left(\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right)$ (c) $\frac{1}{2}\left(\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r\right)$ (d) None of these

271. For all positive integral values of n, the value of $3.1.2+3.2.3+3.3.4+.....+3.n.(n+1)$ is **[Rajasthan PET 1999]**

(a) $n(n+1)(n+2)$ (b) $n(n+1)(2n+1)$ (c) $(n-1)n(n+1)$ (d) $\frac{(n-1)\,n\,(n+1)}{2}$

272. The sum of $(n+1)$ terms of $\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+......$ is **[Rajasthan PET 1999]**

(a) $\frac{n}{n+1}$ (b) $\frac{2n}{n+1}$ (c) $\frac{2}{n(n+1)}$ (d) $\frac{2(n+1)}{n+2}$

273. The sum of $(n-1)$ terms of $1+(1+3)+(1+3+5)+.......$ is **[Rajasthan PET 1999]**

(a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\frac{n^2(n+1)}{4}$ (c) $\frac{n(n-1)(2n-1)}{6}$ (d) n^2

274. The sum $1(1!)+2(2!)+3(3!)+.....+n(n!)$ equals **[AMU 1999]**

(a) $3(n!)+n-3$ (b) $(n+1)!-(n-1)!$ (c) $(n+1)!-1$ (d) $2(n!)-2n-1$

275. Sum of the n terms of the series $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+...$ is **[Pb. CET 1999; Rajasthan PET 2001]**

(a) $\frac{2n}{n+1}$ (b) $\frac{4n}{n+1}$ (c) $\frac{6n}{n+1}$ (d) $\frac{9n}{n+1}$

276. The sum of the series $1+\frac{1.3}{6}+\frac{1.3.5}{6.8}+....\infty$ is **[UPSEAT 2001]**

(a) 1 (b) 0 (c) ∞ (d) 4

277. $11^3+12^3+....+20^3$ **[Pb. CET 1997; Rajasthan PET 2002]**

(a) Is divisible by 5 (b) Is an odd integer divisible by 5

(c) Is an even integer which is not divisible by 5 (d) Is an odd integer which is not divisible by 5

278. The sum of all numbers between 100 and 10,000 which are of form $n^3 (n \in N)$ is equal to **[IIT 1989]**

(a) 55216 (b) 53261 (c) 51261 (d) 53216

279. The cubes of the natural numbers are grouped as $1^3,(2^3,3^3),(4^3,5^3,6^3),.....$ then sum of the numbers in the n^{th} group is

(a) $\frac{1}{8}n^3(n^2+1)(n^2+3)$ (b) $\frac{1}{16}n^3(n^2+16)(n^2+12)$ (c) $\frac{n^3}{12}(n^2+2)(n^2+4)$ (d) None of these

280. The value of the expression $2(1+\omega)(1+\omega^2)+3(2\omega+1)(2\omega^2+1)+4(3\omega+1)(3\omega^2+1)+.....+(n+1)(n\omega+1)(n\omega^2+1)$ is

(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\left[\frac{n(n+1)}{2}\right]^2+n$ (c) $\left[\frac{n(n+1)}{2}\right]^2-n$ (d) None of these

281. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....$up to $\infty=\frac{\pi^2}{6}$, then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....$ equals to

(a) $\pi^2/6$ (b) $\pi^2/16$ (c) $\pi^2/8$ (d) None of these

282. The value of $\sum_{r=1}^{n}\frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}}$ is

(a) $\frac{n}{\sqrt{a}+\sqrt{a+nx}}$ (b) $\frac{\sqrt{a+nx}-\sqrt{a}}{x}$ (c) $\frac{n(\sqrt{a+nx}-a)}{x}$ (d) None of these

283. Let $\sum_{n=1}^{n}r^4=f(n)$. Then $\sum_{r=1}^{n}(2r-1)^4$ is equal to

(a) $f(2n)-16f(n)$,for all $n\in N$ (b) $f(n)-16f\left(\frac{n-1}{2}\right)$, when n is odd

(c) $f(n)-16f\left(\frac{n}{2}\right)$, when n is even (d) None of these

284. The sum to n terms of the series $\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}+.....$ is

(a) $\frac{1}{3(n+1)(n+2)(n+3)}$ (b) $\frac{1}{6(n+2)(n+3)(n+4)}$ (c) $\frac{15}{4n(n+1)(n+5)}$ (d) None of these

Relation between A.P., G.P. and H.P.

