

Kinetic Theory



TOPIC 1 Kinetic Theory of an Ideal Gas and Gas Laws



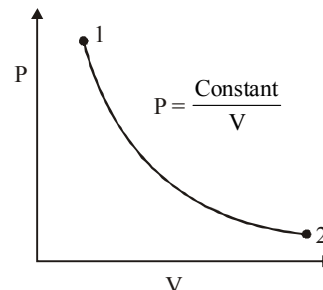
- Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2 . The ratio P_2/P_1 is _____. [NA Sep. 06, 2020 (I)]
- The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm. respectively. If $|\Delta T| = C |\Delta P|$, then value of C in (K/atm.) is _____. [NA Sep. 04, 2020 (II)]
- The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-ar^4}$. Then the total number of molecules is proportional to : [12 April 2019 II]
 - $n_0 \alpha^{-3/4}$
 - $\sqrt{n_0} \alpha^{1/2}$
 - $n_0 \alpha^{1/4}$
 - $n_0 \alpha^{-3}$
- A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1 , and that below the piston is l_2 , such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T . If the piston is stationary, its mass, m , will be given by: (R is universal gas constant and g is the acceleration due to gravity) [12 Jan. 2019 II]

$$(a) \frac{RT}{ng} \left[\frac{l_1 - 3l_2}{l_1 l_2} \right]$$

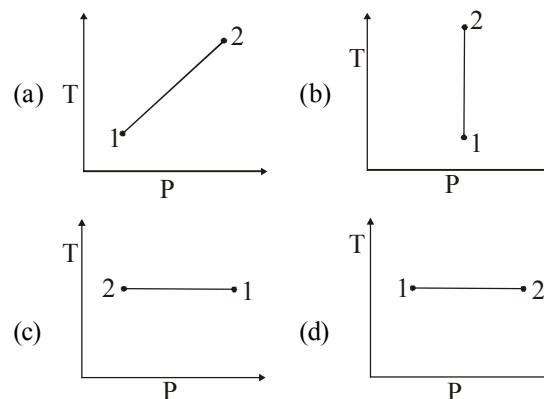
$$(b) \frac{RT}{g} \left[\frac{2l_1 + l_2}{l_1 l_2} \right]$$

$$(c) \frac{nRT}{g} \left[\frac{1}{l_2} + \frac{1}{l_1} \right] \quad (d) \frac{nRT}{g} \left[\frac{l_1 - l_2}{l_1 l_2} \right]$$

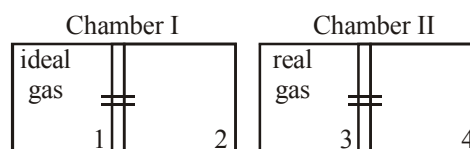
- The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains $1 \times 10^5 \text{ Pa}$. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be : [2017]
 - 2.5×10^{25}
 - -2.5×10^{25}
 - -1.61×10^{23}
 - 1.38×10^{23}
- For the P-V diagram given for an ideal gas,



out of the following which one correctly represents the T-P diagram ? [Online April 9, 2017]



7.



There are two identical chambers, completely thermally insulated from surroundings. Both chambers have a partition wall dividing the chambers in two compartments. Compartment 1 is filled with an ideal gas and Compartment 3 is filled with a real gas. Compartments 2 and 4 are vacuum. A small hole (orifice) is made in the partition walls and the gases are allowed to expand in vacuum.

Statement-1: No change in the temperature of the gas takes place when ideal gas expands in vacuum. However, the temperature of real gas goes down (cooling) when it expands in vacuum.

Statement-2: The internal energy of an ideal gas is only kinetic. The internal energy of a real gas is kinetic as well as potential. **[Online April 9, 2013]**

- (a) Statement-1 is false and Statement-2 is true.
 (b) Statement-1 and Statement-2 both are true. Statement-2 is the correct explanation of Statement-1.
 (c) Statement-1 is true and Statement-2 is false.
 (d) Statement-1 and Statement-2 both are true. Statement-2 is **not** correct explanation of Statement-1.
8. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will **[2002]**
- (a) increase
 (b) decrease
 (c) remain same
 (d) decrease for some, while increase for others

TOPIC 2 Speed of Gas, Pressure and Kinetic Energy



9. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is $4 \times 10^{-14} \text{ erg}$, $g = 980 \text{ cm/s}^2$, density of mercury = 13.6 g/cm^3) **[Sep. 05, 2020 (I)]**
- (a) 4.0×10^{18} (b) 4.0×10^{16}
 (c) 5.8×10^{16} (d) 5.8×10^{18}
10. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is _____. (Molar mass of N_2 gas 28 g); **[NA Sep. 05, 2020 (II)]**
11. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C . At 2 atm pressure and at 227°C , the rms speed of the molecules will be: **[9 April 2019 I]**
- (a) 100 m/s (b) $80\sqrt{5}$ m/s
 (c) $100\sqrt{5}$ m/s (d) 80 m/s
12. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s , the pressure exerted by the gas molecules will be of the order of: **[8 April 2019 I]**

- (a) 10^4 N/m^2 (b) 10^8 N/m^2
 (c) 10^3 N/m^2 (d) 10^{16} N/m^2
13. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to: **[8 April 2019 II]**
- [Boltzmann Constant $k_B = 1.38 \times 10^{-23} \text{ J/K}$
 Avogadro Number $N_A = 6.02 \times 10^{26} / \text{kg}$
 Radius of Earth : $6.4 \times 10^6 \text{ m}$
 Gravitational acceleration on Earth = 10 ms^{-2}]
- (a) 800 K (b) $3 \times 10^5 \text{ K}$
 (c) 10^4 K (d) 650 K
14. A mixture of 2 moles of helium gas (atomic mass = $4u$), and 1 mole of argon gas (atomic mass = $40u$) is kept at 300 K in a container. The ratio of their rms speeds **[9 Jan. 2019 I]**
- $\left[\frac{V_{\text{rms}}(\text{helium})}{V_{\text{rms}}(\text{argon})} \right]$ is close to :
- (a) 3.16 (b) 0.32
 (c) 0.45 (d) 2.24
15. N moles of a diatomic gas in a cylinder are at a temperature T . Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas? **[Online April 9, 2017]**
- (a) $\frac{1}{2}nRT$ (b) 0
 (c) $\frac{3}{2}nRT$ (d) $\frac{5}{2}nRT$
16. In an ideal gas at temperature T , the average force that a molecule applies on the walls of a closed container depends on T as T^q . A good estimate for q is: **[Online April 10, 2015]**
- (a) $\frac{1}{2}$ (b) 2
 (c) 1 (d) $\frac{1}{4}$
17. A gas molecule of mass M at the surface of the Earth has kinetic energy equivalent to 0°C . If it were to go up straight without colliding with any other molecules, how high it would rise? Assume that the height attained is much less than radius of the earth. (k_B is Boltzmann constant). **[Online April 19, 2014]**
- (a) 0 (b) $\frac{273k_B}{2Mg}$
 (c) $\frac{546k_B}{3Mg}$ (d) $\frac{819k_B}{2Mg}$
18. At room temperature a diatomic gas is found to have an r.m.s. speed of 1930 ms^{-1} . The gas is: **[Online April 12, 2014]**
- (a) H_2 (b) Cl_2
 (c) O_2 (d) F_2

