Class X Session 2023-24
Subject - Mathematics (Standard)
Sample Question Paper - 2

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

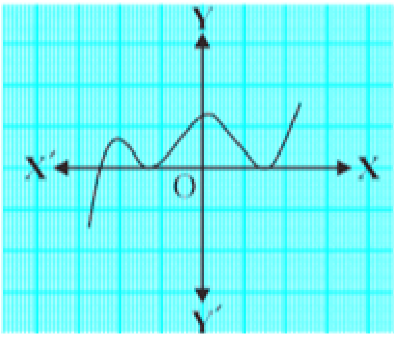
Section A

1. 120 can be expressed as a product of its prime factors as: **[1]**

a) 15×2^3 b) $5 \times 2^3 \times 3$

c) $5 \times 8 \times 3$ d) $10 \times 22 \times 3$

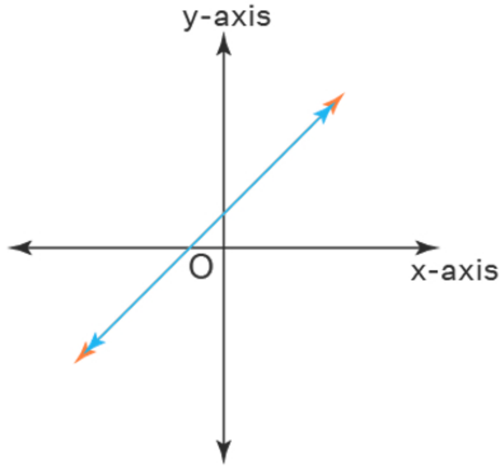
2. The graph of y = p(x) in a figure given below, for some polynomial p(x). Find the number of zeroes of p(x). **[1]**



a) 3 b) 4

c) 2 d) 1

3. The number of solutions of two linear equations representing coincident lines is/are **[1]**



a) infinite solution
b) 0
c) 1
d) 5

4. If x = 3 is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$ then k = ? **[1]**

a) 13
b) -11
c) 11
d) -13

5. In an A.P., if $a_m = \frac{1}{n}$ and $a_n = \frac{1}{m}$, then a_{mn} = **[1]**

a) 1
b) 2
c) -1
d) 0

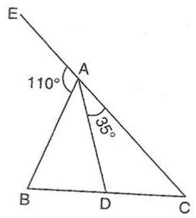
6. Radius of circumcircle of a triangle ABC is $5\sqrt{10}$ units. If point P is equidistant from A (1, 3), B(-3, 5) and C(5, -1) then AP = **[1]**

a) $5\sqrt{10}$ units
b) 25 units
c) $5\sqrt{5}$ units
d) 5 units

7. If the point R(x, y) divides the join of $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the given ratio $m_1 : m_2$, then the coordinates of the point R are **[1]**

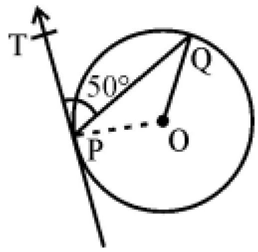
a) $\left(\frac{m_2x_1-m_1x_2}{m_1+m_2}, \frac{m_2y_1-m_1y_2}{m_1+m_2}\right)$
b) $\left(\frac{m_2x_1-m_1x_2}{m_1-m_2}, \frac{m_2y_1-m_1y_2}{m_1-m_2}\right)$
c) $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$
d) None of these

8. In the adjoining figure if exterior $\angle EAB = 110^0, \angle CAD = 35^0$, AB = 5cm, AC = 7cm and BC = 3cm, then CD is equal to **[1]**



a) 1.9 cm
b) 2.25 cm
c) 1.75 cm
d) 2 cm

9. In the figure shown below, O is the centre of the circle. PQ is a chord and PT is tangent at P which makes an angle of 50^o with PQ. Then $\angle$POQ is: **[1]**



a) 75^o b) 100^o

c) 90^o d) 130^o

10. If PA and PB are tangents to the circle with centre O such that ∠APB = 50°, then ∠OAB is equal to **[1]**

a) 50^o b) 40^o

c) 25^o d) 30^o

11. If $\sin\theta - \cos\theta = 0$ then the value of $\left(\sin^4\theta + \cos^4\theta\right)$ is **[1]**

a) $\frac{1}{2}$ b) 1

c) $\frac{3}{4}$ d) $\frac{1}{4}$

12. If x = r sin θ cos ϕ, y = r sin θ sin ϕ and z = r cos θ, then **[1]**

a) $x^2 + y^2 + z^2 = r^2$ b) $x^2 - y^2 + z^2 = r^2$

c) $z^2 + y^2 - x^2 = r^2$ d) $x^2 + y^2 - z^2 = r^2$

13. If the angles of elevation of a tower from two points distant a and b (a > b) from its foot and in the same straight line from it are 30° and 60°, then the height of the tower is **[1]**

a) $\sqrt{a+b}$ b) $\sqrt{a-b}$

c) $\sqrt{\frac{a}{b}}$ d) $\sqrt{ab}$

14. Area of a sector of angle p (in degrees) of a circle with radius R is **[1]**

a) $\frac{p}{360} \times 2\pi R$ b) $\frac{\mathrm{p}}{180} \times \pi \mathrm{R}^2$

c) $\frac{p}{180} \times 2\pi \mathrm{R}$ d) $\frac{p}{720} \times 2\pi R^2$

15. The length of the minute hand of a clock is 21 cm. The area swept by the minute hand in 10 minutes is **[1]**

a) $252\ cm^2$ b) $126\ cm^2$

c) $231\ cm^2$ d) $210\ cm^2$

16. Two dice are thrown together. The probability of getting the same number on both dice is **[1]**

a) $\frac{1}{6}$ b) $\frac{1}{12}$

c) $\frac{1}{3}$ d) $\frac{1}{2}$

17. Cards, each marked with one of the numbers 6,7,8 ,..., 15, are placed in a box and mixed thoroughly. One card is drawn at random from the box, What is the probability of getting a card with number less than 10 **[1]**

a) $\frac{2}{5}$ b) $\frac{1}{2}$

c) $\frac{1}{3}$ d) $\frac{3}{5}$

18. The mean of 1, 3, 4, 5, 7, 4 is m. The numbers 3, 2, 2, 4, 3, 3, p have mean m - 1 and median q. Then, p + q = **[1]**

a) 5 b) 7

c) 4 d) 6

19. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 300 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + lh)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Sum of first n terms in an A.P. is given by the formula: S_n = 2n $\times$[2a + (n - 1)d] **[1]**

