DAY THREE

Scalar and Vector

Learning & Revision for the Day

- Scalar and Vector Quantities
- Laws of Vector Addition
- Subtraction of Vectors
- Multiplication or Division of a Vector by a Scalar
- Product of Vectors
- Resolution of a vector
- Relative velocity
- Motion in a Plane
- Projectile Motion

Scalar and Vector Quantities

A **scalar quantity** is one whose specification is completed with its magnitude only. e.g. mass, distance, speed, energy, etc.

A **vector quantity** is a quantity that has magnitude as well as direction. e.g. Velocity, displacement, force, etc.

Position and Displacement Vectors

A vector which gives position of an object with reference to the origin of a coordinate system is called position vector.

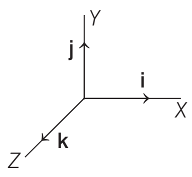
The vector which tells how much and in which direction on object has changed its position in a given interval of time is called displacement vector.

General Vectors and Notation

- **Zero Vector** The vector having zero magnitude is called **zero vector** or **null vector**. It is written as 0. The initial and final points of a zero vector overlap, so its direction is arbitrary (not known to us).
- **Unit Vector** A vector of unit magnitude is known as an **unit vector**. Unit vector for **A** is $\hat{\mathbf{A}}$ (read as A cap).

$$\mathbf{A} = \underbrace{A}_{\text{Magnitude}}\ \overbrace{\hat{\mathbf{A}}}^{\text{Direction}}$$

- **Orthogonal Unit Vectors** The unit vectors along X-axis,s, Y-axis and Z-axis are denoted by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. These are the orthogonal unit vectors.



$$\hat{\mathbf{i}} = \frac{\mathbf{x}}{x}, \hat{\mathbf{j}} = \frac{\mathbf{y}}{y}, \hat{\mathbf{k}} = \frac{\mathbf{z}}{z}$$

- **Parallel Vector** Two vectors are said to be parallel, if they have same direction but their magnitudes may or may not be equal.
- **Antiparallel Vector** Two vectors are said to be anti-parallel when
  (i) both have opposite direction
  (ii) one vectors is scalar non zero negative multiple of another vector.
- **Collinear Vector** Collinear vector are those which act along same line.
- **Coplanar Vector** Vector which lies on the same plane are called coplanar vector.
- **Equal Vectors** Two vectors **A** and **B** are equal, if they have the same magnitude and the same direction.

Laws of Vector Addition

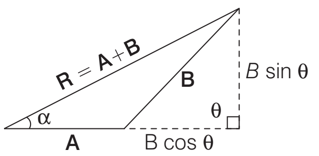
1. Triangle Law

If two non-zero vectors are represented by the two sides of a triangle taken in same order than the resultant is given by the closing side of triangle in opposite order, i.e.

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

The resultant R can be calculated as

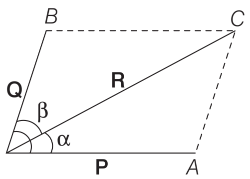
$$|\mathbf{A} + \mathbf{B}| = R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$



If resultant R makes an angle α with vector A, then

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

2. Parallelogram Law



According to parallelogram law of vector addition, if two vector acting on a particle are represented in magnitude and direction by two adjacent side of a parallelogram, then the diagonal of the parallelogram represents the magnitude and direction of the resultant of the two vector acting as the particle.

i.e. $\mathbf{R} = \mathbf{P} + \mathbf{Q}$

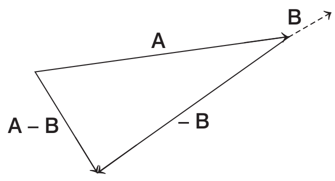
Magnitude of the resultant **R** is given by

$$|\mathbf{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} \quad \Rightarrow \tan\beta = \frac{P\sin\theta}{Q + P\cos\theta}$$

Subtraction of Vectors

Vector subtraction makes use of the definition of the negative of a vector. We define the operation $\mathbf{A} - \mathbf{B}$ as vector $-\mathbf{B}$ added to vector **A**. $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$



Thus, vector subtraction is really a special case of vector addition. The geometric construction for subtracting two vectors is shown in the above figure.

If θ be the angle between A and B,

then $|\mathbf{A} - \mathbf{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

If the vectors form a closed n sided polygon with all the sides in the same order, then the resultant is zero.

Multiplication or Division of a Vector by a Scalar

The multiplication or division of a vector by a scalar gives a vector. For example, if vector **A** is multiplied by the scalar number 3, the result, written as 3**A**, is a vector with a magnitude three times that of **A**, pointing in the same direction as **A**. If we multiply vector **A** by the scalar -3, the result is $-3\mathbf{A}$, a vector with a magnitude three times that of **A**, pointing in the direction opposite to **A** (because of the negative sign).

Products of Vectors

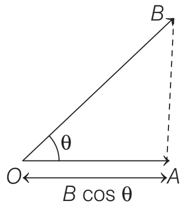
The two types of products of vectors are given below

Scalar or Dot Product

The scalar product of two vectors A and B is defined as the product of magnitudes of A and B multiplied by the cosine of smaller angle between them. i.e. $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$

Properties of Dot Product

- Dot product or scalar product of two vectors gives the scalar two vectors given the scalar quantity.
- It is commutative in nature.
  i.e. $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$.
- Dot product is distributive over the addition of vectors.
  i.e. $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$
- $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$, because angle between two equal vectors is zero.
- If two vectors A and B are perpendicular vectors, then $\mathbf{A} \cdot \mathbf{B} = AB\cos 90^\circ = 0$ and $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$



Component of vector *B* along *A*

The Vector Product

The vector product of **A** and **B**, written as $\mathbf{A} \times \mathbf{B}$, produces a third vector **C** whose magnitude is $C = AB\sin\theta$. where, θ is the smaller of the two angles between **A** and **B**.

Because of the notation, $\mathbf{A} \times \mathbf{B}$ is also known as the **cross product**, and it is spelled as '**A** cross **B**'.

Properties of Cross Product

- Vector or cross product of two vectors gives the vector quantity.
- Cross product of two vectors does not obey the commutative law. i.e. $\mathbf{A}\times\mathbf{B}\neq\mathbf{B}\times\mathbf{A}$;

  Here, $\mathbf{A}\times\mathbf{B}=-\mathbf{B}\times\mathbf{A}$
- Cross product of two vectors is distributive over the addition of vectors.

$$\mathbf{A}\times(\mathbf{B}+\mathbf{C})=\mathbf{A}\times\mathbf{B}+\mathbf{A}\times\mathbf{C}$$

- Cross product of two equal vectors is given by $\mathbf{A}\times\mathbf{A}=0$

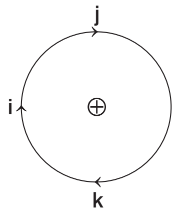
  Similarly,
$$\hat{\mathbf{i}}\times\hat{\mathbf{i}}=(1\times1\times\sin 0^\circ)\,\hat{\mathbf{n}}=0$$
$$\hat{\mathbf{j}}\times\hat{\mathbf{j}}=(1\times1\times\sin 0^\circ)\,\hat{\mathbf{n}}=0$$
$$\hat{\mathbf{k}}\times\hat{\mathbf{k}}=(1\times1\times\sin 0^\circ)\,\hat{\mathbf{n}}=0$$

- Cross product of two perpendicular vectors is given as $\mathbf{A}\times\mathbf{B}=(AB\sin 90^\circ)\,\hat{\mathbf{n}}=(AB)\,\hat{\mathbf{n}}$
- For two vectors $\mathbf{A}=a_x\hat{\mathbf{i}}+a_y\hat{\mathbf{j}}+a_z\hat{\mathbf{k}}$

  and $\mathbf{B}=b_x\hat{\mathbf{i}}+b_y\hat{\mathbf{j}}+b_z\hat{\mathbf{k}}$.

