

Relations and Functions

Question1

The function $f : \mathbb{N} - \{1\} \rightarrow \mathbb{N}$; defined by $f(n)$ = the highest prime factor of n , is :

[27-Jan-2024 Shift 1]

Options:

A.

both one-one and onto

B.

one-one only

C.

onto only

D.

neither one-one nor onto

Answer: D

Solution:

$$f : \mathbb{N} - \{1\} \rightarrow \mathbb{N}$$

$f(n)$ = The highest prime factor of n .

$$f(2) = 2$$

$$f(4) = 2$$

\Rightarrow many one

4 is not image of any element

\Rightarrow into

Hence many one and into

Neither one-one nor onto.

Question2

Let $f : \mathbb{R} - \left\{ \frac{-1}{2} \right\} \rightarrow \mathbb{R}$ and $g : \mathbb{R} - \left\{ \frac{-5}{2} \right\} \rightarrow \mathbb{R}$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then the domain of the function $f \circ g$ is :

[27-Jan-2024 Shift 2]

Options:

A.

$$\mathbb{R} - \left\{ -\frac{5}{2} \right\}$$

B.

\mathbb{R}

C.

$$\mathbb{R} - \left\{ -\frac{7}{4} \right\}$$

D.

$$\mathbb{R} - \left\{ -\frac{5}{2}, -\frac{7}{4} \right\}$$

Answer: A

Solution:

$$f(x) = \frac{2x+3}{2x+1}; x \neq -\frac{1}{2}$$

$$g(x) = \frac{|x|+1}{2x+5}; x \neq -\frac{5}{2}$$

Domain of $f(g(x))$

$$f(g(x)) = \frac{2g(x)+3}{2g(x)+1}$$

$$x \neq -\frac{5}{2} \text{ and } \frac{|x|+1}{2x+5} \neq -\frac{1}{2}$$

$$x \in \mathbb{R} - \left\{ -\frac{5}{2} \right\} \text{ and } x \in \mathbb{R}$$

$$\therefore \text{Domain will be } \mathbb{R} - \left\{ -\frac{5}{2} \right\}$$

Question3

Consider the function $f : [1/2, 1] \rightarrow \mathbb{R}$ defined by $f(x) = 4\sqrt{2x} - 3\sqrt{2x-1}$. Consider the statements

(I) The curve $y = f(x)$ intersects the x-axis exactly at one point

(II) The curve $y = f(x)$ intersects the x-axis at $x = \cos \pi/12$

Then

[29-Jan-2024 Shift 1]

Options:

A.

Only (II) is correct

B.

Both (I) and (II) are incorrect

C.

Only (I) is correct

D.

Both (I) and (II) are correct

Answer: D

Solution:

$$f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} \geq 0 \text{ for } \left[\frac{1}{2}, 1 \right]$$

$$f\left(\frac{1}{2}\right) < 0$$

$f(1) > 0 \Rightarrow$ (A) is correct.

$$f(x) = \sqrt{2}(4x^3 - 3x) - 1 = 0$$

Let $\cos \alpha = x$,

$$\cos 3\alpha = \cos \frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{12}$$

$$x = \cos \frac{\pi}{12}$$

(4) is correct.

Question4

$$\text{If } f(x) = \begin{cases} 2 + 2x & -1 \leq x < 0 \\ 1 - \frac{x}{3} & 0 \leq x \leq 3 \end{cases}$$

$$g(x) = \begin{cases} -x & -3 \leq x \leq 0 \\ x & 0 < x \leq 1 \end{cases} \dots,$$

then range of $(f \circ g(x))$ is

[29-Jan-2024 Shift 1]

Options:

A.

$(0, 1]$

B.

$[0, 3)$

C.

$[0, 1]$

D.

$[0, 1)$

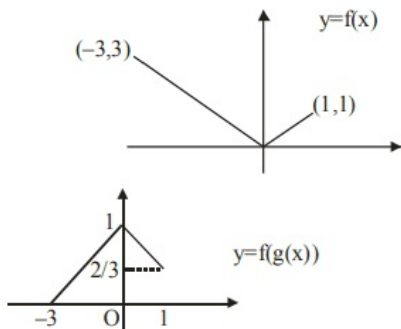
Answer: C

Solution:

$$f(g(x)) = \begin{cases} 2 + 2g(x), & -1 \leq g(x) < 0 \dots\dots (1) \\ 1 - \frac{g(x)}{3}, & 0 \leq g(x) \leq 3 \dots\dots (2) \end{cases}$$

By (1) $x \in \phi$

And by (2) $x \in [-3, 0]$ and $x \in [0, 1]$



Range of $f(g(x))$ is $[0, 1]$

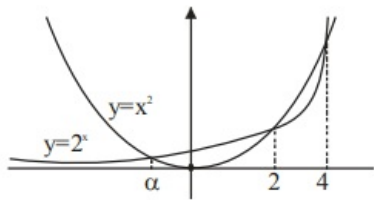
Question5

Let $f(x) = 2^x - x^2$, $x \in \mathbb{R}$. If m and n are respectively the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersects the x -axis, then the value of $m + n$ is

