Short Answer Type Questions – I

[2 marks]

Que 1. In which of the following situations, does the list of numbers involved to make an AP? If yes, give reason.
(i) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.
(ii) The amount of money in the account every year, when ₹ 10,000 is deposited at simple interest at 8% per annum.

Sol. (i) The numbers imvolved are 150, 200, 250, 300, …
Here 200 - 150 = 250 - 200 = 300 - 250 and so on
∴ It forms an AP with a = 150, d = 50
(ii) The numbers involved are 10,800, 11,600, 12,400, …
which forms an AP with a = 10,800 and d = 800.

Que 2. Find the 20th term from the last term of the AP, 3, 8, 13,…, 253.

Sol. We have, last term = l = 253
And, common difference = d = 2nd term - 1st term = 8 – 3 = 5
Therefore, 20th term from end = l - (20 - 1) × d = 253 – 19 x 5 = 253 – 95 = 158.

Que 3. If the sum of the first p terms of an AP is $ap^2 + bp$, find its common difference.

Sol. $a_p = S_{p-1} = (ap^2 + bp) – [a (p – 1)^2 + b (p – 1)]$

$= ap^2 + bp – (ap^2 + a – 2ap + bp – b)$

$= ap^2 + bp – ap^2 – a + 2ap – bp + b = 2ap + b – a$

∴ $a_1 = 2a + b – a = a + b$ and $a_2 = 4a + b – a = 3a + b$

⇒ $d = a_2 – a_1 = (3a + b) – (a + b) = 2a$

Que 4. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Sol. Let the first term be 'a' and common difference be 'd'.
Given, $a = 5$, $T_n = 45$, $S_n = 400$
$T_n = a + (n – 1)d$ ⇒ $45 = 5 + (n – 1)d$
⇒ $(n – 1)d = 40$ … (i)
$S_n = \frac{n}{2}(a + T_n)$ ⇒ $400 = \frac{n}{2}(5 + 45)$
⇒ $n = 2 \times 8 = 16$

Substituting the value of n in (i)
$(16 – 1)d = 40$ ⇒ $d = \frac{40}{15} = \frac{8}{3}$

Que 5. Find the number of natural number between 101 and 999 which are divisible by both 2 and 5.

Sol. Given: $a_1 = 110$, $d = 10$, $a_n = 990$

We know, $a_n = a_1 + (n - 1)d$

$990 = 110 + (n - 1)10$

$(n - 1) = \frac{990 - 110}{10} \Rightarrow n = 88 + 1 = 89$

Que 6. The sum of the first n terms of an AP is $3n^2 + 6n$. Find the nth terms of the AP.

Sol. Given: $S_n = 3n^2 + 6n$

$S_{n-1} = 3(n - 1)^2 + 6(n - 1) = 3(n^2 + 1 - 2n) + 6n - 6$

$= 3n^2 + 3 - 6n + 6n - 6 = 3n^2 - 3$

The n^{th} terms will be a_n

$Sn = S_{n-1} + a_n$

$a_n = S_n - S_{n-1} = 3n^2 + 6n - 3n^2 + 3 = 6n + 3$

Que 7. How many terms of the AP 18, 16, 14, ... be taken so that their sum is zero?

Sol. Here, $a = 18$, $d = -2$, $S_n = 0$

Therefore, $\frac{n}{2}[36 + (n - 1) - 2] = 0$

$\Rightarrow$ $n(36 - 2n + 2) = 0 \Rightarrow n = 19$

Que 8. The 4th term of an AP is zero. Prove that the 25th term of the AP is three times its 11th term.

Sol. $\therefore$ $a_4 = 0$ (Given)

$\Rightarrow a + 3d = 0 \Rightarrow a = -3d$

$a_{25} = a + 24d = -3d + 24d = 21d$

$3a_{11} = 3(a + 10d) = 3(7d) = 21d$

$\therefore a_{25} = 3a_{11}$ Hence proved.

Que 9. If the ratio of sum of the first m and n term of an AP is $m^2 : n^2$, show that ratio of its mth and nth terms is (2m – 1) : (2n – 1).

Sol. $\frac{s_m}{s_n} = \frac{m^2}{n^2} = \frac{\frac{m}{2}}{\frac{n}{2}} \frac{(2a + (m-1)d)}{(2a + (n-1)d)}$

$\Rightarrow \frac{m}{n} = \frac{2a+(m-1)d}{2a+(n-1)d}$ $\Rightarrow$ 2am + mnd – md = 2an + mnd – nd

$\Rightarrow$ a (2m – 2n) = d (m – n) $\Rightarrow$ 2a = d

$$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+2(m-1)a}{a+2(n-1)a} = \frac{2m-1}{2n-1}$$ Hence proved.