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}$$

- Cross product of vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are following cyclic rules as follows $\hat{\mathbf{i}}\times\hat{\mathbf{j}}=\hat{\mathbf{k}}$, $\hat{\mathbf{j}}\times\hat{\mathbf{k}}=\hat{\mathbf{i}}$ and $\hat{\mathbf{k}}\times\hat{\mathbf{i}}=\hat{\mathbf{j}}$



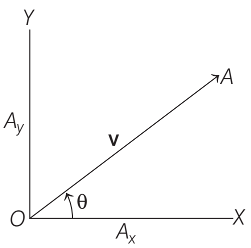
Cyclic representation for unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$

NOTE
- Vector triple product is given by

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=\mathbf{B}\,(\mathbf{A}\cdot\mathbf{C})-\mathbf{C}\,(\mathbf{A}\cdot\mathbf{B})$$

Resolution of a Vector

The process of splitting of a single vector into two or more vectors in different direction is called resolution of a vector. Consider a vector A in the X-Y plane making an angle θ with the X-axis. The X and Y components of A are A_x and A_y respectively.



Thus $\mathbf{A}_x=\mathbf{A}_{xi}=(A\cos\theta)\hat{\mathbf{i}}$ along X-direction

$\mathbf{A}_y=\mathbf{A}_{yj}=(A\sin\theta)\hat{\mathbf{j}}$ along Y-direction

From triangle law of vector addition

$$|\mathbf{A}|=|\mathbf{A}_{xi}+\mathbf{A}_{yj}|=\sqrt{A_x^2+A_y^2}$$

and
$$\tan\theta=\frac{A_y}{A_x}=\theta=\tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Relative Velocity

The time rate of change of relative position of one object with respect to another is called relative velocity.

Different Cases

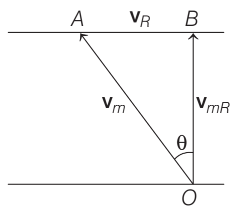
Case I If both objects A and B move along parallel straight lines in the opposite direction, then relative velocity of B w.r.t. A is given as,

$$\mathbf{v}_{BA}=\mathbf{v}_B-(-\mathbf{v}_A)=\mathbf{v}_B+\mathbf{v}_A$$

If both objects A and B move along parallel staight lines in the same direction, then

$$\mathbf{v}_{AB}=\mathbf{v}_B-\mathbf{v}_A$$

Case II Crossing the River To cross the river over shortest distance, i.e. to cross the river straight, the man should swim upstream making an angle θ with **OB** such that, **OB** gives the direction of resultant velocity ($\mathbf{v}_{mR}$) of velocity of swimmer $\mathbf{v}_M$ and velocity of river water $\mathbf{v}_R$ as shown in figure. Let us consider



In ΔOAB, $\sin\theta=\dfrac{v_R}{v_m}$ and $v_{mR}=\sqrt{v_m^2-v_R^2}$

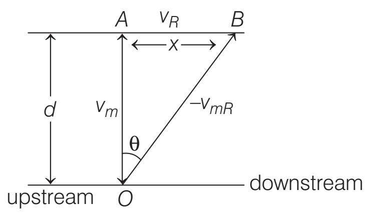
The time taken to cross the river given by

$$t_1=\frac{d}{v_{mR}}=\frac{d}{\sqrt{v_m^2-v_R^2}}$$

Case III To cross the river in possible shortest time The man should go along OA. Now, the swimmer will be going along OB, which is the direction of resultant velocity $\mathbf{v}_{mR}$ of v_m and v_R.

In ΔOAB, $\tan\theta=\dfrac{AB}{OA}=\dfrac{v_R}{v_m}$

and
$$v_{mR}=\sqrt{v_m^2+v_R^2}$$



Time of crossing the river,

$$t=\frac{d}{v_m}=\frac{OB}{v_{mR}}=\frac{\sqrt{x^2+d^2}}{\sqrt{v_m^2+v_R^2}}$$

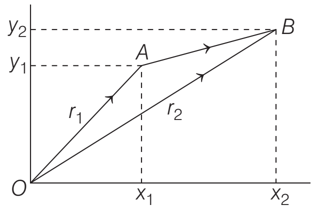
The boat will be reaching the point B instead of point A. If $AB=x$,

then,
$$\tan\theta=\frac{v_R}{v_m}=\frac{x}{d}\Rightarrow x=\frac{dv_R}{v_m}$$

Motion in a Plane

Let the object be at position A and B at timing t_1 and t_2, where $OA = \mathbf{r_1}$, and $OB = \mathbf{r_2}$

Suppose O be the origin for measuring time and position of the object (see figure).



- Displacement of an object form position A to B is

$$AB = \mathbf{r} = \mathbf{r_2} - \mathbf{r_1} = (x_2 - x_1)\,\hat{\mathbf{i}} - (y_2 - y_1)\,\hat{\mathbf{j}}$$

- Velocity, $\mathbf{v} = \dfrac{\mathbf{r_2} - \mathbf{r_1}}{t_2 - t_1}$
- A particle moving in X-Y plane (with uniform velocity) then, its equation of motion for X and Y axes are

$$\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}},\ \mathbf{r_0} = x_0\hat{\mathbf{i}} + y_0\hat{\mathbf{j}} \text{ and } \mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$x = x_0 + v_x t,\ y = y_0 + v_y t$$

- A particle moving in xy-plane (with uniform acceleration), then its equation of motion for X and Y-axes are

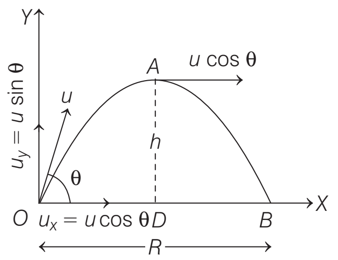
$$v_x = u_x + a_x t,\ \ v_y = u_y + a_y t$$

$$x = x_0 + u_x t + \frac{1}{2}a_x t^2,\ y = y_0 + u_y t + \frac{1}{2}a_y t^2$$

$$\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{i}}$$

Projectile Motion

Projectile is an object which once projected in a given direction with given velocity and is then free to move under gravity alone. The path described by the projectile is called its trajectory.



Let a particle is projected at an angle θ from the ground with initial velocity u.

Resolving u in two components, we have

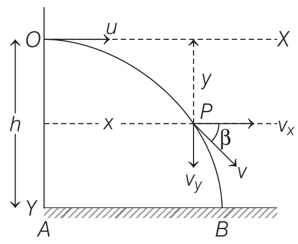
$u_x = u\cos\theta,\ u_y = u\sin\theta,\ a_x = 0,\ a_y = -g.$

- Equation of trajectory, $y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}\,x^2$
- Vertical height covered, $h = \dfrac{u^2\sin^2\theta}{2g}$
- Horizontal range, $R = OB = u_x\ T,\ R = \dfrac{u^2\sin 2\theta}{g}$

NOTE Maximum range occurs when $\theta = 45°$

Projectile Motion in Horizontal Direction From Height (h)

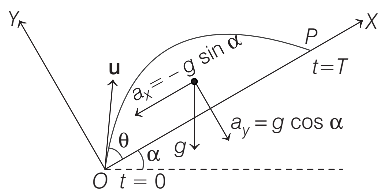
Let a particle be projected in horizontal direction with speed u from height h.



- Equation of trajectory, $y = \dfrac{gx^2}{2u^2}$
- Time of flight, $T = \dfrac{\sqrt{2h}}{g}$
- Horizontal range, $R = u\sqrt{\dfrac{2h}{g}}$
- Velocity of projectile at any time, $v = \sqrt{u^2 + g^2t^2}$

Projectile Motion Up an Inclined Plane

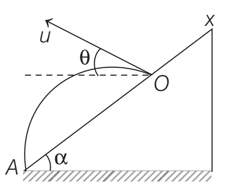
Let a particle be projected up with speed u from an inclined plane which makes an angle α with the horizontal and velocity of projection makes an angle θ with the inclined plane.

