9. COLUMNS

STRUT

Structural member subjected to axial compressive load is called strut.

- **Column:** Vertical structural member fixed at both ends and subjected to axial compressive load is called column.

BUCKLING FAILURE : EULER'S THEORY

- Assumptions in Euler's Theory
  (i) Axis of column is *perfectly straight* when *unloaded*.
  (ii) Load passes through axis
  (iii) Stress in structure are within *elastic limit*.
  (iv) Flexural rigidity is constant.
  (v) Material is isotropic, homogeneous and linear elastic.
  (vi) Column is long and prismatic and it fails only in buckling.
- Limitation of Euler's Formula
  (i) There is always *crookedness* in the column and the load may not be exactly axial.
  (ii) This formula does not take into account the axial stress and the buckling load given by this formula may be much more than the actual buckling load.

$$P_e = \frac{\pi^2 EI_{min}}{l_e^2}$$

P_e = Buckling load
I_{min} = Min. Moment of inertia about centroidal axis
l_e = ***Effective*** length

> It is applicable for long column. Effect of crushing is neglected.

	Column	Fails in
1.	Short column	Crushing
2.	Long column	Buckling
3.	Intermediate column	Combined Crushing and Buckling

EULER'S LOAD FOR DIFFERENT COLUMN WITH DIFFERENT END CONDITION

End condition	Both end hinged	One end fixed other free	Both end fixed	One end fixed and other hinged
Effective length(l_e)	L	2L	$\frac{L}{2}$	$\frac{L}{\sqrt{2}}$

SLENDERNESS RATIO (λ)

Slenderness ratio of a compression member is defined as the ratio of its effective length to least radius of gyration.

$$\lambda = \frac{L_e}{r_{min}}$$

L_e = Effective length
r_{min} = Least radius of gyration

$$r_{min} = \sqrt{\frac{I_{min}}{A}}$$

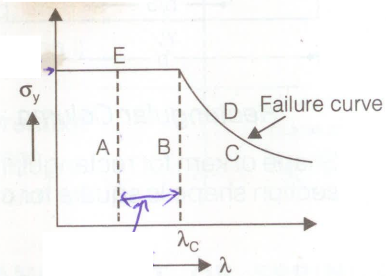
∴ Buckling stress $(\sigma_b) = \frac{P_e}{A} = \frac{\pi^2 E}{\lambda^2}$

For validity of Euler's theory

$$\sigma_e \le \sigma_y$$

$$\Rightarrow \lambda \ge \sqrt{\frac{\pi^2 E}{\sigma_y}}$$

$$\lambda_c = 90 \text{ for Mild steel}$$



Here, σ_y = Permissible stress
D = Unsafe long column
A = Safe short column
λ_c = Critical slenderness ratio
C = Safe long column
B = Intermediate safe column
E = Unsafe short column

RANKINE'S FORMULA

$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_e}$$

Rankine load = P_R
Crushing load = $P_C = \sigma_c \times A$

Buckling load = $P_e = \frac{\pi^2 EI_{min}}{L_e^2}$, $P_e = \frac{\pi^2 EA}{\lambda^2}$

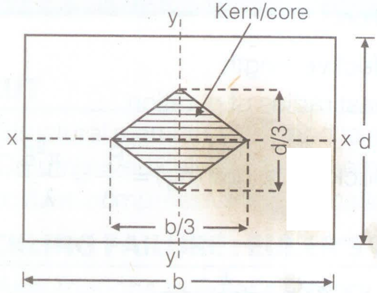
$$\therefore \quad P_r = \frac{A\sigma_c}{1+\left(\frac{\sigma_c}{\pi^2 E}\right)\lambda^2} \rightarrow P_r = \frac{\sigma_c A}{1+\alpha\lambda^2}$$

Here, A = Area of column

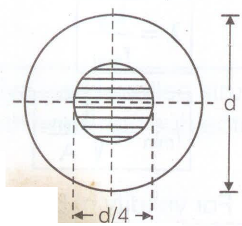
$\alpha = \frac{\sigma_c}{\pi^2 E}$ = Rankine's constant

> This formula is applicable to any column.
> Effect of both crushing and buckling is considered in this formula.

SHAPE OF KERN IN ECCENTRIC LOADINGS



Rectangular Column



Circular Column

Shape of kern for rectangular and I-section is Rhombus and for square section shape is square for circular section shape is circular.

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