Long Answer Type Questions

[4 MARKS]

Que 1. Using Basic proportionality Theorem, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

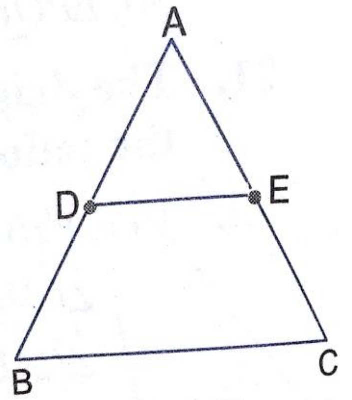

Fig. 7.35

Sol. Given: A ΔABC in which D is the mid-point of AB and DE is drawn parallel to BC, which meets AC at E.

To prove: AE = EC

Proof: In ΔABC, DE || BC

∴ By Basic proportionality Theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \ldots(i)$$

Now, since D is the mid-point of AB

⇒ AD = BD …(ii)

From (i) and (ii), we have

$$\frac{BD}{BD} = \frac{AE}{EC} \quad \Rightarrow \quad 1 = \frac{AE}{EC}$$

⇒ AE = EC

Hence, E is the mid-point of AC.

Que 2. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Sol. Given: ABCD is a trapezium, in which AB || DC and its diagonals intersect each other at the point O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through O, draw OE || AB i.e., OE || DC.

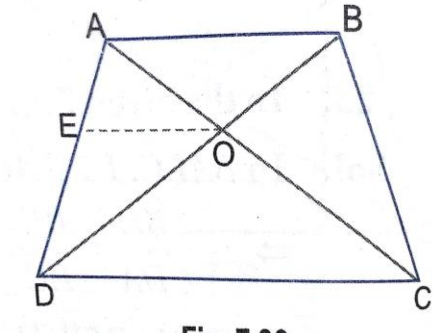

Fig. 7.36

Proof: In ΔADC, we have OE || AB (Construction)

∴ By Basic proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO} \qquad \text{....(i)}$$

Now, in ΔABD, we have OE || AB (Construction)

∴ By Basic proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO} \quad \Rightarrow \quad \frac{AE}{ED} = \frac{BO}{DO} \qquad \text{...(ii)}$$

From (i) and (ii), we have

$$\frac{AO}{CO} = \frac{BO}{DO} \quad \Rightarrow \quad \frac{AO}{BO} = \frac{CO}{DO}$$

Que 3. If AD and PM are medians of triangles ABC and PQR respectively, where ΔABC ~ ΔPQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

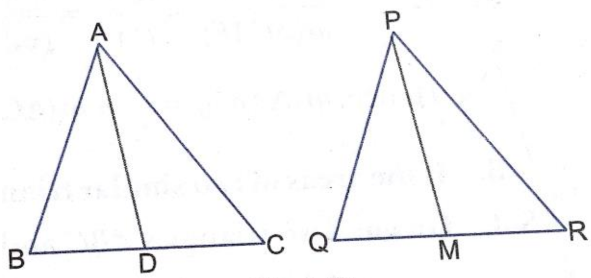

Fig. 7.37

Sol. In ΔABD and ΔPQM, we have

$$\angle B = \angle Q \qquad (\because \ \Delta ABC \sim \Delta PQR) \qquad \text{...(i)}$$

$$\frac{AB}{PQ} = \frac{BC}{QR} (\because \ \Delta ABC \sim \Delta PQR)$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} \quad \Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} \qquad \text{....(ii)}$$

[Since AD and PM are the medians of ΔABC and ΔPQR respectively]

From (i) and (ii) it is proved that

$$\Delta ABD \sim \Delta PQM \qquad \text{(By SAS criterion of similarity)}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \qquad \Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM}$$

Que 4. In Fig. 7.38, ABCD is a trapezium with AB || DC. If ΔAED is similar to ΔBEC, prove that AD = BC.