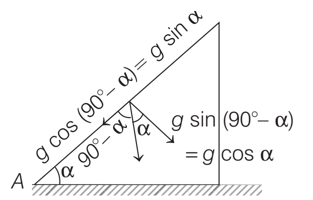


- Time of flight on an inclined plane, $T = \dfrac{2u\sin\theta}{g\sin\alpha}$
- Maximum height, $h = \dfrac{u^2\sin^2\theta}{2g\cos\alpha}$
- Horizontal range, $R = \dfrac{2u^2}{g}\,\dfrac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$
- Maximum range occurs when $\theta = \dfrac{\pi}{2} - \dfrac{\alpha}{2}$
- $R_{\max} = \dfrac{u^2}{g(1+\sin\alpha)}$ when projectile is thrown upwards.
- $R_{\max} = \dfrac{u^2}{g\,(1-\sin\alpha)}$ when projectile is thrown downwards.

Projectile Motion Down an Inclined Plane

A projectile is projected down the plane from the point O with an initial velocity u at an angle θ with horizontal. The angle of inclination of plane with horizontal α. Then,





- Time of flight down an inclined plane, $T = \dfrac{2u\sin(\theta+\alpha)}{g\cos\alpha}$
- Horizontal range, $R = \dfrac{u^2}{g\cos^2\alpha}\,[\sin(2\theta+\alpha) + \sin\alpha]$

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 If **A** and **B** are two non-zero vectors having equal magnitude, the angle between the vectors **A** and **A** − **B** is

(a) 0° (b) 90° (c) 180°
(d) dependent on the orientation of **A** and **B**

2 A vector having magnitude of 30 unit, makes equal angles with each of the X, Y and Z-axes. The components of the vector along each of X, Y and Z-axes are

(a) $10\sqrt{3}$ unit (b) $\frac{10}{\sqrt{3}}$ unit (c) $15\sqrt{3}$ unit (d) 10 unit

3 A particle has an initial velocity $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and an acceleration of $0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}}$. Its speed after 10 s is

(a) 10 units (b) $7\sqrt{2}$ units (c) 7 units (d) 8.5 units

4 Unit vector perpendicular to vector $\mathbf{A} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ both, is

(a) $\pm \frac{3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{11}}$ (b) $\pm \frac{(\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{11}}$
(c) $\pm \frac{-\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{13}}$ (d) $\pm \frac{\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{13}}$

5 A force $\mathbf{F} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ N is applied over a particle which displaces it from its origin to the point $\mathbf{r} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}})$ m. The work done on the particle (in joule) is

(a) − 7 (b) + 7 (c) + 10 (d) + 13

6 If $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{A}$, then the angle between **A** and **B** is

(a) π (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

7 A ball rolls off the top of a stair way with a horizontal velocity of u ms^{-1}. If the steps are h metre high and b metre wide, the ball will hit the edge of the nth step, where n is

(a) $\frac{2hu}{gb^2}$ (b) $\frac{2hu^2}{gb^2}$ (c) $\frac{2hu^2}{gb}$ (d) $\frac{hu^2}{gb^2}$

8 Two paper screens A and B are separated by a distance of 200 m. A bullet pierces A and B. The hole in B is 40 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is

(a) 200 ms^{-1} (b) 400 ms^{-1} (c) 600 ms^{-1} (d) 700 ms^{-1}

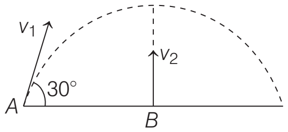
9 A projectile is fired at an angle of 30° with the horizontal such that the vertical component of its initial velocity is 80 ms^{-1}. Its time of flight is T. Its velocity at $t = \frac{T}{4}$ has a magnitude of nearly (take, $g = 10$ ms^{-2})

(a) 180 ms^{-1} (b) 155 ms^{-1} (c) 145 ms^{-1} (d) 140 ms^{-1}

10 A ball is thrown from the ground with a velocity of $20\sqrt{3}$ ms^{-1} making an angle of 60° with the horizontal. The ball will be at a height of 40 m from the ground after a time t equal to (take, $g = 10$ ms^{-2})

(a) $\sqrt{2}$ s (b) $\sqrt{3}$ s (c) 2 s (d) 3 s

11 A body is projected with a velocity v_1 from the point A as shown in the figure. At the same time, another body is projected vertically upwards from B with velocity v_2. The point B lies vertically below the highest point. For both the bodies to collide, $\frac{v_2}{v_1}$ should be



(a) 2 (b) 0.5 (c) $\sqrt{\frac{3}{2}}$ (d) 1

12 A person aims a gun at a bird from a point, at a horizontal distance of 100 m. If the gun can induce a speed of 500 ms^{-1} to the bullet, at what height above the bird must he aim his gun in order to hit it? (take, $g = 10$ ms^{-2})

(a) 10 cm (b) 20 cm (c) 50 cm (d) 100 cm

13 A cannon ball has the same range R on a horizontal plane for two angles of projection. If h_1 and h_2 are the greatest heights in the two paths for which this is possible, then

(a) $R = (h_1h_2)^{1/4}$ (b) $R = 3\sqrt{h_1h_2}$
(c) $R = 4\sqrt{h_1h_2}$ (d) $R = \sqrt{h_1h_2}$

14 A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of 147 ms^{-1}. Then, the time after which its inclination with the horizontal is 45°, is

(a) 15 s (b) 10.98 s (c) 5.49 s (d) 2.745 s

15 A projectile projected with a velocity u at an angle θ passes through a given height h two times at t_1 and t_2. Then,

(a) $t_1 + t_2 = T$ (time of flight) (b) $t_1 + t_2 = \frac{T}{2}$
(c) $t_1 + t_2 = 2T$ (d) $\sqrt{t_1t_2} = T$

16. A particle is projected at angle of 60° with the horizontal having a kinetic energy K. The kinetic energy at the highest point is

(a) K (b) zero
(c) $K/4$ (d) $K/2$

17 A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 ms^{-1} at an angle of 30° with the horizontal. How far from the throwing point, will the ball be at the height of 10 m from the ground?

(take, $g = 10\ ms^{-2}$, $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$)

(a) 5.20 m (b) 4.33 m (c) 2.60 m (d) 8.66 m

18 A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is **→ JEE Main (Online) 2013**

(a) 4.4 m (b) 2.4 m (c) 3.6 m (d) 1.6 m

19 Neglecting the air resistance, the time of flight of a projectile is determined by

(a) U_{vertical} (b) $U_{\text{horizontal}}$
(c) $U = U^2_{\text{vertical}} + U^2_{\text{horizontal}}$ (d) $U = (U^2_{\text{vertical}} + U^2_{\text{horizontal}})^{1/2}$

20 The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Its angle of projection will be

(a) 45° (b) 60° (c) 90° (d) 30°

21 A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection is

(a) 60° (b) $\tan^{-1}(\sqrt{3}/2)$
(c) $\tan^{-1}(1/2)$ (d) 45°

22 A man can swim with a speed of 4 kmh^{-1} in still water.How long does he take to cross a river 1km wide, if the river flows steadily 3 kmh^{-1} and he makes his strokes normal to the river current.How far down the river does he go, when he reaches the other bank?