Basic Level

285. If a and b are two different positive real numbers, then which of the following relations is true **[MP PET 1982,2002]**

(a) $2\sqrt{ab}>(a+b)$ (b) $2\sqrt{ab}<(a+b)$ (c) $2\sqrt{ab}=(a+b)$ (d) None of these

286. If a, b, c are in A.P. as well as in G.P., then **[MNR 1981; AMU 1998]**

(a) $a=b\neq c$ (b) $a\neq b=c$ (c) $a\neq b\neq c$ (d) $a=b=c$

287. If three numbers be in G.P., then their logarithms will be in **[BIT 1992]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

288. If the arithmetic, geometric and harmonic means between two distinct positive real numbers be A, G and H respectively, then the relation between them **[MP PET 1984; Roorkee 1995]**

(a) $A>G>H$ (b) $A>G<H$ (c) $H>G>A$ (d) $G>A>H$

289. If the arithmetic, geometric and harmonic means between two positive real numbers be A,G and H, then

[AMU 1979,82; MP PET 1993]

(a) $A^2=GH$ (b) $H^2=AG$ (c) $G=AH$ (d) $G^2=AH$

290. If a, b, c are in A.P. then $\frac{a}{bc},\frac{1}{c},\frac{2}{b}$ are in **[MNR 1982; MP PET 2002]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

291. The geometric mean of two numbers is 6 and their arithmetic mean is 6.5. The numbers are **[MP PET 1994]**

(a) (3, 12) (b) (4, 9) (c) (2, 18) (d) (7, 6)

292. In the four numbers first three are in G.P. and last three in A.P. whose common difference is 6. If the first and last numbers are same, then first will be **[IIT 1974]**

(a) 2 (b) 4 (c) 6 (d) 8

293. If A_1, A_2 are the two A.M.'s between two numbers a and b and G_1, G_2 be two G.M.'s between same two numbers, then $\frac{A_1 + A_2}{G_1 . G_2} =$

[Roorkee 1983; DCE 1998]

(a) $\frac{a+b}{ab}$ (b) $\frac{a+b}{2ab}$ (c) $\frac{2ab}{a+b}$ (d) $\frac{ab}{a+b}$

294. If the A.M. and H.M. of two numbers is 27 and 12 respectively, then G.M. of the two numbers will be **[Rajasthan PET 19**

(a) 9 (b) 18 (c) 24 (d) 36

295. The A.M., H.M. and G.M. between two numbers are $\frac{144}{15}$, 15 and 12, but necessarily in this order. Then H.M., G.M. and A.M. respectively are

(a) $15, 12, \frac{144}{15}$ (b) $\frac{144}{15}, 12, 15$ (c) $12, 15, \frac{144}{15}$ (d) $\frac{144}{15}, 15, 12$

296. If G.M. =18 and A.M.=27, then H.M. is **[Rajasthan PET 1996]**

(a) $\frac{1}{18}$ (b) $\frac{1}{12}$ (c) 12 (d) $9\sqrt{6}$

297. If sum of A.M. and H.M. between two numbers is 25 and their G.M. is 12, then sum of numbers is

(a) 9 (b) 18 (c) 32 (d) 18 or 32

298. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx} (x \neq 0)$, then a, b, c, d are in **[Rajasthan PET 1986]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

299. The numbers 1,4, 16 can be three terms (not necessarily consecutive) of

(a) No A.P. (b) Only one G.P. (c) Infinite number of A.P's. (d)Infinite numbers of G.P's.

