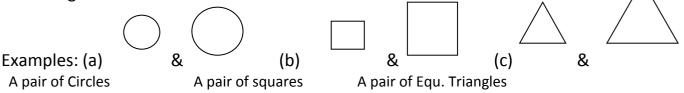
Triangles

<u>Key Points</u>

Similar Figures: Two figures having similar shapes (size may or may not same), called Similar figures.



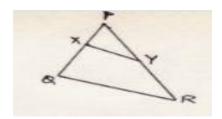
- Pairs of all regular polygons, containing equal number of sides are examples of Similar Figures.
- **<u>Similar Triangles:</u>** Two Triangles are said to be similar if
 - (a) Their corresponding angles are equal (also called Equiangular Triangles)
 - (b) Ratio of their corresponding sides are equal/proportional
- All congruent figures are similar but similar figures may /may not congruent
- Conditions for similarity of two Triangles
 - (a) AAA criterion/A-A corollary
 - (b) SAS similarity criterion
 - (c) SSS similarity criterion (where 'S' stands for ratio of corresponding sides of two Triangles)

Important Theorems of the topicTriangles

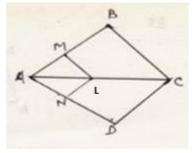
- (a) Basic Proportionality Theorem (B.P.T.)/Thale's Theorem
- (b) Converse of B.P.T.
- (c) Area related theorem of Similar Triangles
- (d) Pythagoras Theorem
- (e) Converse of Pythagoras Theorem

<u>Level I</u>

(1) In the figure XY // QR , PQ/ XQ = 7/3 and PR =6.3cm then find YR



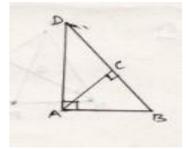
- (2) If $\triangle ABC \sim \triangle DEF$ and their areas be $64 \text{cm}^2 \& 121 \text{cm}^2$ respectively, then find BC if EF =15.4 cm
- (3) ABC is an isosceles Δ , right angled at C then prove that $AB^2 = 2AC^2$
- (4) If $\triangle ABC \sim \triangle DEF$, $\Box A=46^{\circ}$, $\Box E=62^{\circ}$ then the measure of $\Box C=72^{\circ}$. Is it true? Give reason.
- (5) The ratio of the corresponding sides of two similar triangles is 16:25 then find the ratio of their perimeters.
- (6) A man goes 24 km in due east and then He goes 10 km in due north. How far is He from the starting Point?
- (7) The length of the diagonals of a rhombus is 16cm & 12cm respectively then find the perimeter of the rhombus.
- (8) In the figure LM //CB and LN // CD then prove that AM/AB = AN /AD



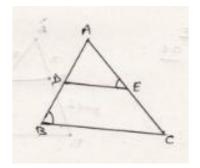
(9) Which one is the sides of a right angled triangles among the following (a) 6cm,8cm & 11cm (b) 3cm,4cm & 6cm (c) 5cm , 12cm & 13cm

Level II

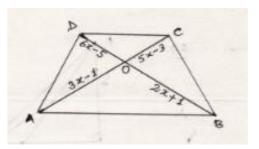
 In the figure ABD is a triangle right angled at A and AC is perpendicular to BD then show that AC²= BC x DC



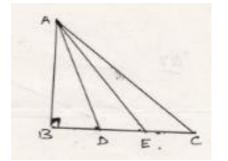
- (2) Two poles of height 10m & 15 m stand vertically on a plane ground. If the distance between their feet is 5v3m then find the distance between their tops.
- (3) D & E are the points on the sides AB & AC of \triangle ABC, as shown in the figure. If $_B = _$ AED then show that \triangle ABC \sim \triangle AED



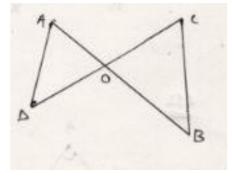
(4) In the adjoining figure AB // DC and diagonal AC & BD intersect at point O. If AO = (3x-1)cm, OB=(2x+1)cm, OC=(5x-3)cm and OD=(6x-5)cm then find the value of x.