[29-Jan-2024 Shift 1]

Answer: 5

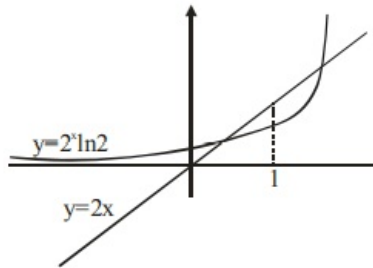
Solution:



$$\therefore m = 3$$

$$f'(x) = 2^x \ln 2 - 2x = 0$$

$$2^x \ln 2 = 2x$$



$$\therefore n = 2$$

$$\Rightarrow m + n = 5$$

Question6

If the domain of the function $f(x) = \cos^{-1} (2 - |x|/4) + (\log_e(3 - x))^{-1}$ is $[-\alpha, \beta) - \{y\}$, then $\alpha + \beta + \gamma$ is equal to :

[30-Jan-2024 Shift 1]

Options:

A.

12

B.

9

C.

11

D.

8

Answer: C

Solution:

$$-1 \leq \left| \frac{2-|x|}{4} \right| \leq 1$$

$$\Rightarrow \left| \frac{2-|x|}{4} \right| \leq 1$$

$$-4 \leq 2-|x| \leq 4$$

$$-6 \leq -|x| \leq 2$$

$$-2 \leq |x| \leq 6$$

$$|x| \leq 6$$

$$\Rightarrow x \in [-6, 6] \dots\dots(1)$$

$$\text{Now, } 3-x \neq 1$$

$$\text{And } x \neq 2 \dots\dots(2)$$

$$\text{and } 3-x > 0$$

$$x < 3 \dots\dots(3)$$

$$\text{From (1), (2) and (3)}$$

$$\Rightarrow x \in [-6, 3) - \{2\}$$

$$\alpha = 6$$

$$\beta = 3$$

$$\gamma = 2$$

$$\alpha + \beta + \gamma = 11$$

Question7

Let $A = \{1, 2, 3, \dots, 7\}$ and let $P(A)$ denote the power set of A . If the number of functions $f : A \rightarrow P(A)$ such that $a \in f(a), \forall a \in A$ is m, m and $n \in \mathbb{N}$ and m is least, then $m + n$ is equal to_____

[30-Jan-2024 Shift 1]

Answer: 44

Solution:

$$f : A \rightarrow P(A)$$

$$a \in f(a)$$

That means 'a' will connect with subset which contain element 'a'.

Total options for 1 will be 2^6 . (Because 2^6 subsets contains 1)

Similarly, for every other element

$$\text{Hence, total is } 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 = 2^{42}$$

$$\text{Ans. } 2 + 42 = 44$$

Question8

If the domain of the function $f(x) = \log_e \left(\frac{2x+3}{4x^2+x-3} \right) + \cos^{-1} \left(\frac{2x-1}{x+2} \right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to

[30-Jan-2024 Shift 2]

Options:

A.

10

B.

12

C.

11

D.

9

Answer: B

Solution:

$$\begin{aligned} \frac{2x+3}{4x^2+x-3} > 0 \text{ and } -1 \leq \frac{2x-1}{x+2} \leq 1 \\ \frac{2x+3}{(4x-3)(x+1)} > 0 \quad \frac{3x+1}{x+2} \geq 0 \text{ \& } \frac{x-3}{x+2} \leq 0 \\ \begin{array}{ccccccc} & - & & + & & - & & + \\ \hline & -3/2 & & -1 & & 3/4 & & \end{array} \\ (-\infty, -2) \cup \left[\frac{-1}{3}, \infty \right) \dots\dots (1) \\ (-2, 3] \dots\dots (2) \\ \left[\frac{-1}{3}, 3 \right] \dots\dots (3) \quad (1) \cap (2) \cap (3) \\ \left(\frac{3}{4}, 3 \right] \\ \alpha = \frac{3}{4}\beta = 3 \\ 5\beta - 4\alpha = 15 - 3 = 12 \end{aligned}$$

Question9

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined $f(x) = \frac{x}{(1+x^4)^{1/4}}$ and $g(x) = \int_0^{\sqrt{2\sqrt{5}}} x^2 g(x) \, dx$
 $f(f(f(f(x))))$ then 18

Options:

A.

33

B.

36

C.

42

D.