19. In the isothermal expansion of 10g of gas from volume V to $2V$ the work done by the gas is 575J. What is the root mean square speed of the molecules of the gas at that temperature? [Online April 25, 2013]

(a) 398m/s (b) 520m/s
(c) 499m/s (d) 532m/s

20. A perfect gas at 27°C is heated at constant pressure so as to double its volume. The final temperature of the gas will be, close to [Online May 7, 2012]

(a) 327°C (b) 200°C
(c) 54°C (d) 300°C

21. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ . It is moving with speed v and it's suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by: [2011]

(a) $\frac{(\gamma-1)}{2\gamma R} Mv^2 K$ (b) $\frac{\gamma M^2 v}{2R} K$
(c) $\frac{(\gamma-1)}{2R} Mv^2 K$ (d) $\frac{(\gamma-1)}{2(\gamma+1)R} Mv^2 K$

22. Three perfect gases at absolute temperatures T_1 , T_2 and T_3 are mixed. The masses of molecules are m_1 , m_2 and m_3 and the number of molecules are n_1 , n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is : [2011]

(a) $\frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$ (b) $\frac{n_1 T_1^2 + n_2 T_2^2 + n_3 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$
(c) $\frac{n_1^2 T_1^2 + n_2^2 T_2^2 + n_3^2 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$ (d) $\frac{(T_1 + T_2 + T_3)}{3}$

23. One kg of a diatomic gas is at a pressure of $8 \times 10^4 \text{ N/m}^2$. The density of the gas is 4 kg/m^3 . What is the energy of the gas due to its thermal motion? [2009]

(a) $5 \times 10^4 \text{ J}$ (b) $6 \times 10^4 \text{ J}$
(c) $7 \times 10^4 \text{ J}$ (d) $3 \times 10^4 \text{ J}$

24. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1} . The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal) [2008]

(a) 1421 ms^{-1} (b) 500 ms^{-1}
(c) 650 ms^{-1} (d) 330 ms^{-1}

25. At what temperature is the r.m.s velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C ? [2002]

(a) 80 K (b) -73 K
(c) 3 K (d) 20 K

TOPIC 3

Degree of Freedom, Specific Heat Capacity, and Mean Free Path



26. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T .

The total internal energy, U of a mole of this gas, and the value of $\gamma \left(= \frac{C_p}{C_v} \right)$ are given, respectively, by:

[Sep. 06, 2020 (I)]

(a) $U = \frac{5}{2} RT$ and $\gamma = \frac{6}{5}$ (b) $U = 5RT$ and $\gamma = \frac{7}{5}$
(c) $U = \frac{5}{2} RT$ and $\gamma = \frac{7}{5}$ (d) $U = 5RT$ and $\gamma = \frac{6}{5}$

27. In a dilute gas at pressure P and temperature T , the mean time between successive collisions of a molecule varies with T is : [Sep. 06, 2020 (II)]

(a) T (b) $\frac{1}{\sqrt{T}}$
(c) $\frac{1}{T}$ (d) \sqrt{T}

28. Match the C_p/C_v ratio for ideal gases with different type of molecules : [Sep. 04, 2020 (I)]

Column-I

Molecule Type

Column-II

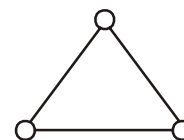
 C_p/C_v

- (A) Monatomic (I) 7/5
(B) Diatomic rigid molecules (II) 9/7
(C) Diatomic non-rigid molecules (III) 4/3
(D) Triatomic rigid molecules (IV) 5/3
(a) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(b) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(c) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(d) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

29. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to _____.

[NA Sep. 04, 2020 (I)]

- 30.

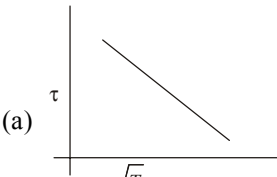


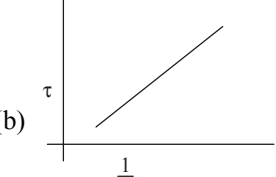
Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is :

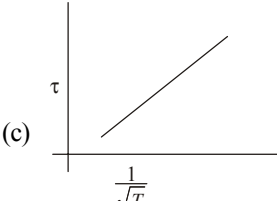
[Sep. 03, 2020 (I)]

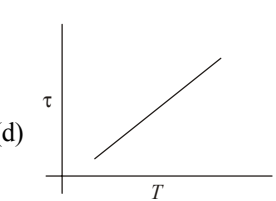
(a) $\frac{5}{2} RT$ (b) $\frac{3}{2} RT$
(c) $\frac{9}{2} RT$ (d) $3RT$

31. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant

- volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)? [Sep. 03, 2020 (II)]
- (a) 5 (b) 6
(c) 3 (d) 7
32. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T . Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is : [Sep. 02, 2020 (I)]
- (a) 15 (b) 13
(c) 20 (d) 11
33. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true? [Sep. 02, 2020 (II)]
- (1) The mean free path of the molecules decreases
(2) The mean collision time between the molecules decreases
(3) The mean free path remains unchanged
(4) The mean collision time remains unchanged
- (a) (2) and (3) (b) (1) and (2)
(c) (3) and (4) (d) (1) and (4)
34. Consider two ideal diatomic gases A and B at some temperature T . Molecules of the gas A are rigid, and have a mass m . Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats (C_V^A and C_V^B) of gas A and B, respectively is: [9 Jan 2020 I]
- (a) 7 : 9 (b) 5 : 9
(c) 3 : 5 (d) 5 : 7
35. Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to: [9 Jan 2020 II]
- (a) 3.67 (b) 1.83
(c) 2.3 (d) 4.67
36. The plot that depicts the behavior of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale) [8 Jan. 2020 I]
- (a) 