Reason (R): Sum of first 15 terms of 2 , 5 , 8 ... is 345.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

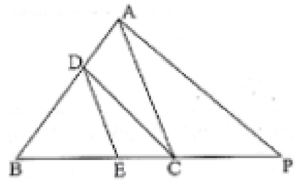
c) A is true but R is false.

d) A is false but R is true.

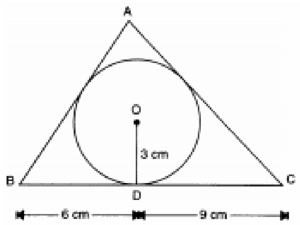
Section B

21. Can two numbers have 15 as their HCF and 175 as their LCM ? Give reasons. **[2]**

22. In the given Fig. DE || AC and DC || AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$ **[2]**



23. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of $\triangle$ABC is 54 square centimeter, then find the lengths of sides AB and AC. **[2]**



24. Evaluate $2\sin^2 30^\circ \tan 60^\circ - 3\cos^2 60^\circ \sec^2 30^\circ$. **[2]**

OR

Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2cosec\theta$

25. In a circle with centre O and radius 5 cm, AB is a chord of length $5\sqrt{3}$ cm. Find the area of sector AOB. **[2]**

OR

A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze?

Section C

26. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point? **[3]**

27. If α and β are the zeros of the polynomial f(x) = x^2 - 5x + k such that $\alpha - \beta$ = 1, find the value of k. **[3]**

28. Two candles of equal height but different thickness are lighted. First candle burns off in 6 hours and the second candle in 8 hours. How long, after lighting both, will the first candle be half the height of the second ? **[3]**

OR

Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.

$x - 5y = 6$, $2x - 10y = 12$

29. If two tangents are drawn to a circle from an external point, show that they subtend equal angles at the centre. **[3]**

OR

ABC is a right triangle in which $\angle B = 90^o$. If AB = 8 cm and BC = 6 cm, find the diameter of the circle inscribed in the triangle.

30. If $\sin\theta + 2\cos\theta = 1$ prove that $2\sin\theta - \cos\theta = 2$. **[3]**

31. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them. **[3]**

Monthly consumption (in units)	Number of Consumers
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	8
185-205	4

Section D

32. If the factory kept increasing its output by the same percentage every year. Find the percentage, if it is known that the output doubles in the last two years. **[5]**

OR

A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

33. In the figure, OB is the perpendicular bisector of the line segment DE, FA $\perp$ OB and F E intersect OB at point C. Prove that $\frac{1}{OA} + \frac{1}{OB} = \frac{2}{OC}$. **[5]**

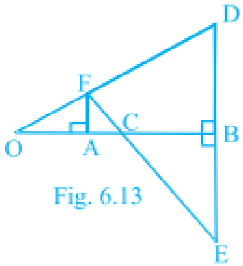


Fig. 6.13

34. A solid toy is in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of the base is 7 cm. Determine the volume of the toy. Also find the area of the coloured sheet required to cover the toy. (Use $\pi = \frac{22}{7}$ and $\sqrt{149}$ = 12.2) **[5]**

OR

A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone.

35. The monthly income of 100 families are given as below: **[5]**

Income in (in ₹.)	Number of families
0-5000	8
5000-10000	26
10000-15000	41
15000-20000	16
20000-25000	3
25000-30000	3
30000-35000	2
35000-40000	1

Calculate the modal income.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(i) Find the production during first year.

(ii) Find the production during 8th year.

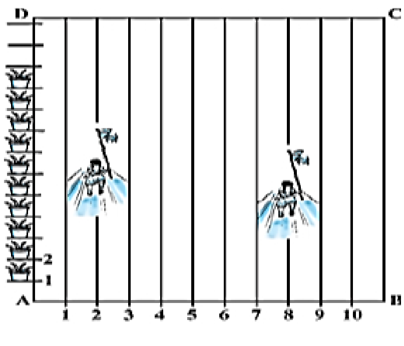
OR

In which year, the production is ₹ 29,200.

(iii) Find the production during first 3 years.

37. **Read the text carefully and answer the questions:** **[4]**

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Fig. Sarika runs the distance AD on the 2nd line and posts a green flag. Priya runs the distance AD on the eighth line and posts a red flag. (take the position of feet for calculation)



(i) What co-ordinates you will use for Green Flag?

(ii) What is the distance between the green flag and the red flag?

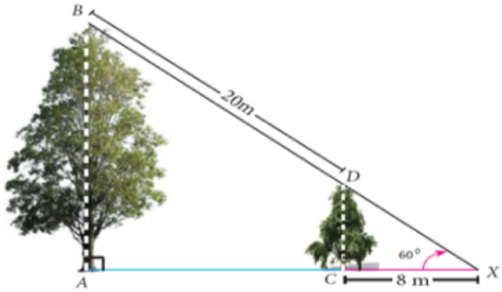
OR

What is the distance between green and blue flag?

(iii) If Monika wants to post a blue flag adjacently in between these two flags. Where she will post a blue flag?

38. **Read the text carefully and answer the questions:** **[4]**

Two trees are standing on flat ground. The angle of elevation of the top of Both the trees from a point X on the ground is 60^o. If the horizontal distance between X and the smaller tree is 8 m and the distance of the top of the two trees is 20 m.