39

Answer: D

Solution:

$$\begin{aligned} f(x) &= \frac{x}{(1+x^4)^{1/4}} \\ f \circ f(x) &= \frac{f(x)}{(1+f(x)^4)^{1/4}} = \frac{\frac{x}{(1+x^4)^{1/4}}}{\left(1 + \frac{x^4}{1+x^4}\right)^{1/4}} = \frac{x}{(1+2x^4)^{1/4}} \\ f(f(f(x))) &= \frac{x}{(1+4x^4)^{1/4}} \\ 18 \int_0^{\sqrt[4]{2\sqrt{5}}} \frac{x^3}{(1+4x^4)^{1/4}} dx \\ \text{Let } 1+4x^4 &= t^4 \\ 16x^3 dx &= 4t^3 dt \\ \frac{18}{4} \int_1^3 \frac{t^3 dt}{t} \\ &= \frac{9}{2} \left(\frac{t^3}{3} \right)_1^3 \\ &= \frac{3}{2} [26] = 39 \end{aligned}$$

Question10

If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ and $(f \circ f)(x) = g(x)$, where $g : \mathbb{R} - \left\{ \frac{2}{3} \right\} \rightarrow \mathbb{R} - \left\{ \frac{2}{3} \right\}$, then $(g \circ g \circ g)(4)$ is equal to

[31-Jan-2024 Shift 1]

Options:

A.

$$-\frac{19}{20}$$

B.

19/20

C.

-4

D.

4

Answer: D

Solution:

$$f(x) = \frac{4x+3}{6x-4}$$

$$g(x) = \frac{4\left(\frac{4x+3}{6x-4}\right) + 3}{6\left(\frac{4x+3}{6x-4}\right) - 4} = \frac{34x}{34} = x$$

$$g(x) = x \therefore g(g(g(4))) = 4$$

Question11

If the function $f : (-\infty, -1] \rightarrow (a, b]$ defined by $f(x) = e^{x^3-3x+1}$ is one-one and onto, then the distance of the point

$P(2b + 4, a + 2)$ from the line $x + e^{-3}y = 4$ is :

[31-Jan-2024 Shift 2]

Options:

A.

$$2\sqrt{1+e^6}$$

B.

$$4\sqrt{1+e^6}$$

C.

$$3\sqrt{1+e^6}$$

D.

$$\sqrt{1+e^6}$$

Answer: A

Solution:

$$f(x) = e^{x^3 - 3x + 1}$$

$$f'(x) = e^{x^3 - 3x + 1} \cdot (3x^2 - 3)$$

$$= e^{x^3 - 3x + 1} \cdot 3(x - 1)(x + 1)$$

For $f'(x) \geq 0$

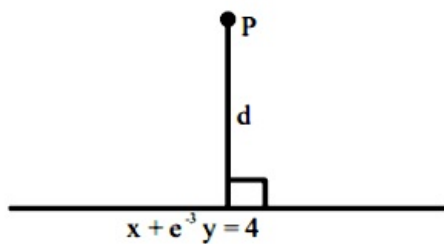
$\therefore f(x)$ is increasing function

$$\therefore a = e^{-\infty} = 0 = f(-\infty)$$

$$b = e^{-1+3+1} = e^3 = f(-1)$$

$$P(2b + 4, a + 2)$$

$$\therefore P(2e^3 + 4, 2)$$



$$d = \frac{(2e^3 + 4) + 2e^{-3} - 4}{\sqrt{1 + e^{-6}}} = 2\sqrt{1 + e^6}$$

Question 12

Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be defined as

$$f(x) = \begin{cases} \log_e x & , \quad x > 0 \\ e^{-x} & , \quad x \leq 0 \end{cases}.$$

and

$$g(x) = \begin{cases} x & , \quad x \geq 0 \\ e^x & , \quad x < 0 \end{cases}.$$

Then, $g \circ f: R \rightarrow R$ is :

[1-Feb-2024 Shift 1]

Options:

A.

one-one but not onto

B.

neither one-one nor onto

C.

onto but not one-one

D.

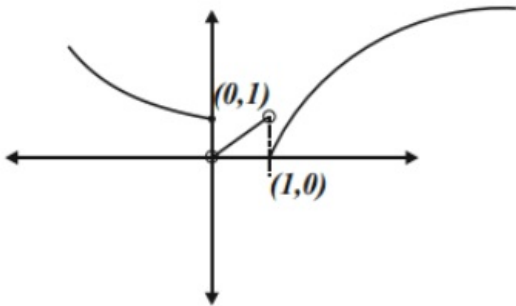
both one-one and onto

Answer: B

Solution:

$$g(f(x)) = \begin{cases} f(x) & f(x) \geq 0 \\ e^{f(x)} & f(x) < 0 \end{cases}.$$

$$g(f(x)) = \begin{cases} e^{-x} & (-\infty, 0] \\ e^{\ln x} & (0, 1) \\ \ln x & [1, \infty) \end{cases}.$$



Graph of $g(f(x))$

$g(f(x)) \Rightarrow$ Many one into

Question13

If the domain of the function $f(x) = \frac{\sqrt{x^2 - 25}}{(4 - x^2)} + \log_{10}(x^2 + 2x - 15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to :

[1-Feb-2024 Shift 2]

Options:

A.

140

B.

175

C.

150

D.

