Sample Question Paper - 10
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
3. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
4. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speeds of the two trains. **[2]**

OR

Solve the quadratic equation by factorization:

$a(x^2 + 1) - x(a^2 + 1) = 0$

2. A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with conical ends each of axis length 9 cm. Determine the capacity of the tank. **[2]**

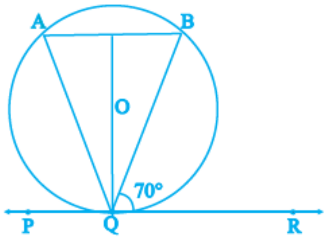
3. Find the value of p from the following data, if its mode is 48. **[2]**

Class	**Frequency**
0 - 10	7
10 - 20	14
20 - 30	13
30 - 40	12
40 - 50	p
50 - 60	18
60 - 70	15
70 - 80	8

4. Find the sum of all multiples of 5 lying between 101 and 999. **[2]**

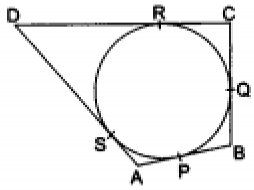
5. Find the mean of a distribution if its mode is 36 and median is 43. Also, write the relation between the mean, mode and median. **[2]**

6. In figure PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70^0$. Find $\angle AQB$. **[2]**



OR

A quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure. Prove that AB + CD = AD + BC.



Section B

7. The first and the last terms of an A.P are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum? **[3]**

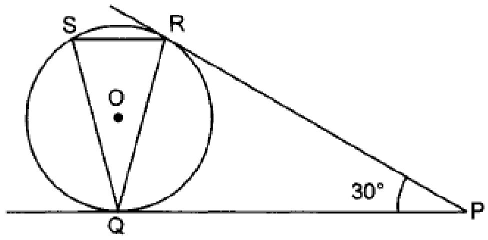
8. As observed from the top of a light-house, 100 m high above sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 60^0 . Determine the distance travelled by the ship during the period of observation. (Use $\sqrt{3}$ = 1.732) **[3]**

OR

From the top of a building 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60° respectively. Find

i. The horizontal distance between the building and the lamp post.
ii. The height of the lamp post. use($\sqrt{3}$ = 1.732)

9. In figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$. **[3]**



10. Solve: **[3]**

$$x = \frac{1}{2-\frac{1}{2-\frac{1}{2-x}}}, x \neq 2$$

Section C

11. Draw two concentric circles of radii 2 cm and 5 cm. Taking a point on the outer circle, construct a pair of tangents to the other. Measure the lengths of the two tangents. Also, verify the measurement by actual calculation. **[4]**

OR

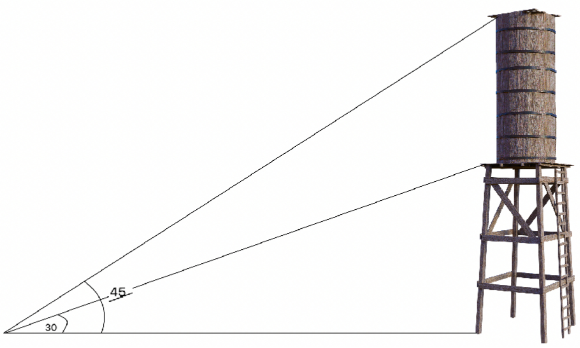
Draw a circle of radius 3 cm. Take a point A on its extended diameter at a distance of 7 cm from its centre. Draw two tangents to the circle from A.

12. The median of the following data is 525. Find the values of x and y, if the total frequency is 100. **[4]**

Class interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

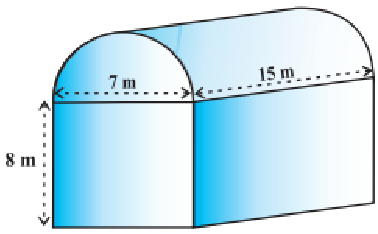
13. In a society, there are many multistorey buildings. The RWA of the society wants to install a tower and a watertank so that all the households can get water without using water pumps. For this they have measured the height of the tallest building in their society and now they want to install a tower that will be taller than that so that the level of water must be higher than the tallest building in their society. Here is one solution they have found and now they want to check if it will work or not. **[4]**

From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 30^0. the angle of elevation of the top of a water tank (on the top of the tower) is 45^0.



i. Find the height of the tower.
ii. Find the depth of the tank.

14. Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. **[4]**



i. If the base of the shed is 7 m by 15 m, and height of tire cuboidal portion is 8 m, find the volume of the air that the shed can hold.
ii. If the industry requires machinery which would occupy a total space of 300 m^3 and there are 20 workers each of whom would occupy 0.08 m^3 space on an average, how much air would be in the shed when it is working? (Take π = 22/7).

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Section A

1. Let the speed of the slow train be x km/hr

   Then, the speed of the fast train = (x+10) km/hr

   As we know that Time = $\frac{Distance}{Speed}$

   Time taken by the fast train to cover 600 km = $\frac{600}{x+10}$ hrs

   Time taken by the slow train to cover 600 km = $\frac{600}{x}$ hrs

   $\therefore \frac{600}{x} - \frac{600}{x+10} = 3$

   $\Rightarrow \frac{600(x+10)-600x}{x(x+10)} = 3$

   $\Rightarrow \frac{6000}{x^2+10x} = 3$

   $\Rightarrow 3x^2 + 30x - 6000 = 0$

   $\Rightarrow 3(x^2 + 10x - 2000) = 0\, or\, x^2 + 10x - 2000 = 0$

   $\Rightarrow x^2 + 50x - 40x - 2000 = 0$

   $\Rightarrow x(x+50) - 40(x+50) = 0$

   $\Rightarrow (x+50)(x-40) = 0$

   Either x = -50 or x = 40

   But the speed of the train cannot be negative. So, x= 40

   Hence, the speed of the two trains are 40km/hr and 50km/hr respectively.

   OR

   We have,

   a(x^2 + 1) - x(a^2 + 1) = 0

   $\Rightarrow$ ax^2 + a - xa^2 - x = 0

   $\Rightarrow$ ax^2 - xa^2 + a - x = 0

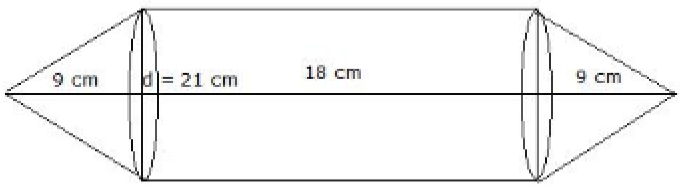
   $\Rightarrow$ ax(x - a) - 1(x - a) = 0

   $\Rightarrow$ (x - a)(ax - 1) = 0

   $\Rightarrow$ either x - a = 0 or, ax - 1 = 0

   $\Rightarrow$ x = a or $x = \frac{1}{a}$

   Hence, the roots of given quadratic equation are a and $\frac{1}{a}$

2. 

   Diameter of common base = 21 cm

   Radius of common base = $\frac{21}{2}$ cm

   Height of cone(h) = 9 cm

   Height of cylinder(H) = 18 cm

   $\therefore$ Capacity of tank = Volume of cylinder + 2$\times$Volume of cone

   = $\pi r^2 H + 2 \times \frac{1}{3}\pi r^2 h$

   $= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 18 + 2 \times \frac{1}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 9$

   $= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\left[18 + \frac{18}{3}\right]$

   $= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 24$

   = 8316 cm^3

3. Here, mode = 48

   Modal Class = 40 - 50

l = 40, f_0 = 12, f_1 = p, f_2 =18, h = 10

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\Rightarrow \quad 48 = 40 + \frac{p-12}{2p-12-18} \times 10$