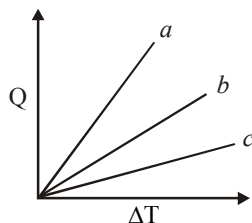
(b) 

(c) 

(d) 
37. Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_p/C_V value will be: [8 Jan. 2020 II]
- (a) 19/13 (b) 67/45
(c) 40/27 (d) 23/15
38. Two moles of an ideal gas with $\frac{C_p}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_p}{C_V} = \frac{4}{3}$. The value of $\frac{C_p}{C_V}$ for the mixture is: [7 Jan. 2020 I]
- (a) 1.45 (b) 1.50
(c) 1.47 (d) 1.42
39. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? ($R = 8.3 \text{ J/mol K}$) [12 April 2019 I]
- (a) 19.7 J/mol L (b) 15.7 J/mol K
(c) 17.4 J/mol K (d) 21.6 J/mol K
40. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process? [12 April 2019 II]
- (a) 25 J (b) 35 J
(c) 30 J (d) 40 J
41. A $25 \times 10^{-3} \text{ m}^3$ volume cylinder is filled with 1 mol of O_2 gas at room temperature (300 K). The molecular diameter of O_2 , and its root mean square speed, are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule? [10 April 2019 I]
- (a) $\sim 10^{12}$ (b) $\sim 10^{11}$
(c) $\sim 10^{10}$ (d) $\sim 10^{13}$
42. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT . The heat required to produce the same change in temperature, at a constant pressure is : [10 April 2019 II]
- (a) $\frac{2}{3}Q$ (b) $\frac{5}{3}Q$
(c) $\frac{7}{5}Q$ (d) $\frac{3}{2}Q$
43. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is \bar{v} , m is its mass and k_B is Boltzmann constant, then its temperature will be: [9 April 2019 I]
- (a) $\frac{m\bar{v}^2}{6k_B}$ (b) $\frac{m\bar{v}^2}{3k_B}$
(c) $\frac{m\bar{v}^2}{7k_B}$ (d) $\frac{m\bar{v}^2}{5k_B}$

44. The specific heats, C_p and C_v of a gas of diatomic molecules, A, are given (in units of $\text{J mol}^{-1} \text{K}^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then: [9 April 2019 II]
 (a) A is rigid but B has a vibrational mode.
 (b) A has a vibrational mode but B has none.
 (c) A has one vibrational mode and B has two.
 (d) Both A and B have a vibrational mode each.
45. An ideal gas occupies a volume of 2 m^3 at a pressure of $3 \times 10^6 \text{ Pa}$. The energy of the gas: [12 Jan. 2019 I]
 (a) $9 \times 10^6 \text{ J}$ (b) $6 \times 10^4 \text{ J}$
 (c) 10^8 J (d) $3 \times 10^2 \text{ J}$
46. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is $6 \times 10^{-8} \text{ s}$. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to: [12 Jan. 2019 II]
 (a) $2 \times 10^{-7} \text{ s}$ (b) $4 \times 10^{-8} \text{ s}$
 (c) $0.5 \times 10^{-8} \text{ s}$ (d) $3 \times 10^{-6} \text{ s}$
47. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is: [11 Jan. 2019 I]
 (a) 15 RT (b) 12 RT
 (c) 4 RT (d) 20 RT
48. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation $VT = K$, where K is a constant. In this process the temperature of the gas is increased by ΔT . The amount of heat absorbed by gas is (R is gas constant): [11 Jan. 2019 II]
 (a) $\frac{1}{2} R \Delta T$ (b) $\frac{1}{2} K R \Delta T$
 (c) $\frac{3}{2} R \Delta T$ (d) $\frac{2K}{3} \Delta T$
49. Two kg of a monoatomic gas is at a pressure of $4 \times 10^4 \text{ N/m}^2$. The density of the gas is 8 kg/m^3 . What is the order of energy of the gas due to its thermal motion? [10 Jan 2019 II]
 (a) 10^3 J (b) 10^5 J
 (c) 10^4 J (d) 10^6 J
50. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C . Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about: [Take $R = 8.3 \text{ J/K mole}$] [9 Jan. 2019 II]
 (a) 0.9 kJ (b) 6 kJ
 (c) 10 kJ (d) 14 kJ
51. Two moles of an ideal monoatomic gas occupies a volume V at 27°C . The gas expands adiabatically to a volume 2 V. Calculate (1) the final temperature of the gas and (2) change in its internal energy. [2018]
 (a) (1) 189 K (2) 2.7 kJ
 (b) (1) 195 K (2) -2.7 kJ
 (c) (1) 189 K (2) -2.7 kJ
 (d) (1) 195 K (2) 2.7 kJ
52. Two moles of helium are mixed with n with moles of hydrogen. If $\frac{C_p}{C_v} = \frac{3}{2}$ for the mixture, then the value of n is [Online April 16, 2018]
 (a) 3/2 (b) 2
 (c) 1 (d) 3
53. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that $C_p - C_v = a$ for hydrogen gas
 $C_p - C_v = b$ for nitrogen gas
 The correct relation between a and b is: [2017]
 (a) $a = 14 b$ (b) $a = 28 b$
 (c) $a = \frac{1}{14} b$ (d) $a = b$
54. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_p) and at constant volume (C_v) is: [Online April 8, 2017]
 (a) 6 (b) $\frac{7}{2}$
 (c) $\frac{5}{2}$ (d) $\frac{7}{5}$
55. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by $PV^n = \text{constant}$, then n is given by (Here C_p and C_v are molar specific heat at constant pressure and constant volume, respectively): [2016]
 (a) $n = \frac{C_p - C_v}{C - C_v}$ (b) $n = \frac{C - C_v}{C - C_p}$
 (c) $n = \frac{C_p}{C_v}$ (d) $n = \frac{C - C_p}{C - C_v}$
56. Using equipartition of energy, the specific heat (in $\text{J kg}^{-1} \text{K}^{-1}$) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27) [Online April 11, 2015]
 (a) 410 (b) 25
 (c) 1850 (d) 925
57. Modern vacuum pumps can evacuate a vessel down to a pressure of $4.0 \times 10^{-15} \text{ atm}$. at room temperature (300 K). Taking $R = 8.0 \text{ JK}^{-1} \text{ mole}^{-1}$, $1 \text{ atm} = 10^5 \text{ Pa}$ and $N_{\text{Avogadro}} = 6 \times 10^{23} \text{ mole}^{-1}$, the mean distance between molecules of gas in an evacuated vessel will be of the order of: [Online April 9, 2014]
 (a) 0.2 μm (b) 0.2 mm
 (c) 0.2 cm (d) 0.2 nm

58. Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M), diatomic (D) and a polyatomic (P) gas. The initial state of all the gases are the same and the scales for the two axes coincide. Ignoring vibrational degrees of freedom, the lines a , b and c respectively correspond to : **[Online April 9, 2013]**



- (a) P, M and D (b) M, D and P
(c) P, D and M (d) D, M and P
59. A given ideal gas with $\gamma = \frac{C_p}{C_v} = 1.5$ at a temperature T . If the gas is compressed adiabatically to one-fourth of its initial volume, the final temperature will be **[Online May 12, 2012]**
- (a) $2\sqrt{2}T$ (b) $4T$
(c) $2T$ (d) $8T$

60. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then **[2007]**

(a) $C_p - C_v = 28R$ (b) $C_p - C_v = R/28$
(c) $C_p - C_v = R/14$ (d) $C_p - C_v = R$

61. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio $\frac{C_p}{C_v}$ of the mixture is **[2005]**

(a) 1.62 (b) 1.59
(c) 1.54 (d) 1.4

62. One mole of ideal monatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture? γ Denotes the ratio of specific heat at constant pressure, to that at constant volume **[2004]**

(a) 35/23 (b) 23/15
(c) 3/2 (d) 4/3

63. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_p/C_v for the gas is **[2003]**

(a) $\frac{4}{3}$ (b) 2
(c) $\frac{5}{3}$ (d) $\frac{3}{2}$



Hints & Solutions



1. (5) Using ideal gas equation, $PV = nRT$

$$\Rightarrow P_1 V_1 = nR \times 250 \quad [\because T_1 = 250 \text{ K}] \quad \dots(i)$$

$$P_2 (2V_1) = \frac{5n}{4} R \times 2000 \quad [\because T_2 = 2000 \text{ K}] \quad \dots(ii)$$

Dividing eq. (i) by (ii),

$$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000} \Rightarrow \frac{P_1}{P_2} = \frac{1}{5}$$

$$\therefore \frac{P_2}{P_1} = 5.$$

2. (150) In first case,
From ideal gas equation

$$PV = nRT$$

$$P\Delta V + V\Delta P = 0 \quad (\text{As temperature is constant})$$

$$\Delta V = -\frac{\Delta P}{P} V \quad \dots(i)$$

In second case, using ideal gas equation again

$$P\Delta V = -nR\Delta T$$

$$\Delta V = -\frac{nR\Delta T}{P} \quad \dots(ii)$$

Equating (i) and (ii), we get

$$\frac{nR\Delta T}{P} = -\frac{\Delta P}{P} V \Rightarrow \Delta T = \Delta P \frac{V}{nR}$$

Comparing the above equation with $|\Delta T| = C |\Delta P|$, we have

$$C = \frac{V}{nR} = \frac{\Delta T}{\Delta P} = \frac{300 \text{ K}}{2 \text{ atm}} = 150 \text{ K/atm}$$

3. (a) $N = \int \rho(dv)$

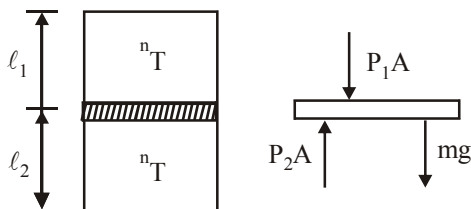
$$= \int_0^r n_0 e^{-\alpha r^4} \times 4\pi r^2 dr = 4\pi n_0 \int_0^r r^2 (e^{-\alpha r^4}) dr$$

$$\propto n_0 a^{-3/4}$$

4. (d) Clearly from figure,

$$P_2 A = P_1 A + mg$$

$$\text{or, } \frac{nRT \cdot A}{A\ell_2} = \frac{nRT \cdot A}{A\ell_1} + mg$$



$$\Rightarrow nRT \left(\frac{1}{\ell_2} - \frac{1}{\ell_1} \right) = mg$$

$$\therefore m = \frac{nRT}{g} \left(\frac{\ell_1 - \ell_2}{\ell_1 \cdot \ell_2} \right)$$

5. (b) Given: Temperature $T_i = 17 + 273 = 290 \text{ K}$

Temperature $T_f = 27 + 273 = 300 \text{ K}$

Atmospheric pressure, $P_0 = 1 \times 10^5 \text{ Pa}$

Volume of room, $V_0 = 30 \text{ m}^3$

Difference in number of molecules, $n_f - n_i = ?$

Using ideal gas equation, $PV = nRT(N_0)$,

$N_0 = \text{Avogadro's number}$

$$\Rightarrow n = \frac{PV}{RT} (N_0)$$

$$\therefore n_f - n_i = \frac{P_0 V_0}{R} \left(\frac{1}{T_f} - \frac{1}{T_i} \right) N_0$$

$$= \frac{1 \times 10^5 \times 30}{8.314} \times 6.023 \times 10^{23} \left(\frac{1}{300} - \frac{1}{290} \right)$$

$$= -2.5 \times 10^{25}$$

6. (c) From P-V graph,

$$P \propto \frac{1}{V}, \quad T = \text{constant and Pressure is increasing from 2}$$

to 1 so option (3) represents correct T-P graph.

7. (a) In ideal gases the molecules are considered as point particles and for point particles, there is no internal excitation, no vibration and no rotation. For an ideal gas the internal energy can only be translational kinetic energy and for real gas both kinetic as well as potential energy.

8. (c) The centre of mass of gas molecules also moves with lorry with uniform speed. As there is no relative motion of gas molecule. So, kinetic energy and hence temperature remain same.

