Mathematics
Class – XII

Time allowed: **3** hours | Maximum Marks: **100**

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A
(1 marks)

1. If $\vec{a}$ and $\vec{b}$ are reciprocal vector, then $\vec{a}\cdot\vec{b}$ =?

2. If R = {(a, a^3): a is a prime no. less than 5} be a relation. Find the range of R.

3. write the value of $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

4. what is the principal value of $\tan^{-1}(-1)$?

Section B
(2 marks)

5. ABCD is a parallelogram with $\vec{AC} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{BD} = \hat{i} + 2\hat{j} - 5\hat{k}$ find area of this parallelogram?

6. Let $\vec{a}, \vec{b}, \vec{c}$ any three vectors. Then $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right]$ is always equal to?

7. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$, write the value of (a – 2b)

8. If f(x) = 2 + x^3 and g(x) = $x^{1/3}$, the find got (x).

9. If A and B are two event such that $P\left(\frac{A}{B}\right) = P,\ P(A) = P$

$P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{9}$ then P =?

10. Prove that $\cos^{-1}(x) + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\} = \frac{\pi}{3}$

11. The radius of a circle is increasing at rate of 0.7cm/s. what is the rate of increase of its circumference?

12. Given, $\int e^x (\tan x + 1) \sec x dx = e^x f(x) + c.$ write f(x) satisfying above.

Section C
(4 marks)

13. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2};\ x \in \left(0, \frac{\pi}{4}\right)$

14. using prop. Determinants, P·t· $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

15. Evaluate $\int (x-3)\ \sqrt{x^2 + 3x - 18}\ dx$

16. Find the vector and the Cartesian equations of the lines that pass through the original and (5, -2, 3)

17. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find value of $A^2 - 3A + 2I$

18. If y log x = x – y, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

19. If x = a (cosθ + θ sinθ) and y = b(sinθ – θ cosθ). Find $\frac{d^2y}{dx^2}$.

20. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

21. Probability of solving specific problem p independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively, if both try to solve the problem independently, find the probability that

(i) the problem is solved (ii) exactly one solved.

22. Evaluate $\int \frac{1}{1-\tan x} dx$

23. The sum of the perimeters of a circle and square is k, where k is some constant. Prove that the sum of their areas is least, when the side of the square is double the radius of the circle.

Section D
(6 marks)

24. Find the area bounded by curve $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$

25. In a hostel, 60% of students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English new papers. A student is selected at random.

(a) find the probability that she read neither Hindi nor English.

(b) If she read Hindi newspaper, find the probability she reads English newspaper.

(c) If she read English newspaper, find the probability that she reads to Hindi newspaper.

26. Show that the semi – vertical angle of the cone of the maximum volume and of given slant height is tan-1 $\sqrt{2}$?

27. Solve equation

$2x + 3y + 10z = 4$

$4x - 6y + 5z = 1$

$6x + 9y - 20z = 1$

28. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$

29. A diction wishes to mix together two kinds of food x and y in such a way that mixture contains at least 10 units of vitamin A, 12 unit of vitamin B and 8unit of vitamin c. The vitamin content of one kg food is Given

Food	Vitamin A	B	C
X	1	2	3
Y	2	2	1

Cost of x = 16, cost of y = 20. Find the least cost of the mixture which will produce the required diet.

Sol.1 If $\vec{a}$ and $\vec{b}$ are reciprocal, then $\vec{a}=\lambda\vec{b}$, $\lambda \in R^{+}$ and $|\vec{a}||\vec{b}|=1$

$\Rightarrow \quad |\vec{a}||\lambda||\vec{b}|$

$\Rightarrow \quad |\lambda|=\frac{|\vec{a}|}{|\vec{b}|}=\frac{1}{|\vec{b}|^{2}}$

$\Rightarrow \quad |\vec{a}|=\frac{1}{|\vec{b}|^{2}}\vec{b}$

$\Rightarrow \quad \vec{a}\cdot\vec{b}=\frac{1}{|\vec{b}|^{2}}|\vec{b}||\vec{b}|\cos 0=1$

Sol.2 Given, R = {(a, a^3)! A is prime number less than 5}
We know that, 2 and 3 are the prime No. less than 5.
∴ a can take values 2 and 3
Then R = {(2, 2^3), (3, 3^3)**}**
= {(2, 8), (3, 27)}
Hence, the range of R is {8, 27}

Sol.3 Let

$$\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

On taking 3x common from R_3 we get

$$\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$

$=\Delta=0$

[∵ R_1 and R_3 are identical].

