Definite Integration

EXERCISE 6.1 [PAGE 145]

Exercise 6.1 | Q 1 | Page 145

Evaluate the following definite integrals: $\int_4^9 \frac{1}{\sqrt{x}} \cdot dx$

Solution:

Let I = $\int_4^9 \frac{1}{\sqrt{x}} \cdot dx$

= $\int_4^9 x^{\frac{1}{2}} \cdot dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^9$

= $2\left[\sqrt{x}\right]_4^9$

= $2\left(\sqrt{9} - \sqrt{4}\right)$

= 2 (3 – 2)

∴ I = 2.

Exercise 6.1 | Q 2 | Page 145

Evaluate the following definite integrals: $\int_{-2}^3 \frac{1}{x+5} \cdot dx$

Solution:

Let I = $\int_{-2}^3 \frac{1}{x+5} \cdot dx$

= $[\log|x+5|]_{-2}^3$

= [log |3 + 5| – log |–2 + 5|]

= log 8 – log 3

∴ I = $\log\left(\frac{8}{3}\right)$.

Exercise 6.1 | Q 3 | Page 145

Evaluate the following definite integrals: $\int_2^3 \frac{x}{x^2 - 1} \cdot dx$

Solution:

Let I = $\int_2^3 \frac{x}{x^2 - 1} \cdot dx$

Put $x^2 - 1 = t$

∴ 2x·dx = dt

∴ x·dx = $\frac{1}{2} \cdot dt$

When x = 2, t = $2^2 - 1 = 3$

When x = 3, t = $3^2 - 1 = 8$

∴ I = $\int_3^8 \frac{1}{\mathrm{t}} \cdot \frac{\mathrm{dt}}{2}$

= $\frac{1}{2} \int_3^8 \frac{\mathrm{dt}}{\mathrm{t}}$

= $\frac{1}{2} [\log|\mathrm{t}|]_3^8$

= $\frac{1}{2} (\log 8 - \log 3)$

∴ I = $\frac{1}{2} \log\left(\frac{8}{3}\right)$.

Exercise 6.1 | Q 4 | Page 145

Evaluate the following definite integrals: $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}} \cdot dx$

Solution:

Let I = $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}} \cdot dx$

= $\int_0^1 \left(\frac{x^2+3x+2}{x^{\frac{1}{2}}}\right) \cdot dx$

= $\int_0^1 \left(\frac{x^2}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{1}{2}}}\right) \cdot dx$

= $\int_0^1 \left(x^{\frac{1}{2}} + 3x^{\frac{1}{2}} + 2x^{\frac{1}{2}}\right) \cdot dx$

= $\int_0^1 x^{\frac{3}{2}} \cdot dx + 3\int_0^1 x^{\frac{1}{2}} \cdot dx + 2\int_0^1 x^{\frac{1}{2}} \cdot dx$

= $\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^1 + 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + 2\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^1$

= $\frac{2}{5}(1-0)\ 3 \times \frac{2}{3}(1-0) + 2 \times 2(1-0)$

= $\frac{2}{5}\ 2 + 4$

∴ I = $\frac{32}{5}$.

Exercise 6.1 | Q 5 | Page 145

Evaluate the following definite integrals: $\int_2^3 \frac{x}{(x+2)(x+3)} \cdot dx$

Solution:

Let I = $\int_{2}^{3} \frac{x}{(x+2)(x+3)} \cdot dx$

Let $\frac{x}{(x+2)x+3} = \frac{\mathrm{A}}{x+2} + \frac{\mathrm{B}}{x+3}$ ...(i)

∴ x = A(x + 3) + B(x + 2) ...(ii)

Putting x = – 3 in (ii) we get

– 2 = A

∴ B = 3

Putting x = – 2 in (ii),we get

– 2 = A

∴ A = – 2

From (i), we get

$$\frac{x}{(x+2(x+3))} = \frac{-2}{x+2} + \frac{3}{x+3}$$

∴ I = $\int_{2}^{3} \left[\frac{-2}{x+2} + \frac{3}{x+3}\right] \cdot dx$

= $-2\int_{2}^{3} \frac{1}{x+2} \cdot dx + 3\int_{2}^{3} \frac{1}{x+3} \cdot dx$

= $-2[\log|x+2|]_{2}^{3} + 3[\log|x+3|]_{2}^{3}$

= $-2\log[\log 5 - \log 4] + 3[\log 6 - \log 5]$

= $-2\left[\log\left(\frac{5}{4}\right)\right] + 3\left[\log\left(\frac{6}{5}\right)\right]$

= $3\log\left(\frac{6}{5}\right) - 2\log\left(\frac{5}{4}\right)$

$$= \log\left(\frac{6}{5}\right)^2 - 2\log\left(\frac{5}{4}\right)^2$$

$$= \log\left(\frac{216}{125}\right) - \log\left(\frac{25}{16}\right)$$

$$= \log\left(\frac{216}{125} \times \frac{16}{25}\right)$$

$$\therefore \text{I} = \log\left(\frac{3456}{3125}\right).$$

Exercise 6.1 | Q 6 | Page 145

Evaluate the following definite integrals: $\int_1^2 \frac{dx}{x^2 + 6x + 5}$

Solution:

Let I = $\int_1^2 \frac{dx}{x^2 + 6x + 5}$

$$= \int_1^2 \frac{dx}{x^2 + 6x + 9 - 9 + 5}$$

$$= \int_1^2 \frac{dx}{(x+3)^2 - (2)^2}$$

$$= \frac{1}{2x2}\left[\log\left|\frac{x+3-2}{x+3+2}\right|\right]_1^2$$

$$= \frac{1}{4}\left[\log\left|\frac{x+1}{x+5}\right|\right]_1^2$$

$$= \frac{1}{4}\left[\log\frac{3}{7} - \log\frac{2}{6}\right]$$

= $\frac{1}{4}\log\left(\frac{3}{7}\times\frac{6}{2}\right)$

∴ I $\frac{1}{4}\log\left(\frac{9}{7}\right)$.

Exercise 6.1 | Q 7 | Page 145

Evaluate the following definite integrals: If $\int_0^a (2x+1)\cdot dx$ = 2, find the real value of a.

Solution:

Given, $\int_0^a (2x+1)\cdot dx$ = 2

∴ $\left[\frac{2x^2}{2}+x\right]_0^a$ = 2

∴ $\left[x^2+x\right]_0^a$ = 2

∴ $[(a^2 + a) - (0)] = 2$

∴ $a^2 + a = 2$

∴ $a^2 + a - 2 = 0$

∴ $a^2 + 2a - a - 2 = 0$

∴ a(a + 2) – 1(a + 2) = 0

∴ (a +)(a – 1) = 0

∴ a + 2 = 0 or a – 1 = 0

∴ a = – 2 or a = 1.

Exercise 6.1 | Q 8 | Page 145

Evaluate the following definite integrals: if $\int_1^{a}\left(3x^2+2x+1\right)\cdot dx$ = 11, find a.

Solution:

Given, $\int_1^{a}\left(3x^2+2x+1\right)\cdot dx$ = 11

$\therefore \left[\frac{3x^3}{3}+\frac{2x^2}{2}+x\right]_1^{a}$ = 11

$\therefore \left[x^3+x^2+x\right]_1^{a}$ = 11

$\therefore (a^3 + a^2 + a) - (1 + 1 + 1) = 11$

$\therefore a^3 + a^2 + a - 3 = 11$

$\therefore a^3 + a^2 + a - 14 = 0$

$\therefore (a - 2)(a^2 + 3a + 7) = 0$

$\therefore a = 2$ or $a^2 + 3a + 7 = 0$

But $a^2 + 3a + 7 = 0$ does not have eal roots.