300. In a G.P. of alternately positive and negative terms, any terms is the A.M. of the next two terms . Then the common ratio is

(a) – 1 (b) – 3 (c) – 2 (d) $-\frac{1}{2}$

301. If a, b, c are in A.P., then $a + \frac{1}{bc}, b + \frac{1}{ca}, c + \frac{1}{ab}$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

302. The A.M. of two given positive numbers is 2. If the larger number is increased by 1, the G.M. of the numbers becomes equal to the A.M. of the given numbers. Then the H.M. of the given numbers is

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) None of these

Advance Level

303. If p^{th}, q^{th}, r^{th} and s^{th} terms of an A.P. be in G.P., then $(p-q), (q-r), (r-s)$ will be in

(a) G.P. (b) A.P. (c) H.P. (d) None of these

304. If a, b, c are the positive integers, then $(a+b)(b+c)(c+a)$ is **[DCE 2000]**

(a) $< 8abc$ (b) $> 8abc$ (c) $= 8abc$ (d) None of these

305. If a, b, c are in A.P., then $3^a, 3^b, 3^c$ shall be in **[Pb. CET 1990]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

306. If a, b, c, d and p are different real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$, then a, b, c, d are in

[IIT 1987]

(a) A.P. (b) G.P. (c) H.P. (d) $ab = cd$

307. If the first and $(2n-1)^{th}$ terms of an A.P., G.P. and H.P. are equal and their n^{th} terms are respectively a, b and c, then **[IIT 1985,88]**

(a) $a \geq b \geq c$ (b) $a + c = b$ (c) $ac - b^2 = 0$ (d) (a) and (c) both

308. If the $(m+1)^{th}, (n+1)^{th}$ and $(r+1)^{th}$ terms of an A.P. are in G.P. and m, n, r in H.P., then the value of the ratio of the common difference to the terms of the A.P. is **[MNR 1989; Roorkee 1994]**

(a) $-\frac{2}{n}$ (b) $\frac{2}{n}$ (c) $-\frac{n}{2}$ (d) $\frac{n}{2}$

309. Given $a^x = b^y = c^z = d^u$ and *a, b, c, d* are in G.P., then *x, y, z, u* are in **[Dhanbad Engg. 1972; Roorkee 1984; Rajasthan PET**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

310. If *a, b, c* are in G.P. and $\log a - \log 2b, \log 2b - \log 3c$ and $\log 3c - \log a$ are in A.P., then *a, b, c* are the length of the sides of a triangle which is

(a) Acute angled (b) Obtuse angled (c) Right angled (d) Equilateral

311. If *a, b, c* are in A.P., *b, c, d* are in G.P. and *c, d, e* are in H.P., then *a, c, e* are in **[AMU 1988,2001; MP PET 1993]**

(a) No particular order (b) A.P. (c) G.P. (d) H.P.

312. If *a, b, c* are in A.P. and a^2, b^2, c^2 are in H.P., then **[MNR 1986,88; IIT 1977,2003]**

(a) $a = b = c$ (b) $2b = 3a + c$ (c) $b^2 = \sqrt{(ac/8)}$ (d) None of these

313. The harmonic mean of two numbers is 4 and the arithmetic and geometric means satisfy the relation $2A + G^2 = 27$, the numbers are

[MNR 1987; UPSEAT 1999,2000]

(a) 6, 3 (b) 5, 4 (c) 5, – 2.5 (d) – 3, 1

314. In a G.P. the sum of three numbers is 14, if 1 is added to first two numbers and subtracted from third numbers, the series becomes A.P., then the greatest number is **[Roorkee 1973]**

(a) 8 (b) 4 (c) 24 (d) 16

315. If *a, b, c* are in G.P. and *x, y* are the arithmetic means between *a, b* and *b, c* respectively, then $\frac{a}{x} + \frac{c}{y}$ is equal to **[Roorkee 1969]**

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

316. If *a, b, c* are in A.P. and *a, b, d* in G.P., then *a, a – b, d – c* will be in **[Ranchi BIT 1968]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

317. If *x*, 1, *z* are in A.P. and *x*, 2, *z* are in G.P., then *x*, 4, *z* will be in **[IIT 1965]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

318. $x + y + z = 15$, if $9, x, y, z, a$ are in A.P.; while $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$ if $9, x, y, z, a$ are in H.P., then the value of *a* will be **[IIT 1978]**

(a) 1 (b) 2 (c) 3 (d) 9

319. If 9 A.M.'s and H.M.'s are inserted between the 2 and 3 and if the harmonic mean *H* is corresponding to arithmetic mean *A*, then $A + \frac{6}{H} =$ **[Dhanbad Engg. 1987]**