9. (c) Given: $\text{K.E.}_{\text{mean}} = \frac{3}{2} kT = 4 \times 10^{-14}$

$$P = 2 \text{ cm of Hg, } V = 4 \text{ cm}^3$$

$$N = \frac{PV}{KT} = \frac{P\rho g V}{KT} = \frac{2 \times 13.6 \times 980 \times 4}{\frac{8}{3} \times 10^{-14}} \approx 4 \times 10^{18}$$

10. (41) Room mean square speed is given by

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Here, M = Molar mass of gas molecule

T = temperature of the gas molecule

We have given $v_{N_2} = v_{H_2}$

$$\therefore \sqrt{\frac{3RT_{N_2}}{M_{N_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\Rightarrow \frac{T_{H_2}}{2} = \frac{573}{28} \Rightarrow T_{H_2} = 41 \text{ K}$$

11. (c) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \frac{(273+127)}{(273+237)} = \sqrt{\frac{400}{500}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\therefore v_2 = \frac{\sqrt{5}}{2} v_1 = \frac{\sqrt{5}}{2} \times 200 = 100\sqrt{5} \text{ m/s.}$$

12. (Bouns) Rate of change of momentum during collision

$$= \frac{mv - (-mv)}{\Delta t} = \frac{2mv}{\Delta t} N$$

$$\text{so pressure } P = \frac{N \times (2mv)}{\Delta t \times A}$$

$$= \frac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1 \times 1} = 2 \text{ N/m}^2$$

13. (c) $v_{rms} = v_e$

$$\sqrt{\frac{3RT}{M}} = 11.2 \times 10^3$$

$$\text{or } \sqrt{\frac{3kT}{m}} = 11.2 \times 10^3$$

$$\text{or } \sqrt{\frac{3 \times 1.38 \times 10^{-23} T}{2 \times 10^{-3}}} = 11.2 \times 10^3 \quad \therefore T = 10^4 \text{ K}$$

14. (a) Using $\frac{V_{rms}}{V_{2rms}} = \sqrt{\frac{M_2}{M_1}}$

$$\frac{V_{rms}(\text{He})}{V_{rms}(\text{Ar})} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = 3.16$$

15. (a) Energy associated with N moles of diatomic gas,

$$U_i = N \frac{5}{2} RT$$

Energy associated with n moles of monoatomic gas

$$= n \frac{3}{2} RT$$

Total energy when n moles of diatomic gas converted into

$$\text{monoatomic } (U_f) = 2n \frac{3}{2} RT + (N-n) \frac{5}{2} RT$$

$$= \frac{1}{2} nRT + \frac{5}{2} NRT$$

Now, change in total kinetic energy of the gas

$$\Delta U = Q = \frac{1}{2} nRT$$

16. (c) Pressure, $P = \frac{1}{3} \frac{mN}{V} V_{rms}^2$

$$\text{or, } P = \frac{(mN)T}{V}$$

If the gas mass and temperature are constant then

$$P \propto (V_{rms})^2 \propto T$$

$$\text{So, force} \propto (V_{rms})^2 \propto T$$

$$\text{i.e., Value of } q = 1$$

17. (d) Kinetic energy of each molecule,

$$\text{K.E.} = \frac{3}{2} K_B T$$

In the given problem,

$$\text{Temperature, } T = 0^\circ\text{C} = 273 \text{ K}$$

Height attained by the gas molecule, $h = ?$

$$\text{K.E.} = \frac{3}{2} K_B (273) = \frac{819 K_B}{2}$$

$$\text{K.E.} = \text{P.E.}$$

$$\Rightarrow \frac{819 K_B}{2} = Mgh$$

$$\text{or } h = \frac{819 K_B}{2Mg}$$

18. (a) $\therefore C = \sqrt{\frac{3RT}{M}}$

$$(1930)^2 = \frac{3 \times 8.314 \times 300}{M}$$

$$M = \frac{3 \times 8.314 \times 300}{1930 \times 1930} \approx 2 \times 10^{-3} \text{ kg}$$

The gas is H_2 .

19. (c) $v_{rms} = \sqrt{\frac{3pv}{\text{mass of the gas}}}$

20. (a) Given, $V_1 = V$

$$V_2 = 2V$$

$$T_1 = 27^\circ + 273 = 300 \text{ K}$$

$$T_2 = ?$$

From charle's law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (\because \text{Pressure is constant})$$

$$\text{or, } \frac{V}{300} = \frac{2V}{T_2}$$

$$\therefore T_2 = 600 \text{ K} = 600 - 273 = 327^\circ\text{C}$$

21. (c) As, work done is zero.

So, loss in kinetic energy = heat gain by the gas

$$\frac{1}{2} mv^2 = nC_v \Delta T = n \frac{R}{\gamma - 1} \Delta T$$

$$\frac{1}{2} mv^2 = \frac{m}{M} \frac{R}{\gamma - 1} \Delta T$$

$$\therefore \Delta T = \frac{Mv^2(\gamma - 1)}{2R} K$$

22. (a) Number of moles of first gas = $\frac{n_1}{N_A}$

Number of moles of second gas = $\frac{n_2}{N_A}$

Number of moles of third gas = $\frac{n_3}{N_A}$

If there is no loss of energy then

$$P_1 V_1 + P_2 V_2 + P_3 V_3 = PV$$

$$\frac{n_1}{N_A} RT_1 + \frac{n_2}{N_A} RT_2 + \frac{n_3}{N_A} RT_3$$

$$= \frac{n_1 + n_2 + n_3}{N_A} RT_{\text{mix}}$$

$$T_{\text{mix}} = \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$$

23. (a) Given, mass = 1 kg

Density = 4 kg m⁻³

$$\text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{1}{4} \text{ m}^3$$

Internal energy of the diatomic gas

$$= \frac{5}{2} PV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 J$$

Alternatively:

$$\text{K.E} = \frac{5}{2} nRT = \frac{5}{2} \frac{m}{M} RT = \frac{5}{2} \frac{m}{M} \times \frac{PM}{d} \quad [\because PM = dRT]$$

$$= \frac{5}{2} \frac{mP}{d} = \frac{5}{2} \times \frac{1 \times 8 \times 10^4}{4} = 5 \times 10^4 J$$

24. (a) The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma RT}{M}}$

$$\therefore v \propto \sqrt{\frac{\gamma}{M}} \quad [\text{As } R \text{ and } T \text{ is constant}]$$

$$\therefore \frac{v_{\text{O}_2}}{v_{\text{He}}} = \sqrt{\frac{\gamma_{\text{O}_2}}{\gamma_{\text{He}}} \times \frac{M_{\text{He}}}{M_{\text{O}_2}}}$$

$$= \sqrt{\frac{1.4}{32} \times \frac{4}{1.67}} = 0.3237$$

$$\therefore v_{\text{He}} = \frac{v_{\text{O}_2}}{0.3237} = \frac{460}{0.3237} = 1421 \text{ m/s}$$