$\therefore a = 2$.

Exercise 6.1 | Q 9 | Page 145

Evaluate the following definite integrals: $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}}\cdot dx$

Solution:

Let I = $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}}\cdot dx$

$$= \int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} \times \frac{\sqrt{1+x}-\sqrt{x}}{\sqrt{1+x}-\sqrt{x}} \cdot dx$$

$$= \int_0^1 \frac{\sqrt{1+x}-\sqrt{x}}{\left(\sqrt{1+x}\right)^2 - \left(\sqrt{x}^2\right) \cdot dx}$$

$$= \int_0^1 \frac{\sqrt{1+x}-\sqrt{x}}{1+x-x} \cdot dx$$

$$= \int_0^1 \left[(1+x)^{\frac{1}{2}} - x^{\frac{1}{2}}\right] \cdot dx$$

$$= \int_0^1 (1+x)^{\frac{1}{2}} \cdot dx - \int_0^1 x^{\frac{1}{2}} \cdot dx$$

$$= \left[\frac{(1+x)^{\frac{1}{2}}}{\frac{3}{2}}\right]_0^1 - \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{3}\left[(2)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right] - \frac{2}{3}\left[(1)^{\frac{3}{2}} - 0\right]$$

$$= \frac{2}{3}\left(2\sqrt{2}-1\right) - \frac{2}{3}(1)$$

$$= \frac{4\sqrt{2}}{3} - \frac{2}{3} - \frac{2}{3}$$

$\therefore$ I = $\frac{4}{2}\left(\sqrt{2}-1\right)$.

Exercise 6.1 | Q 10 | Page 145

Evaluate the following definite integrals: $\int_1^2 \frac{3x}{(9x^2-1)} \cdot dx$

Solution:

Let I = $\int_1^2 \frac{3x}{(9x^2 - 1)} \cdot dx$

= $3\int_1^2 \frac{x}{9x^2 - 1} \cdot dx$

Put $9x^2 - 1 = t$

$\therefore$ $18x \cdot dx = dt$

$\therefore$ $x \cdot dx = \frac{1}{18} \cdot dx$

When $x = 1$, $t = 9(1)^2 - 1 = 8$

When $x = 2$, $t = 9(2)^2 - 1 = 35$

$\therefore$ I = $3\int_8^{35} \frac{1}{t} \cdot \frac{dt}{18}$

= $\frac{1}{6}\int_8^{35} \frac{dt}{t}$

= $\frac{1}{6}[\log|t|]_8^{35}$

= $\frac{1}{6}(\log 35 - \log 8)$

$\therefore$ I = $\frac{1}{6}\log\left(\frac{35}{8}\right)$.

Exercise 6.1 | Q 11 | Page 145

Evaluate the following definite integrals: $\int_1^3 \log x \cdot dx$

Solution:

Let I = $\int_1^3 \log x \cdot dx$

$$= \int_1^3 \log x \cdot 1 dx$$

$$= \left[\log x \int 1 \cdot dx\right]_1^3 \left[\frac{d}{dx}(\log x) \int 1 \cdot dx\right] \cdot dx$$

$$= [\log x \cdot (x)]_1^3 - \int_1^3 \frac{1}{x} x \cdot dx$$

$$= [x \log x]_1^3 - \int_1^3 1 \cdot dx$$

$$= (3 \log 3 - 1 \log 1) - [x]_1^3$$

= (3 log 3 – 0) – (3 – 1)

= 3 log 3 – 2

= log 3^3 – 2

∴ I = log 27 – 2.

EXERCISE 6.2 [PAGE 148]

Exercise 6.2 | Q 1 | Page 148

Evaluate the following integrals : $\int_{-9}^{9} \frac{x^3}{4 - x^2} \cdot dx$

Solution:

Let I = $\int_{-9}^{9} \frac{x^3}{4 - x^2} \cdot dx$

Let f(x) = $\frac{x^3}{4 - x^2}$

∴ f(– x) = $\frac{(-x)^2}{4 - (-x)^2}$

$= -\dfrac{x^3}{4-x^2}$

= – f(x)

∴ f(x) is an odd function.

$\therefore \int_{-9}^{9} \dfrac{x^3}{4-x^2} \cdot dx = 0. \quad ...\left[\because \int_{a}^{a} f(x) = 0, \quad \text{if} \quad f(x)\text{odd function}\right]$

Exercise 6.2 | Q 2 | Page 148

Evaluate the following integrals : $\int_0^{a} x^2(a-x)^{\frac{3}{2}} \cdot dx$

Solution:

Let I = $\int_0^{a} x^2(a-x)^{\frac{3}{2}} \cdot dx$

$= \int_0^{a} (a-x)^2[a-(a-x)]^{\frac{3}{2}} \cdot dx \qquad ...\left[\because \int_0^{a} f(x)\cdot dx = \int_0^{a} f(a-x)\cdot dx\right]$

$= \int_0^{a} (a^2-2ax+x^2)x^{\frac{3}{2}} \cdot dx$

$= \int_0^{a} \left(a^2x^{\frac{3}{2}} - 2ax^{\frac{5}{2}} + x^{\frac{7}{2}}\right) \cdot dx$

$= a^2\int_0^{a} x^{\frac{3}{2}} \cdot dx - 2a\int_0^{a} x^{\frac{5}{2}} \cdot dx + \int_0^{a} x^{\frac{7}{2}} \cdot dx$

$= a^2\left[\dfrac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^{a} - 2a\left[\dfrac{x^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^{a} + \left[\dfrac{x^{\frac{9}{2}}}{\frac{9}{2}}\right]_0^{a}$

$= \dfrac{2a^2}{5}\left[(a)^{\frac{5}{2}} - 0\right] - \dfrac{4a}{7}\left[(a)^{\frac{7}{2}} - 0\right] + \dfrac{2}{9}\left[(a)^{\frac{9}{2}} - 0\right]$

$= \dfrac{2}{5}a^{\frac{9}{2}} - \dfrac{4}{7}a^{\frac{9}{2}} + \dfrac{2}{9}a^{\frac{9}{2}}$

$= \left(\dfrac{2}{5} - \dfrac{4}{7} + \dfrac{2}{9}\right)a^{\frac{9}{2}}$

$$= \left(\frac{126 - 180 + 70}{315}\right)a^{\frac{9}{2}}$$

$$\therefore I = \frac{16}{315}a^{\frac{9}{2}}.$$

Exercise 6.2 | Q 3 | Page 148

Evaluate the following integrals : $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} \cdot dx$

Solution:

Let I = $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} \cdot dx$ ...(i)

$= \int_1^3 \frac{\sqrt[3]{(1+3-x)+5}}{\sqrt[3]{(1+3-x)+5} + \sqrt[3]{9-(1+3-x)}} \cdot dx$ $\quad \ldots\left[\because \int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx\right]$

$\therefore$ I = $\int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{5+x}} \cdot dx$ ...(ii)

Adding (i) and (ii), we get

2I = $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} \cdot dx + \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{5+x}} \cdot dx$

$= \int_1^3 \frac{\sqrt[3]{x+5} + \sqrt[3]{9-x}}{\sqrt[3]{x+5} - \sqrt[3]{9-x}} \cdot dx$

$= \int_1^3 1 \cdot dx$

$= [x]_1^3$

$\therefore$ 2I = 3 – 1 = 2

$\therefore$ I = I.