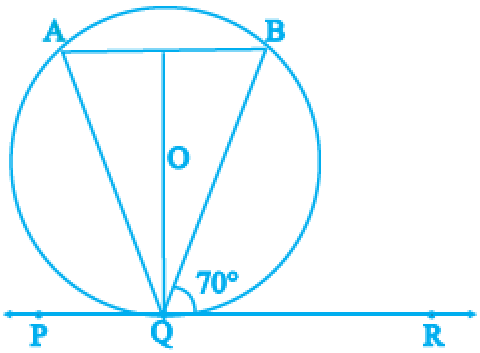
$8 = \frac{10p-120}{2p-30}$

$\Rightarrow$ 16p - 240 = 10p - 120

$\Rightarrow$ 6p = 120

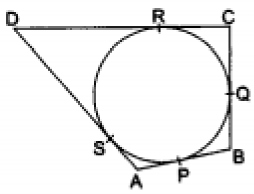
$\Rightarrow$ p = 20

4. Multiple of 5 lying between 101 and 999 are 105,110,115,..., 995 which are in AP.
   Here a = 105 and d = 5. where a is first term and d is common difference
   a_n = a + (n - 1)d
   $\Rightarrow$ 995 = 105 + (n - 1)5
   $\Rightarrow$ 890 = 5n - 5
   $\Rightarrow$ 895 = 5n
   $\therefore$ n = 179
   $\therefore$ S_n = $\frac{n}{2}[a + l]$ = $\frac{179}{2}$[105 + 995] = $\frac{179}{2} \times$ 1100 = 98450.
5. Given, mode = 36 , median = 43
   We know that, relation between mean , median & mode is:-
   3 median = mode + 2 mean.......(1)
   Or, -2 mean = mode - 3 median
   Or, -2 mean = 36 - 3 × 43
   Or, - 2 mean = 36 - 129
   Or, - 2 mean = - 93
   Thus, Mean = 46.5
6. Given,



PQR is a tangent
AB II PR
∠ BQR = 70
so ∠BAQ = ∠BQR = 70 (By Alternate segment theorem)
and ∠ABQ = ∠BQR = 70 (Alternate interior angles)
By summing all the angles
∠BAQ + ∠ABQ +∠AQB= 180
∠AQB = 180 - 70 -70
= 40

OR

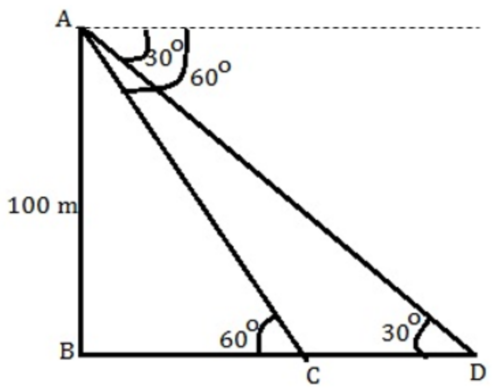


We know that the lengths of tangents drawn from an exterior point to a circle are equal.
AP = AS, ... (i) [tangents from A]
BP = BQ, ... (ii) [tangents from B]

CR = CQ, ... (iii) [tangents from C]
DR = DS. ... (iv) [tangents from D]
AB + CD = (AP + BP) + (CR + DR)
= (AS + BQ) + (CQ + DS) [using (i), (ii), (iii), (iv)]
= (AS + DS) + (BQ + CQ)
= AD + BC.
Hence, AB + CD = AD + BC.

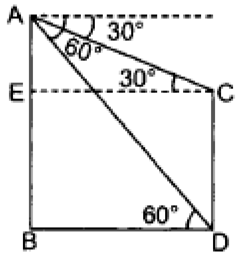
Section B

7. Let their be n terms in the given AP.
First term, a = 17
Last term, l = 350
Common difference, d=9
Now, T_n = 350
$\Rightarrow$ a + (n-1)d = 350
$\Rightarrow$ 17 + (n-1) 9 = 350
$\Rightarrow$ (n-1)(9) = 333
$\Rightarrow$ n-1 = 37
$\Rightarrow$ n = 38
Therefore,there are 38 terms in the AP
Now, $S_n = \frac{n}{2}$ [a+l]
$\Rightarrow S_{38} = \frac{38}{2}$ [17+350] = 19$\times$367 = 6973.

8. 

Height of the tower = 100 m
Let BC = x and BD = y
Consider the $\triangle ABC$,
$\frac{\text{AB}}{\text{BC}} = \tan 60^\circ$
$\Rightarrow \frac{100}{\text{x}} = \sqrt{3}$
$\Rightarrow \text{x} = \frac{100}{\sqrt{3}}\text{m}$
Consider the $\triangle ABD$,
$\frac{\text{AB}}{\text{BD}} = \tan 30^\circ$
$\frac{1}{\sqrt{3}} = \frac{100}{y}$
$y = 100\sqrt{3}$
We know that ,
BD = BC + CD
y = x + CD
CD = y - x
= $100\sqrt{3} - \frac{100}{\sqrt{3}}$
$= \frac{200}{\sqrt{3}}\text{m}$
$= \frac{200\sqrt{3}}{3}\text{m}$
$= 115.466\text{m}$

OR



Let AB = 60 m is height of building and CD is lamp post.