Sol.4 we know that the principal value branch of tan x is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$\therefore \quad \tan^{-1}(-1)=\tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$=\tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right]$

$=\frac{-\pi}{4}\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

Hence, $\tan^{-1}(-1) = \frac{-\pi}{4}$

Sol.5 Area of parallelogram

$= \frac{1}{2}(\vec{AC} \times \vec{BD})$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ -1 & 2 & -5 \end{vmatrix}$$

$= \frac{1}{2}(8\hat{i} + 4\hat{j})$

$= 4\hat{i} + 2\hat{j}$

$\therefore$ Area of parallelogram

$= |4\hat{i} + 2\hat{j}| = 2\sqrt{5} sq.units.$

Sol.6 $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}]$

$= (\vec{a}+\vec{b}) \cdot \left[(\vec{b}+\vec{c}) \times (\vec{c}+\vec{a})\right]$

$= (\vec{a}+\vec{b}) \cdot \left[\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a}\right]$

$= \left[\vec{a} \cdot \vec{b}\, \vec{c}\right] \cdot \left[\vec{b}\, \vec{c}\, \vec{a}\right]$

$= 2\left[\vec{a}\, \vec{b}\, \vec{c}\right]$

Sol.7 Given

$$= \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$$

We know that two matrices are equal, if its corresponding elements are equal

$\therefore$ a + 4 = 2a + 2 ……(1)

3b = b + 2 …..(2)

On solving equation 1, 2 and 3, we get

a = 2 and b = 1

now a – 2b = 0

Sol.8 Given, f(x) = $27x^3$ and g(x) = $x^{\frac{1}{3}}$

$\therefore$ g of (x) = g[f(x)]

= g ($27x^3$)

$= (27x^3)^{\frac{1}{3}}$

$$= (27^3)^{\frac{1}{3}} \cdot (x^3)^{\frac{1}{3}}$$

$$= (3^3)^{\frac{1}{3}} \cdot (x^3)^{\frac{1}{3}}$$

$$= 3x$$

$\therefore$ g of (x) = 3x

Sol.9 $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$P = \frac{P(A \cap B)}{\frac{1}{3}}$$

$$\frac{P}{3} = P(A \cap B) \qquad \text{......(1)}$$

$$P(AUB) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{9} = \frac{P}{1} + \frac{1}{3} - \frac{P}{3}$$

$$\frac{5}{9} = \frac{3P + 1 - P}{3}$$

$$\frac{5}{3} - 1 = 2P$$

$$\frac{5-3}{3} = 2P$$

$$\frac{2}{3} = 2P$$

$$P = \frac{2}{6} = \frac{1}{3}$$

Sol.10 we have to prove

$$\cos^{-1} + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\} = \frac{\pi}{3}$$

L.H.S.

$$= \cos^{-1}(x) + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\}$$

Let $\cos^{-1}(x) = \alpha \Rightarrow x = \cos\alpha$

Then, L.H.S. = $\propto$ + $\cos^{-1}$

$$\left[\cos\alpha \cdot \cos\frac{\pi}{3} + \frac{\sqrt{3}}{2}\sqrt{1-\cos^2\alpha}\right]$$

$$= \alpha + \cos^{-1}\left[\cos\frac{\pi}{3}\cos\alpha + \sin\frac{\pi}{3}\sin\alpha\right]$$

$$\left[\because \quad \sin\alpha = \sqrt{1-\cos^2\alpha}, \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\right]$$

$$= \alpha + \cos^{-1}\left[\cos\left(\frac{\pi}{3} - \alpha\right)\right]$$

[∵ cos(A – B) = cosAcosB + sinAsinB]

$$= \alpha + \frac{\pi}{3} - \alpha \quad \Rightarrow \quad \frac{\pi}{3}$$

Sol.11 The circumference of a circle with radius is given by

C = 2πr

Therefore, the rate of change of circumference with respect to time is given by :-