(a) 1 (b) 3 (c) 5 (d) 6

320. If the p^{th}, q^{th} and r^{th} term of a G.P. and H.P. are *a, b, c,* then $a(b-c)\log a + b(c-a)\log b + c(a-b)\log c =$ **[Dhanbad Engg. 1976]**

(a) – 1 (b) 0 (c) 1 (d) Does not exist

321. If the product of three terms of G.P. is 512. If 8 added to first and 6 added to second term, so that number may be in A.P., then the numbers are **[Roorkee 1964]**

(a) 2, 4, 8, (b) 4, 8, 16 (c) 3, 6, 12 (d) None of these

322. If the ratio of H.M. and G.M. between two numbers *a* and *b* is $4:5$, then ratio of the two numbers will be **[IIT 1992; MP**

(a) $1:2$ (b) $2:1$ (c) $4:1$ (d) $1:4$

323. If the A.M. and G.M. of roots of a quadratic equations are 8 and 5 respectively, then the quadratic equation will be **[Pb.CET 1990]**

(a) $x^2 - 16x - 25 = 0$ (b) $x^2 - 8x + 5 = 0$ (c) $x^2 - 16x + 25 = 0$ (d) $x^2 + 16x - 25 = 0$

324. Let $a_1, a_2, ... a_{10}$ be in A.P. and $h_1, h_2, ..., h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is **[IIT 1999]**

(a) 2 (b) 3 (c) 5 (d) 6

325. If $\ln(a+c), \ln(c-a), \ln(a-2b+c)$ are in A.P., then **[IIT Screening 1994; Rajasthan PET 1999]**

(a) *a, b, c* are in A.P. (b) a^2, b^2, c^2 are in A.P. (c) *a, b, c* are in G.P. (d) *a, b, c* are in H.P.

326. If $A_1, A_2; G_1, G_2$ and H_1, H_2 be two A.M's, G.M's and H.M's between two numbers respectively, then $\frac{G_1 G_2}{H_1 H_2} \times \frac{H_1 + H_2}{A_1 + A_2}$ equals

[Rajasthan PET 1997; AMU 2000]

(a) 1 (b) 0 (c) 2 (d) 3

327. If $x>1, y>1, z>1$ are in G.P., then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in **[IIT 1998; UPSEAT 2001]**

(a) A.P. (b) H.P. (c) G.P. (d) None of these

328. If *p, q, r* are in one geometric progression and *a, b, c* in another geometric progression, then cp, bq, ar are in **[Roorkee Qualifying 1998]**

(a) A.P. (b) H.P. (c) G.P. (d) None of these

329. If first three terms of sequence $\frac{1}{16}, a, b, \frac{1}{6}$ are in geometric series and last three terms are in harmonic series, then the value of a and b will be **[UPSEAT 1999]**

(a) $a=-\frac{1}{4}, b=1$ (b) $a=\frac{1}{12}, b=\frac{1}{9}$ (c) (*a*) and (*b*) both are true (d) None of these

330. If $a^x=b^y=c^z$ and a, b, c are in G.P., then x, y, z are in **[Pb. CET 1993; DCE 1999; AMU 1999]**

(a) A. P. (b) G. P. (c) H. P. (d) None of these

331. If G_1 and G_2 are two geometric means and A the arithmetic mean inserted between two numbers, then the value of $\frac{G_1^2}{G_2}+\frac{G_2^2}{G_1}$ is

[DCE 1999]

(a) $\frac{A}{2}$ (b) A (c) $2A$ (d) None of these

332. If $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P., then a, b, c are in **[Rajasthan PET 1999]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

333. If *a, b, c* are in A.P., then $\frac{1}{\sqrt{a}+\sqrt{b}}, \frac{1}{\sqrt{a}+\sqrt{c}}, \frac{1}{\sqrt{b}+\sqrt{c}}$ are in **[Roorkee 1999]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

334. The sum of three decreasing numbers in A.P. is 27. If $-1, -1, 3$ are added to them respectively, the resulting series is in G.P. The numbers are **[AMU 1999]**