(i) Calculate the distance between the point X and the top of the smaller tree.

(ii) Calculate the horizontal distance between the two trees.

OR

Find the height of small tree.

(iii) Find the height of big tree.

Solution

Section A

1.

   (b) $5 \times 2^3 \times 3$

   Explanation: We have,

   $120 = 5 \times 2^3 \times 3$

2. **(a)** 3

   Explanation: The number of zeroes is 3 as the graph given in the question intersects the x-axis at 3 points.

3. **(a)** infinite solution

   Explanation: The number of solutions of two linear equations representing coincident lines are ∞ because two linear equations representing coincident lines has infinitely many solutions.

4.

   (b) -11

   Explanation: $3x^2 + (k-1)x + 9 = 0$

   x = 3 is a solution of the equation means it satisfies the equation

   Put x = 3, we get

   $3(3)^2 + (k - 1) 3 + 9 = 0$

   27 + 3 k - 3 + 9 = 0

   27 + 3 k + 6 = 0

   3 k = - 33

   k = - 11

5. **(a)** 1

   Explanation: Given: $a_m = \frac{1}{n}$

   $\Rightarrow a + (m - 1)d = \frac{1}{n}$ ... (i)

   And $a_n = \frac{1}{m}$

   $\Rightarrow a + (n - 1)d = \frac{1}{m}$ ... (ii)

   Subtracting eq. (ii) from eq. (i), we get,

   $(m - 1)d - (n - 1)d = \frac{1}{n} - \frac{1}{m}$

   $\Rightarrow d(m - 1 - n + 1) = \frac{m-n}{mn}$

   $\Rightarrow d(m - n) = \frac{m-n}{mn}$

   $\Rightarrow d = \frac{1}{mn}$

   Putting the value of d in eq. (i), we get

   $a + (m - 1)\frac{1}{mn} = \frac{1}{n}$

   $\Rightarrow a = \frac{1}{n} - \frac{m-1}{mn} = \frac{1}{mn}$

   $\therefore a_{mn} = a + (mn - 1)d = \frac{1}{mn} + (mn-1)\frac{1}{mn} = \frac{1}{mn} \times mn = 1$

6. **(a)** $5\sqrt{10}$ units

   Explanation: Since P is equidistant from A, B and C.

   Therefore, P is centre of circumcircle of triangle ABC.

   Hence, AP = Radius of circumcircle = $5\sqrt{10}$ units

7.

   (c) $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$

   Explanation: If the point R(x, y) divides the join of $P(x_1, y_2)$ and $Q(x_2, y_2)$

   internally in the given ratio $m_1 : m_2$,

   then the coordinates of the point R are $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$

8.

(c) 1.75 cm

Explanation: $\angle BAD = 180^\circ - (\angle EAB + \angle ADC)$ = {180^o - 110^o - 35^o = 35^o

Since AD bisect $\angle$A, then

$\frac{AB}{AC} = \frac{BD}{CD}$ [Internal bisector of an angle divides opposite sides in the ratio of the sides containing the angle]

$\Rightarrow \frac{5}{7} = \frac{3-CD}{CD}$

$\Rightarrow$ 5CD = 21 - 7CD

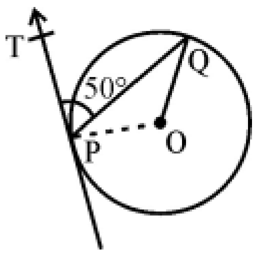
$\Rightarrow$ 12CD = 21

$\Rightarrow$ CD = 1.75 cm

9.

(b) 100^o

Explanation: In the given figure shown below, PQ is a chord of a circle with centre O and PT is a tangent at P to the circle such that $\angle QPT = 50^o$.



Then, we have to calculate $\angle$POQ.

PT is the tangent and OP is the radius

OP $\perp$ PT

$\Rightarrow \angle OPT = 90^o$

$\angle OPQ = \angle OPT - \angle QPT = 90^o - 50^o = 40^o$

In $\triangle$OPQ,

OP = OQ (radii of the same circle)

$\angle OPQ = \angle OQP = 40^o$

and $\angle POQ = 180^o - (\angle OPQ + \angle OQP)$

$= 180^o - (40^o + 40^o)$

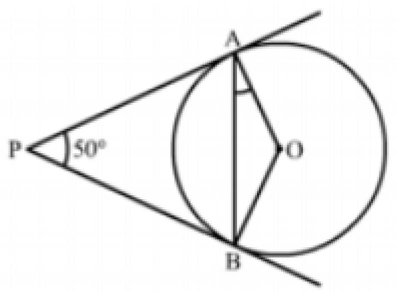
$= 180^o - 80^o = 100^o$

therefore, $\angle POQ = 100^o$

10.

(c) 25^o

Explanation:



Given, PA and PB are tangent lines.

PA = PB [Since, the length of tangents drawn from a point are equal]

$\angle PBA = \angle PAB = \theta$ (say)

In ΔPAB

$\angle P + \angle A + \angle B = 180^\circ$

[since, sum of angles of a triangle $= 180^\circ$

$50^\circ + \theta + \theta = 180^\circ$

$2\theta = 180^\circ - 50^\circ = 130^\circ$

$\theta = 65^\circ$

Also, $\mathrm{OA} \perp \mathrm{PA}$

[Since, tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\angle\mathrm{PAO} = 90^\circ$

$\Rightarrow \angle PAB + \angle OAB = 90^\circ$

$\Rightarrow 65^\circ + \angle\mathrm{BAO} = 90^\circ$

$\Rightarrow \angle OAB = 90^\circ - 65^\circ = 25^\circ$

11. **(a)** $\frac{1}{2}$

Explanation: It is given that,

$\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \tan\theta = \tan 45^\circ$

$\Rightarrow \theta = 45^\circ$

$\therefore \sin^4\theta + \cos^4\theta$

$= \sin^4 45^\circ + \cos^4 45^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