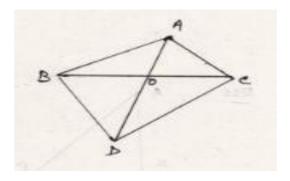
(5) In the figure D &E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$



(6) In the figure OA/OC = OD /OB then prove that $\bot A = \bot C$

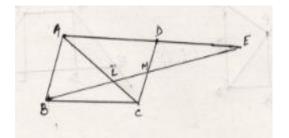


- (7) Using converse of B.P.T. prove that the line joining the mid points of any two sides of a triangle is parallel to the third side of the triangle.
- (8) In the given figure $\triangle ABC \& \triangle DBC$ are on the same base BC. if AD intersect BC at O then prove that $ar(\triangle ABC)/ar(\triangle DBC) = AO/DO$

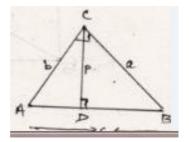


Level III

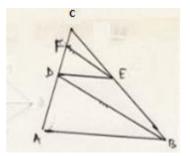
- (1) A point O is in the interior of a rectangle ABCD, is joined with each of the vertices A, B, C & D. Prove that $OA^2 + OC^2 = OB^2 + OD^2$
- (2) In an equilateral triangle ABC, D is a point on the base BC such that BD= 1/3 BC ,then show that $9AD^2=7AB^2$
- (3) Prove that in a rhombus, sum of squares of the sides is equal to the sum of the squares of its diagonals
- (4) In the adjoining figure ABCD is a parallelogram. Through the midpoint M of the side CD, a line is drawn which cuts diagonal AC at L and AD produced at E. Prove that EL =2BL



- (5) ABC & DBC are two triangles on the same base BC and on the same side of BC with $\bot A = \bot D = 90^{\circ}$. If CA & BD meet each other at E then show that AE x EC = BE x ED
- (6) ABC is a Triangle, right angle at C and p is the length of the perpendicular drawn from C to AB. By expressing the area of the triangle in two ways show that (i) pc = ab (ii) $1/p^2 = 1/a^2 + 1/b^2$

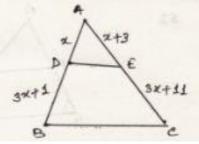


- (7) Prove that the ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.
- (8) In the figure AB|| DE and BD|| EF. Prove that $DC^2 = CF \times AC$

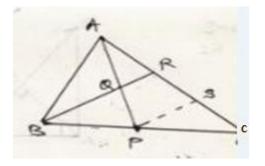


Self-Evaluation Questions including Board Questions & Value Based Questions

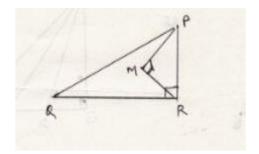
(1) Find the value of x for which DE ||BC in the adjoining figure



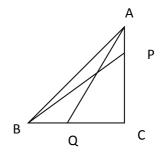
- (2) In an equilateral triangle prove that three times the square of one side is equal to four times the square of one of its altitude.
- (3) The perpendicular from A on the side BC of a triangle ABC intersect BC at D such that DB = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$
- (4) In the adjoining figure P is the midpoint of BC and Q is the midpoint of AP. If BQ when produced meets AC at R , then prove that RA = 1/3 CA



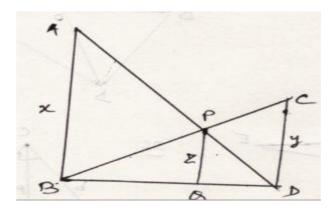
- (5) BL and CM are medians of triangle ABC , right angled at A then prove that $4(BL^2+CM^2)=5BC^2$
- (6) In \triangle ABC if AB =6 \vee 3cm , AC =12cm and BC=6cm then show that $_B = 90^{\circ}$
- (7) In the adjoining figure $\angle QRP = 90^{\circ}$, $\angle PMR = 90^{\circ}$, QR = 26 cm, PM = 8 cm and MR = 6 cm then find the area of $\triangle PQR$



- (8) If the ratio of the corresponding sides of two similar triangles is 2:3 then find the ratio of their corresponding altitudes.
- (9) In the adjoining figure ABC is a Δ right angled at C. P& Q are the points on the sides CA & CB respectively which divides these sides in the ratio 2:1, then prove that $9(AQ^2 + BP^2) = 13 AB^2$

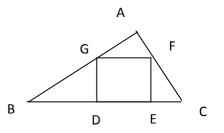


(10) In the adjoining figure AB || PQ ||CD, AB =x unit, CD = y unit & PQ = z unit then prove that 1/x + 1/y = 1/z



(11)State and prove Pythagoras theorem. Using this theorem find the distance between the tops of two vertical poles of height 12m & 18m respectively fixed at a distance of 8m apart from each other.

(12) in the adjoining figure DEFG is a square & \bot BAC= 90⁰ then prove that (a) \triangle AGF ~ \triangle DBG (B) \triangle AGF ~ \triangle EFC (C) \triangle DBG ~ \triangle EFC (D)DE² = BD X EC



(13) A man steadily goes 4 m due east and then 3m due north .Find

- (a) Distance from initial point to last point.
- (b) What mathematical concept is used in this problem?