25. (d) RMS velocity of a gas molecule is given by

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Let T be the temperature at which the velocity of hydrogen molecule is equal to the velocity of oxygen molecule.

$$\therefore \sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R \times (273 + 47)}{32}}$$

$$\Rightarrow T = 20K$$

26. (c) Total degree of freedom $f = 3 + 2 = 5$

$$\text{Total energy, } U = \frac{nfRT}{2} = \frac{5RT}{2}$$

$$\text{And } \gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$$

27. (b) Mean free path, $\lambda = \frac{1}{\sqrt{2} \pi n d^2}$

where, d = diameter of the molecule

n = number of molecules per unit volume

But, mean time of collision, $\tau = \frac{\lambda}{v_{\text{rms}}}$

$$\text{But } v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$\therefore \tau = \frac{\lambda}{\sqrt{\frac{3kT}{m}}} \Rightarrow \tau \propto \frac{1}{\sqrt{T}}$$

28. (c) As we know,

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}, \text{ where } f = \text{degree of freedom}$$

(A) Monatomic, $f = 3$

$$\therefore \gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

(B) Diatomic rigid molecules, $f = 5$

$$\therefore \gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

(C) Diatomic non-rigid molecules, $f = 7$

$$\therefore \gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

(D) Triatomic rigid molecules, $f = 6$

$$\therefore \gamma = 1 + \frac{2}{6} = \frac{4}{3}$$

29. (266.67) Here work done on gas and heat supplied to the gas are zero.

Let T be the final equilibrium temperature of the gas in the vessel.

Total internal energy of gases remain same.

$$\text{i.e., } u_1 + u_2 = u'_1 + u'_2$$

$$\text{or, } n_1 C_v \Delta T_1 + n_2 C_v \Delta T_2 = (n_1 + n_2) C_v T$$

$$\Rightarrow (0.1) C_v (200) + (0.05) C_v (400) = (0.15) C_v T$$

$$\therefore T = \frac{800}{3} = 266.67 \text{ K}$$

30. (d) Here degree of freedom, $f = 3 + 3 = 6$ for triatomic non-linear molecule.

Internal energy of a mole of the gas at temperature T ,

$$U = \frac{f}{2} nRT = \frac{6}{2} RT = 3RT$$

31. (b) Let C_p and C_v be the specific heat capacity of the gas at constant pressure and volume.

At constant pressure, heat required

$$\Delta Q_1 = nC_p \Delta T$$

$$\Rightarrow 160 = nC_p \cdot 50 \quad \dots(i)$$

At constant volume, heat required

$$\Delta Q_2 = nC_v \Delta T$$

$$\Rightarrow 240 = nC_v \cdot 100 \quad \dots(ii)$$

Dividing (i) by (ii), we get

$$\frac{160}{240} = \frac{C_p}{C_v} \cdot \frac{50}{100} \Rightarrow \frac{C_p}{C_v} = \frac{4}{3}$$

$$\gamma = \frac{C_p}{C_v} = \frac{4}{3} = 1 + \frac{2}{f} \quad (\text{Here, } f = \text{degree of freedom})$$

$$\Rightarrow f = 6.$$

32. (a) Total energy of the gas mixture,

$$\begin{aligned} E_{\text{mix}} &= \frac{f_1 n_1 R T_1}{2} + \frac{f_2 n_2 R T_2}{2} \\ &= 3 \times \frac{5}{2} R T + \frac{5}{2} \times 3 R T = 15 R T \end{aligned}$$

33. (a) As we know mean free path

$$\lambda = \frac{1}{\sqrt{2} \left(\frac{N}{V} \right) \pi d^2}$$

Here, N = no. of molecule
 V = volume of container
 d = diameter of molecule

$$\text{But } PV = nRT = nNKT$$

$$\Rightarrow \frac{N}{V} = \frac{P}{KT} = n$$

$$\lambda = \frac{1}{\sqrt{2}} \frac{KT}{\pi d^2 P}$$

For constant volume and hence constant number density

n of gas molecules $\frac{P}{T}$ is constant.

So mean free path remains same.

As temperature increases no. of collision increases so relaxation time decreases.

34. (d) Specific heat of gas at constant volume

$$C_v = \frac{1}{2} f R; f = \text{degree of freedom}$$

For gas A (diatomic)

$f = 5$ (3 translational + 2 rotational)

$$\therefore C_v^A = \frac{5}{2} R$$

For gas B (diatomic) in addition to (3 translational + 2 rotational) 2 vibrational degree of freedom.

$$\therefore C_v^B = \frac{7}{2} R \quad \text{Hence } \frac{C_v^A}{C_v^B} = \frac{\frac{5}{2} R}{\frac{7}{2} R} = \frac{5}{7}$$

35. (Bonus) Mean free path of a gas molecule is given by

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 n}$$

Here, n = number of collisions per unit volume

d = diameter of the molecule

If average speed of molecule is v then

$$\text{Mean free time, } \tau = \frac{\lambda}{v}$$

$$\Rightarrow \tau = \frac{1}{\sqrt{2} \pi n d^2 v} = \frac{1}{\sqrt{2} \pi n d^2} \sqrt{\frac{M}{3RT}}$$

$$\left(\because v = \sqrt{\frac{3RT}{M}} \right)$$

$$\therefore \tau \propto \frac{\sqrt{M}}{d^2} \therefore \frac{\tau_1}{\tau_2} = \frac{\sqrt{M_1}}{d_1^2} \times \frac{d_2^2}{\sqrt{M_2}}$$

$$= \sqrt{\frac{40}{140}} \times \left(\frac{0.1}{0.07} \right)^2 = 1.09$$

36. (c) Relaxation time (τ) $\propto \frac{\text{mean free path}}{\text{speed}} \Rightarrow \tau \propto \frac{1}{v}$

$$\text{and, } v \propto \sqrt{T}$$

$$\therefore \tau \propto \frac{1}{\sqrt{T}}$$

Hence graph between τ v/s $\frac{1}{\sqrt{T}}$ is a straight line which is correctly depicted by graph shown in option (c).