$$\frac{dc}{dt} = \frac{dc}{dr}\cdot\frac{dr}{dt}$$

$$= \frac{d}{dr}(2\pi r)\frac{dr}{dt}$$

$$= 2\pi\cdot\frac{dt}{dt}$$

$$= 2\pi(0.7)$$

$$= 1.4\pi \text{ cm/s.}$$

Sol.12 Given that,

$$\int e^x(\tan x + 1)\sec x)dx = e^x \cdot f(x) + c$$

$$\Rightarrow \int e^x(\sec x + \sec x\tan x)dx = e^x f(x) + c$$

$$\Rightarrow e^x \cdot \sec x + c = e^x f(x) + c$$

$$\left[\because \quad e^x\left\{f(x) + f^1(x)\right\}dx = e^x f(x) \text{ and } \frac{d}{dx}(\sec x) = \sec\tan x\right]$$

On comparing both sides, we get

F(x) = secx.

Sol.13 L.H.S:-

$$\cot\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}} \times \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right]$$

$$= \cot^{-1}\left[\frac{(\sqrt{1+\sin x}+\sqrt{1-\sin x})^2}{(\sqrt{1+\sin x})^2 - (\sqrt{1-\sin x})^2}\right]$$

$$= \cot^{-1}\left[\frac{1+\sin x+1-\sin x+2\sqrt{1-\sin^2 x}}{1+\sin x-1+\sin x}\right]$$

$$= \cot^{-1}\left[\frac{2+2\cos x}{2\sin x}\right]$$

$$= \cot^{-1}\left[\frac{1+\cos x}{\sin x}\right] = \cot^{-1}\left[\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$$

$$\left[\because \cos x = 2\cos^2\frac{x}{2}-1 \text{ and } \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}\right]$$

$$= \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2} = R.H.S$$

Sol.14 $$L.H.S. = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

On applying $C_1 \to C_1 + C_2 + C_3$, we get

$$L.H.S. = \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix}$$

On taking common $(1 + x + x^2)$, from

$$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$$

On applying $R_2 \to R_2 - R_1$, $R_3 + R_3 - R_1$

$$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x & x(1-x) \\ 0 & x(x-1) & 1-x^2 \end{vmatrix}$$

On expanding along c_1, we get

$$= (1+x+x^2)\begin{vmatrix} 1-x & x(1-x) \\ -x(1-x) & (1-x)(1+x) \end{vmatrix}$$

Taking common

$$= (1+x+x^2)(1-x)^2(1+x+x^2)$$

$$= \left[(1+x+x^2)(1-x)\right]^2$$

$$= (1-x^3)^2 = \text{R.H.S}$$

Sol.15 Let

$$\int (x-3)\sqrt{x^2+3x-18}dx$$

Here, we can write $x-3 = A\frac{d}{dx}(x^2+3x-18)+B$

$\Rightarrow \quad x-3 = A(2x+3)+B$

On equation the coefficients of x and constant term from both sides, we get

2A = 1 and 3A + B = -3

$\Rightarrow \quad A = \frac{1}{2} \text{ and } 3\times\frac{1}{2}+B=-3$

$\Rightarrow \quad A = \frac{1}{2} \text{ and } B = \frac{-9}{2}$

Thus, the given integral reduces in the following form

$$I = \int \left\{\frac{1}{2}(2x+3-\frac{9}{2}\right\}\sqrt{x^2+3x-18}dx$$

$$\Rightarrow I = \frac{1}{2}\int (2x+3)\sqrt{x^2+3x-18}dx - \frac{9}{2}\int\sqrt{x^2+3x-18}dx$$

$$= \frac{1}{2}I_1 - \frac{9}{2}I_2$$

Where $= I_1 = \int (2x+3)\sqrt{x^2+3x-18}dx$

Put $x^2+3x-18=t$

$\Rightarrow \quad (2x+3)dx = dt$

$$\therefore \quad I_1 = \int t^{\frac{1}{2}}dt = \frac{2}{3}t^{\frac{3}{2}}+C_1$$