Answer: C**Solution:**

$$f(x) = \frac{\sqrt{x^2 - 25}}{4 - x^2} + \log_{10}(x^2 + 2x - 15)$$

$$\text{Domain} : x^2 - 25 \geq 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$$

$$4 - x^2 \neq 0 \Rightarrow x \neq \{-2, 2\}$$

$$x^2 + 2x - 15 > 0 \Rightarrow (x + 5)(x - 3) > 0$$

$$\Rightarrow x \in (-\infty, -5) \cup (3, \infty)$$

$$\therefore x \in (-\infty, -5) \cup [5, \infty)$$

$$\alpha = -5; \beta = 5$$

$$\therefore \alpha^2 + \beta^3 = 150$$

Question14

Let $A = \{1, 2, 3, 4, \dots, 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times A : 2(a - b)^2 + 3(a - b) \in B\}$ is _____.

[6-Apr-2023 shift 1]

Answer: 18**Solution:****Solution:**

$$A = \{1, 2, 3, \dots, 10\}$$

$$B = \{0, 1, 2, 3, 4\}$$

$$R = \{(a, b) \in A \times A : 2(a - b)^2 + 3(a - b) \in B\}$$

$$\text{Now } 2(a - b)^2 + 3(a - b) = (a - b)(2(a - b) + 3)$$

$$\Rightarrow a = b \text{ or } a - b = -2$$

$$\text{When } a = b \Rightarrow 10 \text{ order pairs}$$

$$\text{When } a - b = -2 \Rightarrow 8 \text{ order pairs}$$

$$\text{Total} = 18$$

Question15

Let $A = \{0, 34, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{x, y) \in A \times A : x - y \text{ is odd positive integer or } x - y = 2\}$. The minimum number of elements that must be added to the relation R , so that it is a symmetric relation, is equal to _____.

[8-Apr-2023 shift 1]

Answer: 19

Solution:

Solution:

$A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ 3, 7, 9 \rightarrow odd

$R = \{x - y = \text{odd} + \text{ve or } x - y = 2\}$ 0, 4, 6, 8, 10 \rightarrow even

${}^3C_1 \cdot {}^5C_1 = 15 + (6, 4), (8, 6), (10, 8), (9, 7)$

Min^m ordered pairs to be added must be

$: 15 + 4 = 19$

Question16

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Then the relation $R = \{(x, y) \in A \times A : x + y = 7\}$ is
[8-Apr-2023 shift 2]

Options:

- A. Symmetric but neither reflexive nor transitive
- B. Transitive but neither symmetric nor reflexive
- C. An equivalence relation
- D. Reflexive but neither symmetric nor transitive

Answer: A

Solution:

Solution:

$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

Question17

Let $A = \{2, 3, 4\}$ and $B = \{8, 9, 12\}$. Then the number of elements in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1 \text{ divides } b_2 \text{ and } a_2 \text{ divides } b_1\}$ is
[10-Apr-2023 shift 2]

Options:

- A. 18
- B. 24

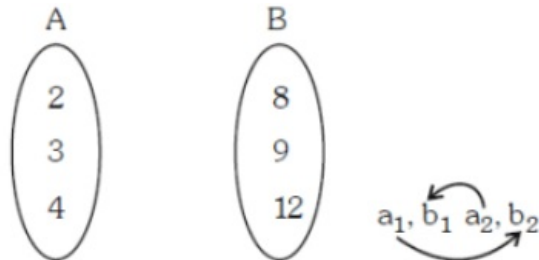
C. 12

D. 36

Answer: D

Solution:

Solution:



a_1 divides b_2

Each elements has 2 choices

$\Rightarrow 3 \times 2 = 6$

a_2 divides b_1

Each elements has 2 choices

$\Rightarrow 3 \times 2 = 6$

Total = $6 \times 6 = 36$

Question18

Let $A = \{1, 3, 4, 6, 9\}$ and $B = \{2, 4, 5, 8, 10\}$. Let R be a relation defined on $A \times B$ such that $R = \{ (a_1, b_1), (a_2, b_2, \dots)) : a_1 \leq b_2 \text{ and } b_1 \leq a_2 \}$. Then the number of elements in the set R is [11-Apr-2023 shift 2]

Options:

A. 52

B. 160

C. 26

D. 180

Answer: B

Solution:

Solution:

Let $a_1 = 1 \Rightarrow 5$ choices of b_2

$a_1 = 3 \Rightarrow 4$ choices of b_2

$a_1 = 4 \Rightarrow 4$ choices of b_2

$a_1 = 6 \Rightarrow 2$ choices of b_2

$a_1 = 9 \Rightarrow 1$ choices of b_2

For (a_1, b_2) 16 ways .

Similarly, $b_1 = 2 \Rightarrow 4$ choices of a_2

$b_1 = 4 \Rightarrow 3$ choices of a_2

$b_1 = 5 \Rightarrow 2$ choices of a_2

$b_1 = 8 \Rightarrow 1$ choices of a_2
Required elements in $R = 160$

Question19

The number of the relations, on the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 3)$, which are reflexive and transitive but not symmetric, is _____.
[12-Apr-2023 shift 1]

Answer: 3

Solution:

Solution:

$A = \{1, 2, 3\}$

For Reflexive $(1, 1)(2, 2), (3, 3) \in R$

For transitive : $(1, 2)$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$

Not symmetric : $(2, 1)$ and $(3, 2) \notin R$

$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(2, 1)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(2, 1)\}$