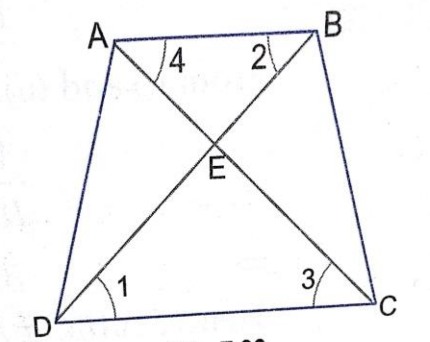

Fig. 7.38

Sol. In ΔEDC and ΔEBA, we have

$\angle 1 = \angle 2$ [Alternate angles]

$\angle 3 = \angle 4$ [Alternate angles]

and $\angle CED = \angle AEB$ [Vertically opposite angles]

$\therefore \quad \Delta EDC \sim \Delta EBA$ [By AA criterion of similarity]

$$\Rightarrow \quad \frac{ED}{EB} = \frac{EC}{EA} \quad \Rightarrow \quad \frac{ED}{EC} = \frac{EB}{EA} \qquad \text{....(i)}$$

It is given that ΔAED ~ ΔBEC

$$\therefore \quad \frac{ED}{EC} = \frac{EA}{EB} = \frac{AD}{BC} \qquad \text{....(ii)}$$

From (i) and (ii), we get

$$\frac{EB}{EA} = \frac{EA}{EB} \quad \Rightarrow \quad (EB)^2 = (EA)^2 \quad \Rightarrow \quad EB = EA$$

Substituting EB = EA in (ii), we get

$$\frac{EA}{EA} = \frac{AD}{BC} \quad \Rightarrow \quad \frac{AD}{BC} = 1 \quad \Rightarrow \quad AD = BC$$

Que 5. Prove that the area of an equilateral triangle described on a side of a right-angled isosceles triangle is half the area of the equilateral triangle describe on its hypotenuse.

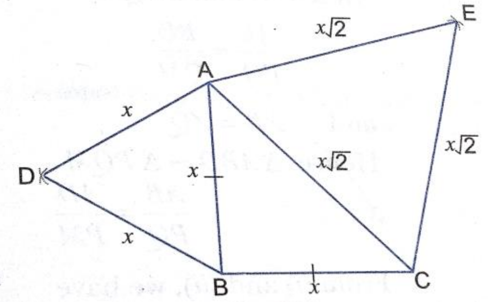

Fig. 7.39

Sol. Given: ΔABC in which $\angle ABC = 90° \; and \; AB = BC. \Delta ABD$ and ΔACE are equilateral triangles.

To Prove: ar (ΔABD) = $\frac{1}{2} \times ar(\Delta CAE)$

Proof: Let AB = BC = x units.

$\therefore \quad hyp.\; CA = \sqrt{x^2 + x^2} \; = \; x\sqrt{2}\, units.$

Each of the ΔABD and ΔCAE being equilateral, has each angle equal to 60°

$\therefore \qquad \Delta ABD \sim \Delta CAE$

But, the ratio of the areas of two similar triangles in equal to the ratio of the squares of their corresponding sides.

$$\therefore \qquad \frac{ar(\Delta ABD)}{ar(\Delta CAE)} = \frac{AB^2}{CA^2} = \frac{x^2}{(x\sqrt{2})^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$

Hence, ar (ΔABD) = $\frac{1}{2} \times ar(\Delta CAE)$

Que 6. If the areas of two similar triangles are equal, prove that they are congruent.

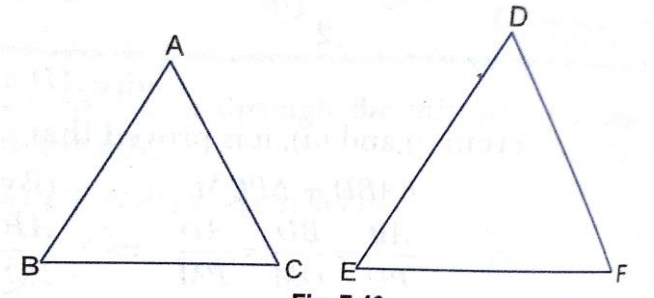

Fig. 7.40

Sol. Given: Two triangles ABC and DEF, such that
ΔABC ~ ΔDEF and area (ΔABC) = area (ΔDEF)
To prove: ΔABC ≅ ΔDEF
Proof: ΔABC ~ ΔDEF

$\Rightarrow \qquad \angle A = \angle D, \angle B = \angle C = \angle F$

And $\qquad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Now, ar (ΔABC) = ar (ΔDEF) (Given)

$\therefore \qquad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = 1$ ….(ii)

And $\qquad \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = \frac{ar(\Delta ABC)}{ar(\Delta DEF)} (\because \Delta ABC - \Delta DEF)$ ….(ii)

From (i) and (ii), we have

$$\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1 \quad \Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$$

Hence, $\Delta ABC \cong \Delta DEF$ (By SSS criterion of congruency)