(a) 800 m (b) 900 m
(c) 400 m (d) 750 m

23 A swimmer crosses a flowing stream of width d to and fro in time t_1. The time taken to cover the same distance up and down the stream is t_2. Then, the time the swimmer would take to swim across a distance 2 d in still water is

(a) $\frac{t_1^2}{t_2}$ (b) $\frac{t_2^2}{t_1}$ (c) $\sqrt{t_1 t_2}$ (d) $(t_1 + t_2)$

24 A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 kmh^{-1}. He finds that the raindrops are hitting his head vertically. The actual speed of raindrops is

(a) 20 kmh^{-1} (b) $10\sqrt{3}$ kmh^{-1}
(c) $20\sqrt{3}$ kmh^{-1} (d) 10 kmh^{-1}

25 A passenger train is moving at 5 ms^{-1} . An express train is travelling at 30 ms^{-1}, on the same track and rear side of the passenger train at some distance. The driver in express train applied brakes to avoid collision. If the retardation due to brakes is 4 ms^{-2}, the time in which the accident is avoided after the application of brakes is

(a) 4.25 s (b) 5.25 s (c) 6.25 s (d) 7.25 s

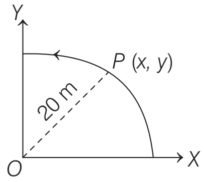
26 A boat takes 2 h to travel 8 km and back in a still water lake. If the velocity of water is 4 kmh^{-1}, the time taken for going upstream of 8 km and coming back is

(a) 2 h (b) 2 h and 40 min
(c) 1 h and 20 min
(d) Cannot be estimated from the given information

27 A car is travelling with a velocity of 10 kmh^{-1} on a straight road. The driver of the car throws a parcel with a velocity of $10\sqrt{2}$ kmh^{-1} when the car is passing by a man standing on the side of the road. If the parcel is to reach the man, the direction of throw makes the following angle with the direction of the car.

(a) 135° (b) 45° (c) $\tan^{-1}(\sqrt{2})$ (d) $\tan\left(\frac{1}{2}\right)$

28 A point P moves in counter- clockwise direction on a circular path as shown in the figure.



The movement of p is such that it sweeps out a length $s = t^3 + 5$, where, s is in metre and t is in second.The radius of the path is 20 m. The acceleration of P when t = 2s is nearly

(a) 13 ms^{-2} (b) 12 ms^{-2} (c) 7.2 ms^{-2} (d) 14 ms^{-2}

29 For a particle in uniform circular motion the acceleration **a** at a point $P(R, \theta)$ on the circle of radius R is (here, θ is measured from the X-axis)

(a) $-\frac{v^2}{R}\cos\theta\,\hat{\mathbf{i}} + \frac{v^2}{R}\sin\theta\,\hat{\mathbf{j}}$ (b) $-\frac{v^2}{R}\sin\theta\,\hat{\mathbf{i}} + \frac{v^2}{R}\cos\theta\,\hat{\mathbf{j}}$
(c) $-\frac{v^2}{R}\cos\theta\,\hat{\mathbf{i}} - \frac{v^2}{R}\sin\theta\,\hat{\mathbf{j}}$ (d) $\frac{v^2}{R}\hat{\mathbf{i}} + \frac{v^2}{R}\hat{\mathbf{j}}$

Direction (Q. Nos. 30-34) *Each of these questions contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below :*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

30 **Statement I** Rain is falling vertically downwards with a velocity of 3 kmh^{-1}. A man walks with a velocity of 4 kmh^{-1}. Relative velocity of rain w.r.t. man is 5 kmh^{-1}.

Statement II Relative velocity of rain w.r.t. man is given by $\mathbf{v}_{rm} = \mathbf{v}_r - \mathbf{v}_m$

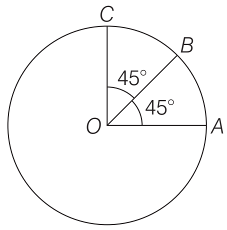
31 **Statement I** For the projection angle $\tan^{-1}(4)$, the horizontal and maximum height of a projectile are equal.

Statement II The maximum range of a projectile is directly proportional to the square of velocity and inversely proportional to the acceleration due to gravity.

32 **Statement I** In order to hit a target, a man should point his rifle in the same direction as the target.

Statement II The horizontal range of bullet is dependent on the angle of projection with the horizontal.

33 **Statement I** The resultant of three vectors **OA**, **OB** and **OC** as shown in the figure is $R(1+\sqrt{2})$. R is the radius of the circle.



Statement II $\mathbf{OA} + \mathbf{OC}$ is acting along **OB** and $(\mathbf{OA} + \mathbf{OC}) + \mathbf{OB}$ is acting along **OB**.

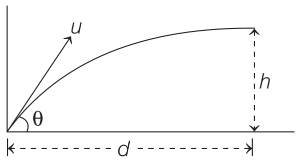
34 **Statement I** Angle between $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\hat{\mathbf{i}}$ is 45°.

Statement II $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is equally include to both $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ and the angle between $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ is 90°.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 If $\mathbf{F}_1$ and $\mathbf{F}_2$ are two vectors of equal magnitude F such that $|\mathbf{F}_1 \cdot \mathbf{F}_2| = |\mathbf{F}_1 \times \mathbf{F}_2|$, then $|\mathbf{F}_1 + \mathbf{F}_2|$ is equal to



(a) $\sqrt{(2+\sqrt{2})}F$ (b) $2F$
(c) $F\sqrt{2}$ (d) None of these

2 If a stone is to hit at a point which is at a distance d away and at a height h above the point from where the stone starts, then what is the value of initial speed u if stone is launched at an angle θ?

(a) $\dfrac{g}{\cos\theta}\sqrt{\dfrac{d}{2(d\tan\theta - h)}}$ (b) $\dfrac{d}{\cos\theta}\sqrt{\dfrac{g}{2(d\tan\theta - h)}}$

(c) $\sqrt{\dfrac{gd^2}{h\cos^2\theta}}$ (d) $\sqrt{\dfrac{gd^2}{(d-h)}}$

3 A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the time of flight in the two cases, then the product of the two time of flight is directly proportional to

(a) R (b) $\dfrac{1}{R}$

(c) $\dfrac{1}{R^2}$ (d) R^2

4 A projectile is given an initial velocity of $(\mathbf{i} + 2\mathbf{j})$ ms^{-1} where, **i** is along the ground and **j** is along the vertical. If $g = 10\ ms^{-2}$, the equation of its trajectory is

→ **JEE Main 2013**

(a) $Y = X - 5X^2$ (b) $Y = 2X - 5X^2$
(c) $4Y = 2X - 5X^2$ (d) $4Y = 2X - 25X^2$

5 A particle of mass m is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is

(a) $\dfrac{\sqrt{3}mv^2}{2g}$ (b) zero

(c) $\dfrac{mv^3}{\sqrt{2}g}$ (d) $\dfrac{\sqrt{3}}{16}\dfrac{mv^3}{g}$

6 The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by

(a) $t^2\sqrt{\alpha^2 + \beta^2}$ (b) $\sqrt{\alpha^2 + \beta^2}$
(c) $3t\sqrt{\alpha^2 + \beta^2}$ (d) $3t^2\sqrt{\alpha^2 + \beta^2}$

7 A ball whose kinetic energy is E, is projected at an angle of 45° with respect to the horizontal. The kinetic energy of the ball at the highest point of its flight will be

(a) E (b) $\dfrac{E}{\sqrt{2}}$

(c) $\dfrac{E}{2}$ (d) zero

8 A river is flowing from West to East with a speed of 5 m - min^{-1}. A man on the South bank of the river, is capable of swimming at 10 m - min^{-1} in still water, he wants to swim across the river in the shortest time. He should swim in a direction

(a) due to North (b) 30° East of North
(c) 30° West of North (d) 60° East of North

9 A ship A is moving Westwards with a speed of 10 kmh^{-1} and a ship B, 100 km South of A is moving Northwards with a speed of 10 kmh^{-1}. The time after which the distance between them is shortest and the the shortest distance between them are

(a) 0 h, 100 km (b) 5h, $50\sqrt{2}$ km
(c) $5\sqrt{2}$ h, 50 km (d) $10\sqrt{2}$ h, $50\sqrt{2}$ km

10 A particle is moving Eastwards with a velocity of 5 ms^{-1}. In 10s, the velocity changes to 5 ms^{-1} Northwards. The average acceleration in this time is