(c) What is its value?

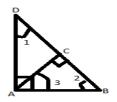
Solutions

<u>Level I</u>

(1) By B.P.T. PQ/XQ=PR/YR⇒ 7/3=6.3/YR ⇒ YR= 3x6.3/7=2.7 So YR=2.7cm (2) By theorem Ar of $\Delta ABC/Ar$ of $\Delta DEF = BC^2/15.4^2$ $\Rightarrow 64/121 = BC^2/15.4^2 \Rightarrow$ solving BC = 11.2 cm (3) By Pythagoras theorem $AB^2 = AC^2 + BC^2 \Rightarrow AB^2 = AC^2 + AC^2$ (given that AC = BC) So AB²=2AC² (4) $\triangle ABC \sim \triangle DEF \Rightarrow \angle A = \angle D = 46^\circ$, $\angle B = \angle E = 62^\circ$ so $\angle C = 180 \cdot (46 + 62) = 72^\circ$ So it is true. (5) Let $\triangle ABC \sim \triangle DEF$ then AB/DE= BC/EF=AC/DF= perimeter of Δ ABC/Perimeter of Δ DEF \Rightarrow AB/DE=perimeter of \triangle ABC/Perimeter of \triangle DEF So perimeter of $\Delta ABC/Perimeter$ of $\Delta DEF=16:25$ (6) By Pythagoras theorem , Distance = $\sqrt{24^2 + 10^2}$ On Solving , distance =26km (7) In $\triangle AOD$, by Pythagoras theorem $AD = \sqrt{6^2 + 8^2}$ ⇒AD= 10cm So perimeter of Rhombus = 4x10cm = 40cm (8) In $\triangle ABC$, LM//BC so by BPT AM/AB=AL/AC-----(i) Similarly in ΔACD , LN//DC, so by BPT AN/AD = AL/AC-----(ii) Comparing results I & ii we get AM/AB= AN/AD

Using Pythagoras thermo ,finding the value of p²+b²&h² separately in each case , it comes equal in case of c where p²+b² comes equal to h² So sides given in question c is the sides of right triangle

Level II

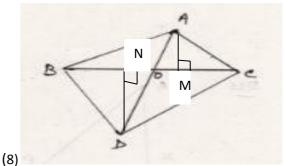


(1)<u>In</u> \triangle ABDABC $\angle 2 + \angle 3 = 90^{\circ}$ $\Rightarrow \angle 1 + \angle 2 = \angle 2 + \angle 3$ $\Rightarrow \angle 1 = \angle 3$ $\triangle ACD \sim \triangle BCA$ $\Rightarrow AC/BC = CD/AC$ So $AC^2 = BC \times CD$ (2) Using Pythagoras theorem Distance between their tops = $\sqrt{5^2 + (5\sqrt{3})^2}$ $\sqrt{25+75}$ Distance between their tops= 10m (3)In $\triangle AED \& \triangle ABC$ $\angle AED = \angle ABC(given)$ $\angle A = \angle A(\text{common})$ By AA corollary Δ ABC ~ Δ AED (4)Diagonals of a trapezium divide each proportionally So AO/OC = BO/OD3x-1/5x-3 = 2x+1/6x-5 $\Rightarrow 8x^2 - 20x + 8 = 0$ Solving we get x=2 & 1/2(na)So x=2(5) BD=DE=EC=P(let)BE=2P &BC=3P In Rt \triangle ABD , $AD^2 = AB^2 + BD^2$ $=AB^2+p^2$ In Rt \triangle ABE, AE²= AB²+BE² $=AB^{2}+(2p)^{2}$ $=AB^2+4p^2$ In Rt \triangle ABC,AC²=AB²+BC² $=AB^{2}+(3p)^{2}$ $=AB^2+9p^2$ Now taking RHS 3AC²+5AD² =3(AB2+9p2)+5(AB2+p2) $=8AB^{2}+32p^{2}$ $=8(AB^{2}+4p^{2})$ $=8AE^2$ =LHS(6)OA/OC=OD/OB(given) ⇒OA/OD=OC/OB $\& \angle AOD = \angle BOC(v.o. \angle s)$ By SAS similarity condition $\triangle AOD \sim \triangle COB$ ⇒∠A=∠C A (7)D Е

