Factorisation

- **Factorization** is the decomposition of an algebraic expression into product of factors. Factors of an algebraic term can be numbers or algebraic variables or algebraic expressions.

  For example, the factors of $2a^2b$ are 2, a, a, b, since $2a^2b = 2 \times a \times a \times b$
  The factors, 2, a, a, b, are said to be irreducible factors of $2a^2b$ since they cannot be expressed further as a product of factors.

  Also, $2a^2b = 1 \times 2 \times a \times a \times b$
  Therefore, 1 is also a factor of $2a^2b$. In fact, 1 is a factor of every term. However, we do not represent 1 as a separate factor of any term unless it is specially required.
  For example, the expression, $2x^2(x + 1)$, can be factorized as $2 \times x \times x \times (x + 1)$.
  Here, the algebraic expression $(x + 1)$ is a factor of $2x^2(x + 1)$.

- **Factorization of expressions by the method of common factors**

  This method involves the following steps.

  Step 1: Write each term of the expression as a product of irreducible factors.

  Step 2: Observe the factors, which are common to the terms and separate them.

  Step 3: Combine the remaining factors of each term by making use of distributive law.

  Example: Factorize $12p^2q + 8pq^2 + 18\,pq$.

  Solution: We have,
  $12p^2q = 2 \times 2 \times 3 \times p \times p \times q$
  $8pq^2 = 2 \times 2 \times 2 \times p \times q \times q$
  $18pq = 2 \times 3 \times 3 \times p \times q$
  The common factors are 2, p, and q.
  $\therefore 12p^2q + 8pq^2 + 18pq$
  $= 2 \times p \times q\,[(2 \times 3 \times p) + (2 \times 2 \times q) + (3 \times 3)]$
  $= 2pq\,(6p + 4q + 9)$

- **Factorization by regrouping terms**

  Sometimes, all terms in a given expression do not have a common factor. However, the terms can be grouped by trial and error method in such a way that all the terms in each group have a common factor. Then, there happens to occur a common factor amongst each group, which leads to the required factorization.

Example: Factorize $2a^2 - b + 2a - ab$.

Solution: $2a^2 - b + 2a - ab = 2a^2 + 2a - b - ab$
The terms, $2a^2$ and $2a$, have common factors, 2 and a.
The terms, $-b$ and $-ab$ have common factors, -1 and b.
Therefore,
$2a^2 - b + 2a - ab = 2a^2 + 2a - b - ab$
$= 2a(a + 1) - b(1 + a)$
$= (a + 1)(2a - b)$ (As the factor, $(1 + a)$, is common to both the terms)
Thus, the factors of the given expression are $(a + 1)$ and $(2a - b)$.

- **Some of the expressions can also be factorized by making use of the following identities.**

1. $a^2 + 2ab + b^2 = (a + b)^2$
2. $a^2 - 2ab + b^2 = (a - b)^2$
3. $a^2 - b^2 = (a + b)(a - b)$

For example, the expression $4x^2 + 12xy + 9y^2 - 4$ can be factorized as follows:
$4x^2 + 12xy + 9y^2 - 4$
$= (2x^2) + 2(2x)(3y) + (3y)^2 - 4$
$= (2x + 3y)^2 - 4$ [Using the identity, $a^2 + 2ab + b^2 = (a + b)^2$]
$= (2x + 3y)^2 - (2)^2$
$= (2x + 3y + 2)(2x + 3y - 2)$ [Using the identity, $a^2 - b^2 = (a + b)(a - b)$]

- **Factorization by using the identity, $x^2 + (a + b)x + ab = (x + a)(x + b)$.**
To apply this identity in an expression of the type $x^2 + px + q$, we observe the coefficient of x and the constant term.
Two numbers, a and b, are chosen such that their product is q and their sum is p.
i.e., $a + b = p$ and $ab = q$
Then, the expression, $x^2 + px + q$, becomes $(x + a)(x + \mathrm{b})$.

Example: Factorize $a^2 - 2a - 8$.

Solution: Observe that, $-8 = (-4) \times 2$ and $(-4) + 2 = -2$
Therefore, $a^2 - 2a - 8 = a^2 - 4a + 2a - 8$
$= a(a - 4) + 2(a - 4)$
$= (a - 4)(a + 2)$

- Division of any polynomial by a monomial is carried out either by dividing each term of the polynomial by the monomial or by the common factor method.

For example, $(8x^3 + 4x^2\mathrm{y} + 6x\mathrm{y}^2)$ can be divided by $2x$ as follows:

$$\left(8x^3 + 4x^2y + 6xy^2\right) \div 2x = \frac{8x^3+4x^2y+6xy^2}{2x}$$

$$= \frac{8x^3}{2x} + \frac{4x^2y}{2x} + \frac{6xy^2}{2x}$$

$$= 4x^2 + 2xy + 3y^2$$

Or,

$$\left(8x^3 + 4x^2y = 6xy^2\right) \div 2x = \frac{2\times x\left(4x^2+2xy+3y^2\right)}{2\times x} = 4x^2 + 2xy$$