(a) $\frac{1}{\sqrt{2}}$ ms^{-2} towards North-East
(b) $\frac{1}{2}$ ms^{-2} towards North
(c) zero
(d) $\frac{1}{\sqrt{2}}$ ms^{-2} towards North-West

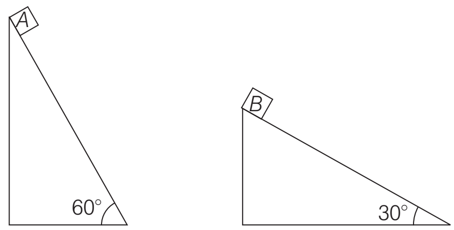
11 A boy can throw a stone upto a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone upto will be

(a) $20\sqrt{2}$ m (b) 10 m
(c) $10\sqrt{2}$ m (d) 20 m

12 A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$
(c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

13 Two fixed frictionless inclined plane making an angle 30° and 60° with the vertical are as shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



(a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) zero
(d) 4.9 ms^{-2} in vertical direction

14 A particle is moving with velocity $v = k(Y\,\mathbf{i} + X\,\mathbf{j})$, where k is a constant.The general equation for its path is

(a) $Y = X^2$ + constant (b) $Y^2 = X$ + constant
(c) XY = constant (d) $Y^2 = X^2$ + constant

15 The maximum range of a bullet fired from a toy pistol, mounted on a car at rest is $R_0 = 40$ m. What will be the acute angle of inclination of the pistol for maximum range when the car is moving in the direction of firing with uniform velocity $v = 20\,\text{ms}^{-1}$, on a horizontal surface? (take, $g = 10\,\text{ms}^{-2}$) **→ JEE Main (Online) 2013**

(a) 30° (b) 60°
(c) 75° (d) 45°

ANSWERS

SESSION 1

1 (d)	**2** (a)	**3** (b)	**4** (b)	**5** (b)	**6** (a)	**7** (b)	**8** (d)	**9** (c)	**10** (c)
11 (b)	**12** (b)	**13** (c)	**14** (c)	**15** (a)	**16** (c)	**17** (d)	**18** (c)	**19** (a)	**20** (d)
21 (c)	**22** (d)	**23** (a)	**24** (a)	**25** (c)	**26** (b)	**27** (b)	**28** (d)	**29** (c)	**30** (a)
31 (b)	**32** (d)	**33** (a)	**34** (a)						

SESSION 2

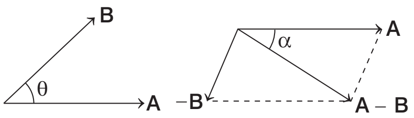
1 (a)	**2** (b)	**3** (a)	**4** (b)	**5** (d)	**6** (d)	**7** (c)	**8** (a)	**9** (b)	**10** (d)
11 (d)	**12** (a)	**13** (d)	**14** (d)	**15** (b)					

Hints and Explanations

SESSION 1

1 Suppose angle between two vectors **A** and **B** of equal magnitude is θ. Then, angle between **A** and **A** − **B** will be $\frac{180° - \theta}{2}$ or $90° - \frac{\theta}{2}$.

Hence, this angle will depend on the angle between **A** and **B** or θ.



2 $A_x = A_y = A_z$

Now, $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{3}\, A_x$

$\therefore \quad A_x = \frac{A}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3}$

Similarly $A_y = A_x = 10\sqrt{3}$ unit

3 $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j};\ \mathbf{a} = 0.4\,\hat{\mathbf{i}} + 0.3\,\hat{\mathbf{j}}$

$$\mathbf{v} = \mathbf{u} + \mathbf{a}\,t$$

$= 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + (0.4\hat{\mathbf{i}} + 0.3\,\hat{\mathbf{j}})10$

$= 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$

$= 7\,\hat{\mathbf{i}} + 7\,\hat{\mathbf{j}}$

Speed $= \sqrt{7^2 + 7^2} = 7\sqrt{2}$ units

4 Given $\mathbf{A} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}},\ \mathbf{B} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

The unit vector in the normal direction is

$$\hat{\mathbf{n}} = \pm\frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}|\ |\mathbf{B}|\sin\theta}$$

Here, $\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & 0 \\ 2 & -1 & -5 \end{vmatrix}$

$$= -5\hat{\mathbf{i}} + 15\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

$|\mathbf{A}| = \sqrt{3^2 + 1^2} = \sqrt{10}$

$|\mathbf{B}| = \sqrt{(2)^2 + (-1)^2 + (-5)^2} = \sqrt{30}$

$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}|\,|\mathbf{B}|}$

$= \frac{5}{10\sqrt{3}} - \frac{1}{2\sqrt{3}}$

$\therefore \quad \sin\theta = \sqrt{1 - \cos^2\theta} = \frac{\sqrt{11}}{2\sqrt{3}}$

$\therefore \quad \hat{\mathbf{n}} = \pm\frac{-5\hat{\mathbf{i}} + 15\hat{\mathbf{j}} - 5\hat{\mathbf{k}}}{\sqrt{10}\cdot\sqrt{30}\cdot\frac{\sqrt{11}}{2\sqrt{3}}}$

$= \pm\frac{(\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{11}}$

5 Work done in displacing the particle

$W = \mathbf{F}\cdot\mathbf{r} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}} - \hat{\mathbf{j}})$

$= 5\times 2 + 3\times(-1) + 2\times 0 = 10 - 3$

$= 7$ J

6 $(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\times\mathbf{A})$ (given)

$\Rightarrow \quad (\mathbf{A}\times\mathbf{B}) - (\mathbf{B}\times\mathbf{A}) = \mathbf{0}$

or $\quad (\mathbf{A}\times\mathbf{B}) + (\mathbf{A}\times\mathbf{B}) = \mathbf{0}$

$[\because (\mathbf{B}\times\mathbf{A}) = -(\mathbf{A}\times\mathbf{B})]$

or $\quad 2(\mathbf{A}\times\mathbf{B}) = \mathbf{0}$

$\Rightarrow \quad 2\,AB\sin\theta = 0$

or $\quad \sin\theta = 0$

$[\because |\mathbf{A}| = A \neq 0, |\mathbf{B}| = B \neq 0]$

$\Rightarrow \quad \theta = 0$ or π

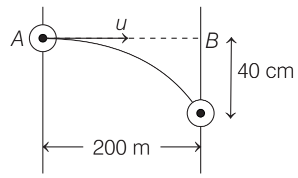
7 Let the ball strike the, nth step after t second. Vertical distance travelled by the ball $= nh = \frac{1}{2}gt^2$ ...(i)

Horizontal distance travelled by the ball $= nb = ut$ or $t = \frac{nb}{u}$

So from Eq (i), we get

$nh = \frac{1}{2}g\left(\frac{nb}{u}\right)^2$ or $n = \frac{2u^2h}{gb^2}$

8 Refering to the figure,



$$200 = ut \text{ or } t = \frac{200}{u}$$

Also, $\quad \frac{40}{100} = \frac{1}{2}\times 9.8\left(\frac{200}{u}\right)^2$

On solving, $\quad u = 700\ \text{ms}^{-1}$

9 Vertical component of the initial velocity,

$u_y = u\sin 30°$

or $u = \frac{u_y}{\sin 30°} = \frac{80}{1/2} = 160\ \text{ms}^{-1}$

$t = \frac{T}{4} = \frac{2u\sin 30°}{4\times g} = \frac{2\times 80}{4\times 10} = 4$ s

$v_x = u\cos 30° = 160\times\frac{\sqrt{3}}{2}$

$= 80\sqrt{3}\ \text{ms}^{-1}$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{u_x^2 + (u_y - gt)^2}$

$= \sqrt{(80\sqrt{3})^2 + (80 - 10\times 4)^2}$

$= 144.3\ \text{ms}^{-1}$

$= 145\ \text{ms}^{-1}$

10 As, $s = u\sin\theta t - \frac{1}{2}gt^2$

So, $40 = 20\sqrt{3}\times\frac{\sqrt{3}t}{2} - \frac{1}{2}\times 10\times t^2$

or $\quad 5t^2 - 30t + 40 = 0$

or $\quad t^2 - 6t + 8 = 0$ or $t = 2$ or 4

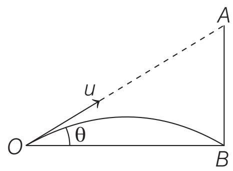
The minimum time $t = 2$ s

11 The two bodies will collide at the highest point if both cover the same vertical height in the same time. So,

$$\frac{v_1^2\sin^2 30°}{2g} = \frac{v_2^2}{2g}$$

or $\quad \frac{v_2}{v_1} = \sin 30° = \frac{1}{2} = 0.5$

12 Let the gun be fired with a velocity u from the point O on the bird at B, making an angle θ with the horizontal direction. Therefore, the height of the aim of the person be at height BA (h) above the bird.