12. **(a)** $x^2 + y^2 + z^2 = r^2$

Explanation: $x = r\sin\theta\cos\phi \Rightarrow \frac{x}{r} = \sin\theta\cos\phi$ ...(i)

$y = r\sin\theta\sin\phi \Rightarrow \frac{y}{r} = \sin\theta\sin\phi$ ...(ii)

$z = r\cos\theta \Rightarrow \frac{z}{r} = \cos\theta$ ...(iii)

Squaring and adding (i) and (ii)

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi$

$= \sin^2\theta\left(\cos^2\phi + \sin^2\phi\right)$

$= \sin^2\theta \times 1 \quad \left\{\sin^2\theta + \cos^2\theta = 1\right\}$

$= \sin^2\theta$

Now adding (iii) in it

$\frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} = \sin^2\theta + \cos^2\theta = 1$

Hence $\frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} = 1$

$\Rightarrow \frac{x^2+y^2+z^2}{r^2} = 1$

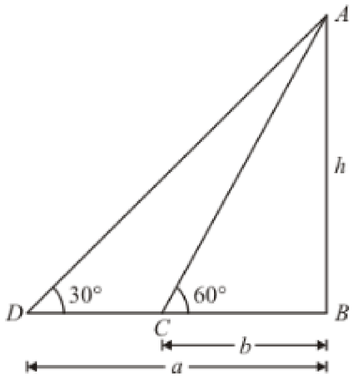
$\Rightarrow x^2 + y^2 + z^2 = r^2$

13.

(d) $\sqrt{ab}$

Explanation:

Let h be the height of tower AB



Given that: angle of elevation are $\angle C = 60^{o}$ and $\angle D = 30^{o}$.

Distance BC = b and BD = a

Here, we have to find the height of tower.

So we use trigonometric ratios.

In a triangle ABC,

$\Rightarrow \tan C = \frac{AB}{BC}$

$\Rightarrow \tan 60^{\circ} = \frac{AB}{BC}$

$\Rightarrow \tan 60^{\circ} = \frac{h}{b}$

Again in a triangle ABD,

$\Rightarrow \tan D = \frac{AB}{BD}$

$\Rightarrow \tan 30^{\circ} = \frac{h}{a}$

$\Rightarrow \tan(90^{\circ} - 60^{\circ}) = \frac{h}{a}$

$\Rightarrow \cot 60^{\circ} = \frac{h}{a}$

$\Rightarrow \frac{1}{\tan 60^{\circ}} = \frac{h}{a}$

$\Rightarrow \frac{b}{h} = \frac{h}{a}$ put $\tan 60^{o} = \frac{h}{b}$

$\Rightarrow h^2 = ab$

$\Rightarrow h = \sqrt{ab}$

14.

(d) $\frac{p}{720} \times 2\pi R^2$

Explanation: Area of the sector of angle p of a circle with radius R

$= \frac{\theta}{360} \times \pi r^2 = \frac{p}{360} \times \pi R^2$

$= \frac{p}{2(360)} \times 2\pi R^2 = \frac{p}{720} \times 2\pi R^2$

15.

(c) 231 cm^2

Explanation: Area swept by minute hand in 60 minutes = πR^2

Area swept by it in 10 minutes

$= \left(\frac{\pi R^2}{60} \times 10\right) \text{cm}^2 = \left(\frac{22}{7} \times 21 \times 21 \times \frac{1}{6}\right) \text{cm}^2$

= 231 cm^2

16. **(a)** $\frac{1}{6}$

Explanation: Here 2 dice are thrown together.

$\therefore$ Number of total outcomes = 6 $\times$ 6 = 36

Number which should come together are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) = 6 pairs

Therefore, probablity = $\frac{1}{6}$

17. **(a)** $\frac{2}{5}$

Explanation: Total number of cards = 15 - 5 = 10.

Number of cards with number less than 10 = 4.

P (getting a card with number less than 10) = $\frac{4}{10} = \frac{2}{5}$.

18.

(b) 7

Explanation: Mean of 1, 3, 4, 5, 7, 4 is m

$\therefore \frac{1+3+4+5+7+4}{6}$ = m

$\Rightarrow \frac{24}{6}$ = m $\Rightarrow$ m = 4

Mean of 3, 2, 2, 4, 3, 3, p is m - 1

$\Rightarrow \frac{3+2+2+4+3+3+p}{7}$ = m - 1

$\Rightarrow \frac{17+p}{7} = 4 - 1 \Rightarrow \frac{17+p}{7} = 3$

$\Rightarrow$ 17 + p = 21 $\Rightarrow$ p = 21 - 17 = 4

Median of 3, 2, 2, 4, 3, 3, p is q

3, 2, 2, 4, 3, 3, 4 is q

Arranging in order, we get

4, 4, 3, 3, 3, 2, 2

Here n = 7

$\therefore$ Median = $\frac{7+1}{2}$th term = 4th term = 3

i.e, q = 3

$\therefore$ p + q = 4 + 3 = 7

19.

(d) A is false but R is true.

Explanation: A is false but R is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. $\frac{175}{15} = 11.667$

Hence 175 is not divisible by 15

But LCM of two numbers should be divisible by their HCF.

$\therefore$ Two numbers cannot have their HCF as 15 and LCM as 175.

22. Given: $\triangle$ABP in which DE $\parallel$ AC and DC $\parallel$ AP.

To prove: $\frac{BE}{EC} = \frac{BC}{CP}$

Proof: In $\triangle$ BDC and $\triangle$ ABP

DC $\parallel$ AP ....[Given]

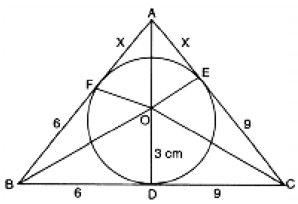
$\Rightarrow \frac{BD}{AD} = \frac{BC}{CP}$ ....(ii).........[By BPT]

Agin in $\triangle$ BDE and $\triangle$ BAC,

DE $\parallel$ AC ....[Given]

$\Rightarrow \frac{BD}{AD} = \frac{BE}{EC}$ ....(ii).........[By BPT]

From (i) and (ii), we have.$\Rightarrow \frac{BE}{EC} = \frac{BC}{CP}$ Hence Proved

23. 