$$= \frac{2}{3}(x^2 3x-18)^{\frac{3}{2}}+C_1$$

and $I_2 = \int\sqrt{x^2+3x-18}dx$

$$= \int\sqrt{\left(x+\frac{3}{2}\right)^2-18-\frac{9}{4}}dx$$

$$= \int\sqrt{\left(x+\frac{3}{2}\right)^2-\frac{81}{4}}dx$$

$$= \int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)^2}dx$$

$$= \frac{x+\frac{3}{2}}{2}\sqrt{x^2+3x-18} - \frac{81}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x-18}\right|+c_2$$

$$= \frac{2x+3}{4}\sqrt{x^2+3x-18} - \frac{81}{8}\log\left|\frac{2x+3}{2}+\sqrt{x^2+3x-18}\right|$$

On putting the volume of I_1, and I_2 in I then

$$I = \frac{1}{2}\left[\frac{2}{3}(x^2+3x-18)^{\frac{3}{2}}+c_1\right]-\frac{9}{2}\left[\frac{2x+3}{4}\sqrt{x^2+3x-18}\right.$$

$$\left.-\frac{81}{2}\log\left|\frac{2x+3}{2}+\sqrt{x^2+3x-18}\right|+c_2\right.$$

$$\Rightarrow I = \frac{1}{3}(x^3+3x-18)^{\frac{3}{2}}-\frac{9}{8}(2x+3)\sqrt{x^3+3x-18)^{\frac{3}{2}}}$$

$$+\frac{729}{16}\log\left|\frac{2x+3}{2}+\sqrt{x^2+3x-18}\right|+c$$

Where $c = \frac{c_1}{2}-\frac{9c_2}{2}$

Sol.16 The required line – passes through the origin. Therefore, its position vector is given by,

$\vec{a} = \vec{o}$ .....(1)

The dir. Ratios of the line through origin and (5, -2, 3) are (5 – 0) = 5, (-2, -0) = -2

(3 – 0) = 3

The line is parallel to the vector given by the equation of the $\hat{b} = 5\hat{i}-2\hat{j}+3\hat{k}$.

The equation of the line in vector form through a point with position vector $\vec{a}$ and parallel to $\hat{b}$ is $\vec{r} = \vec{a}+\lambda\vec{b}; \lambda \in R$

$\Rightarrow \quad \vec{r} = 0+\lambda(5\hat{i}-2\hat{j}+3\hat{k})$

$\Rightarrow \quad \vec{r} = \lambda(5\hat{i}-2\hat{j}+3\hat{k})$

The equation of the line passes through the point (x_1 y_1 z_1) and dir. Ratios the point a, b, c is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Therefore, the equation of the required line in the Cartesian form is

$$\frac{x-0}{5} = \frac{y-0}{-2} = \frac{z-0}{3}$$

$$\Rightarrow \quad \frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$$

Sol.17 Given

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

We have to find the value of $A^2 - 3A + 2I$.

Now $A^2 = A : A$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix}$$

$$\text{And } 2I = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$\therefore$ $A^2 - 3A + 2I$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -4 \\ -3 & 2 & 0 \end{bmatrix}$$

Sol.18 Given $y \log x = x - y$ ....(1)

diff. w·r·t 'x'

$$y \times \frac{1}{x} + \log x \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{y}{x} + \log x \frac{dy}{dx} + \frac{dy}{dx} + \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(1 + \log x) = 1 - \frac{y}{x} = \frac{x - y}{x}$$

$$\frac{dy}{dx}(1 + \log x) = \frac{x - y}{x} \quad .....(2)$$

From 1st $y \log x = x - y$

Use in 2nd

$$\frac{dy}{dx}(1+\log x) = \frac{y\log x}{x}$$

Solve equation Ist

$$y\log x + y = x$$

$$y(\log x + 1) = x$$

$$(1+\log x) = \frac{x}{y} \qquad ......(3)$$

Use value of $\frac{x}{y}$ in equation

$$\frac{dy}{dx}(1+\log x) = \frac{\log x}{(1+\log x)}$$

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

= R.H.S

Sol.19 $x = a(\cos\theta + \theta\sin\theta)$

Diff. w·r·t 'θ'

