CBSE Test Paper 03

CH-12 Three Dimensional Geometry

1. The medians of a triangle are concurrent at the point called
    a. incentre
    b. orthocentre
    c. centroid
    d. circumcentre
2. The direction cosines of X -axis are
    a. $< 0 , 1 , 1 >$
    b. $< 0 , 0 , 1 >$
    c. $< 1 , 0 , 0 >$
    d. $< 0 , 1 , 0 >$
3. The distance of the point (x , y , z) from the XY –plane is
    a. x
    b. y
    c. z
    d. |z|
4. The distance of the point (3, 4, 5) from X- axis is
    a. 3
    b. 5
    c. $\sqrt{41}$
    d. $\sqrt{34}$
5. G is the centroid of triangle ABC . If P.V. of the points G , A , B , C are respectively , $3\hat{i} + 7\hat{j} + \hat{k}, 2\hat{i} + \hat{j} + \hat{k}, x\hat{i} + 3\hat{j} + \hat{k}, and 4\hat{i} + y\hat{j} + \hat{k}, then\, (x, y) =$
    a. (3 , 3)
    b. (17 , 3)
    c. (3 , 17)
    d. (7 , 17)
6. Fill in the blanks:

    If the distance between the point (a, 2, 1) and (1, -1, 1) is 5, then a = ________.

7. Fill in the blanks:

   The coordinates of a point are the perpendicular distance from the ________ on the respectives axes.

8. A point is on the X-axis. What are its y and z-coordinates?

9. Name the octants in which the following points lie
   (1, 2, 3), (4, -2, 3), (4, -2, -5), (-4, 2, -5), (-4, 2, 5), (-4, 2, 5), (-3, -1, 6), (2, -4, -7)

10. Are the points A(3, 6, 9), B(10, 20, 30) and C(25, -41, 5), the vertices of a right-angled triangle?

11. Let L, M, N are the feet of the perpendiculars drawn from the point P (3, 4, 5) on the XY, YZ, and ZX-planes, respectively. Find the distance of these points L, M, N from the point P.

12. Show that the three points A (2, 3, 4), B (-1, 2 - 3) and C (-4,1,-10) are collinear and find the ratio in which C divides AB.

13. Find the locus of the point which is equidistant from the points A (0,2,3) and (2, -2,1)

14. Show that the coordinates of the centroid of a triangle with vertices A(x_1, y_1, z_1), B(x_2, y_2, z_2) and C(x_3, y_3, z_3) are $\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right]$

15. If the origin is the centriod of the triangle with vertices A(3a, 4, -5), B(-2, 4b, 6), C (6, 10, c). Find the value of a, b, c.

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Solution

1. (c) centroid

   Explanation: The centroid is the point of concurrency of the medians of the triangle.it is a point of centre of gravity of triangle
2. (c) $< 1 , 0 , 0 >$

   Explanation:

   As we know that if a lines makes angles ***a*** , ***b*** and ***c*** with X-axis , Y-axis and Z-axis respectively then direction cosines are given by < **cos a , cos b ,cos c** >

   In our case line is X-axis itself which we know makes angle of 0° , 90° , 90° with X-axis , Y-axis and Z-axis respectively then direction cosine will be

   <cos 0° , cos 90°, cos 90°>

   = < 1 0 0 >

3. (d) |z|

   Explanation: Let L be the foot of perpendicular segment from the point P (x,y,z) on XY plane

   Now since L is the foot of perpendicular on XY plane so z coordinate will be zero so the point L will be (x,y,0)

   then distance between these two will be $\sqrt{(x-x)^2+(y-y)^2+(z-0)^2}$ = $\sqrt{z^2}$

   = |z|
4. (c) $\sqrt{41}$

   Explanation:

   The distance of the point (3, 4, 5) from X- axis is

   let L be the foot of perpendicular from the point (3, 4, 5) to X axis ,then coordinate of L will be (3,0,0) [because on X axis y and z coordinate are zero]

   then distance of the point (3, 4, 5) from X- axis i.e. from L (3,0,0) is given by

$\sqrt{(3-3)^2+(4-0)^2+(5-0)^2}$ = $\sqrt{41}$

5. (c) (3 , 17)

   Explanation:

   $\vec{g}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$ centroid formula
   $3(3\hat{i}+7\hat{j}+\hat{k})$=$2\hat{i}+\hat{j}+\hat{k}+x\hat{i}+3\hat{j}+\hat{k}+4\hat{i}+y\hat{j}+\hat{k}$
   $9\hat{i}+21\hat{j}+3\hat{k}$=$(6+x)\hat{i}+(4+y)\hat{j}+3\hat{k}$

   9=6+x and 21=4+y

   x=3 and y=17

6. a = 5 or -3

7. given point

8. Coordinates of any point on the X-axis is (x, 0, 0). So, its y and z-coordinates are zero.

9. Point (1, 2, 3) lies in Ist Octant.

   Point (4, - 2, 3) lies in IVth Octant. Point (4, -2, -5) lies in VIIIth Octant.