Let, AF = AE = x

ar $\triangle$ABC = ar$\triangle$AOB + ar$\triangle$BOC + ar$\triangle$AOC

ar $\triangle$ABC = $\frac{1}{2}(15)(3) + \frac{1}{2}(6+x)(3) + \frac{1}{2}(9+x)(3)$

$\frac{1}{2}[15 + 6 + x + 9 + x].3 = 54$

45 + 3x - 54

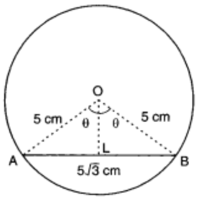
x = 3

$\therefore$ AB = 9 cm, AC = 12 cm
and BC = 15 cm.

24. We have,
$\sin 30^\circ = \frac{1}{2},\ \tan 60^\circ = \sqrt{3},\ \cos 60^\circ = \frac{1}{2}\ and\ \sec 30^\circ = \frac{2}{\sqrt{3}}$
therefore,
$2\sin^2 30^\circ \tan 60^\circ - 3\cos^2 60^\circ \sec^2 30^\circ$
$= 2(\sin 30^\circ)^2 \tan 60^\circ - 3(\cos 60^\circ)^2 (\sec 30^\circ)^2$
$= 2 \times \left(\frac{1}{2}\right)^2 \times \sqrt{3} - 3 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2$
$= 2 \times \frac{1}{4} \times \sqrt{3} - 3 \times \frac{1}{4} \times \frac{4}{3} = \frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{3}-2}{2}$

OR

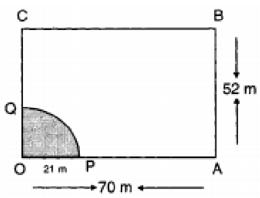
We have, $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$
$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$
$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)}$
$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$
= 2 cosec θ

25. 

It is given that AB = $5\sqrt{3}$ cm.
$\Rightarrow$ AL = BL = $\frac{5\sqrt{3}}{2}$ cm
Let $\angle AOB = 2\theta$. Then, $\angle$AOL = $\angle$BOL =θ
In $\triangle$OLA, we have
$\sin\theta = \frac{AL}{OA} = \frac{\frac{5\sqrt{3}}{2}}{5} = \frac{\sqrt{3}}{2}$
$\Rightarrow \theta = 60^\circ$
$\Rightarrow \angle AOB = 120^\circ$
$\therefore$ Area of sector AOB = $\frac{120}{360} \times \pi \times 5^2 \text{cm}^2 = \frac{25\pi}{3} \text{cm}^2$

OR

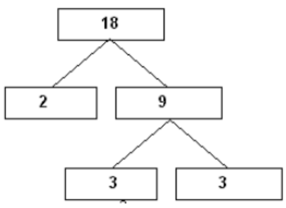
Shaded portion indicates the area which the horse can graze. Clearly, shaded area is the area of a quadrant of a circle of radius r - 21 m.



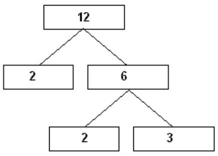
$\therefore$Required Area = $\frac{1}{4}\pi r^2$
$\therefore$Required Area = $\left\{\frac{1}{4} \times \frac{22}{7} \times (21)^2\right\} \text{cm}^2 = \frac{693}{2}\text{cm}^2$ = 346.5 cm^2

Section C

26. By taking LCM of time taken (in minutes) by Sonia and Ravi, We can get the actual number of minutes after which they meet again at the starting point after both start at the same point and at the same time, and go in the same direction.
$18 = 2 \times 3 \times 3 = 2 \times 3^2$



12 = 2 $\times$ 2 $\times$ 3 = $2^2 \times 3$



LCM $(18, 12) = 2^2 \times 3^2 = 36$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

27. Since α, β are the zeros of the polynomial f(x) = x^2 - 5x + k.

Compare f(x) = x^2 - 5x + k with ax^2 + bx + c.

So, a = 1 , b = -5 and c = k

$\alpha + \beta = -\frac{(-5)}{1}$ = 5

$\alpha\beta = \frac{k}{1} = k$

Given, $\alpha - \beta$ = 1

Now, $(\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$

$\Rightarrow$ $(5)^2$ = $(1)^2$ + 4k

$\Rightarrow$ 25 = 1 + 4k

$\Rightarrow$ 4k = 24

$\Rightarrow$ k = 6

Hence the value of k is 6.

28. Let height of each candle = $x\ unit.$

First candle burns off in 6 hours.

Second candle burns off in 8 hours.

Height of 1st candle after burning for 1 hr = $\frac{x}{6} unit$

and height of 2nd candle after burning for 1 hr = $\frac{x}{8} unit$

Let the required time $= y\ hrs.$

Length of 1st candle burnt after y hrs = $\frac{y \times x}{6} unit$

Height of 1st candle left = $\left(x - \frac{xy}{6}\right)$

Length of 2nd candle burnt after y hrs = $\left(\frac{y \times x}{8}\right) unit$

Height of 2nd candle left = $\left(x - \frac{xy}{8}\right)$

According to the question,

Height of 1st candle = $\frac{1}{2} \times$ Height of 2nd candle

$\Rightarrow \quad x - \frac{xy}{6} = \frac{1}{2}\left(x - \frac{xy}{8}\right)$

$\Rightarrow \quad x\left(1 - \frac{y}{6}\right) = \frac{1}{2}x\left(1 - \frac{y}{8}\right)$