Question20

Let $A = \{-4, -3, -2, 0, 1, 3, 4\}$ and $R = \{(a, b) \in A \times A : b = |a| \text{ or } b^2 = a + 1\}$ be a relation on A . Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is _____.
[13-Apr-2023 shift 2]

Answer: 7

Solution:

Solution:

$R = \{(-4, 4), (-3, 3), (3, -2), (0, 1), (0, 0), (1, 1), (4, 4), (3, 3)\}$

For reflexive, add $\Rightarrow (-2, -2), (-4, -4), (-3, -3)$

For symmetric, add $\Rightarrow (4, -4), (3, -3), (-2, 3), (1, 0)$

Question21

Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set $A \times A$ defined by $R = \{ (a, b, (c, d)) : 2a + 3b = 4c + 5d \}$. Then the number of elements in R is _____
[15-Apr-2023 shift 1]

Answer: 6

Solution:

Solution:

$$A = \{1, 2, 3, 4\}$$

$$R = \{(a, b), (c, d)\}$$

$$2a + 3b = 4c + 5d = \alpha \text{ let}$$

$$2a = \{2, 4, 6, 8\} \quad 4c = \{4, 8, 12, 16\}$$

$$3b = \{3, 6, 9, 12\} \quad 5d = \{5, 10, 15, 20\}$$

$$2a + 3b = \left\{ \begin{array}{cccc} 5 & 8 & 11 & 14 \\ 7 & 10 & 13 & 16 \\ 9 & 12 & 15 & 18 \\ 11 & 14 & 17 & 20 \end{array} \right\} \quad 4c + 5d = \left\{ \begin{array}{cccc} 9 & 14 & 19 & 24 \\ 13 & 18 & & \\ 17 & 22 & & \\ 21 & 26 & & \end{array} \right\}$$

Possible value of $\alpha = 9, 13, 14, 14, 17, 18$

Pairs of $\{(a, b), (c, d)\} = 6$

Question22

Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$. Then $18 \int_1^2 f(x) dx$ is equal to :
[6-Apr-2023 shift 1]

Options:

A. $10\log_e 2 - 6$

B. $10\log_e 2 + 6$

C. $5\log_e 2 - 3$

D. $5\log_e 2 + 3$

Answer: A

Solution:

Solution:

$$5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3 \dots (1)$$

$$x \rightarrow \frac{1}{x}$$

$$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3 \dots (2)$$

$$(1) \times 5 - (2) \times 4$$

$$\Rightarrow f(x) = \frac{5}{9x} - \frac{4}{9}x + \frac{1}{3}$$

$$\Rightarrow 18 \int_1^2 f(x) dx = 18 \left(\frac{5}{9} \ln 2 - \frac{4}{9} \times \frac{3}{2} + \frac{1}{3} \right)$$

$$= 10 \ln 2 - 6$$

Question23

Let $A = \{x \in \mathbb{R} : [x + 3] + [x + 4] \leq 3\}$,

$B = \left\{ x \in \mathbb{R} : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r} \right)^{x-3} < 3^{-3x} \right\}$, where $[t]$ denotes greatest

integer function. Then,

[6-Apr-2023 shift 1]

Options:

A. $A \subset B, A \neq B$

B. $A \cap B = \varnothing$

C. $A = B$

D. $B \subset C, A \neq B$

Answer: C

Solution:

Solution:

$$A = \{x \in \mathbb{R} : [x + 3] + [x + 4] \leq 3\}$$

$$2[x] + 7 \leq 3$$

$$2[x] \leq -4$$

$$[x] \leq -2 \Rightarrow x < -1 \dots (A)$$

$$B = \left\{ x \in \mathbb{R} : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r} \right)^{x-3} < 3^{-3x} \right\}$$

$$3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r} \right)^{x-3} < 3^{-3x}$$

$$3^{2x-3} \left(\frac{10}{10} \right)^{x-3}$$

$$\Rightarrow \left(\frac{1}{9} \right)^{x-3} < 3^{-5x}$$

$$\Rightarrow 3^{6-2x} < 3^{3-5x}$$

$$\Rightarrow 6-2x < 3-5x$$

$$\Rightarrow 3 < -3x$$

$$\Rightarrow \frac{1}{10}^x < -1 \dots (B)$$

$$A = B$$

Question24

Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{[x] - x}}$, where $[x]$ denotes the smallest integer greater than or equal to x. Then among the statements :

(S1) : $A \cap B = (1, \infty) - \mathbb{N}$ and

(S2) : $A \cup B = (1, \infty)$
[6-Apr-2023 shift 2]

Options:

- A. only (S1) is true
- B. neither (S1) nor (S2) is true
- C. only (S2) is true
- D. both (S1) and (S2) are true

Answer: A

Solution:

Solution:

$$f(x) = \frac{1}{\sqrt{[x] - x}}$$

If $x \in I$ $[x] = [x]$ (greatest integer function)

If $x \notin I$ $[x] = [x] + 1$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{[x] - x}} & x \in I \\ \frac{1}{\sqrt{[x] + 1 - x}} & x \notin I \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{-\{x\}}} & x \in I \text{ (does not exist)} \\ \frac{1}{\sqrt{1 - \{x\}}} & x \notin I \end{cases}$$