(a) 5, 9, 13 (b) 15, 9, 3 (c) 13, 9, 5 (d) 17, 9, 1

335. If in the equation $ax^2+bx+c=0,$ the sum of roots is equal to sum of square of their reciprocals, then $\frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in

[Rajasthan PET 2000]

(a) A.P. (b) G.P. (c) H.P. (d) None of these

336. If *a, b, c* are in A.P., then $2^{ax+1}, 2^{bx+1}, 2^{cx+1}, x \neq 0$ are in **[DCE 2000; Pb. CET 2000]**

(a) A.P. (b) G.P. only when $x>0$ (c) G.P. if $x<0$ (d) G.P. for all $x \neq 0$

337. If $b+c,\ c+a,\ a+b$ are in H.P., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in **[Rajasthan PET 2000]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

338. The common difference of an A.P. whose first term is unity and whose second, tenth and thirty fourth terms are in G.P., is

[AMU 2000]

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$

339. The sum of three consecutive terms in a geometric progression is 14. If 1 is added to the first and the second terms and 1 is subtracted from the third, the resulting new terms are in arithmetic progression. Then the lowest of the original term is

[MP PET 2001]

(a) 1 (b) 2 (c) 4 (d) 8

340. a, g, h are arithmetic mean, geometric mean and harmonic mean between two positive numbers *x* and *y* respectively. Then identify the correct statement among the following **[Karnataka CET 2001]**

(a) *h* is the harmonic mean between *a* and *g* (b) No such relation exists between *a*, *g* and *h*

(c) g is the geometric mean between a and h (d) a is the arithmetic mean between g and h

341. Let the positive numbers *a*, *b*, *c*, *d* be in A.P., then *abc, abd, acd, bcd* are **[IIT Screening 2001]**

(a) Not in A.P./G.P./H.P. (b) In A.P. (c) In G.P. (d) In H.P.

342. If $(y-x)$, $2(y-a)$ and $(y-z)$ are in H.P., then $x-a, y-a, z-a$ are in **[Rajasthan PET 2001]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

343. If *A* and *G* are arithmetic and geometric means and $x^2-2Ax+G^2=0$, then **[UPSEAT 2001]**

(a) $A=G$ (b) $A>G$ (c) $A<G$ (d) $A=-G$

344. If A is the A.M. of the roots of the equation $x^2-2ax+b=0$ and G is the G.M. of the roots of the equation $x^2-2bx+a^2=0$, then

[UPSEAT 2001]

(a) $A>G$ (b) $A\neq G$ (c) $A=G$ (d) None of these

345. If a,b,c are three unequal numbers such that a,b,c are in A.P. and *b* – *a,* c – *b, a* are in G.P., then *a* : *b* : *c* is **[UPSEAT 2**

(a) 1 : 2 : 3 (b) 2: 3 : 1 (c) 1 : 3 : 2 (d) 3 : 2 : 1

346. If a,b,c are in A.P. and a^2,b^2,c^2 are in H.P., then **[UPSEAT 2001]**

(a) $a\neq b\neq c$ (b) $a^2=b^2=\frac{c^2}{2}$ (c) a,b,c are in G.P. (d) $\frac{-a}{2},b,c$ are in G.P.

347. Let a_1,a_2,a_3 be any positive real numbers, then which of the following statement is not true **[Orissa JEE 2002]**

(a) $3a_1a_2a_3\leq a_1^3+a_2^3+a_3^3$ (b) $\frac{a_1}{a_2}+\frac{a_2}{a_3}+\frac{a_3}{a_1}\geq 3$

(c) $(a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)\geq 9$ (d) $(a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)^3\leq 27$

348. If $a_1,a_2,....a_n$ are positive real numbers whose product is a fixed number *c*, then the minimum value of $a_1+a_2+...+a_{n-1}+2a_n$ is

[IIT Screening 2002]

(a) $n(2c)^{1/n}$ (b) $(n+1)c^{1/n}$ (c) $2nc^{1/n}$ (d) $(n+1)(2c)^{1/n}$

349. Suppose a,b,c are in A.P. and a^2,b^2,c^2 are in G.P. If *a* < *b* < *c* and $a+b+c=\frac{3}{2}$, then the value of *a* is

[IIT Screening 2002]

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$ (c) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

350. Two sequences $\{t_n\}$ and $\{s_n\}$ are defined by $t_n=\log\left(\frac{5^{n+1}}{3^{n-1}}\right), s_n=\left[\log\left(\frac{5}{3}\right)\right]^n$, then **[AMU 2002]**

(a) $\{t_n\}$ is an A.P., $\{s_n\}$ is a G.P. (b) $\{t_n\}$ and $\{s_n\}$ are both G.P.