   Point (4, 2, -5) lies in Vth Octant. Point (-4, 2, -5) lies in VIth octant.

   Point (-4, 2, 5) lies in IInd Octant. Point (-3, -1, 6) lies in VIIIth octant.

   Point (2, -4, -7) lies in VIIIth Octant.

10. Given: A(3, 6, 9), B(10, 20, 30) and C(25, -41, 5),

    According to the distance formula, we have

    $AB^2 = (10 - 3)^2 + (20 - 6)^2 + (30 - 9)^2$

    = 49 + 196 + 441 = 686

    $BC^2 = (25 - 10)^2 + (-41 - 20)^2 + (5 - 30)^2$

    = 225 + 3721 + 625 = 4571

    and $CA^2 = (3 - 25)^2 + (6 + 41)^2 + (9 - 5)^2$

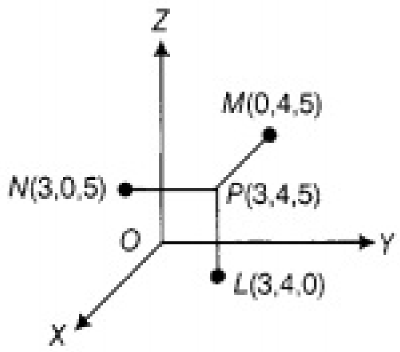
    = 484 + 2209 + 16 = 2709

    We observe that, $CA^2 + AB^2 \neq BC^2$

    Hence, the $\triangle$ABC is not a right angled triangle.

11. L is the foot of the perpendicular drawn from the point P (3, 4, 5) to the XY-plane.

Therefore, the coordinates of the point L is (3, 4, 0).



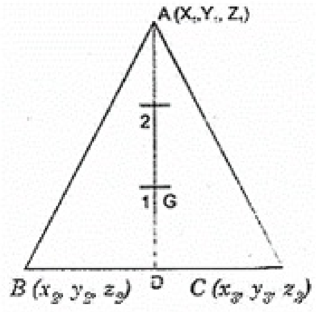
The distance between the points(3, 4, 5) and (3, 4, 0) is 5. Similarly, the lengths of the foot of perpendiculars on YZ and ZX planes are 3 and 4 units, respectively.

12. If points are collinear then all points lie on the same line and Direction Ratio's should be proportional
A(2,3,4), B (−1,2,−3) and C(−4,1,−10)
DR's of AB = (3, 1, 7)
DR's of BC = (3, 1, 7)
So A, B, C are collinear
Length of AC = $\sqrt{36+4+196} = \sqrt{236}$=2$\sqrt{59}$
Length of BC = $\sqrt{9+1+49} = \sqrt{59}$
Ratio is AC: AB = 2:1
So, C divides AB in Ratio 2:1 externally.

13. Let P(x,y,z) be any point which is equidistant from A(0,2,3) and B(2,-2,1),
PA=PB
⇒PA^2=PB^2
$\Rightarrow \sqrt{(x-0)^2 + (y-2)^2 + (z-3)^2} = \sqrt{(x-2)^2 + (y+2)^2 + (z-1)^2}$
$\Rightarrow 4x - 8y - 4z + 4 = 0 \; or \; x - 2y - z + 1 = 0$
Hence the required locus in x-2y-z+1=0

14. If A(x_1, y_1, z_1), B(x_2, y_2, z_2) and C(x_3, y_3, z_3) be three vertices of ΔABC then coordinates of point D are $\left[\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right]$.
Let G be the centroid of ΔABC. Then G divides AD in the ratio 2 : 1.
So the coordinates of G are

$$\left[\frac{x_1+2\left(\frac{x_2+x_3}{2}\right)}{1+2}, \frac{y_1+2\left(\frac{y_2+y_3}{2}\right)}{1+2}, \frac{z_1+2\left(\frac{z_2+z_3}{2}\right)}{1+2}\right]$$
$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$



15. If A(3a, 4, -5), B(-2, 4b, 6) and C (6, 10, c) be three vertices of $\Delta\mathrm{AB}C$, then coordinates of centriod are $\left[\frac{3a-2+6}{3}, \frac{4+4b+10}{3}, \frac{-5+6+c}{3}\right]$.

But it is given that coordinates of centriod are (0, 0, 0).

$\therefore \frac{3a-2+6}{3} = 0 \Rightarrow 3a = -4 \Rightarrow a = -\frac{4}{3}$

$\frac{4+4b+10}{3} = 0 \Rightarrow 4b = -14 \Rightarrow b = -\frac{7}{2}$

$\frac{-5+6+c}{3} = 0 \Rightarrow c = -1$