i. In ΔABD ,

$\frac{AB}{BD} = \tan 60^\circ$

$\Rightarrow \frac{60}{BD} = \sqrt{3} \Rightarrow \frac{60}{\sqrt{3}} = BD$

$\Rightarrow BD = \frac{60\times\sqrt{3}}{3} = 20\sqrt{3}\text{m}$

$\Rightarrow \text{BD} = 20 \times 1.732 = 34.64\text{m}$

ii. In Δ AEC ,

$\frac{\text{AE}}{\text{EC}} = \tan 30^\circ$

$\Rightarrow \frac{\text{AE}}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ AE = 20 m

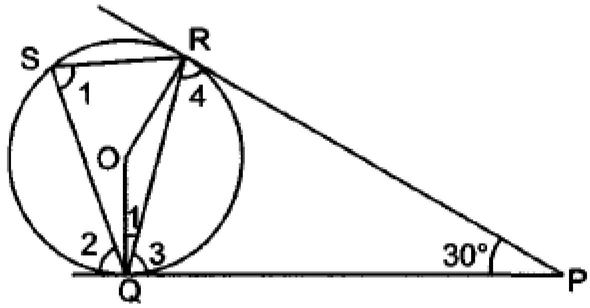
and

$EB = AB - AE = 60 - 20 = 40m$

Also ,

$EB = CD$

$\Rightarrow CD = 40m$

9. 

In $\triangle RQP$, $QP = RP$

$\therefore \angle 3 = \angle 4$

Now $\angle 3 + \angle 4 + 30^\circ = 180^\circ$

$\Rightarrow 2\angle 3 = 150^\circ \Rightarrow \angle 3 = 75^\circ$

Now $\angle \text{QOR} + \angle \text{QPR} = 180^\circ$

$\Rightarrow \angle \text{QOR} = 150^\circ$

Now, $\angle 1 = \frac{1}{2}\angle \text{QOR} \Rightarrow \angle 1 = 75^\circ$

Also $SR \,||\, QP$

$\therefore \quad \angle 1 = \angle 2$ [Alternate interior angles]

$\Rightarrow \angle 2 = 75^\circ$

$\angle 2 + \angle RQS + \angle 3 = 180^\circ$

$\Rightarrow \angle \text{ROS} = 180^\circ - 150^\circ = 30^\circ$

10. Given,

$x = \frac{1}{2-\frac{1}{2-\frac{1}{2-x}}}$

Now, $2 - \frac{1}{2-x} \Rightarrow \frac{2(2-x)-1}{(2-x)} \Rightarrow \frac{4-2x-1}{2-x} \Rightarrow \frac{3-2x}{2-x}$

and $2 - \frac{1}{2-\frac{1}{2-x}} \Rightarrow 2 - \frac{2-x}{3-2x} \Rightarrow \frac{2(3-2x)-(2-x)}{3-2x} \Rightarrow \frac{4-3x}{3-2x}$

$Hence,\ x = \frac{3-2x}{4-3x}$

Cross multiplication,

$\Rightarrow x(4 - 3x) = (3 - 2x)$

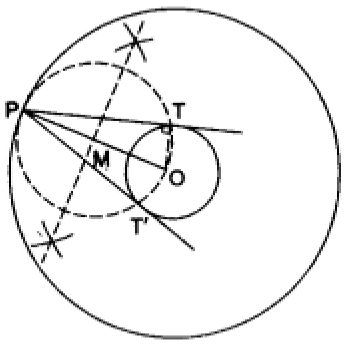
$\Rightarrow 4x - 3x^2 = 3 - 2x$

$\Rightarrow 3x^2 - 6x + 3 = 0 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1, 1$

Section C

11. **STEPS OF CONSTRUCTION**

1. Mark a point O on the paper.
2. With O as centre and radii 2 cm and 5 cm, draw two concentric circles
3. Mark a point P on the outer circle.



4. Join OP and bisect it at a point M.
5. Draw a circle with M as the centre and radius equal to MP, to intersect the inner circle in points, T and T'.
6. Join PT and PT'.