(c) $\{t_n\}$ and $\{s_n\}$ are both A.P. (d) $\{s_n\}$ is a G.P., $\{t_n\}$ is neither A.P. nor G.P.

351. If $\begin{vmatrix} a & b & a\alpha-b \\ b & c & b\alpha-c \\ 2 & 1 & 0 \end{vmatrix}$ = 0 and $\alpha\neq 1/2$, then *a, b, c* are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

352. If *x, y, z* are in G.P. and $\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ are in A.P., then

(a) $x=y=z$ or $y\neq 1$ (b) $z=1/x$

(c) $x=y=z,$ but their common value is not necessarily zero (d) $x=y=z=0$

353. If in a progression $a_1, a_2, a_3 ......$, etc., $(a_r - a_{r+1})$ bears a constant ratio with $a_r . a_{r+1}$ then the terms of the progression are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

354. If $\frac{a_2 a_3}{a_1 a_4} = \frac{a_2 + a_3}{a_1 + a_4} = 3\left(\frac{a_2 - a_3}{a_1 - a_4}\right)$ then a_1, a_2, a_3, a_4 are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

355. If $a, a_1, a_2, a_3, ... a_{2n-1}, b$ are in A.P., $a, b_1, b_2, b_3, ... b_{2n-1}, b$ are in G.P. and $a, c_1, c_2, c_3, ... c_{2n-1}, b$ are in H.P., where *a, b* are positive, then the equation $a_n x^2 - b_n x + c_n = 0$ has its roots

(a) Real and unequal (b) Real and equal (c) Imaginary (d) None of these

356. If *a, x, b,* are in A.P., *a, y, b* are in G.P. and *a, z, b* are in H.P. such that $x = 9z$ and $a > 0, b > 0$ then

(a) $|y| = 3z$ (b) $x = 3|y|$ (c) $2y = x + z$ (d) None of these

357. If *a, b, c* are in G.P. and *a, p, q* in A.P. such that $2a, b + p, c + q$ are in G.P. then the common difference of the A.P. is

(a) $\sqrt{2}a$ (b) $(\sqrt{2} + 1)(a - b)$ (c) $\sqrt{2}(a + b)$ (d) $(\sqrt{2} - 1)(b - a)$

Applications of Progressions

Basic Level

358. If *x, y, z* are positive then the minimum value of $x^{\log y - \log z} + y^{\log z - \log x} + z^{\log x - \log y}$ is

(a) 3 (b) 1 (c) 9 (d) 16

359. *a, b, c* are three positive numbers and abc^2 has the greatest value $\frac{1}{64}$. Then

(a) $a = b = \frac{1}{2}, c = \frac{1}{4}$ (b) $a = b = \frac{1}{4}, c = \frac{1}{2}$ (c) $a = b = c = \frac{1}{3}$ (d) None of these

360. If $a > 0,\ b > 0, c > 0$ and the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$ is λabc, then the λ is

(a) 2 (b) 1 (c) 6 (d) 3

361. If *x, y, z* are three real numbers of the same sign then the value of $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ lies in the interval

(a) $[2, +\infty)$ (b) $[3, +\infty)$ (c) $(3, +\infty)$ (d) $(-\infty, 3)$

362. The sum of the products of the ten numbers $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5$ taking two at a time is

(a) 165 (b) – 55 (c) 55 (d) None of these

363. Let $S_1, S_2 .....$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 *cm,* then for which of the following values of *n* is the area of S_n less then 1 *sq cm* **[IIT 1999]**

(a) 7 (b) 8 (c) 9 (d) 10

364. Jairam purchased a house in Rs. 15000 and paid Rs. 5000 at once. Rest money he promised to pay in annual installment of Rs. 1000 with 10% per annum interest. How much money is to be paid by Jairam **[UPSEAT 1999]**