Here, horizontal range

$$= \frac{u^2\sin 2\theta}{g} = 100$$

or $\quad \frac{(500)^2\sin 2\theta}{10} = 100$

or $\quad \sin 2\theta = \frac{100\times 10}{(500)^2} = \frac{1}{250} = \sin 14'$

or $\quad 2\theta = 14'$ or $\theta = 7'$

$= \frac{7}{60}\times\frac{\pi}{180}$ rad

As, angle $= \frac{\text{arc}}{\text{radius}}$

$\therefore \quad \theta = \frac{AB}{OB}$

or $\quad AB = \theta\times OB$

$= \frac{7}{60}\times\frac{\pi}{180}\times(100\times 100)$ cm

$= 20$ cm

13 The cannon ball will have the same horizontal range for the angle of projection θ and (90° − θ) . So,

$h_1 = \frac{u^2\sin^2\theta}{2g}$ and $h_2 = \frac{u^2\cos^2\theta}{2g}$

$h_1h_2 = \frac{1}{4}\left(\frac{u^2\sin\theta\cos\theta}{g}\right)^2 = \frac{1}{4}\times\frac{R^2}{4}$

or $R = 4\sqrt{h_1h_2}$

14 Horizontal component of the velocity at an angle 60°
= Horizontal component of the velocity at an angle 45°.

i.e. $u\cos 60° = v\cos 45°$

$$147 \times \frac{1}{2} = v \times \frac{1}{\sqrt{2}}$$

or $$v = \frac{147}{\sqrt{2}}\ \text{ms}^{-1}$$

Vertical component of $u = u\sin 60°$

$$= \frac{147\sqrt{3}}{2}\ \text{m}$$

Vertical component of $v = v\sin 45°$

$$= \frac{147}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{147}{2}\ \text{m}$$

But $v_y = u_y + at$

$$\therefore \quad \frac{147}{2} = \frac{147\sqrt{3}}{2} - 9.8\,t$$

or $$9.8\,t = \frac{147}{2}(\sqrt{3} - 1)$$

$$\therefore \quad t = 5.49\ \text{s}$$

15 For a projectile fired with a velocity u inclined at an angle θ with the horizontal.

$$h = u\sin\theta\, t - \frac{1}{2}g t^2$$

$$\Rightarrow \quad g t^2 - 2u\sin\theta t + 2h = 0$$

$$\therefore \quad t = \frac{2u\sin\theta \pm \sqrt{4u^2\sin^2\theta - 8gh}}{2g}$$

$$\Rightarrow t_1 = \frac{2u\sin\theta + \sqrt{4u^2\sin^2\theta - 8gh}}{2g}$$

and $$t_2 = \frac{2u\sin\theta - \sqrt{4u^2\sin^2\theta - 8gh}}{2g}$$

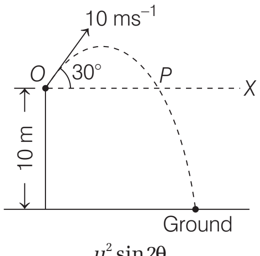
$$\therefore \quad t_1 + t_2 = \frac{2u\sin\theta}{g} = T$$

16 Kinetic energy at highest point,

$$(\text{KE})_H = \frac{1}{2}mv^2\cos^2\theta = K\cos^2\theta$$

$$= K(\cos 60°)^2 = \frac{K}{4}$$

17



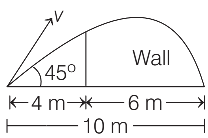
$$OP = R = \frac{u^2\sin 2\theta}{g}$$

$$= \frac{10^2 \times \sin(2 \times 30°)}{10}$$

$$= \frac{10\sqrt{3}}{2} = 5\sqrt{3} = 8.66\ \text{m}$$

18 As, range $= 10 = \dfrac{u^2\sin 2\theta}{g}$

$$\Rightarrow \quad u^2 = 10\,g$$



$\therefore\ u = 10\text{ms}^{-1}$ (as, $g = 10\text{ms}^{-2}$)

$$Y = x\tan\theta - \frac{1}{2}\frac{gx^2}{2v_0^2\cos^2\theta}$$

$$= 4\tan 45° - \frac{1}{2}\frac{g \times 16}{2\,v_0^2\cos^2 45°}$$

$$= 4 \times 1 - \frac{1}{2}\frac{10 \times 16}{2 \times 10 \times 10 \times \frac{1}{2}}$$

$$= 4 - 0.8 = 3.2 \approx 3.6\ \text{m}$$

19 Time of flight (T) is $2t$

$$\therefore \quad T = 2t = \frac{2u\sin\theta}{g}$$

$$= \frac{2}{g} \times U_{\text{vertical}}$$

20 Let u be initial velocity of projection at angle θ with the horizontal.
Then horizontal range,

$$R = \frac{u^2\sin 2\theta}{g}$$

and maximum height $H = \dfrac{u^2\sin^2\theta}{2g}$

Given, $R = 4\sqrt{3}\,H$

$$\therefore \quad \frac{u^2\sin 2\theta}{g} = 4\sqrt{3}\frac{u^2\sin^2\theta}{2g}$$

$$\therefore \ 2\sin\theta\cos\theta = 2\sqrt{3}\sin^2\theta$$

or $$\frac{\cos\theta}{\sin\theta} = \sqrt{3}$$

or $$\cot\theta = \sqrt{3} = \cot 30°$$

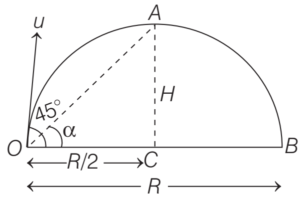
$$\theta = 30°$$

21 Maximum height,

$$H = \frac{u^2\sin^2 45°}{2\,g} = \frac{u^2}{4\,g} = AC$$

Horizontal range,

$$R = \frac{u^2\sin 2 \times 45°}{g} = \frac{u^2}{g}$$

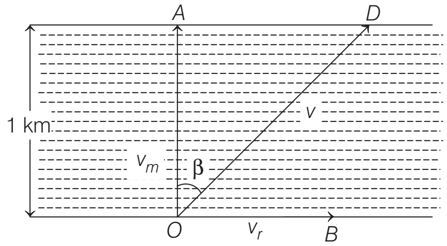


$$\therefore \quad OC = R/2 = u^2/(2g)$$

$$\tan\alpha = \frac{AC}{OC} = \frac{u^2/4\,g}{u^2/2\,g} = \frac{1}{2}$$

$$\therefore \quad \alpha = \tan^{-1}(1/2)$$

22 Given, speed of man $v_m = 4$ km/h



Speed of river, $v_r = 3\ \text{kmh}^{-1}$
Width of the river $d = 1$ km
Time taken by the man to cross the river,

$$t = \frac{\text{Width of the river}}{\text{Speed of the man}} = \frac{1\text{km}}{4\ \text{kmh}^{-1}}$$

$$= \frac{1}{4}\text{h} = \frac{1}{4} \times 60 = 15\ \text{min}$$

Distance travelled along the river
$= v_r \times t$

$$= 3 \times \frac{1}{4} = \frac{3}{4}\text{km} = \frac{3000}{4} = 750\ \text{m}$$