$1 - \frac{y}{6} = \frac{1}{2}\left(1 - \frac{y}{8}\right)$

$\Rightarrow \quad 2 - \frac{y}{3} = 1 - \frac{y}{8}$

$2 - 1 = \frac{y}{3} - \frac{y}{8}$

1 = $\frac{8y - 3y}{24}$

$\Rightarrow 24 = 5y$

$\Rightarrow y = \frac{24}{5}$

$y = 4.8 hours = 4\ hours\ 48\ minutes.$

OR

Given, x - 5y = 6 or x = 6 + 5y

x	6	1	-4
y	0	-1	-2

Thus when x = 6, y = 0

when x = 1, y = -1

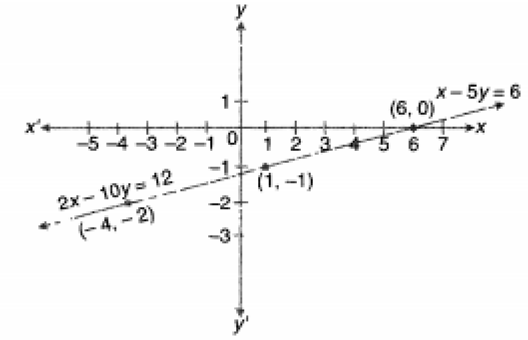
when x = -4, y = -2

and 2x - 10y = 12 or x = 5y + 6

x	6	1	-4

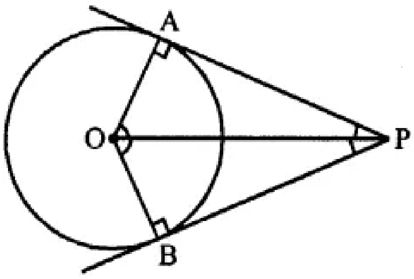
y	0	-1	-2

when x = 6, y =0
when x = 1, y = -1
when x = -4,y = -2



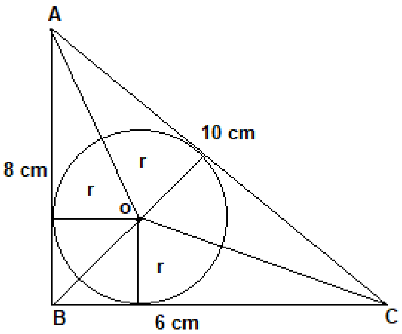
Since the lines are coincident, so the system of linear equations is consistent with infinite many solutions

29. Given : In a circle from an external point P, PA and PB are the tangents to the circle
OP, OA and OB are joined.



To prove: ∠POA = ∠POB
Proof: OA and OB are the radii of the circle and PA and PB are the tangents to the circle
OA ⊥ AP and OB ⊥ BP
∠OAP = ∠= 90°
Now, in right △OAP △OBP,
Hyp. OP = OP (common)
Side OA = OB (radii of the same circle)
△OAP = △OBP (RHS axiom)
∠POA = ∠POB (c.p.c.t.)
Hence proved.

OR



By pythagoras theorem
AC = $\sqrt{AB^2 + BC^2}$
= $\sqrt{8^2 + 6^2}$
= $\sqrt{64 + 36}$
= $\sqrt{100}$ = 10 cm
Area of △ ABC = Area of △ AOB + Area of △BOC + Area of △ AOC
$\frac{1}{2} \times b \times h = \frac{1}{2} \times b_1 \times h_1 + \frac{1}{2} \times b_2 \times h_2 \times \frac{1}{2} \times b_3 \times h_3$
$\frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 8 \times r + \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r$
24 = 4r + 3r + 5r
24 = 12 r

r = 2 cm

Hence the radius is 2 cm.

30. Given, $\sin\theta + 2\cos\theta = 1$

We have,

$(\sin\theta + 2\cos\theta)^2 + (2\sin\theta - \cos\theta)^2$

$= (\sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta) + (4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta)$

$= \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta + 4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta$

$= 5\sin^2\theta + 5\cos^2\theta$

$\Rightarrow 5\left(\sin^2\theta + \cos^2\theta\right)$

= 5

$\Rightarrow 1^2 + (2\sin\theta - \cos\theta)^2 = 5$

$\Rightarrow (2\sin\theta - \cos\theta)^2 = 4$

$\Rightarrow 2\sin\theta - \cos\theta = \pm 2$

$\Rightarrow 2\sin\theta - \cos\theta = 2$

31. First, we will convert the graph into tabular form given below:

Monthly consumption (in units)	**Number of consumers (f_i)**	**Class mark (x_i)**	**$d_i = x_i$ - 135**	$u_i = \frac{x_i - 135}{5}$	**f_iu_i**	**Cumulative Frequency**
65-85	4	75	–60	–3	–12	4
85-105	5	95	–40	–2	–10	9
105-125	13	115	–20	–1	–13	22
125-145	20	135	0	0	0	42
145-165	14	155	20	1	14	56
165-185	8	175	40	2	16	64
185-205	4	195	60	3	12	68
Total	$\sum f_i = 68$				$\sum f_iu_i = 7$	

i. Let a = 135.

Now, h = 20

Using the step-deviation method,

$Mean, \overline{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h = 135 + \left(\frac{7}{68}\right) \times 20$

$= 135 + \frac{35}{17}$ = 135 + 2.05 = 137.05

ii. Now, N = 68

So, $\frac{N}{2} = \frac{68}{2} = 34$

This observation lies in class 125-145.

Therefore, 125-145 is the median class.

So, l = 125, CF = 22, f = 20

$\therefore Median = l + \left(\frac{\frac{N}{2} - CF}{f}\right) \times h$

$= 125 + \left(\frac{34-22}{20}\right) \times 20$ = 125 + 12 = 137

iii. Mode = 3 Median - 2 Mean

= 3×137 - 2×137.05 = 136.9

Section D

32. Let P be the initial production (2 yr ago) and the increase in production in every year be x%.

Then, production at the end of the first year.