Exercise 6.2 | Q 4 | Page 148

Evaluate the following integrals : $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}} \cdot dx$

Solution:

Let I = $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}} \cdot dx$ ...(i)

= $\int_2^5 \frac{\sqrt{2+5-x}}{\sqrt{2+5-x}+\sqrt{7-(2+5-x)}} \cdot dx$ ...$\left[\because \int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx\right]$

∴ I = $\int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}} \cdot dx$ ...(ii)

Adding (i) and (ii), we get

2I = $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}} \cdot dx + \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}} \cdot dx$

= $\int_2^5 \frac{\sqrt{x}+\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}} \cdot dx$

= $\int_2^5 1 \cdot dx$

= $[x]_2^5$

∴ 2I = 5 – 2 = 3

∴ I = $\frac{3}{2}$.

Exercise 6.2 | Q 5 | Page 148

Evaluate the following integrals : $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} \cdot dx$

Solution:

Let I = $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} \cdot dx$ ...(i)

$$= \int_1^2 \frac{\sqrt{1+2-x}}{\sqrt{3-(1+2-x)}+\sqrt{1+2-x}} \cdot dx \quad \ldots\left[\because \int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx\right]$$

$$\therefore I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} \cdot dx \qquad \ldots(ii)$$

Adding (i) and (ii), we get

$$2I = \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} \cdot dx + \int_1^2 \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} \cdot dx$$

$$= \int_1^2 \frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} \cdot dx$$

$$= \int_1^2 1 \cdot dx$$

$$= [x]_1^2$$

$\therefore$ 2I = 2 – 1 = 1

$$\therefore I = \frac{1}{2}.$$

Exercise 6.2 | Q 6 | Page 148

Evaluate the following integrals : $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx$

Solution:

$$\text{Let } I = \int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx \qquad \ldots(i)$$

$$= \int_2^7 \frac{\sqrt{2+7-x}}{\sqrt{2+7-x}+\sqrt{9-(2+7-x)}} \cdot dx \quad \ldots\left[\because \int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx\right]$$

$\therefore$ I = $\int_2^7 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}} \cdot dx$ ...(ii)

Adding (i) and (ii), we get

2I = $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx + \int_2^7 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}} \cdot dx$

= $\int_2^7 \frac{\sqrt{x}+\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx$

= $\int_2^7 1 \cdot dx$

= $[x]_2^7$

$\therefore$ 2I = 7 – 2 = 5

$\therefore$ I = $\frac{5}{2}$.

Exercise 6.2 | Q 7 | Page 148

Evaluate the following integrals : $\int_0^1 \log\left(\frac{1}{x}-1\right) \cdot dx$

Solution:

Let I = $\int_0^1 \log\left(\frac{1}{x}-1\right) \cdot dx$

$\therefore$ I = $\int_0^1 \log\left(\frac{1-x}{x}\right) \cdot dx$ ...(i)

= $\int_0^1 \log\left[\frac{1-(1-x)}{1-x}\right] \cdot dx$ ...$\left[\because \int_0^a f(x) \cdot dx = \int_0^a f(a-x) \cdot dx\right]$

I = $\int_0^a \log\left(\frac{x}{1-x}\right) \cdot dx$ ...(ii)

Adding (i) and (ii), we get

2I = $\int_0^1 \log\left(\frac{1-x}{x}\right) \cdot dx + \int_0^1 \log\left(\frac{x}{1-x}\right) \cdot dx$

= $\int_0^1 \left[\log\left(\frac{1-x}{x}\right) + \log\left(\frac{x}{1-x}\right)\right] \cdot dx$

= $\int_0^1 \log\left(\frac{1-x}{x} \times \frac{x}{1-x}\right) \cdot dx$

= $\int_0^1 \log 1 \cdot dx$

∴ 2I = $\int_0^1 0 \cdot dx$

∴ I = 0.

Exercise 6.2 | Q 8 | Page 148

Evaluate the following integrals : $\int_0^1 x(1-x)^5 \cdot dx$

Solution:

Let I = $\int_0^1 x(1-x)^5 \cdot dx$

= $\int_0^1 (1-x)[1-(1-x)]^5 \cdot dx$ ...$\left[\because \int_0^a f(x) \cdot dx = \int_0^a f(\mathrm{a}-x) \cdot dx\right]$

$$= \int_0^1 (1 - x)x^5 \cdot dx$$

$$= \int_0^1 \left(x^5 - x^6\right) \cdot dx$$

$$= \int_0^1 x^5 \cdot dx - \int_0^1 x^6 \cdot dx$$

$$= \left[\frac{x^6}{6}\right]_0^1 - \left[\frac{x^7}{7}\right]_0^1$$

$$= \frac{1}{6}\left(1^6 - 0\right) - \frac{1}{7}\left(1^7 - 0\right)$$

$$= \frac{1}{6} - \frac{1}{7}$$

$\therefore$ I = $\dfrac{1}{42}$.

MISCELLANEOUS EXERCISE 6 [PAGES 148 - 150]

Miscellaneous Exercise 6 | Q 1.01 | Page 148

Choose the correct alternative :

$$\int_{-9}^{9} \frac{x^3}{4 - x^2} \cdot dx =$$

1. **0**
2. 3
3. 9
4. – 9

Solution:

Let I = $\displaystyle\int_{-9}^{9} \frac{x^3}{4 - x^2} \cdot dx$

Let f(x) = $\frac{x^3}{4 - x^2}$

∴ f(– x) = $\frac{(-x)^2}{4 - (-x)^2}$

= $-\frac{x^3}{4 - x^2}$

= – f(x)

∴ f(x) is an odd function.

∴ $\int_{-9}^{9} \frac{x^3}{4 - x^2} \cdot dx = 0.$ $\quad ...\left[\because \int_{a}^{a} f(x) = 0, \quad \text{if} \quad f(x)\text{odd function}\right]$

Miscellaneous Exercise 6 | Q 1.02 | Page 148

Choose the correct alternative :

$\int_{-2}^{3} \frac{dx}{x + 5}$ =

Options

$-\log\left(\frac{8}{3}\right)$

$\log\left(\frac{8}{3}\right)$

$\log\left(\frac{3}{8}\right)$

$-\log\left(\frac{3}{8}\right)$

Solution:

Let I = $\int_{-2}^{3} \frac{1}{x+5} \cdot dx$

= $[\log|x+5|]_{-2}^{3}$

= [log |3 + 5| – log |–2 + 5|]

= log 8 – log 3

∴ **I =** $\log\left(\frac{8}{3}\right)$.

Miscellaneous Exercise 6 | Q 1.03 | Page 148

Choose the correct alternative :

$\int_{2}^{3} \frac{x}{x^2-1} \cdot dx =$

Options

$\log\left(\frac{8}{3}\right)$

$-\log\left(\frac{8}{3}\right)$

$\frac{1}{2}\log\left(\frac{8}{3}\right)$

$\frac{-1}{2}\log\left(\frac{8}{3}\right)$

Solution:

Let I = $\int_2^3 \frac{x}{x^2-1} \cdot dx$

Put $x^2 - 1 = t$

$\therefore 2x \cdot dx = dt$

$\therefore x \cdot dx = \frac{1}{2} \cdot dt$

When x = 2, $t = 2^2 - 1 = 3$

When x = 3, $t = 3^2 - 1 = 8$

$\therefore I = \int_3^8 \frac{1}{t} \cdot \frac{dt}{2}$

$= \frac{1}{2} \int_3^8 \frac{dt}{t}$

$= \frac{1}{2} [\log|t|]_3^8$

$= \frac{1}{2} (\log 8 - \log 3)$

$\therefore$ **I =** $\frac{1}{2} \log\left(\frac{8}{3}\right)$.