23 Let u be the velocity of the swimmer and v be the velocity of the river flow. Then

$$t_1 = \frac{2d}{\sqrt{u^2 - v^2}}$$

and $$t_2 = \frac{d}{u - v} + \frac{d}{u + v} = \frac{2\,ud}{u^2 - v^2}$$

If t is the time taken by the swimmer to swim a distance $2d$ in still water, then

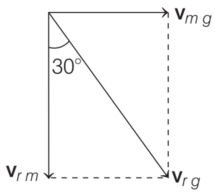
$$t = \frac{2d}{u}$$

$$t_2 \times t = \frac{2ud}{u^2 - v^2} \times \frac{2d}{u} = \frac{4d^2}{u^2 - v^2}$$

From above, $t_2 \times t = t_1^2$ or $t = \dfrac{t_1^2}{t_2}$

24 When the man is at rest w.r.t. to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops w.r.t. to the ground.

Here, $\mathbf{v}_{rg}$ = velocity of the rain w.r.t. the ground.



$$\mathbf{v}_{rm} = \mathbf{v}_{rg} + \mathbf{v}_{gm} = \mathbf{v}_{rg} - \mathbf{v}_{mg} = \mathbf{v}_{rg} + (-\mathbf{v}_{mg})$$

$\mathbf{v}_{mg}$ = velocity of the man w.r.t the ground,

and $\mathbf{v}_{rm}$ = velocity of the rain w.r.t. the man.

Here, $v_{rg} \sin 30° = v_{mg} = 10\,\text{kmh}^{-1}$

$\Rightarrow \quad v_{rg} = \dfrac{10}{\sin 30°} = 20\ \text{kmh}^{-1}$

25 Initial relative velocity of the express train w.r.t. the passenger train,

$u_{ep} = u_e - u_p = 30 - 5 = 25\text{ms}^{-1}$

Final relative velocity of the express train w.r.t. the passenger train, $v_{ep} = 0$ (because express train comes to rest relative to passenger train)

From first equation of motion,

$v_{ep} = u_{ep} - at \Rightarrow 0 = 25 - 4t$

$\Rightarrow \quad 4t = 25 \qquad \Rightarrow \ t = 6.25\,\text{s}$

26 Total distance travelled by the boat in 2 h in still water $= 8 + 8 = 16$ km

Therefore, speed of boat in still water,

$v_b = \dfrac{16}{2} = 8\,\text{km h}^{-1}$

Effective velocity when boat moves upstream $= v_b - v_w = 8 - 4$

$= 4\,\text{km h}^{-1}$,

Therefore, time taken to travel from one end to other $= \dfrac{8}{4} = 2$ h

Effective velocity when boat moves downstream $= v_b + v_w = 8 + 4$

$= 12\,\text{km h}^{-1}$

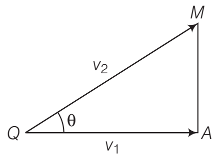
The time taken to travel 8 km distance

$= \dfrac{8}{12} = \dfrac{2}{3}\text{h} = 40\,\text{min}$

Total time taken $= 2\,\text{h} + 40\ \text{min}$

$= 2\,\text{h}$ and 40 min

27 Let v_1 be the velocity of the car and v_2 be the velocity of the parcel. The parcel is thrown at an angle θ from Q, it reaches the man at M.



$\therefore \cos\theta = \dfrac{v_1}{v_2} = \dfrac{10}{10\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \cos 45°$

So, $\quad \theta = 45°$

28 Given, $s = t^3 + 5$

$\therefore \quad$ Speed $v = \dfrac{ds}{dt} = 6t$

and rate of change of speed

$a_t = \dfrac{dv}{dt} = 6t$

$\therefore$ Tangential acceleration at $t = 2$s

$a_t = 6 \times 2 = 12\ \text{ms}^{-2}$

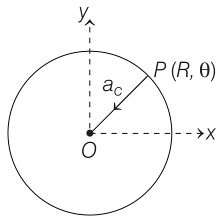
and at $t = 2\text{s}, v = 3(2)^2 = 12\ \text{ms}^{-1}$

$\therefore \quad$ Centripetal acceleration,

$a_c = \dfrac{v^2}{R} = \dfrac{144}{20}\ \text{ms}^{-2}$

Net acceleration $= \sqrt{a_c^2 + a_t^2} \approx 14\ \text{ms}^{-2}$

29 For a particle in uniform circular motion



$\mathbf{a} = \dfrac{v^2}{R}$ towards centre of circle

$\therefore \quad \mathbf{a} = \dfrac{v^2}{R}(-\cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}})$

or $\mathbf{a} = -\dfrac{v^2}{R}\cos\theta\,\hat{\mathbf{i}} - \dfrac{v^2}{R}\sin\theta\,\hat{\mathbf{j}}$

30 Velocity of rain

$\mathbf{v}_r = -3\hat{\mathbf{j}}$ (vertically downward)

Velocity of man

$\mathbf{v}_m = 4\hat{\mathbf{i}}$

$\therefore \quad$ Relative velocity of rain w.r.t. man

$\mathbf{v}_{rm} = \mathbf{v}_r - \mathbf{v}_m = -3\hat{\mathbf{j}} - 4\hat{\mathbf{i}}$

$\therefore \quad |\mathbf{v}_{rm}| = \sqrt{(-3)^2 + (-4)^2}$

$= 5\,\text{kmh}^{-1}$

31 Horizontal range of the projectile,

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R_{\max} = \frac{u^2}{g} \qquad (\theta = 45°)$$

The maximum height attained by the projectile

$$H = \frac{u^2 \sin^2\theta}{2g}$$

If $H = R$, then

$$\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$$

So, $\quad \tan\theta = 4$

$\therefore \quad \theta = \tan^{-1}(4)$

32 To hit a target, the man should aim his rifle at a point higher than the target as the bullet suffers a vertical deflection $(y = \frac{1}{2}gt^2)$ due to acceleration due to gravity.

33 $OA = OC$

$\mathbf{OA} + \mathbf{OC}$ is along $\mathbf{OB}$ (bisector) and its magnitude is

$$2R\cos 45° = R\sqrt{2}$$

$(\mathbf{OA} + \mathbf{OC}) + \mathbf{OB}$ is along $\mathbf{OB}$ and its magnitude is

$$R\sqrt{2} + R = R(1 + \sqrt{2})$$

34 $\cos\theta = \dfrac{(\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot \hat{\mathbf{i}}}{|(\hat{\mathbf{i}} + \hat{\mathbf{j}})| \cdot |\hat{\mathbf{i}}|} = \dfrac{1}{\sqrt{2}} = \cos 45°$

So, $\theta = 45°$

Hence, angle between $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ is 90°.