В

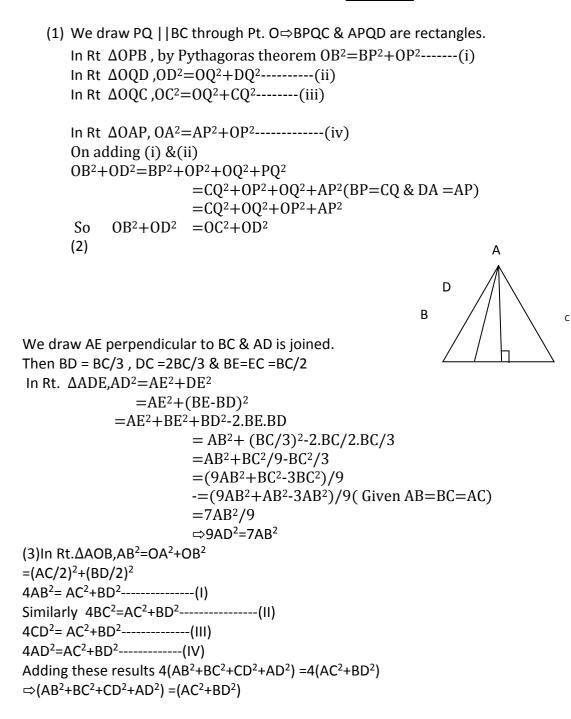
С

Given that AD/DE=1 &AE/EC=1(as D &E are mid points of the sides AB & AC) \Rightarrow AD/DB =AE/EC By converse of BPT DE//BC



We draw perpendiculars AM & DN as shown . $\Delta DON \sim \Delta AOM$ (by AA corollary) DN/AM =OD/OA \Rightarrow AM/DN=OA/OD------(i) Ar of $\Delta ABC/Ar$ of $\Delta DBC = (1/2xBCx AM)/(1/2x BC x DN)$ =AM/DN Ar of $\Delta ABC/Ar$ of $\Delta DBC = AO/OD$ (from (i))

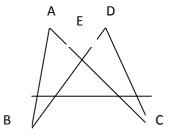
Level III



Е

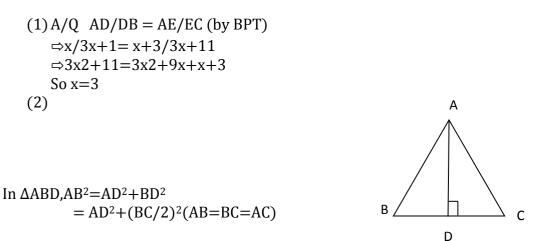
(4) Δ BMC ≅ Δ EDM(by ASA criterion) ⇒ by cpct DE=BC & AD =BC (opp. sides of //gm) Adding above results AD+DE=BC+BC ⇒ AE =2BC Now Δ AEL~ Δ CBL (By AA corollary) EL/BL=AE/BC⇔EL/BL =2BC/BC ⇔EL =2BL