(a) Rs. 21555 (b) Rs. 20475 (c) Rs. 20500 (d) Rs. 20700

365. The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is **[MP PET 2000]**

(a) 2489 (b) 4735 (c) 2317 (d) 2632

366. The product of n positive numbers is unity. Their sum is

(a) A positive integer (b) Equal to $n+\frac{1}{n}$ (c) Divisible by n (d) Never less than n

367. If a,b,c,d are positive real numbers such that $a+b+c+d=2$, then $M=(a+b)(c+d)$ satisfies the relation **[IIT Screening**

(a) $0<M\le 1$ (b) $1\le M\le 2$ (c) $2\le M\le 3$ (d) $3\le M\le 4$

368. The sum of all positive divisors of 960 is **[Karnataka CET 2000]**

(a) 3048 (b) 3087 (c) 3047 (d) 2180

369. $2^{\sin\theta}+2^{\cos\theta}$ is greater than **[AMU 2000]**

(a) $\frac{1}{2}$ (b) $\sqrt{2}$ (c) $2^{\frac{1}{\sqrt{2}}}$ (d) $2^{\left(1-\frac{1}{\sqrt{2}}\right)}$

370. If the altitudes of a triangle are in A.P., then the sides of the triangle are in **[EAMCET 2002]**

(a) A.P. (b) H.P.

(c) G.P. (d) Arithmetico-geometric progression

371. A boy goes to school from his home at a speed of x *km/hour* and comes back at a speed of y *km/hour*, then the average speed is given by

(a) A.M. (b) G.M. (c) H.M. (d) None of these

372. A monkey while trying to reach the top of a pole height 12 *metres* takes every time a jump of 2 *metres* but slips 1 *metre* while holding the pole. The number of jumps required to reach the top of the pole, is

(a) 6 (b) 10 (c) 11 (d) 12

373. Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row of two balls and so on. If 669 more balls are added then all the balls can be arranged in the shape of a square and each of the sides then contains 8 balls less than each side of the triangle did. The initial number of balls is **[Roorkee 1**

(a) 1600 (b) 1500 (c) 1540 (d) 1690

374. If *a, b* and c are three positive real numbers, then the minimum value of the expression $\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}$ is

(a) 1 (b) 2 (c) 3 (d) 6

375. If $x_1>0, i=1,2,.....,50$ and $x_1+x_2+.....+x_{50}=50$, then the minimum value of $\frac{1}{x_1}+\frac{1}{x_2}+.....+\frac{1}{x_{50}}$ equals to

(a) 50 (b) $(50)^2$ (c) $(50)^3$ (d) $(50)^4$

376. If *a, b* and *c* are positive real numbers, then least value of $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ is

(a) 9 (b) 3 (c) 10/3 (d) None of these

377. In the value of 100 ! the number of zeros at the end is

(a) 11 (b) 22 (c) 23 (d) 24

378. If $(1-p)(1+3x+9x^2+27x^3+81x^4+243x^5)=1-p^6, p\neq 1$ then the value of $\frac{p}{x}$ is

(a) 1/3 (b) 3 (c) 1/2 (d) 2

379. Let $f(n)=\left[\frac{1}{2}+\frac{n}{100}\right]$ where[x] denotes the integral part of x. Then the value of $\sum_{n=1}^{100} f(n)$ is

(a) 50 (b) 51 (c) 1 (d) None of these

380. $A_r; r=1,2,3,.....,n$ are n points on the parabola $y^2=4x$ in the first quadrant. If $A_r=(x_r,y_r)$, where $x_1,x_2,x_3,.....,x_n$ are in G.P. and $x_1=1, x_2=2$, then y_n is equal to

(a) $-2^{\frac{n+1}{2}}$ (b) 2^{n+1} (c) $(\sqrt{2})^{n+1}$ (d) $2^{\frac{n}{2}}$

381. The lengths of three unequal edges of a rectangular solid block are in G.P. The volume of the block is $216\ cm^3$ and the total surface area is $252\ cm^2$. The length of the longest edge is