37. (a) Helium is a monoatomic gas and Oxygen is a diatomic gas.

$$\text{For helium, } C_{V1} = \frac{3}{2} R \text{ and } C_{P1} = \frac{5}{2} R$$

$$\text{For oxygen, } C_{V2} = \frac{5}{2} R \text{ and } C_{P2} = \frac{7}{2} R$$

$$\gamma = \frac{N_1 C_{P1} + N_2 C_{P2}}{N_1 C_{V1} + N_2 C_{V2}}$$

$$\Rightarrow \gamma = \frac{n \cdot \frac{5}{2} R + 2n \cdot \frac{7}{2} R}{n \cdot \frac{3}{2} R + 2n \cdot \frac{5}{2} R} = \frac{19nR \times 2}{2(13nR)}$$

$$\therefore \left(\frac{C_P}{C_V} \right)_{\text{mixture}} = \frac{19}{13}$$

38. (d) Using, $\gamma_{\text{mixture}} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}}$

$$\Rightarrow \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_m - 1}$$

$$\Rightarrow \frac{3}{\frac{4}{3} - 1} + \frac{2}{\frac{5}{3} - 1} = \frac{5}{\gamma_m - 1}$$

$$\Rightarrow \frac{9}{1} + \frac{2 \times 3}{2} = \frac{5}{\gamma_m - 1} \Rightarrow \gamma_m - 1 = \frac{5}{12}$$

$$\Rightarrow \gamma_m = \frac{17}{12} = 1.42$$

39. (c) $[C_v]_{\text{min}} = \frac{n_1 [C_{v1}] + n_2 [C_{v2}]}{n_1 + n_2}$

$$= \left[\frac{2 \times \frac{3R}{2} + 3 \times \frac{5R}{2}}{2 + 3} \right]$$

$$= 2.1 R = 2.1 \times 8.3 = 17.4 \text{ J/mol-K}$$

40. (b) $F = \frac{C_v}{C_p} = \frac{1}{r} = \frac{1}{(7/5)} = \frac{5}{7}$

$$\text{or } \frac{W}{Q} = 1 - \frac{5}{7} = \frac{2}{7}$$

$$\text{or } Q = \frac{7}{2} W = \frac{7 \times 10}{2} = 35 \text{ J}$$

41. (c) $V = 25 \times 10^{-3} \text{ m}^3$, $N = 1 \text{ mole of O}_2$

$$T = 300 \text{ K}$$

$$V_{\text{rms}} = 200 \text{ m/s}$$

$$\therefore \lambda = \frac{1}{\sqrt{2N\pi r^2}}$$

$$\text{Average time } \frac{1}{\tau} = \frac{\langle V \rangle}{\lambda} = 200 \cdot N\pi r^2 \cdot \sqrt{2}$$

$$= \frac{\sqrt{2} \times 200 \times 6.023 \times 10^{23}}{25 \times 10^{-3}} \cdot \pi \times 10^{-18} \times 0.09$$

The closest value in the given option is $= 10^{10}$

42. (c) Amount of heat required (Q) to raise the temperature at constant volume

$$Q = nC_v \Delta T \quad \dots (i)$$

Amount of heat required (Q_1) at constant pressure

$$Q_1 = nC_p \Delta T \quad \dots (ii)$$

Dividing equation (ii) by (i), we get

$$\therefore \frac{Q_1}{Q} = \frac{C_p}{C_v}$$

$$\Rightarrow Q_1 = (Q) \left(\frac{7}{5} \right) \left(\because \gamma = \frac{C_p}{C_v} = \frac{7}{5} \right)$$

43. (a) $\frac{1}{2} m \bar{v}^2 = 3k_B T$

$$\text{or } T = \frac{m \bar{v}^2}{6k_B}$$

44. (b) $\gamma_A = \frac{C_p}{C_v} = \frac{29}{22} = 1.32 < 1.4$ (diatomic)

$$\text{and } \gamma_B = \frac{30}{21} = \frac{10}{7} = 1.43 > 1.4$$

Gas A has more than 5-degrees of freedom.

45. (a) Energy of the gas, E

$$= \frac{f}{2} nRT = \frac{f}{2} PV$$

$$= \frac{f}{2} (3 \times 10^6)(2) = f \times 3 \times 10^6$$

Considering gas is monoatomic i.e., $f = 3$

$$\text{Energy, } E = 9 \times 10^6 \text{ J}$$

46. (b) Using, $\tau = \frac{1}{2n\pi d^2 V_{\text{avg}}}$

$$\therefore t \propto \frac{\sqrt{T}}{P} \left[\because n = \frac{\text{no. of molecules}}{\text{Volume}} \right]$$

$$\text{or, } \frac{t_1}{6 \times 10^{-8}} = \frac{\sqrt{500}}{2P} \times \frac{P}{\sqrt{300}} \approx 4 \times 10^{-8}$$

47. (a) $U = \frac{f_1}{2} n_1 RT + \frac{f_2}{2} n_2 RT$

Considering translational and rotational modes, degrees of freedom $f_1 = 5$ and $f_2 = 3$

$$\therefore U = \frac{5}{2} (3RT) + \frac{3}{2} \times 5RT$$

$$U = 15RT$$

48. (a) According to question $VT = K$

we also know that $PV = nRT$

$$\Rightarrow T = \left(\frac{PV}{nR} \right)$$

$$\Rightarrow V \left(\frac{PV}{nR} \right) = K \Rightarrow PV^2 = K$$

$$\therefore C = \frac{R}{1-x} + C_v \text{ (For polytropic process)}$$

$$C = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2}$$

$$\therefore \Delta Q = nC \Delta T$$

$$= \frac{R}{2} \times \Delta T \text{ [here, } n = 1 \text{ mole]}$$

49. (c) Thermal energy of N molecule

$$= N \left(\frac{3}{2} kT \right)$$

$$= \frac{N}{N_A} \frac{3}{2} RT = \frac{3}{2} (nRT) = \frac{3}{2} PV$$

$$= \frac{3}{2} P \left(\frac{m}{\rho} \right) = \frac{3}{2} P \left(\frac{2}{8} \right)$$

$$= \frac{3}{2} \times 4 \times 10^4 \times \frac{2}{8} = 1.5 \times 10^4 \text{ J}$$

therefore, order = 10^4 J

50. (c) Heat transferred,

$Q = nC_v \Delta T$ as gas in closed vessel

To double the rms speed, temperature should be 4 times

i.e., $T' = 4T$ as $v_{rms} = \sqrt{3RT/M}$

$$\therefore Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

$$\left[\therefore \frac{CP}{CV} = \gamma_{diatomic} = \frac{7}{5} \text{ \& } C_p - C_v = R \right]$$

or, $Q = 10000 \text{ J} = 10 \text{ kJ}$

51. (c) In an adiabatic process

$$TV^{\gamma-1} = \text{Constant} \quad \text{or, } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