$P + \frac{Px}{100} = P(1 + \frac{x}{100})$

Production at the end of the second year = $P(1 + \frac{x}{100}) + \frac{x}{100}P[1 + \frac{x}{100}]$

$= P(1 + \frac{x}{100})(1 + \frac{x}{100})$

$= P(1 + \frac{x}{100})^2$

Since, production doubles in the last two years,

$\therefore P(1 + \frac{x}{100})^2 = 2P$

$\Rightarrow (1+\frac{x}{100})^2 = 2$

$\Rightarrow (1+\frac{x}{100}) = \sqrt{2}$

$\Rightarrow \frac{x}{100} = \sqrt{2} - 1 = 1.4142 - 1 = 0.4142$

$\Rightarrow x = 0.4142 \times 100$

$\Rightarrow x = 41.42\%$

OR

Let the usual speed of train be x km/hr

$\frac{300}{x} - \frac{300}{x+5} = 2$

300 (x+5 - x) = 2x(x+5)

150(5) = x^2 + 5x

750 = x^2 + 5x

or, $x^2 + 5x - 750 = 0$

or, $x^2 + 30x - 25x - 750 = 0$

or, (x + 30) (x - 25) = 0

or, x = - 30 or x = 25

Since, speed cannot be negative.

$\therefore \quad x \neq -30, x = 25\text{km/hr}$

$\therefore$ Speed of train = 25 km/hr

33. In $\triangle$AOF and $\triangle$BOD

$\angle$O = $\angle$O (Same angle) and $\angle$A = $\angle$B (each 90°)

Therefore, $\triangle$AOF $\sim$ $\triangle$BOD (AA similarity)

So, $\frac{\text{OA}}{\text{OB}} = \frac{\text{FA}}{\text{DB}}$

Also, in $\triangle$FAC and $\triangle$EBC, $\angle$A = $\angle$B (Each 90°)

and $\angle$FCA = $\angle$ECB (Vertically opposite angles).

Therefore, $\triangle$FAC $\sim$ $\triangle$EBC (AA similarity).

So, $\frac{\text{FA}}{\text{EB}} = \frac{\text{AC}}{\text{BC}}$

But EB = DB (B is mid-point of DE)

So, $\frac{\text{FA}}{\text{DB}} = \frac{\text{AC}}{\text{BC}}$ (2)

Therefore, from (1) and (2), we have:

$\frac{\text{AC}}{\text{BC}} = \frac{\text{OA}}{\text{OB}}$

i.e. $\frac{\text{OC}-\text{OA}}{\text{OB}-\text{OC}} = \frac{\text{OA}}{\text{OB}}$

or OB . OC – OA . OB = OA . OB – OA . OC

or OB . OC + OA . OC = 2 OA . OB

or (OB + OA). OC = 2 OA . OB

or $\frac{1}{\text{OA}} + \frac{1}{\text{OB}} = \frac{2}{\text{OC}}$ [Dividing both the sides by OA . OB . OC]

34. Height of cone (h) = 10 cm

Radius of cone and hemisphere (r) = 7 cm

Slant height of cone (l) = $\sqrt{h^2 + r^2}$

l = $\sqrt{10^2 + 7^2}$ = $\sqrt{100 + 49}$ = $\sqrt{149}$

l = 12.2cm

Volume of toy = volume of cone + volume of hemisphere

Volume of toy = $\pi r^2 h + \frac{2}{3}\pi r^3$

Volume = $\pi r^2 \left(h + \frac{2}{3}r\right) = \frac{22}{7} \times 49 \times \left(10 + \frac{2}{3} \times 7\right)$

Volume = $22 \times 7 \times \left(10 + \frac{14}{3}\right)$ = $\frac{22\times7\times44}{3}$

Volume = 2258.66 cm^3

Volume of toy = 2258.66cm^3

Now,

Surface area of toy = CSA of cone + CSA of hemisphere

Surface area = $\pi rl + 2\pi r^2$

Surface area = $\pi r(l + 2r) = \frac{22}{7} \times 7$ (12.2 + 14)

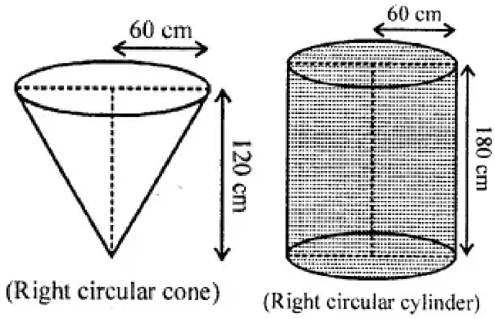
Surface area= 22 $\times$ 26.2

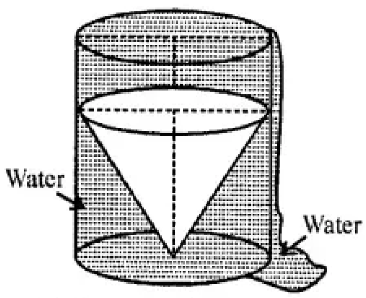
Surface area = 576.4 cm^2

Surface area of coloured sheet required = 576.4cm^2

OR

i. Whenever we placed a solid right circular cone in a right circular cylinder ,cylinder with full of water, then volume of a solid right circular cone is equal to the volume of water filled from the cylinder.
ii. Total volume of water in a cylinder is equal to the volume of the cylinder.
iii. Volume of water left in the cylinder is = Volume of the right circular cylinder - Volume of a right circular cone.