\Rightarrow domain of $f(x) = \mathbb{R} - I$

Now, $f(x) = \frac{1}{\sqrt{1 - \{x\}}}$, $x \notin I$

$\Rightarrow x < \{x\} < 1$

$\Rightarrow 0 < 1 - \{x\} < 1$

$\Rightarrow \frac{1}{\sqrt{1 - \{x\}}} > 1$

$\Rightarrow \text{Range}(1, \infty)$

$\Rightarrow A = \mathbb{R} - I$

$B = (1, \infty)$

So, $A \cap B = (1, \infty) - \mathbb{N}$

$A \cup B \neq (1, \infty)$

$\Rightarrow S1$ is only correct.

Question25

Let $f, g : \mathbb{N} - \{1\} \rightarrow \mathbb{N}$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^α divides a , and $g(a) = a + 1$, for all $a \in \mathbb{N} - \{1\}$. Then, the function $f + g$ is
[27-Jul-2022-Shift-1]

Options:

- A. one-one but not onto
- B. onto but not one-one

C. both one-one and onto

D. neither one-one nor onto

Answer: D

Solution:

Solution:

$f, g : \mathbb{N} - \{1\} \rightarrow \mathbb{N}$ defined as

$f(a) = \alpha$, where α is the maximum power of those primes p such that p^α divides a .

$g(a) = a + 1$

Now,

$f(2) = 1, g(2) = 3 \Rightarrow (f + g)(2) = 4$

$f(3) = 1, g(3) = 4 \Rightarrow (f + g)(3) = 5$

$f(4) = 2, g(4) = 5 \Rightarrow (f + g)(4) = 7$

$f(5) = 1, g(5) = 6 \Rightarrow (f + g)(5) = 7$

$\therefore (f + g)(5) = (f + g)(4)$

$\therefore f + g$ is not one-one

Now, $\therefore f_{\min} = 1, g_{\min} = 3$

So, there does not exist any $x \in \mathbb{N} - \{1\}$ such that $(f + g)(x) = 1, 2, 3$

$\therefore f + g$ is not onto

Question26

If domain of the function $\log_e \left(\frac{6x^2 + 5x + 1}{2x - 1} \right) + \cos^{-1} \left(\frac{2x^2 - 3x + 4}{3x - 5} \right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then, $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to
[8-Apr-2023 shift 2]

Answer: 20

Solution:

Solution:

$$\frac{6x^2 + 5x + 1}{2x - 1} > 0$$

$$\frac{(3x + 1)(2x + 1)}{2x - 1} > 0$$

$$x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right] \cup \left(\frac{5}{3}, \infty \right) \dots (B)$$

$$x < \frac{5}{3} \dots (C)$$

$$A \cap B \cap C \equiv \left(\frac{-1}{2}, \frac{-1}{3} \right) \cup \left(\frac{1}{2}, \frac{1}{\sqrt{2}} \right]$$

$$\begin{aligned} \text{So } 18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2) &= 18 \left(\frac{1}{4} + \frac{1}{9} + \frac{1}{4} + \frac{1}{2} \right) \\ &= 18 + 2 = 20 \end{aligned}$$

Question27

Let $R = \{a, b, c, d, e\}$ and $S = \{1, 2, 3, 4\}$. Total number of onto functions $f : R \rightarrow S$ such that $f(a) \neq 1$, is equal to _____.

[8-Apr-2023 shift 2]

Answer: 180

Solution:

Solution:

Total onto function

$$\frac{5}{1} \times 4 = 240$$

Now when $f(a) = 1$

$$4 + \frac{4}{2} \times 3 = 24 + 36 = 60.$$

$$\text{so required } f^n = 240 - 60 = 180$$

Question28

If the domain of the function $f(x) = \sec^{-1} \left(\frac{2x}{5x+3} \right)$ is $[\alpha, \beta) \cup (\gamma, \delta]$, then

$|3\alpha + 10(\beta + \gamma) + 21\delta|$ is equal to _____.

[10-Apr-2023 shift 2]

Answer: 24

Solution:

Solution:

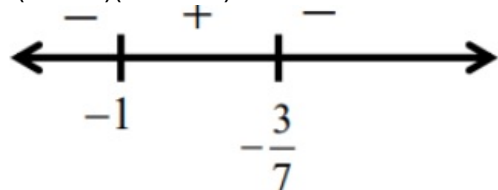
$$f(x) = \sec^{-1} \frac{2x}{5x+3}$$

$$\left| \frac{2x}{5x+3} \right|$$

$$\left| \frac{2x}{5x+3} \right| \geq 1 \Rightarrow |2x| \geq |5x+3|$$

$$(2x)^2 - (5x+3)^2 \geq 0$$

$$(7x+3)(-3x-3) \geq 0$$



$$\therefore \text{domain } \left[-1, -\frac{3}{7} \right) \cup \left(-\frac{3}{7}, 0 \right]$$

$$\alpha = -1, \beta = -\frac{3}{7}, \gamma = -\frac{3}{7}, \delta = 0$$

$$3\alpha + 10(\beta + \gamma) + 21\delta = -3$$

$$-3 + 10 \left(-\frac{6}{7} \right) + \left(-\frac{3}{7} \right) 21 = -24$$

Question29

The domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is (where $[x]$ denotes the greatest integer less than or equal to x)
[11-Apr-2023 shift 2]