For monoatomic gas $\gamma = \frac{5}{3}$

$$(300)V^{2/3} = T_2(2V)^{2/3} \Rightarrow T_2 = \frac{300}{(2)^{2/3}}$$

$T_2 = 189 \text{ K}$ (final temperature)

Change in internal energy $\Delta U = n \frac{f}{2} R \Delta T$

$$= 2 \left(\frac{3}{2} \right) \left(\frac{25}{3} \right) (-111) = -2.7 \text{ kJ}$$

52. (b) Using formula,

$$\gamma_{\text{mixture}} = \left(\frac{C_p}{C_v} \right)_{\text{mix}} = \frac{\frac{n_1 \gamma_1}{\gamma_1 - 1} + \frac{n_2 \gamma_2}{\gamma_2 - 1}}{\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}}$$

Putting the value of $n_1 = 2$, $n_2 = n$, $\left(\frac{C_p}{C_v} \right)_{\text{mix}} = \frac{3}{2}$

$\gamma_1 = \frac{5}{3}$, $\gamma_2 = \frac{7}{5}$ and solving we get, $n = 2$

53. (a) As we know, $C_p - C_v = R$ where C_p and C_v are molar specific heat capacities

$$\text{or, } C_p - C_v = \frac{R}{M}$$

For hydrogen ($M = 2$) $C_p - C_v = a = \frac{R}{2}$

For nitrogen ($M = 28$) $C_p - C_v = b = \frac{R}{28}$

$$\therefore \frac{a}{b} = 14 \quad \text{or, } a = 14b$$

54. (d) The ratio of specific heats at constant pressure (C_p) and constant volume (C_v)

$$\frac{C_p}{C_v} = \gamma = \left(1 + \frac{2}{f} \right)$$

where f is degree of freedom

$$\frac{C_p}{C_v} = \left(1 + \frac{2}{5} \right) = \frac{7}{5}$$

55. (d) For a polytropic process

$$C = C_v + \frac{R}{1-n} \quad \therefore C - C_v = \frac{R}{1-n}$$

$$\therefore 1-n = \frac{R}{C-C_v} \quad \therefore 1 - \frac{R}{C-C_v} = n$$

$$\therefore n = \frac{C - C_v - R}{C - C_v} = \frac{C - C_v - C_p + C_v}{C - C_v}$$

$$= \frac{C - C_p}{C - C_v} \quad (\because C_p - C_v = R)$$

56. (d) Using equipartition of energy, we have

$$\frac{6}{2} KT = mCT$$

$$C = \frac{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{23}}{27 \times 10^{-3}}$$

$$\therefore C = 925 \text{ J/kgK}$$

57. (b)

58. (b) On giving same amount of heat at constant pressure, there is no change in temperature for mono, dia and polyatomic.

$$(\Delta Q)_p = \mu C_p \Delta T \left(\mu = \frac{\text{No. of molecules}}{\text{Avogadro's no.}} \right)$$

$$\text{or } \Delta T \propto \frac{1}{\text{no. of molecules}}$$

59. (c) $TV^{\gamma-1} = \text{constant}$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow T(V)^{1/2} = T_2 \left(\frac{V}{4} \right)^{1/2}$$

$$\left[\because \gamma = 1.5, T_1 = T, V_1 = V \text{ and } V_2 = \frac{V}{4} \right]$$

$$\therefore T_2 = \left(\frac{4V}{V} \right)^{1/2} T = 2T$$

60. (b) According to Mayer's relationship

$C_p - C_v = R$, as per the question $(C_p - C_v) M = R$

$$\Rightarrow C_p - C_v = R/28$$

Here $M = 28 = \text{mass of 1 unit of } N_2$

61. (a) For mixture of gas specific heat at constant volume

$$C_v = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$$

No. of moles of helium,

$$n_1 = \frac{m_{He}}{M_{He}} = \frac{16}{4} = 4$$

Number of moles of oxygen,

$$n_2 = \frac{16}{32} = \frac{1}{2}$$

$$\therefore C_v = \frac{4 \times \frac{3}{2}R + \frac{1}{2} \times \frac{5}{2}R}{\left(4 + \frac{1}{2}\right)} = \frac{6R + \frac{5}{4}R}{\frac{9}{2}}$$

$$= \frac{29R \times 2}{9 \times 4} = \frac{29R}{18} \text{ and}$$

Specific heat at constant pressure

$$C_p = \frac{n_1 C_{p1} + n_2 C_{p2}}{(n_1 + n_2)} = \frac{4 \times \frac{5R}{2} + \frac{1}{2} \times \frac{7R}{2}}{\left(4 + \frac{1}{2}\right)}$$

$$= \frac{10R + \frac{7}{4}R}{\frac{9}{2}} = \frac{47R}{18}$$

$$\therefore \frac{C_p}{C_v} = \frac{47R}{18} \times \frac{18}{29R} = 1.62$$

$$62. \text{ (c) } \gamma_1 = \frac{5}{3} \quad \gamma_2 = \frac{7}{5}$$

$$n_1 = 1, n_2 = 1$$

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\Rightarrow \frac{1+1}{\gamma-1} = \frac{1}{\frac{5}{3}-1} + \frac{1}{\frac{7}{5}-1} = \frac{3}{2} + \frac{5}{2} = 4$$

$$\therefore \frac{2}{\gamma-1} = 4 \Rightarrow \gamma = \frac{3}{2}$$

$$63. \text{ (d) } P \propto T^3 \Rightarrow PT^{-3} = \text{constant} \quad \dots(i)$$

But for an adiabatic process, the pressure temperature relationship is given by

$$P^{1-\gamma} T^\gamma = \text{constant}$$

$$\Rightarrow PT^{\frac{\gamma}{1-\gamma}} = \text{constt.} \quad \dots(ii)$$

$$\text{From (i) and (ii) } \frac{\gamma}{1-\gamma} = -3 \Rightarrow \gamma = -3 + 3\gamma \Rightarrow \gamma = \frac{3}{2}$$