Miscellaneous Exercise 6 | Q 1.04 | Page 149

Choose the correct alternative :

$\int_4^9 \frac{dx}{\sqrt{x}} =$

1. 9
2. 4

3. **2**
4. 0

Solution:

Let I = $\int_4^9 \frac{1}{\sqrt{x}} \cdot dx$

= $\int_4^9 x^{\frac{1}{2}} \cdot dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^9$

= $2\left[\sqrt{x}\right]_4^9$

= $2\left(\sqrt{9} - \sqrt{4}\right)$

= 2 (3 – 2)

∴ **I = 2**.

Miscellaneous Exercise 6 | Q 1.05 | Page 149

Choose the correct alternative :

If $\int_0^a 3x^2 \cdot dx$ = 8, then a = ?

1. **2**
2. 0
3. 8/3
4. a

Solution:

$\int_0^a 3x^2 \cdot dx$ = 8

∴ $3\left[\frac{x^3}{3}\right]_0^a$ = 8

∴ $a^3 = 2^3$

∴ a = 2.

Miscellaneous Exercise 6 | Q 1.06 | Page 149

Choose the correct alternative :

$$\int_2^3 x^4 \cdot dx =$$

1. 12
2. 52
3. 521/1
4. **211/5**

Solution:

$$\int_2^3 x^4 \cdot dx = \left[\frac{x^5}{5}\right]_2^1$$

$$= \frac{1}{5}\left(3^5 - 2^5\right)$$

$$= \frac{1}{5}(243 - 32)$$

$$= \frac{211}{5}.$$

Miscellaneous Exercise 6 | Q 1.07 | Page 149

Choose the correct alternative :

$$\int_0^2 e^x \cdot dx =$$

1. e – 1
2. 1 – e
3. 1 – e^2
4. **e^2 – 1**

Solution:

$$\int_0^2 e^x \cdot dx$$

$= [e^x]_0^2$

$= e^2 - e^0$

$= \mathbf{e^2 - 1}$.

Miscellaneous Exercise 6 | Q 1.08 | Page 149

Choose the correct alternative :

$$\int_a^b f(x) \cdot dx =$$

Options

$$\int_b^a f(x) \cdot dx$$

$$-\int_a^b f(x) \cdot dx$$

$$-\int_b^a f(x) \cdot dx$$

$$\int_0^a f(x) \cdot dx$$

Solution:

$$\int_a^b f(x) \cdot dx = -\int_b^a f(x) \cdot dx.$$

Miscellaneous Exercise 6 | Q 1.09 | Page 149

Choose the correct alternative :

$$\int_{-7}^{7} \frac{x^3}{x^2+7} \cdot dx =$$

1. 7
2. 49
3. **0**
4. 7/2

Solution:

Let f(x) = $\frac{x^3}{x^2+7}$

∴ f(– x) = $\frac{(-x)^3}{(-x)^2+7}$

= $\frac{x^3}{x^2+7}$

= – f(x)

∴ f(x) is an odd function.

Miscellaneous Exercise 6 | Q 1.1 | Page 149

Choose the correct alternative :

$$\int_{2}^{7} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx =$$

1. 7/2
2. **5/2**
3. 7
4. 2

Solution:

Let I = $\int_{2}^{7} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx$ ...(i)

= $\int_2^7 \frac{\sqrt{2+7-x}}{\sqrt{2+7-x}+\sqrt{9-(2+7-x)}} \cdot dx$ ...$\left[\because \int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx\right]$

∴ I = $\int_2^7 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}} \cdot dx$ ...(ii)

Adding (i) and (ii), we get

2I = $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx + \int_2^7 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}} \cdot dx$

= $\int_2^7 \frac{\sqrt{x}+\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx$

= $\int_2^7 1 \cdot dx$

= $[x]_2^7$

∴ 2I = 7 – 2 = 5

∴ **I =** $\frac{5}{2}$.

Miscellaneous Exercise 6 | Q 2.01 | Page 149

Fill in the blank : $\int_0^2 e^x \cdot dx$ = _______

Solution:

$\int_0^2 e^x \cdot dx$

= $[e^x]_0^2$

= $e^2 - e^0$

= $\mathbf{e^2 - 1}$.

Miscellaneous Exercise 6 | Q 2.02 | Page 149

Fill in the blank : $\int_2^3 x^4 \cdot dx$ = ______

Solution:

$\int_2^3 x^4 \cdot dx = \left[\frac{x^5}{5}\right]_2^1$

$= \frac{1}{5}\left(3^5 - 2^5\right)$

$= \frac{1}{5}(243 - 32)$

$= \frac{211}{5}$.

Miscellaneous Exercise 6 | Q 2.03 | Page 149

Fill in the blank : $\int_0^1 \frac{dx}{2x+5}$ = ______

Solution:

Let I = $\int_0^1 \frac{dx}{2x+5}$

Put 2x + 5 = t

∴ 2dx = dt

∴ dx = $\frac{\text{dt}}{2}$

When x = 0, t = 2(0) + 5 = 5

When x = 1, t = 2(1) + 5 = 7

∴ I = $\frac{1}{2}\int_5^7 \frac{\text{dt}}{\text{t}}$

= $\frac{1}{2}[\log|\text{t}|]_5^7$

$= \frac{1}{2}(\log 7 - \log 5)$

$= \frac{1}{2}\log\left(\frac{7}{5}\right).$

Miscellaneous Exercise 6 | Q 2.04 | Page 149

Fill in the blank : If $\int_0^{a} 3x^2 \cdot dx$ = 8, then a = ______

Solution:

$\int_0^{a} 3x^2 \cdot dx$ = 8

$\therefore 3\left[\frac{x^3}{3}\right]_0^{a}$ = 8

$\therefore a^3 = 2^3$

∴ a = **2**.

Miscellaneous Exercise 6 | Q 2.05 | Page 149

Fill in the blank : $\int_4^9 \frac{1}{\sqrt{x}} \cdot dx$ = ______

Solution:

Let I = $\int_4^9 \frac{1}{\sqrt{x}} \cdot dx$

$= \int_4^9 x^{\frac{1}{2}} \cdot dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^9$

$= 2\left[\sqrt{x}\right]_4^9$

$= 2\left(\sqrt{9} - \sqrt{4}\right)$

= 2 (3 – 2)

∴ I = **2**.

Miscellaneous Exercise 6 | Q 2.06 | Page 149

Fill in the blank : $\int_2^3 \frac{x}{x^2 - 1} \cdot dx$ = ______

Solution:

Let I = $\int_2^3 \frac{x}{x^2 - 1} \cdot dx$

Put $x^2 - 1 = t$

∴ 2x·dx = dt

∴ x·dx = $\frac{1}{2} \cdot dt$

When x = 2, t = $2^2 - 1 = 3$

When x = 3, t = $3^2 - 1 = 8$

∴ I = $\int_3^8 \frac{1}{t} \cdot \frac{dt}{2}$

= $\frac{1}{2} \int_3^8 \frac{dt}{t}$

= $\frac{1}{2} [\log|t|]_3^8$

= $\frac{1}{2} (\log 8 - \log 3)$

∴ I = $\frac{1}{2} \log\left(\frac{8}{3}\right)$.