Options:

- A. $(-\infty, -3] \cup [6, \infty)$
- B. $(-\infty, -2) \cup (5, \infty)$
- C. $(-\infty, -3] \cup (5, \infty)$
- D. $(-\infty, -2) \cup [6, \infty)$

Answer: D

Solution:

Solution:

$$F(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$$

$$[x]^2 - 3[x] - 10 > 0$$

$$([x] + 2)([x] - 5) > 0$$



$$[x] < -2 \text{ or } [x] > 5$$

$$[x] \leq -3 \text{ or } [x] \geq 6$$

$$x < -2 \text{ or } x \geq 6$$

$$x \in (-\infty, -2) \cup [6, \infty)$$

Question30

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Then the number of functions $f : A \rightarrow B$ satisfying $f(1) + f(2) = f(4) - 1$ is equal to _____.
[11-Apr-2023 shift 2]

Answer: 360

Solution:

Solution:

$$f(1) + f(2) + 1 = f(4) \leq 6$$

$$f(1) + f(2) \leq 5$$

Case (i) $f(1) = 1 \Rightarrow f(2) = 1, 2, 3, 4 \Rightarrow 4$ mappings

Case (ii) $f(1) = 2 \Rightarrow f(2) = 1, 2, 3 \Rightarrow 3$ mappings

Case (iii) $f(1) = 3 \Rightarrow f(2) = 1, 2 \Rightarrow 2$ mappings

Case (iv) $f(1)4 \Rightarrow f(2) = 1 \Rightarrow 1$ mapping

f(5) & f(6) both have 6 mappings each
 Number of functions = $(4 + 3 + 2 + 1) \times 6 \times 6 = 360$

Question 31

Let D be the domain of the function $f(x) = \sin^{-1} \left(\log_{3x} \left(\frac{6 + 2\log_3 x}{-5x} \right) \right)$. If the range of the function $g : D \rightarrow \mathbb{R}$ defined by $g(x) = x - [x]$, ($[x]$ is the greatest integer function), is (α, β) , then $\alpha^2 + \frac{5}{\beta}$ is equal to
 [12-Apr-2023 shift 1]

Options:

- A. 46
- B. 135
- C. 136
- D. 45

Answer: B

Solution:

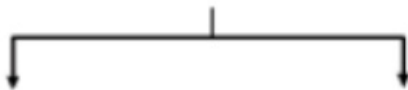
Solution:

$$\frac{6 + 2\log_3 x}{-5x} > 0 \text{ \& } x > 0 \text{ \& } x \neq \frac{1}{3}$$

$$\text{this gives } x \in \left(0, \frac{1}{27} \right) \dots (1)$$

$$-1 \leq \log_{3x} \left(\frac{6 + 2\log_3 x}{-5x} \right) \leq 1$$

$$3x \leq \frac{6 + 2\log_3 x}{-5x} \leq \frac{1}{3x}$$



$$15x^2 + 6 + 2\log_3 x \geq 0 \quad 6 + 2\log_3 x + \frac{5}{3} \geq 0$$

$$x \in \left(0, \frac{1}{27} \right) \dots (2) \quad x \geq 3^{-\frac{23}{6}} \dots (3)$$

from (1), (2) & (3)

$$x \in \left[3^{-\frac{23}{6}}, \frac{1}{27} \right)$$

$\therefore \alpha$ is small positive quantity

$$\&\beta = \frac{1}{27}$$

$\therefore \alpha^2 + \frac{5}{\beta}$ is just greater than 135

Question 32

For $x \in \mathbb{R}$, two real valued functions $f(x)$ and $g(x)$ are such that, $g(x) = \sqrt{x} + 1$ and $f \circ g(x) = x + 3 - \sqrt{x}$. Then $f(0)$ is equal to

[13-Apr-2023 shift 1]

Options:

- A. 5
- B. 0
- C. -3
- D. 1

Answer: A

Solution:

Solution:

$$\begin{aligned}g(x) &= \sqrt{x} + 1 \\f \circ g(x) &= x + 3 - \sqrt{x} \\&= (\sqrt{x} + 1)^2 - 3(\sqrt{x} + 1) + 5 \\&= g^2(x) - 3g(x) + 5 \\&\Rightarrow f(x) = x^2 - 3x + 5 \\&\therefore f(0) = 5\end{aligned}$$

But, if we consider the domain of the composite function $f \circ g(x)$ then in that case $f(0)$ will be not defined as $g(x)$ cannot be equal to zero.