$$\frac{dx}{d\theta}, a(-\sin\theta + \theta\cos\theta + \sin\theta)$$

$$= a\theta\cos\theta$$

$$\frac{dy}{d\theta} = b(\cos\theta + \theta\sin\theta - \cos\theta)$$

$$= b\theta\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{b\theta\sin\theta}{a\theta\cos\theta}$$

$$\frac{dy}{dx} = \frac{b}{a}\tan\theta$$

Diff. w·r·t 'x'

$$\frac{d^2y}{dx^2} = \frac{b}{a}\sec^2\theta\frac{d\theta}{dx}$$

$$= \frac{b}{a}\sec^2\theta \times \frac{1}{a\theta\cos\theta}$$

$$\frac{d^2y}{dx^2} == \frac{b}{a^2\theta}\sec^3\theta$$

Sol.20 The given diff. equation is:

$$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$$

$$\Rightarrow \quad \frac{\sec^2 x \tan dx + \sec^2 y \tan x dy}{\tan x \tan y} = 0$$

$$\Rightarrow \quad \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

$$\Rightarrow \quad \frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$$

Integrating both sides of this equation , we get :-

$$\int \frac{\sec^2}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy \quad .....(1)$$

Let tanx = t

$$\therefore \quad \frac{d}{dx}(\tan x) = \frac{dt}{dx}$$

$$\Rightarrow \quad \sec^2 x = \frac{dt}{dx}, \Rightarrow \quad \sec^2 x dx = dt$$

Now, $\int \frac{\sec^2 x}{\tan x} dx = \int \frac{1}{t} dt$

$= \log t$

$= \log(\tan x)$

Log; $\int \frac{\sec^2 y}{\tan y} dy = \log(\tan y)$

Substituting value in 1st

$$\log(\tan x) = \log(\tan y) + \log c$$

$$\Rightarrow \quad \log(\tan x) = \log\left(\frac{c}{\tan y}\right)$$

$$\Rightarrow \quad \tan x = \frac{c}{\tan y}$$

$$\Rightarrow \quad \tan x \tan y = c$$

This is the required solution of the Given equation

Sol.21 probability of solving the problem by A, $P(A) = \frac{1}{2}$

Probability of solving the problem by B, $P(B) = \frac{1}{3}$

Since the problem is solved independently by A and B,

$$\therefore \; P(AB) = P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$P(A^1) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(B^1) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$$

i) Probability that the problem is solved $= P(A \cup B)$

$$= P(A) + P(B) - P(AB)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6}$$

$$= \frac{2}{3}$$

ii) Probability that exactly one of them solved the problem is given by,

$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{3} + \frac{1}{6}$$

$$= \frac{1}{2}$$

Sol.22 **Let** $I = \int \frac{1}{1 - \tan x} dx$

$$= \int \frac{1}{1 - \frac{\sin x}{cox}} dx$$

$$= \int \frac{\cos x}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int \frac{2\cos x}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int 1dx + \frac{1}{2} \int \frac{\cos x + \sin xdx}{\cos x - \sin x}$$

$$= \frac{x}{2} + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Put cosx – sinx = t

$$(-\sin x - \cos x)dx = dt$$

$$I = \frac{x}{2} + \frac{1}{2} \int -\frac{dt}{t}$$

$$= \frac{x}{2} - \frac{1}{2} \log|t| + c$$

$$= \frac{x}{2} - \frac{1}{2} \log\left|\cos x - \sin x\right| + c$$

Sol.23 Let r be the radius of circle and x be the side of a square then, given that

Perimeter of square + circumference of circle = k

i.e. $4x+2\pi r=k$

$\Rightarrow \quad x=\dfrac{k-2\pi r}{4} \qquad$ ......(1)

Let A denotes the sum of their areas

$\therefore \quad A=x^2+\pi r^2 \qquad$ ......(2)

On putting the value of x we get

$$A=\left(\frac{k-2\pi r}{4}\right)^2+\pi r^2$$

On diff. w·r·t 'r'

$$\frac{dA}{dr}=2\left(\frac{k-2\pi r}{4}\right)\left(\frac{-2\pi}{4}\right)+2\pi r$$

$$=-\frac{\pi}{4}(k-2\pi r)+2\pi r$$

For max. and min. $\dfrac{dA}{dr}=0$

$\Rightarrow \quad -\dfrac{\pi}{4}(k-2\pi)+2\pi r=0$

$\Rightarrow \quad -\dfrac{\pi}{4}k+\dfrac{\pi^2 r}{2}+2\pi r=0$

$\Rightarrow \quad r=\dfrac{k}{2\pi+8} \qquad$ ......(3)