(a) 12 *cm* (b) 6 *cm* (c) 18 *cm* (d) 3 *cm*

382. *ABC* is right-angled triangle in which $\angle B = 90°$ and $BC = a$. If n points $L_1, L_2, \ldots\ldots, L_n$ on *AB* are such that *AB* is divided in $n+1$ equal parts and $L_1M_1, L_2M_2, \ldots\ldots, L_nM_n$ are line segments parallel to *BC* and $M_1, M_2, \ldots\ldots, M_n$ are on *AC* then the sum of the lengths of $L_1M_1, L_2M_2, \ldots\ldots, L_nM_n$ is

(a) $\frac{a(n+1)}{2}$ (b) $\frac{a(n-1)}{2}$

(c) $\frac{an}{2}$ (d) Impossible to find from the given data

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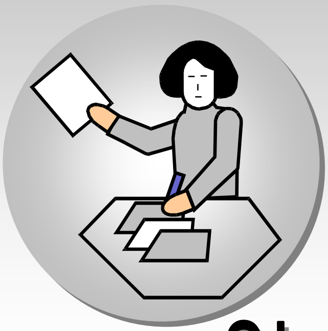

Answer Sheet

Progressions ***Assignment (Basic and Advance Level)***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
c	b	b	a	b	b	a	b	b	a	a	c	a	c	a	c	a	c	b	d
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
c	a	c,d	d	d	b	c	b	c	a	b	a	b	d	d	d	d	d	b	a
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
b	c	b	c	c	b	a	b	d	a	d	b	c	d	d	b	b	c	d	a
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
d	d	b	b	b	b	a	d	d	b	a	c	c	c	b	c	d	a	e	b
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
b	a	a	b	c	c	b	d	c	a	a	b	d	c	b	d	a	a	b	d
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
b	a	a	c	a	a	a,d	c	d	b	a	c	b	b	c	c	c	a	c	b
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
c	a	b	a	a	b	b	a	a	a	a	c	b	b	c	c	b	a	d	a
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
d	d	d	d	d	c	b	a,b	a	d	d	c	c	c	b	d	a	a	d	d
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b	c	a	d	c	c	a	b	c	a	d	a	d	d	c	b	b	c	b	b
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
c	c	a	b	c	d	b	c	a	b	a	c	c	b	a	c	a	a	c	b
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
a	b	b	d	a	d	c	a	b	b	b	b	a	d	c	a	c	b	c	c
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
c	c	b	d	c	c	c	b	b	c	d	b	c	d	d	b	a	a	c	d
241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
c	b	d	c	c	a	b	c	b	b	c	b	d	b	c	d	a	b	b	a
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**	**277**	**278**	**279**	**280**
a	a	b	c	b	d	d	a	b	c	a	d	c	c	c	d	b	b	a	b
281	**282**	**283**	**284**	**285**	**286**	**287**	**288**	**289**	**290**	**291**	**292**	**293**	**294**	**295**	**296**	**297**	**298**	**299**	**300**
c	a,b	a	d	b	d	a	a	d	d	b	d	a	b	b	c	c	b	a	c
301	**302**	**303**	**304**	**305**	**306**	**307**	**308**	**309**	**310**	**311**	**312**	**313**	**314**	**315**	**316**	**317**	**318**	**319**	**320**
a	a	a	b	b	b	d	a	c	b	c	a	a	a	c	b	c	a	c	b
321	**322**	**323**	**324**	**325**	**326**	**327**	**328**	**329**	**330**	**331**	**332**	**333**	**334**	**335**	**336**	**337**	**338**	**339**	**340**
b	c,d	c	d	d	a	b	c	c	c	c	c	a	d	a	d	a	b	b	c
341	**342**	**343**	**344**	**345**	**346**	**347**	**348**	**349**	**350**	**351**	**352**	**353**	**354**	**355**	**356**	**357**	**358**	**359**	**360**
d	b	b	c	a	d	d	a	d	a	b	a	c	c	c	b	b,d	a	b	c
361	**362**	**363**	**364**	**365**	**366**	**367**	**368**	**369**	**370**	**371**	**372**	**373**	**374**	**375**	**376**	**377**	**378**	**379**	**380**
b	b	b,c,d	c	d	d	a	a	d	b	c	c	c	d	a	a	d	b	b	c
381	**382**																		
a	c																		