SESSION 2

1 $FF\cos\theta = FF\sin\theta$

or $\quad \tan\theta = 1$ or $\theta = 45°$

$\therefore |\mathbf{F}_1 + \mathbf{F}_2|$

$= \sqrt{F^2 + F^2 + 2FF\ \cos 45°}$

$= \sqrt{(2 + \sqrt{2})}\ F$

2 Vertical distance covered by a projectile is given by

$$h = (u\sin\theta)t - \frac{1}{2}gt^2 \qquad \text{...(i)}$$

$d = u\cos\theta \times t$

or $\quad t = \dfrac{d}{u\cos\theta}$

From Eq. (i), we get

$$h = u\sin\theta \times \frac{d}{u\cos\theta} - \frac{1}{2}g\frac{d^2}{u^2\cos^2\theta}$$

or $\ u = \dfrac{d}{\cos\theta}\sqrt{\dfrac{g}{2(d\tan\theta - h)}}$

3 We know that the range is

$$T = \frac{2u\sin\theta}{g}$$

According to the question, the range of the projectile is the same for complementary angles,

So, $\quad T_1 = \dfrac{2u\sin\theta}{g}$

$\Rightarrow \quad T_2 = \dfrac{2u\sin(90° - \theta)}{g}$

Again, the range of the projectile is

$$R = \frac{u^2\sin 2\theta}{g}$$

$\therefore \ T_1\ T_2 = \dfrac{2u\sin\theta}{g} \times \dfrac{2u\sin(90° - \theta)}{g}$

$= \dfrac{2u^2\sin 2\theta}{g^2} = \dfrac{2R}{g}$

$\therefore \quad T_1T_2 \propto R$

4 Initial velocity, $\mathbf{u} = \mathbf{i} + 2\,\mathbf{j}\ \text{ms}^{-1}$

Magnitude of velocity

$u = \sqrt{(1)^2 + (2)^2} = \sqrt{5}\,\text{ms}^{-1}$

Equation of trajectory of projectile

$$Y = X\tan\theta - \frac{gX^2}{2u^2}(1 + \tan^2\theta)$$

$$\left[\tan\theta\frac{Y}{X} = \frac{2}{1} = 2\right]$$

$\therefore \ Y = X \times 2 - \dfrac{10(X)^2}{2(\sqrt{5})^2}[1 + (2)^2]$

$= 2X - \dfrac{10(X^2)}{2 \times 5}(1 + 4) = 2X - 5X^2$

5 Angular momentum of the projectile,
$L = mv_h r_\perp = m(v\cos\theta)h$

where, h is the maximum height

$$= m(v\cos\theta)\left(\frac{v^2\sin^2\theta}{2g}\right);$$

$$L = \frac{mv^3\sin^2\theta\cos\theta}{2g} = \frac{\sqrt{3}\,mv^3}{16g}\quad[\because \theta = 30°]$$

6 Since, $x = \alpha t^3$ and $y = \beta t^3$

$\therefore\quad \mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \alpha t^3\hat{\mathbf{i}} + \beta t^3\hat{\mathbf{j}}$

Now, $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = \alpha t^2 \times 3\hat{\mathbf{i}} + \beta t^2 \times 3\hat{\mathbf{j}}$

Thus, $|\mathbf{v}| = \sqrt{(3\alpha t^2)^2 + (3\beta t^2)^2}$

$$= \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

7 At the highest point of its flight, vertical component of velocity is zero and only horizontal component is left which is

$$u_x = u\cos\theta$$

Given, $\theta = 45°$

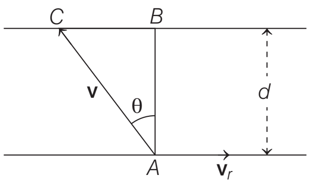
$\therefore\quad u_x = u\cos 45° = \dfrac{u}{\sqrt{2}}$

Hence, at the highest point kinetic energy

$$E' = \frac{1}{2}mu_x^2 = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2}m\left(\frac{u^2}{2}\right) = \frac{E}{2}\quad\left(\because \frac{1}{2}mv^2 = E\right)$$

8 Let the swimmer starts swimming with velocity v along AC in a direction making an angle θ with AB as shown in the figure. If d is the width of the river, time taken by the swimmer to cross the river will be



$$t = d/v\cos\theta$$

As component of AB will be $v\cos\theta$, This time will be minimum, when
$\cos\theta = \max = 1$, i.e. $\theta = 0°$.
So, the swimmer should swim in North direction.

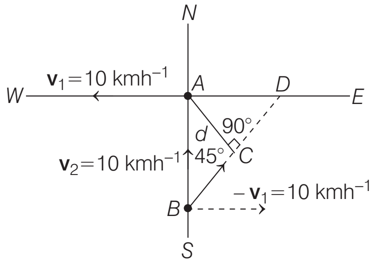
9 Let the ships A and B be at positions as shown in figure when the distance between them is shortest.

Relative velocity of B w.r.t. A is

$$v_r = \sqrt{v_1^2 + v_2^2} = \sqrt{10^2 + 10^2}$$

$$= 10\sqrt{2}\ \text{kmh}^{-1}\ \text{along } BC$$

The shortest distance between A and C is d given by, $d = AC = AB\sin 45°$



$$= 100 \times \frac{1}{\sqrt{2}} = 50\sqrt{2}\ \text{km}$$

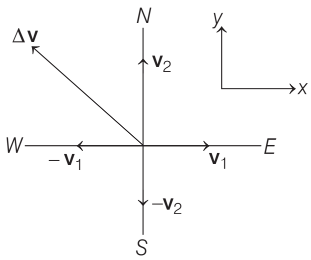
Shortest time, $t = \dfrac{d}{v_r} = \dfrac{50\sqrt{2}}{10\sqrt{2}} = 5\,\text{h}$

10 $\mathbf{v}_1 = +5\hat{\mathbf{i}},\ \mathbf{v}_2 = +5\hat{\mathbf{j}}$

$$\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = 5\hat{\mathbf{j}} - 5\hat{\mathbf{i}}$$

$$|\Delta\mathbf{v}| = 5\sqrt{2}$$

$\therefore\quad a = \dfrac{|\Delta\mathbf{v}|}{t} = \dfrac{5\sqrt{2}}{10} = \dfrac{1}{\sqrt{2}}\,\text{ms}^{-2}$



For direction,

$$\tan\alpha = -\frac{5}{5} = -1$$

Average acceleration is $\dfrac{1}{\sqrt{2}}\,\text{ms}^{-2}$ towards North-West.

11 Maximum speed with which the boy can throw stone is

$$u = \sqrt{2gh} = \sqrt{2 \times 10 \times 10}$$

$$= 10\sqrt{2}\ \text{ms}^{-1}.$$

Range is maximum when projectile is thrown at an angle of 45°.

Thus, $R_{\max} = \dfrac{u^2}{g} = \dfrac{(10\sqrt{2})^2}{10} = 20\,\text{m}$

12 Maximum range of water coming out of the fountain,

$$R_m = \frac{v^2}{g}$$

$\therefore$ Total area around fountain,

$$A = \pi R_m^2 = \pi\frac{v^4}{g^2}$$

13 $mg\sin\theta = ma$

$\therefore\quad a = g\sin\theta$

where, a is along the inclined plane.

$\therefore$ Vertical component of acceleration is $g\sin^2\theta$.

$\therefore$ Relative vertical acceleration of A with respect to B is

$$g(\sin^2 60° - \sin^2 30°) = \frac{g}{2} = 4.9\ \text{ms}^{-2}$$

(in vertical direction)

14 Given, velocity $v = kY\mathbf{i} + kX\mathbf{j}$

$$\frac{dX}{dt} = kY, \frac{dY}{dt} = kX$$

$$\frac{dY}{dX} = \frac{dY}{dt} \times \frac{dt}{dX} = \frac{kX}{kY}$$

$$YdY = XdX;$$

$$Y^2 = X^2 + C$$

where, c = constant.

15 According to question, $\dfrac{u^2}{g} = 40$

$\therefore\quad u = 20\,\text{ms}^{-1}$ and $T = \dfrac{2u\sin 45°}{g}$

$$= 2 \times 20 \times \frac{1}{\sqrt{2}} \times \frac{1}{10}$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

When car is moving with speed,

$v = 20\,\text{ms}^{-1}$, then

$$(v\cos\theta + 20) \times t = 40$$

$$(20\cos\theta + 20) \times 2\sqrt{2} = 40$$

$$\Rightarrow\quad \cos^2\frac{\theta}{2} = \frac{1}{2\sqrt{2}} \approx 60°$$