Now . $\dfrac{d^2A}{dr^2}=2\pi+\dfrac{2\pi^2}{4}$

$$=2\pi+\frac{\pi^2}{2}>0$$

$\therefore \quad \dfrac{d^2A}{dr^2}>0\Rightarrow$A is min.

from equation 3rd, we get

$r=\dfrac{k}{2\pi+8}$

$\Rightarrow \quad 2\pi r+8r=k$

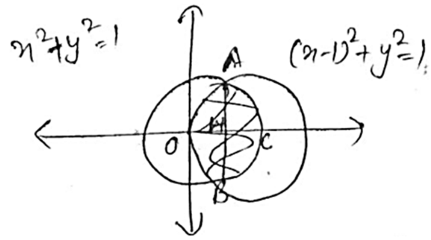
$\Rightarrow \quad 2\pi r+8r=4x+2\pi r$

$\Rightarrow \quad 8r=4x$

Or $x=2r$

Side of sq. = double the radius of circle

Sol.24 The area add by the curves, $(x-1)^2+y^2=1$ and x^2+y^2-1, is represented by



On solving the equation

$(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$

We obtain the point of intersection

$$A = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \text{ and } B = \left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$$

∴ Area OBCAO = 2 × area OCAO

$$= \left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}\,dx + \int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\,dx\right]$$

$$= \left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x\right]_{\frac{1}{2}}^{1}$$

$$= \left[-\frac{1}{4}\sqrt{1-\left(-\frac{1}{2}\right)^2} + \frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2}\sin^{-1}(-1)\right] + \left[\frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\sqrt{1-\left(\frac{1}{2}\right)^2}\right.$$

$$\left. + \frac{1}{2}\sin^{-1}\left[\frac{1}{2}\right]\right.$$

$$= \left[-\frac{\sqrt{3}}{8} + \frac{1}{2}\left(-\frac{\pi}{6}\right) - \frac{1}{2}\left(-\frac{\pi}{2}\right)\right] + \left[\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12}\right]$$

$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2}\right]$$

$$= \left[\frac{2\pi}{6} - \frac{\sqrt{3}}{4}\right]$$

Therefore, required area

$$OBCAO = 2\left(\frac{2\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) unit$$

Sol.25 Let H denotes the students who read Hindi newspaper and E denote the students who read English newspaper.

It is Given that,

$$P(H) = 60\% = \frac{6}{10} = \frac{3}{5}$$

$$P(E) = 40\% = \frac{40}{100} = \frac{2}{5}$$

$$P(H \cap E) = 20\% = \frac{1}{5}$$

i) Probability that a – student reads Hindi or English newspaper is

$$P(H \cup E)^1 = 1 - P(H \cup E)$$

$$= 1 - \{P(H) + P(E) - P(H \cap E)\}$$

$$= 1 - \frac{4}{5} = \frac{1}{5}$$

ii) $P\left(\frac{E}{H}\right) = \frac{P(E \cap H)}{P(H)}$

$$= \frac{\frac{1}{5}}{\frac{3}{5}} \qquad = \frac{1}{3}$$

iii) $P\left(\frac{H}{E}\right) = \frac{P(H \cap E)}{P(E)}$

$$= \frac{\frac{1}{5}}{\frac{2}{5}} \qquad = \frac{1}{2}$$

Sol.26 Let θ be the semi vertical angle of the cone:

It is clear that $\theta = \left[0, \frac{\pi}{2}\right]$

Let r, h and l be the radius height and slant height of the cone respectively.
The slant height of cone is constant.