(Cylinder contained a cone)

Now, given that

Height of a right circular cone = 120cm

Radius of a right circular cone = 60cm

$\therefore$ The volume of a right circular cone = $\left(\frac{1}{3}\right)\pi r^2 \times h$

$=\left(\frac{1}{3}\right) \times\left(\frac{22}{7}\right) \times 60 \times 60 \times 120$

$=\left(\frac{22}{7}\right) \times 20 \times 60 \times 120$

= 14000 π cm^3

$\therefore$ Volume of a right circular cone = Volume of water spilled from the cylinder =144000π cm^3 [from point (i)]

Given that, the height of a right circular cylinder =180cm

and radius of a right circular cylinder = Radius of a right circular cone = 60 cm

$\therefore$ Volume of a right circular cylinder $=\pi \mathrm{r}^2 \times \mathrm{h}$

$=\pi \times 60 \times 60 \times 180=648000 \pi$ cm^3 So, volume of a right circular cylinder = Total volume of water in a cylinder = 648000 π cm^3 [from point (ii)]

From point (iii),

Volume of water left in the cylinder = Total volume of water in a cylinder - Volume of water failed from the cylinder when solid cone is placed in it

$=648000 \pi-144000 \pi$

$=504000 \pi=504000 \times\left(\frac{22}{7}\right)=1584000$ cm^3

$=\left(\frac{1584000}{(10)6}\right) m^3=1.584 m^3$

Hence, the required volume of water left in the cylinder is 1.584 m^3

35. class 10000 - 15000 has the maximum frequency,

so it is the modal class.

$\therefore$ l = 10000, h = 5000, f = 41, f_1 = 26 and f_2 = 16

Mode = l + $\frac{f-f_1}{2f-f_1-f_2} \times$ h

$= 10000 + \frac{41-26}{2(41)-26-16} \times 5000$

$= 10000 + \frac{15}{40} \times 5000$

= 10000 + 1875

= 11875

Section E

36. **Read the text carefully and answer the questions:**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(i) Let 1st year production of TV = x

Production in 6th year = 16000

$t_6 = 16000$

$t_9 = 22,600$

$t_6 = a + 5d$

$t_9 = a + 8d$

16000 = x + 5d ...(i)
22600 = x + 8d ...(ii)
- - -
-6600 = - 3d

d = 2200

Putting d = 2200 in equation ...(i)

$16000 = x + 5 \times (2200)$

16000 = x + 11000

x = 16000 - 11000

x = 5000

$\therefore$ Production during 1st year = 5000

(ii) Production during 8th year is (a + 7d) = 5000 + 7(2200) = 20400

OR

Let in nth year production was = 29,200

$t_n = a + (n - 1)d$

29,200 = 5000 + (n - 1) 2200

29,200 = 5000 + 2200n - 2200

29200 - 2800 = 2200n

26,400 = 2200n

$\therefore n = \frac{26400}{2200}$

n = 12

i.e., in 12th year, the production is 29,200

(iii) Production during first 3 year = Production in (1st + 2nd + 3rd) year

Production in 1st year = 5000

Production in 2nd year = 5000 + 2200

= 7200

Production in 3rd year = 7200 + 2200

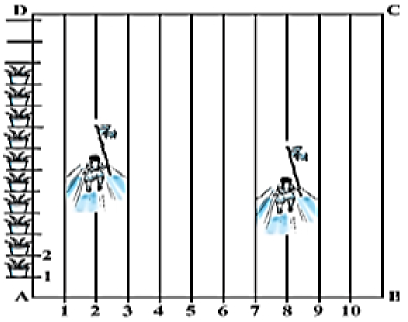
= 9400

∴ Production in first 3 year = 5000 + 7200 + 9400

= 21,600

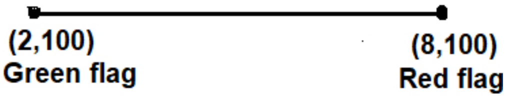
37. **Read the text carefully and answer the questions:**

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Fig. Sarika runs the distance AD on the 2nd line and posts a green flag. Priya runs the distance AD on the eighth line and posts a red flag. (take the position of feet for calculation)



(i) Co-ordinate of green flag = (2,100)

(ii)



distance between Red flag and Green flag

d $= \sqrt{(8-2)^2 + (100-100)^2}$

= $\sqrt{6^2 + 0^2}$

d = 6

∴ distance between Green and Red flag is 6 m.

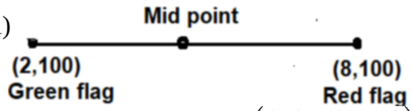
OR

Distance = $\sqrt{(5-2)^2 + (100-100)^2}$

$= \sqrt{9+0}$

= 3 m

(iii)



Position of blue flag = $\left(\frac{2+8}{2}, \frac{100+100}{2}\right)$

= (5,100)

38. **Read the text carefully and answer the questions:**

Two trees are standing on flat ground. The angle of elevation of the top of Both the trees from a point X on the ground is 60^{o}. If the horizontal distance between X and the smaller tree is 8 m and the distance of the top of the two trees is 20 m.

(i) In △DCX

tan 60^{o} = $\frac{DC}{CX}$

$\sqrt{3} = \frac{DC}{8}$

DC = 8$\sqrt{3}$ m

DX = $\sqrt{DC^2 + CX^2}$

$= \sqrt{(8\sqrt{3})^2 + 8^2}$

$= \sqrt{192 + 64}$

$= \sqrt{256}$

= 16 m

Hence, distance between X and top of smaller tree is 16 m.

(ii) In $\triangle$BAX

cos 60^o = $\frac{AX}{BX}$

$\frac{1}{2} = \frac{AC+8}{36}$

36 = 2AC + 16

20 = 2AC

$\frac{20}{2}$ = 10 AC

AC = 10

$\therefore$ horizontal distance between both trees is 10 m.

OR

Height of small tree = CD

In $\triangle$ CDX

tan 60^o = $\frac{CD}{CX}$

$\sqrt{3} = \frac{CD}{8}$

CD = $8\sqrt{3}$ m

(iii) Height of big tree = AB

$\therefore$ In $\triangle$BAX

tan 60^o = $\frac{AB}{AX} = \frac{AB}{18}$

AB = $18\sqrt{3}$ m