Que 7. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

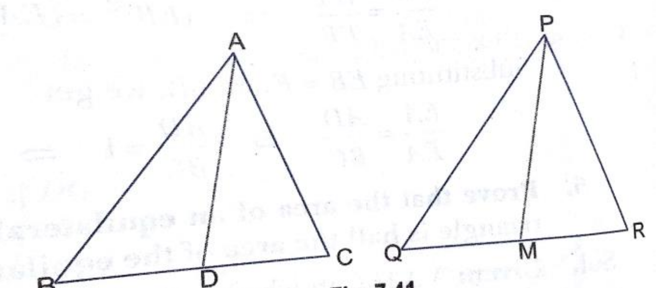

Fig. 7.41

Sol. Let ΔABC and ΔPQR be two similar triangles. AD and PM are the medians of ΔABC and ΔPQR resopectively.

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$

Proof: Since $\Delta ABC \sim \Delta PQR$

$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ...(i)

In ΔABD and ΔPQM

$\frac{AB}{PQ} = \frac{BD}{QM}$ $\left(\because \frac{AB}{PQ} = \frac{BC}{QR} = \frac{1/2\,BC}{1/2\,QR}\right)$

And $\angle B = \angle Q$ $(\because \Delta ABC \sim \Delta PQR)$

Hence, $\Delta ABD \sim \Delta PQM$ (By SAS Similarity criterion)

$\therefore \quad \frac{AB}{PQ} = \frac{AD}{PM}$ ...(ii)

From (i) and (ii), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$$

Que 8. In Fig.7.42, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$
(ii) $AF^2 + BD^2 + CE^2 + AE^2 + CD^2 + BF^2$.

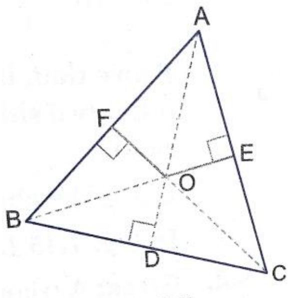

Fig. 7.42

Sol. Join OA, OB and OC.

(i) In right $\Delta's$ OFA, ODB and OEC, we have

$$OA^2 = AF^2 + OF^2 \quad \ldots(i)$$

$$OB^2 = BD^2 + OD^2 \quad \ldots(ii)$$

and $$OC^2 = CE^2 + OE^2 \quad \ldots(iii)$$

Adding (i), (ii) and (iii), we have

$$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$$

$$\Rightarrow \quad OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$$

(ii) We have, $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

$$\Rightarrow \quad (OA^2 - OE^2) + (OB^2 - OF^2) + (OC^2 - OD^2) = AF^2 + BD^2 + CE^2$$

$$\Rightarrow \quad AE^2 + CD^2 + BF^2 = AF^2 + BD^2\ CE^2$$

[Using Pythagoras Theorem in ΔAOE, ΔBOF and ΔCOD]

Que 9. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD
(see Fig. 7.43). Prove that 2 AB^2 = 2 AC^2 + BC^2.

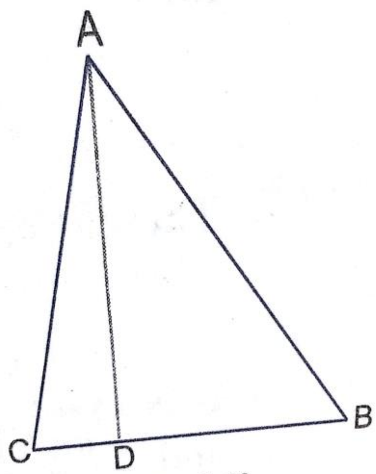

Fig. 7.43

Sol. We have, DB = 3 CD

Now, BC = BD + CD

$\Rightarrow$ BC = 3 CD + CD = 4 CD (Given DB = 3 CD)

$\therefore \quad CD = \frac{1}{4}BC$

And DB = 3 CD = $\frac{3}{4}BC$

Now, in right-angled triangle ABD, we have

$AB^2 = AD^2 + DB^2$ …(i)

Again, in right- angled triangle ΔADC, we have

$AC^2 = AD^2 + CD^2$ …(ii)

Subtracting (ii) from (i), we have

$AB^2 – AC^2 = DB^2 – CD^2$

$$\Rightarrow \ AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \left(\frac{9}{16} - \frac{1}{16}\right) BC^2 = \frac{8}{16} BC^2$$

$$\Rightarrow \ AB^2 - AC^2 = \frac{1}{2} BC^2$$

$$\therefore \ 2AB^2 - 2AC^2 = BC^2 \qquad \Rightarrow \ 2AB^2 = 2AC^2 + BC^2$$