(5) Δ AEB~ Δ DEC(AA corollary) AE/DE=EB/EC ⇔AE X EC= BE X ED

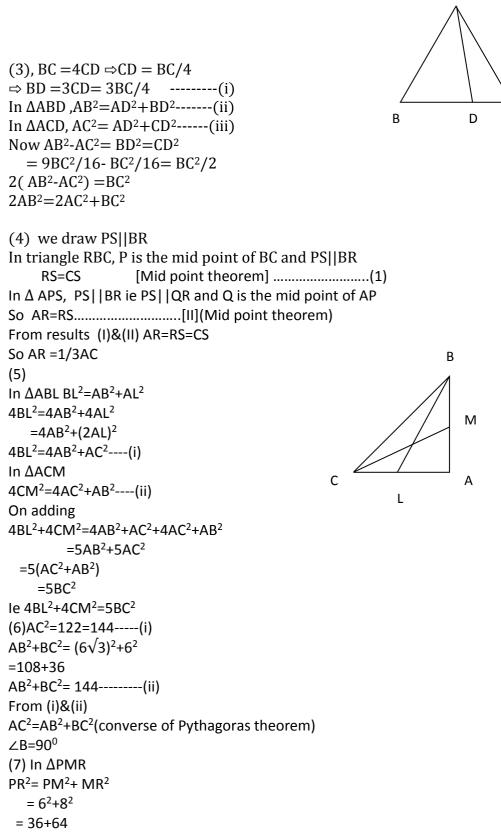


(6)Ar of $\triangle ABC=1/2x AB X DC$ =1/2 X c Xp = pc/2Again Ar of \triangle ABC= $\frac{1}{2}$ x AC X BC $=1/2 \times b \times a$ = ab/2Comparing above two areas ab/2 = pc/2⇒ab=pc Now in Rt \triangle ABC,AB²=BC²+AC₂ $c^2 = a^2 + b^2$ $(ab/p)^2 = a^2 + b^2)(ab = pc \Rightarrow c = ab/p)$ $a^{2}b^{2}/p^{2}=a^{2}+b^{2}$ $1/p^2 = a^2 + b^2/a^2b^2$ $1/p^2 = 1/a^2 + 1/b^2$ (7) Theorem question, as proved (8) In \triangle ABC,AB //DE , by BPT AC/DC BC/CE-----(i) In Δ DBC, EF//BD, by BPT DC/CF = BC/EC -----(ii) Comparing (i) &(ii) AC/DC=DC/CF \Rightarrow DC²=AC X CF

Self-Evaluation Questions



 $-= AD^{2} + AB^{2}/4$ $4AB^2 = 4AD^2 + AB^2$ $4AB^2-AB^2=4AD^2$ $3AB^2 = 4AD^2$



С

А

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= 100
PR= 10cm
In \Delta PQR PQ^2 = QR^2 - PR^2
       = 26^2 - 10^2
    =676-100
  =576
PQ= 24cm
Now Area of \triangle PQR = \frac{1}{2} \times PR \times PQ
     = \frac{1}{2} \times 10 \times 24
   = 120 \text{ cm}^2
                8. Ratio of areas of two similar \Delta^{s} is equal to the ratio of squares of corresponding sides
                        So Ratio of areas of two similar \Delta^{s} = (2x/3x)^2 = 4/9
                        So Ratio of areas of two similar \Delta^{s} = ratio of squares of their corresponding altitudes=
                        4/9
                        So, Ratio of corresponding altitudes = 4/9
                9. P divide CA in the ratio 2 :1 Therefore
                CP = 2/3 AC .....(i)
                QC = 2/3BC .....(ii)
                In Right Triangle ACQ<sup>,</sup>
                AQ^2 = QC^2 + AC^2
                Or, AQ^2 = 4/9 BC^2 + AC^2 (QC = 2/3 BC)
                Or, 9 AQ^2 = 4 BC^2 + 9 AC^2 .....(iii)
                Similarly, In Right Triangle BCP
                  9BP<sup>2</sup>= 9BC<sup>2</sup> + 4 AC<sup>2</sup> .....(iv)
                Adding eq. (iii) & (iv)
                9(AQ^2 + BQ^2) = 13(BC^2 + AC^2)
                9(AQ^2 + BQ^2) = 13AB^2
                10.In triangle ABD,
                PQ !! AB
                 PQ/AB = DQ/BD
                Or, Z/X=DQ/BD.....(i)
                In triangle BCD,
                PQ !! CD
                PQ/CD=BQ/BD
                Or, Z/Y=BQ/BD.....(ii)
                Adding eq. (i) & (ii)
                Z/X + Z/Y = DQ/BD + BQ/BD = DQ + BQ/BD
                Or, Z/X + Z/Y = BD/BD=1
                Or, 1/X + 1/y = 1/Z
                11. State and Prove Pythagoras Theorem
                AP = AB - PB = (18 - 12)m = 6m
                                                        [PB = CD = pm]
                Pc = BD = 8m
                In ∆ACP
                AC = \sqrt{AP^2 + PC^2}
                = \sqrt{(8)^2 + (6)^2}
                =\sqrt{64} + 36 = \sqrt{100} = 10
                AC = 10 m
                (12)DE//GF &AC cuts them
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 $\Rightarrow \angle DAG = \angle FGC(corres. \angle^{s})$ $\angle GDE = 90^{0} \Rightarrow \angle GDA = 90^{0}$ $\Delta ADG \sim \Delta GCF (By AA corollary , shown above)$ (ii) similarly $\Delta FEB \sim \Delta GCF$ Since $\Delta ADG \otimes \Delta FEB$ are both similar to ΔGCF $\Rightarrow \Delta ADG \sim \Delta FEB$ (iii) $\Delta ADG \sim \Delta FEB$ AD/FE = DG/FB $\Rightarrow AD/DG = EF/EB$ (iv) $\Delta ADG \sim \Delta FEB$ AD/FE = DG/FB $\Rightarrow AD/DE = DE/EB(FE = DG = DE)$ $DE^{2} = AD X EB$

(13)(i)distance from the initial point= $\sqrt{3^2+4^2}$ = $\sqrt{25}$ =5m (ii) Pythagoras theorem (iii) To save time & energy