Miscellaneous Exercise 6 | Q 2.07 | Page 149

Fill in the blank : $\int_{-2}^{3} \frac{dx}{x+5}$ = ______

Solution:

Let I = $\int_{-2}^{3} \frac{dx}{x+5} \cdot dx$

= $[\log|x+5|]_{-2}^{3}$

= [log |3 + 5| – log |–2 + 5|]

= log 8 – log 3

∴ I = $\log\left(\frac{8}{3}\right)$.

Miscellaneous Exercise 6 | Q 2.08 | Page 149

Fill in the blank : $\int_{-9}^{9} \frac{x^3}{4-x^2} \cdot dx$ = ______

Solution:

Let I = $\int_{-9}^{9} \frac{x^3}{4-x^2} \cdot dx$

Let f(x) = $\frac{x^3}{4-x^2}$

∴ f(– x) = $\frac{(-x)^2}{4-(-x)^2}$

= $-\frac{x^3}{4-x^2}$

= – f(x)

∴ f(x) is an odd function.

$$\therefore \int_{-9}^{9} \frac{x^3}{4 - x^2} \cdot dx = 0. \quad ...\left[\because \int_{\mathrm{a}}^{\mathrm{a}} f(x) = 0, \quad \text{if} \quad f(x)\text{odd function}\right]$$

Miscellaneous Exercise 6 | Q 3.01 | Page 149

State whether the following is True or False :

$$\int_{\mathrm{a}}^{\mathrm{b}} f(x) \cdot dx = \int_{-\mathrm{b}}^{-\mathrm{a}} f(x) \cdot dx$$

1. **True**
2. False

Solution:

Let I = $\int_{\mathrm{a}}^{\mathrm{b}} f(x) \cdot dx$

Put x = – 1

∴ dx = – dt

When x = a, t = – a

When x = b, t = – b

∴ I = $\int_{-a}^{-b} f(-\mathrm{t})(-\mathrm{dt})$

= $\int_{-\mathrm{b}}^{-\mathrm{a}} f(-\mathrm{t}) \cdot \mathrm{dt} \quad ...\left[\because \int_{\mathrm{a}}^{\mathrm{b}} f(x) \cdot dx = -\int_{\mathrm{b}}^{\mathrm{a}} f(x) \cdot dx\right]$

= $\int_{-\mathrm{b}}^{-\mathrm{a}} f(-x) \cdot dx \quad ...\left[\because \int_{\mathrm{a}}^{\mathrm{b}} f(x) \cdot dx = \int_{\mathrm{a}}^{\mathrm{b}} f(\mathrm{t}) \cdot dt\right].$

Miscellaneous Exercise 6 | Q 3.02 | Page 149

State whether the following is True or False :

$$\int_{a}^{b} f(x)\cdot dx = \int_{a}^{b} f(t)\cdot dt$$

1. **True**
2. False

Solution:

$$\int_{a}^{b} f(x)\cdot dx = \int_{a}^{b} f(t)\cdot dt$$ **True**.

Miscellaneous Exercise 6 | Q 3.03 | Page 149

State whether the following is True or False :

$$\int_{0}^{a} f(x)\cdot dx = \int_{a}^{0} f(a-x)\cdot dx$$

1. True
2. **False**

Solution:

$$\int_{0}^{a} f(x)\cdot dx = \int_{0}^{a} f(a-x)\cdot dx$$ **False**.

Miscellaneous Exercise 6 | Q 3.04 | Page 149

State whether the following is True or False :

$$\int_{a}^{b} f(x)\cdot dx = \int_{a}^{b} f(x-a-b)\cdot dx$$

1. True
2. **False**

Solution:

$$\int_{a}^{b} f(x)\cdot dx = \int_{a}^{b} f(a+b-x)\cdot dx$$ **False**.

Miscellaneous Exercise 6 | Q 3.05 | Page 149

State whether the following is True or False : $\int_{-5}^{5} \frac{x^3}{x^2+7} \cdot dx = 0$

1. **True**
2. False

Solution:

$\frac{x^3}{x^2+7}$ is an odd function **True**.

Miscellaneous Exercise 6 | Q 3.06 | Page 150

State whether the following is True or False :

$$\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} \cdot dx = \frac{1}{2}$$

1. **True**
2. False

Solution:

$$\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} \cdot dx$$

$$= \frac{1}{2}(b-a)$$

Here, f(x) = $\sqrt{x}$, a = 1, b = 2 **True**.

Miscellaneous Exercise 6 | Q 3.07 | Page 150

State whether the following is True or False :

$$\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \cdot dx = \frac{9}{2}$$

1. True
2. **False**

Solution:

Here, f(x) = $\sqrt{x}$, a = 2, b = 7 **False**.

Miscellaneous Exercise 6 | Q 3.08 | Page 150

State whether the following is True or False :

$$\int_4^7 \frac{(11-x)^2}{(11-x)^2+x^2} \cdot dx = \frac{3}{2}$$

1. True
2. False

Solution: Here, f(x) = $(11 - x)^2$, a = 4, b = 7 **True**.

Miscellaneous Exercise 6 | Q 4.01 | Page 150

Solve the following : $\int_2^3 \frac{x}{(x+2)(x+3)} \cdot dx$

Solution:

Let I = $\int_2^3 \frac{x}{(x+2)(x+3)} \cdot dx$

Let $\frac{x}{(x+2)x+3} = \frac{A}{x+2} + \frac{B}{x+3}$ ...(i)

∴ x = A(x + 3) + B(x + 2) ...(ii)

Putting x = – 3 in (ii) we get

– 2 = A

∴ B = 3

Putting x = – 2 in (ii),we get

– 2 = A

∴ A = – 2

From (i), we get

$$\frac{x}{(x+2(x+3))} = \frac{-2}{x+2} + \frac{3}{x+3}$$

$$\therefore \text{I} = \int_2^3 \left[\frac{-2}{x+2} + \frac{3}{x+3}\right] \cdot dx$$

$$= -2\int_2^3 \frac{1}{x+2} \cdot dx + 3\int_2^3 \frac{1}{x+3} \cdot dx$$

$$= -2[\log|x+2|]_2^3 + 3[\log|x+3|]_2^3$$

$$= -2\log[\log 5 - \log 4] + 3[\log 6 - \log 5]$$

$$= -2\left[\log\left(\frac{5}{4}\right)\right] + 3\left[\log\left(\frac{6}{5}\right)\right]$$

$$= 3\log\left(\frac{6}{5}\right) - 2\log\left(\frac{5}{4}\right)$$

$$= \log\left(\frac{6}{5}\right)^2 - 2\log\left(\frac{5}{4}\right)^2$$

$$= \log\left(\frac{216}{125}\right) - \log\left(\frac{25}{16}\right)$$

$$= \log\left(\frac{216}{125} \times \frac{16}{25}\right)$$

$$\therefore \text{I} = \log\left(\frac{3456}{3125}\right).$$

Miscellaneous Exercise 6 | Q 4.02 | Page 150

Solve the following : $\int_1^2 \frac{x+3}{x(x+2)} \cdot dx$

Solution:

Let I = $\int_{1}^{2} \frac{x+3}{x(x+2)} \cdot dx$

Let $\frac{x+3}{x(x+2)} = \frac{\mathrm{A}}{x} + \frac{\mathrm{B}}{x+2}$ ...(i)

∴ x + 3 = A(x + 2) + Bx ...(ii)

Putting x = 0 in (ii), we get

3 = A(0 + 2) + B(0)

∴ 3 = 2A

∴ A = $\frac{3}{2}$

Putting x = – 2 in (ii), we get

– 2 + 3 = A(–2 + 2) + B(– 2)