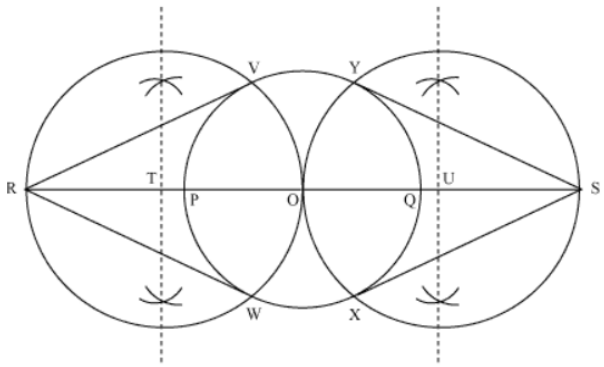
Then, PT and PT' are the required tangents.

Measurement Upon measuring the two tangents, we find PT = PT' = 4.6 cm.

OR

The steps of construction are as follows:

i. Take any point O as centre and draw a circle of 3 cm radius.
ii. Let PQ be one of its diameters. Extend it on both sides. Locate two points on this diameter such that OR = OS = 7 CM
iii. Bisect OR and OS. Let T and U be the mid-points of OR and OS respectively.
iv. Taking T and U as its centre and with TO and UO as radius, draw two circles. These two circles will intersect the circle with centre O at points V, W, X, Y respectively. Join RV, RW, SX, and SY. These are the required tangents.



12.

Class intervals	Frequency (f)	Cumulative frequency (cf/F)
0-100	2	2
100-200	5	7
200-300	x	7 + x
300-400	12	19 + x
400-500	17	36 + x
500-600	20	56 + x

600-700	y	56 + x + y
700-800	9	65 + x + y
800-900	7	72 + x + y
900-1000	4	76 + x + y
		Total = 76 + x + y

We have,
N = Σf_i = 100
$\Rightarrow$ 76 + x + y = 100
$\Rightarrow$ x + y = 24
It is given that the median is 525. Clearly, it lies in the class 500 - 600
$\therefore$ l = 500, h = 100, f = 20, F = 36 + x and N = 100
Now, Median = l + $\frac{\frac{N}{2}-F}{f}\times$ h
$\Rightarrow$ 525 = 500 + $\frac{50-(36+x)}{20}\times$ 100
$\Rightarrow$ 525 - 500 = (14 - x)5
$\Rightarrow$ 25 = 70 - 5x
$\Rightarrow$ 5x = 45
$\Rightarrow$ x = 9
Putting x = 9 in x + y = 24, we get y = 15
Hence, x = 9 and y = 15

13. Let BC be the tower of height h metre and CD be the water tank of height h_1 metre.
In ΔABD, we have
$\tan 45^\circ = \frac{BD}{AB}$
$\Rightarrow \quad 1 = \frac{h+h_1}{40}$
$\Rightarrow \quad h + h_1 = 40\text{m}$
In ΔABC, we have
$\tan 30^\circ = \frac{BC}{AB}$
$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40}$
$\Rightarrow \quad h = \frac{40}{\sqrt{3}}\text{m} = \frac{40\sqrt{3}}{3}\text{m} = 23.1\text{m}$
Substituting the value of h in (i), we have
23.1 + h_1 = 40
$\Rightarrow$ h_1 = (40 - 23.1) m = 16.9 m
Hence, the height of the tower is h = 23.1 m and the depth of the tank is h_1 = 16.9 m.

14. Total volume = volume of cuboid + 1/2 $\times$ volume of cylinder.
For cuboidal part we have
length = 15 m, breadth = 7 m and height = 8 m
$\therefore$ Volume of cuboidal part = l × b ×h$= 15 \times 7 \times 8\text{m}^2 = 840\text{m}^3$
Clearly,
r = Radius of half-cylinder = $\frac{1}{2}$ (Width of the cuboid) = $\frac{7}{2}$m
and, h = Height (length) of half-cylinder = Length of cuboid = 15 m
$\therefore$ Volume of half-cylinder = $\frac{1}{2}\pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15\text{m}^3 = \frac{1155}{4}\text{m}^3 = 288.75\text{m}^3$
Volume of air inside the shed when there is no people or machinery = (840 + 288.75) m^3 = 1128.75 m^3
Now, Total space occupied by 20 workers = $20 \times 0.08\text{m}^3 = 1.6\text{m}^3$
Total space occupied by the machinery = 300 m^3
$\therefore$ Volume of the air inside the shed when there are machine and workers inside it
= (1128.75 - 1.6 - 300) m^3

$= (1128.75 - 301.6)\ m^3 = 827.15\ m^3$

Hence, volume of air when there are machinery and workers is $827.15\ m^3$