R = l sinθ, h = l cosθ

$$v = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left[l^2 \sin^2\theta)(l\cos\theta\right]$$

$$= \frac{1}{3}\pi l^3 \sin^2\theta\cos\theta$$

$$\therefore \qquad \frac{dx}{d\theta} = \frac{l^3\pi}{3}\left[-\sin^3\theta + 2\sin\theta\cos^2\theta\right]$$

$$\frac{d^2v}{d\theta^2} = \frac{l^3\pi}{3}\left[2\cos^3\theta - 7\sin^2\theta\cos\theta\right]$$

Now $\frac{du}{d\theta} = 0$

$$\Rightarrow \qquad \sin^3\theta = 2\sin\theta\cos^2\theta$$

$$\Rightarrow \qquad \tan^2\theta = 2$$

$$\Rightarrow \qquad \tan\theta = \sqrt{2}$$

$$\Rightarrow \qquad \theta = \tan^{-1}\sqrt{2}$$

When $\theta = \tan^{-1}\sqrt{2}$, then $\tan^2\theta = 2$ or $\sin^2\theta = 2\cos^2\theta$

$$\frac{d^2v}{d\theta^2} = \frac{l^2\pi}{3}\left[2\cos^3\theta - 14\cos^3\theta\right]$$

$$= -4\pi l^3\cos^3\theta < 0 \text{ for } \theta \in \left[0, \frac{\pi}{2}\right]$$

By 2nd derivative test,

The volume is maximum

$\theta = \tan^{-1}\sqrt{2}$.

Hence, for a given height the semi-vertical angle of the cone of the maximum volume is $\tan^{-1}\sqrt{2}$.

Sol.27 Given that

2x + 3y + 10z = 4

4x – 6y + 5z = 1

Ax = B where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, \ x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$|A| = 150 + 330 + 720 = 1200$$

$A_{11} = 75$, $A_{12} = 110$,

$A_{13} = 72$, $A_{21} = 150$, $A_{22} = -100$

$A_{23} = 0$, $A_{31} = 75$

$A_{32} = 30$, $A_{33} = -24$.

$$\therefore \quad A^{-1} = \frac{1}{|A|} adj\ A$$

$$= \frac{1}{1200}\begin{vmatrix} 75 & 100 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{vmatrix}$$

Now

$X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

Sol.28 The equation of the plane through the point (-1, 3, 2) is

$a(x + 1) + b(y - 3) + C(z - 2) = 0$ ......(1)

where a, b, c are the direction ratios of normal to the plane

It is known that 2 Planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$

Are $\perp$, if $a_1a_2 + b_1b_2+c_1c_2 = 0$ Plane (1) if perpendicular to plane

$3x + 3y + 3 = 0$

$\therefore a \times 3 + b \times 3 + c \times 1 = 0$

$\Rightarrow 3a + 3b + c = 0$ ....(2)

Perpendicular to plane $x + 2y + 3z = 5$

$\therefore a\cdot 1 + b\cdot 2 + c\cdot 3 = 0$

$\Rightarrow a + 2b + 3c = 0$ ......(3)

From 2 & 3 equation

$$\frac{a}{-7} = \frac{b}{8} = \frac{c}{-3} = k$$

$\Rightarrow a = -7k, b = 8k, c = -3k$

Use a, b, c in equation 1st $- 7k(x + 1) + 8k(y - 3) -3k(z - 2) = 0$

$\Rightarrow -7x + 8y - 3z - 25 = 0$

$\Rightarrow 7x - 8y + 3z + 25 = 0$

This is required equation of plane.

Sol.29 Let the mixture contain xkg of food and ykg of food minimize

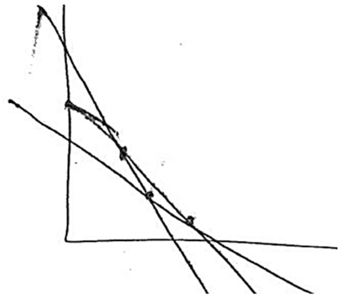
$Z = 16x + 20y$ .....(1)

$X + 2y \geq 10$ .....(2)

$X + y \geq 6$ .....(3)

$3x + y \geq 8$ .....(4)

$X, y \geq 0$ .....(5)



The corner point of the feasible region A(10, 0) B (2, 4) C(1, 5), D=(0, 8)

at $Z = 16x + 20y$

A(10, 0)	160
B(2, 4)	112 Minimum
C(1, 5)	116
D(0, 8)	160

Minimum value of z = 112

At (2, 4)