∴ 1 = – 2B

∴ B = $\frac{1}{2}$

From (i), we get

$$\frac{x+3}{x(x+2)} = \frac{3}{2} \cdot \frac{1}{x} - \frac{1}{2(x+2)}$$

∴ I = $\int_{1}^{2} \left[\frac{3}{2x} - \frac{1}{2(x+2)}\right] \cdot dx$

= $\frac{3}{2} \int_{1}^{2} \frac{1}{x} \cdot dx - \frac{1}{2} \int_{1}^{2} \frac{1}{x+2} \cdot dx$

= $\frac{3}{2} [\log|x|]_{1}^{2} - \frac{1}{2} [\log|x+2|]_{1}^{2}$

= $\frac{3}{2} [\log|2| - \log|1|] - \frac{1}{2} [\log|2+2| - \log|1+2|]$

$= \frac{3}{2}(\log 2 - 0) - \frac{1}{2}(\log 4 - \log 3)$

$= \frac{3}{2}\log 2 - \frac{1}{2}\left(\log \ \frac{4}{3}\right)$

$= \frac{1}{2}\left(3\log 2 - \log \ \frac{4}{3}\right)$

$= \frac{1}{2}\log\left(2^3 \times \frac{3}{4}\right)$

$= \frac{1}{2}\log\left(\frac{8 \times 3}{4}\right)$

$\therefore$ I = $\frac{1}{2}\log 6$.

Miscellaneous Exercise 6 | Q 4.03 | Page 150

Solve the following : $\int_1^3 x^2 \log x \cdot dx$

Solution:

Let I = $\int_1^3 x^2 \log x \cdot dx$

$= \left[\log x \int x^2 \cdot dx\right]_1^3 - \int_1^3 \left[\frac{d}{dx}(\log x) \int x^2 \cdot dx\right] \cdot dx$

$= \left[\log x \cdot \frac{x^3}{3}\right]_1^3 - \int_1^3 \frac{1}{x} \cdot \frac{x^3}{3} \cdot dx$

$= \left[9\log 3 - \log 1 \cdot \frac{1}{3}\right] - \frac{1}{3}\int_1^3 x^2 \cdot dx$

$= [9\log 3 - 0] - \frac{1}{3}\left[\frac{x^3}{3}\right]_1^3$

$= 9 \log 3 - \frac{1}{3}\left(\frac{27}{3} - \frac{1}{3}\right)$

$= 9 \log 3 - \frac{1}{3}\left(\frac{26}{3}\right)$

$\therefore$ I = $9 \log 3 - \frac{26}{9}$.

Miscellaneous Exercise 6 | Q 4.04 | Page 150

Solve the following : $\int_0^1 e^{x^2} \cdot x^3 dx$

Solution:

Let I = $\int_0^1 e^{x^2} \cdot x^3 dx$

$= \int_0^1 e^{x^2} \cdot x^2 \cdot x dx$

Put $x^2 = t$

$\therefore$ 2x·dx = dt

$\therefore$ x·dx = $\frac{1}{2} \cdot \mathrm{dt}$

When x = 0, t = 0

When x = 1, t = 1

$\therefore$ I = $\frac{1}{2} \int_0^1 e^{\mathrm{t}} \cdot \mathrm{tdt}$

$= \frac{1}{2}\left\{\left[\mathrm{t} \int e^{\mathrm{t}} \cdot \mathrm{dt}\right]_0^1 - \int_0^1 \left[\frac{d}{\mathrm{dt}}(\mathrm{t}) \int e^{\mathrm{t}} \cdot \mathrm{dt}\right] \mathrm{dt}\right\}$

$$= \frac{1}{2}\left[\left[t \cdot e^{t}\right]_{0}^{1} - \int_{0}^{1} 1 \cdot e^{t} dt\right]$$

$$= \frac{1}{2}\left\{\left(1 \cdot e^{1} - 0\right) - \left[e^{t}\right]_{0}^{1}\right\}$$

$$= \frac{1}{2}\left[e - \left(e^{1} - e^{0}\right)\right]$$

$$= \frac{1}{2}(e - e + 1)$$

$$\therefore I = \frac{1}{2}.$$

Miscellaneous Exercise 6 | Q 4.05 | Page 150

Solve the following : $\int_{1}^{2} e^{2x}\left(\frac{1}{x} - \frac{1}{2x^2}\right) \cdot dx$

Solution:

$$\text{Let } I = \int_{1}^{2} e^{2x}\left(\frac{1}{x} - \frac{1}{2x^2}\right) \cdot dx$$

$$= \int_{1}^{2} e^{2x} \cdot \frac{1}{x} dx - \int_{1}^{2} e^{2x} \cdot \frac{1}{2x^2} dx$$

$$= \left[\frac{1}{x}\int e^{2x} \cdot dx\right]_{1}^{2} - \int_{1}^{2}\left[\frac{d}{dx}\left(\frac{1}{x}\right)\int e^{2x} \cdot dx\right] dx - \frac{1}{2}$$

$$= \left[\frac{1}{x} \cdot \frac{e^{2x}}{2}\right]_{1}^{2} - \int_{1}^{2}\left(-\frac{1}{x^2}\right) \cdot \frac{e^{2x}}{2} dx - \frac{1}{2}\int_{1}^{2} e^{2x} \frac{1}{x^2} \cdot dx$$

$$= \left(\frac{1}{4}e^{4} - \frac{e^{2}}{2}\right) + \frac{1}{2}\int_{1}^{2} e^{2x} \cdot \frac{1}{x^2} dx - \frac{1}{2}\int_{1}^{2} e^{2x} \cdot \frac{1}{x^2} dx$$

$$\therefore I = \frac{e^{4}}{4} - \frac{e^{2}}{2}.$$

Miscellaneous Exercise 6 | Q 4.06 | Page 150

Solve the following : $\int_4^9 \frac{1}{\sqrt{x}} \cdot dx$

Solution:

Let I = $\int_4^9 \frac{1}{\sqrt{x}} \cdot dx$

= $\int_4^9 x^{\frac{1}{2}} \cdot dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^9$

= $2\left[\sqrt{x}\right]_4^9$

= $2\left(\sqrt{9} - \sqrt{4}\right)$

= 2 (3 – 2)

∴ I = 2.

Miscellaneous Exercise 6 | Q 4.07 | Page 150

Solve the following : $\int_{-2}^3 \frac{1}{x+5} \cdot dx$

Solution:

Let I = $\int_{-2}^3 \frac{1}{x+5} \cdot dx$

= $[\log|x+5|]^3_{-2}$

= [log |3 + 5| – log |–2 + 5|]

= log 8 – log 3

∴ I = $\log\left(\frac{8}{3}\right)$.

Miscellaneous Exercise 6 | Q 4.08 | Page 150

Solve the following : $\int_2^3 \frac{x}{x^2 - 1} \cdot dx$

Solution:

Let I = $\int_2^3 \frac{x}{x^2 - 1} \cdot dx$

Put $x^2 - 1 = t$

∴ $2x \cdot dx = dt$

∴ $x \cdot dx = \frac{1}{2} \cdot dt$

When $x = 2$, $t = 2^2 - 1 = 3$

When $x = 3$, $t = 3^2 - 1 = 8$

∴ I = $\int_3^8 \frac{1}{t} \cdot \frac{dt}{2}$

$= \frac{1}{2} \int_3^8 \frac{dt}{t}$

$= \frac{1}{2} [\log|t|]_3^8$

$= \frac{1}{2} (\log 8 - \log 3)$

∴ I = $\frac{1}{2} \log\left(\frac{8}{3}\right)$.

Miscellaneous Exercise 6 | Q 4.09 | Page 150

Solve the following : $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}} \cdot dx$

Solution:

Let I = $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}} \cdot dx$

= $\int_0^1 \left(\frac{x^2+3x+2}{x^{\frac{1}{2}}}\right) \cdot dx$

= $\int_0^1 \left(\frac{x^2}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{1}{2}}}\right) \cdot dx$

= $\int_0^1 \left(x^{\frac{1}{2}} + 3x^{\frac{1}{2}} + 2x^{\frac{1}{2}}\right) \cdot dx$

= $\int_0^1 x^{\frac{3}{2}} \cdot dx + 3\int_0^1 x^{\frac{1}{2}} \cdot dx + 2\int_0^1 x^{\frac{1}{2}} \cdot dx$

= $\left[\frac{\frac{x^5}{2}}{\frac{5}{2}}\right]_0^1 + 3\left[\frac{\frac{x^3}{2}}{\frac{3}{2}}\right]_0^1 + 2\left[\frac{\frac{x^1}{2}}{\frac{1}{2}}\right]_0^1$

= $\frac{2}{5}(1-0)\ 3 \times \frac{2}{3}(1-0) + 2 \times 2(1-0)$

= $\frac{2}{5}\ 2+4$

$\therefore$ I = $\frac{32}{5}$.

Miscellaneous Exercise 6 | Q 4.1 | Page 150

Solve the following : $\int_3^5 \frac{dx}{\sqrt{x+4}+\sqrt{x-2}}$

Solution:

Let I = $\int_3^5 \frac{dx}{\sqrt{x+4}+\sqrt{x-2}}$

= $\int_3^5 \frac{1}{\sqrt{x+4}+\sqrt{x-2}} \times \frac{\sqrt{x+4}-\sqrt{x-2}}{\sqrt{x+4}-\sqrt{x-2}} \cdot dx$

= $\int_3^5 \frac{\sqrt{x+4}-\sqrt{x-2}}{\left(\sqrt{x+4}\right)^2-\left(\sqrt{x-2}\right)^2} \cdot dx$

= $\int_3^5 \frac{\sqrt{x+4}-\sqrt{x-2}}{x+4-(x-2)} \cdot dx$

= $\int_3^5 \frac{\sqrt{x+4}-\sqrt{x-2}}{6} \cdot dx$

= $\frac{1}{6}\int_3^5 (x+4)^{\frac{1}{2}} \cdot dx - \frac{1}{6}\int_3^5 (x-2)^{\frac{1}{2}} \cdot dx$

= $\frac{1}{6}\left[\frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}}\right]_3^5 - \frac{1}{6}\left[\frac{(x-2)^{\frac{3}{2}}}{\frac{3}{2}}\right]_3^5$

= $\frac{1}{9}\left[(9)^{\frac{3}{2}}-(7)^{\frac{3}{2}}\right] - \frac{1}{9}\left[(3)^{\frac{3}{2}}-(1)^{\frac{3}{2}}\right]$

= $\frac{1}{9}\left(27-7\sqrt{7}\right) - \frac{1}{9}\left(3\sqrt{3}-1\right)$

= $\frac{1}{9}\left(27-7\sqrt{7}-3\sqrt{3}+1\right)$

∴ I = $\frac{1}{9}\left(28-3\sqrt{3}-7\sqrt{7}\right)$.

Miscellaneous Exercise 6 | Q 4.11 | Page 150

Solve the following : $\int_2^3 \frac{x}{x^2+1} \cdot dx$

Solution:

Let I = $\int_2^3 \frac{x}{x^2+1} \cdot dx$

Put $x^2 + 1 = t$

$\therefore 2x \cdot dx = dt$

$\therefore x \cdot dx = \frac{dt}{2}$

When x = 2, $t = 2^2 + 1 = 5$

When x = 3, $t = 3^2 + 1 = 10$

$\therefore I = \int_5^{10} \frac{1}{t} \cdot \frac{dt}{2}$

$= \frac{1}{2} \int_5^{10} \frac{dt}{t}$

$= \frac{1}{2} [\log|t|]_5^{10}$

$= \frac{1}{2} (\log 10 - \log 5)$

$= \frac{1}{2} \log\left(\frac{10}{5}\right)$

$\therefore I = \frac{1}{2} \log 2$

$= \log 2^{\frac{1}{2}}$

$= \log \sqrt{2}.$

Miscellaneous Exercise 6 | Q 4.12 | Page 150

Solve the following : $\int_{1}^{2} x^2 \cdot dx$

Solution:

Let I = $\int_{1}^{2} x^2 \cdot dx$

= $\left[\frac{x^3}{3}\right]_1^2$

= $\frac{1}{3}\left(2^3 - 1^3\right)$

= $\frac{1}{3}(8 - 1)$

∴ I = $\frac{7}{3}$.

Miscellaneous Exercise 6 | Q 4.13 | Page 150

Solve the following : $\int_{-4}^{-1} \frac{1}{x} \cdot dx$

Solution:

Let I = $\int_{-4}^{-1} \frac{1}{x} \cdot dx$

= $[\log|x|]_{-4}^{-1}$

= log | – 1| – log | – 4|

= log 1 – log 4

= 0 – log 4

∴ I = – log 4

Miscellaneous Exercise 6 | Q 4.14 | Page 150

Solve the following : $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} \cdot dx$

Solution:

Let I = $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} \cdot dx$

= $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} \times \frac{\sqrt{1+x}-\sqrt{x}}{\sqrt{1+x}-\sqrt{x}} \cdot dx$

= $\int_0^1 \frac{\sqrt{1+x}-\sqrt{x}}{\left(\sqrt{1+x}\right)^2 - \left(\sqrt{x}^2\right) \cdot dx}$

= $\int_0^1 \frac{\sqrt{1+x}-\sqrt{x}}{1+x-x} \cdot dx$

= $\int_0^1 \left[(1+x)^{\frac{1}{2}} - x^{\frac{1}{2}}\right] \cdot dx$

= $\int_0^1 (1+x)^{\frac{1}{2}} \cdot dx - \int_0^1 x^{\frac{1}{2}} \cdot dx$

= $\left[\frac{(1+x)^{\frac{1}{2}}}{\frac{3}{2}}\right]_0^1 - \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$

= $\frac{2}{3}\left[(2)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right] - \frac{2}{3}\left[(1)^{\frac{3}{2}} - 0\right]$

= $\frac{2}{3}\left(2\sqrt{2}-1\right) - \frac{2}{3}(1)$

$= \frac{4\sqrt{2}}{3} - \frac{2}{3} - \frac{2}{3}$

$\therefore$ I = $\frac{4}{2}\left(\sqrt{2} - 1\right)$.

Miscellaneous Exercise 6 | Q 4.15 | Page 150

Solve the following : $\int_0^4 \frac{1}{\sqrt{x^2 + 2x + 3}} \cdot dx$

Solution:

Let I = $\int_0^4 \frac{1}{\sqrt{x^2 + 2x + 3}} \cdot dx$

$= \int_0^4 \frac{1}{\sqrt{x^2 + 2x + 1 - 1 + 3}} \cdot dx$

$= \int_0^4 \frac{1}{\left(\sqrt{x + 1}\right)^2 + 2} \cdot dx$

$= \int_0^4 \frac{1}{\sqrt{(x + 1)^2 + \left(\sqrt{2}\right)^2}} \cdot dx$

$= \left[\log \mid x + 1 + \sqrt{(x + 1)62 + \left(\sqrt{2}\right)^2}\right|_0^4$

$= \log\left|5 + \sqrt{27}\right| - \log\left|1 + \sqrt{3}\right|$

$= \log\left|5 + 3\sqrt{3}\right| - \log\left|1 + \sqrt{3}\right|$

$\therefore$ I = $\log\left|\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right|$.

Miscellaneous Exercise 6 | Q 4.16 | Page 150

Solve the following : $\int_2^4 \frac{x}{x^2+1} \cdot dx$

Solution:

Let I = $\int_2^4 \frac{x}{x^2+1} \cdot dx$

Put $x^2 + 1 = t$

$\therefore$ 2x·dx = dt

$\therefore$ x·dx = $\frac{\text{dt}}{2}$

When x = 2, t = $2^2 + 1 = 5$

When x = 4, t = 42 + 1 = 17

$\therefore$ I = $\int_5^{17} \frac{1}{\text{t}} \cdot \frac{\text{dt}}{2}$

= $\frac{1}{2}\int_5^{17} \frac{\text{dt}}{\text{t}}$

= $\frac{1}{2}[\log|\text{t}|]_5^{17}$

= $\frac{1}{2}(\log 17 - \log 5)$

$\therefore$ I = $\frac{1}{2}\log\left(\frac{17}{5}\right)$.

Miscellaneous Exercise 6 | Q 4.17 | Page 150

Solve the following : $\int_0^1 \frac{1}{2x-3} \cdot dx$

Solution:

Let I = $\int_0^1 \frac{1}{2x-3} \cdot dx$

Put 2x – 3 =t

∴ 2·dx = dt

∴ dx = $\frac{\text{dt}}{2}$

When x = 0t = 2(0) – 3 = – 3

When x = 1, t = 2(1) – 3 = – 1

∴ I = $\int_{-3}^{-1} \frac{1}{\text{t}} \cdot \frac{\text{dt}}{2}$

= $\frac{1}{2} \int_{-3}^{-1} \frac{\text{dt}}{\text{t}}$

= $\frac{1}{2} [\log|\text{t}|]_{-3}^{-1}$

= $\frac{1}{2} [\log|-1| - \log|-3|]$

= $\frac{1}{2} (\log 1 - \log 3)$

= $\frac{1}{2} (0 - \log 3)$

∴ I = $-\frac{1}{2} \log 3$.

Miscellaneous Exercise 6 | Q 4.18 | Page 150

Solve the following : $\int_1^2 \frac{5x^2}{x^2+4x+3} \cdot dx$

Solution:

Let I = $\int_1^2 \frac{5x^2}{x^2+4x+3} \cdot dx$

= $5\int_1^2 \frac{x^2}{x^2+4x+3} \cdot dx$

Dividing numerator by denominator, we get

$$\begin{array}{r|l} & 1 \\ x^2+4x+3 & x^2 \\ & x^2 + 4x + 3 \\ & - \quad - \quad - \\ \hline & -4x - 3 \end{array}$$

$\therefore$ I = $5\int_1^2 \left(1 - \frac{4x+3}{x^2+4x+3}\right) \cdot dx$

= $5\int_1^2 1 \cdot dx - 5\int_1^2 \frac{4x+3}{x^2+4x+3} \cdot dx$

= $5\int_1^2 1 \cdot dx - 5\int_1^2 \frac{4x+3}{(x+3)(x+1)} \cdot dx$

Let $\frac{4x+3}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1}$ ...(i)

$\therefore$ 4x + 3 =A(x + 1) + B(x + 3) ...(ii)

Putting x = – 1 in (ii), we get

– 4 + 3 = A (– 1 + 1) + B(– 1 + 3)

$\therefore$ – 1 = 2B

$\therefore$ B = $-\frac{1}{2}$

Putting x = –3 in (ii), we get

– 12 + 3 = A(– 3 + 1) + B(– 3 + 3)

$\therefore$ – 9 = – 2A

$\therefore$ A = $\frac{9}{2}$

From (i), we get

$$\frac{4x+3}{(x+3(x+1))}$$

$$= \frac{\frac{9}{2}}{x+3} + \frac{\left(-\frac{1}{2}\right)}{x+1}$$

$$\therefore I = 5\int_1^2 1 \cdot dx - 5\int_1^2 \left[\frac{\frac{9}{2}}{x+3} + \frac{\left(-\frac{1}{2}\right)}{x+1}\right] \cdot dx$$

$$= 5[x]_1^2 - 5\left[\frac{9}{2}\int_1^2 \frac{1}{x+3} \cdot dx - \frac{1}{2}\int_1^2 \frac{1}{x+1} \cdot dx\right]$$

$$= 5(2-1) - 5\left\{\frac{9}{2}[\log|x+3|]_1^2 \ - \frac{1}{2}[\log|x+1|]_1^2\right\}$$

$$= 5 - 5\left[\frac{9}{2}(\log 5 - \log 4) \ - \frac{1}{2}(\log 3 - \log 2)\right]$$

$$= 5 - \frac{5}{2}\left[9\left(\log 5 - \log 2^2\right) - (\log 3 - \log 2)\right]$$

$$= 5 - \frac{5}{2}[9(\log 5 - 2\log 2) \ \log 3 + \log 2]$$

$$= 5 - \frac{5}{2}(-\log 3 - 17\log 2 + 9\log 5)$$

$$\therefore I \ = 5 + \frac{1}{2}(5\log 3 + 85\log 2 - 45\log 5).$$

Miscellaneous Exercise 6 | Q 4.19 | Page 150

Solve the following : $\int_1^2 \frac{dx}{x(1+\log x)^2}$

Solution:

Let I = $\int_1^2 \frac{dx}{x(1+\log x)^2}$

Put 1 + log x = t

$\therefore \frac{1}{x} \cdot dx$ = dt

When x = 1, t = 1 + log 1

= 1 + 0 = 1

When x = 2, t = 1 + log 2

$\therefore$ I = $\int_1^{1+\log 2} \frac{\text{dt}}{\text{t}^2}$

= $\left[-\frac{1}{\text{t}}\right]_1^{1+\log 2}$

= $-\left(\frac{1}{1+\log 2} - 1\right)$

= $-\left(\frac{1-1-\log 2}{1+\log 2}\right)$

$\therefore$ I = $\frac{\log 2}{1+\log 2}$.

Miscellaneous Exercise 6 | Q 4.2 | Page 150

Solve the following : $\int_0^9 \frac{1}{1+\sqrt{x}} \cdot dx$

Solution:

Let I = $\int_0^9 \frac{1}{1+\sqrt{x}} \cdot dx$

Put $1+\sqrt{x}$ = t

∴ x = $(t - 1)^2$

∴ dx = 2(t – 1)dt

When x = 0, t = 1 + 0 = 1

When x = 9, t = $1+\sqrt{9}$

= 1 + 3 = 4

∴ I = $\int_1^4 \frac{2(t-1)}{\mathrm{t}} \cdot \mathrm{dt}$

= $2\int_1^4 \left(1-\frac{1}{\mathrm{t}}\right) \cdot \mathrm{dt}$

= $2]\mathrm{t} - \log|\mathrm{t}|]_1^4$

= 2 [(4 – log |4|) – (1 – log |1|)]

= 2 [4 – log 4 – (1 – 0)]

= 2 [4 – log 2^2 – 1)

= 2 (3 – 2log 2)

∴ I = 6 – 4 log 2.