Question33

For the differentiable function $f : \mathbb{R} - \{0\} \rightarrow \mathbb{R}$, let

$3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$, then $\left| f(3) + f'\left(\frac{1}{4}\right) \right|$ is equal to

[13-Apr-2023 shift 1]

Options:

- A. 13
- B. $\frac{29}{5}$
- C. $\frac{33}{5}$
- D. 7

Answer: A

Solution:

Solution:

$$\begin{aligned}\left[3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10 \right] \times 3 \\ \left[2f(x) + 3f\left(\frac{1}{x}\right) = x - 10 \right] \times 2 \\ 5f(x) = \frac{3}{x} - 2x - 10 \\ f(x) = \frac{1}{5} \left(\frac{3}{x} - 2x - 10 \right)\end{aligned}$$

$$f'(x) = \frac{1}{5} \left(-\frac{3}{x^2} - 2 \right)$$

$$\left| f(3) + f' \left(\frac{1}{4} \right) \right| = \left| \frac{1}{5}(1 - 6 - 10) + \frac{1}{5}(-48 - 2) \right|$$

$$= | -3 - 10 | = 13$$

Question34

The range of $f(x) = 4\sin^{-1} \left(\frac{x^2}{x^2 + 1} \right)$ is
[13-Apr-2023 shift 2]

Options:

- A. $[0, \pi)$
- B. $[0, \pi]$
- C. $[0, 2\pi)$
- D. $[0, 2\pi]$

Answer: C

Solution:

Solution:

$$f(x) = 4\sin^{-1} \left(\frac{x^2}{1 + x^2} \right)$$

$$0 \leq \frac{x^2}{1 + x^2} < 1$$

$$\Rightarrow 0 \leq \sin^{-1} \left(\frac{x^2}{1 + x^2} \right) < \frac{\pi}{2}$$

$$\Rightarrow 0 \leq 4\sin^{-1} \left(\frac{x^2}{1 + x^2} \right) < 2\pi$$

Range : $[0, 2\pi)$

Question35

If the domain of the function

$f(x) = \log_e(4x^2 + 11x + 6) + \sin^{-1}(4x + 3) + \cos^{-1} \left(\frac{10x + 6}{3} \right)$ is $(\alpha, \beta]$, then
36 | $\alpha + \beta$ | is equal to
[15-Apr-2023 shift 1]

Options:

- A. 72
- B. 63
- C. 45

D. 54

Answer: C

Solution:

Solution:

$$f(x) = \ln(4x^2 + 11x + 6) + \sin^{-1}(4x + 3)$$

$$+ \cos^{-1}\left(\frac{10x+6}{3}\right)$$

$$(i) \quad 4x^2 + 11x + 6 > 0$$

$$4x^2 + 8x + 3x + 6 > 0$$

$$(4x + 3)(x + 2) > 0$$

$$x \in (-\infty, -2) \cup \left(-\frac{3}{4}, \infty\right)$$

$$(ii) \quad 4x + 3 \in [-1, 1]$$

$$x \in [-1, -1/2]$$

$$(iii) \quad \frac{10x+6}{3} \in [-1, 1]$$

$$x \in \left[-\frac{9}{10}, -\frac{3}{10}\right]$$

$$x \in \left[-\frac{3}{4}, -\frac{1}{2}\right] \quad \alpha = -\frac{3}{4}, \beta = -\frac{1}{2}$$

$$\alpha + \beta = -\frac{5}{4}$$

$$36 |\alpha + \beta| = 45$$

Question36

**The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$ is:
[24-Jan-2023 Shift 1]**

Options:

A. transitive but not reflexive

B. symmetric but not transitive

C. reflexive but not symmetric

D. neither symmetric nor transitive

Answer: D

Solution:

Solution:

Reflexive : $(a, a) \quad \gcd(a, a) = a$

Which is not true for every $a \in \mathbb{Z}$.

Symmetric:

Take $a = 2, b = 1 \quad \gcd(2, 1) = 1$

Also $2a = 4 \neq b$

Now when $a = 1, b = 2 \quad \gcd(1, 2) = 1$

Also now $2a = 2 = b$

Hence $a = 2b$

$\Rightarrow R$ is not Symmetric

Transitive:

Let $a = 14, b = 19, c = 21$

$\gcd(a, b) = 1$

$\gcd(b, c) = 1$

$\gcd(a, c) = 7$

Hence not transitive
R is neither symmetric nor transitive.

Question37

Let R be a relation defined on N as a R b is $2a + 3b$ is a multiple of 5, $a, b \in \mathbb{N}$. Then R is
[29-Jan-2023 Shift 2]

Options:

- A. not reflexive
- B. transitive but not symmetric
- C. symmetric but not transitive
- D. an equivalence relation

Answer: D

Solution:

Solution:

$a R a \Rightarrow 5a$ is multiple of 5

So reflexive

$a R b \Rightarrow 2a + 3b = 5\alpha$,

Now $b R a$

$$2b + 3a = 2b + \left(\frac{5\alpha - 3b}{2} \right) \cdot 3$$

$$= \frac{15}{2}\alpha - \frac{5}{2}b = \frac{5}{2}(3\alpha - b)$$

$$= \frac{5}{2}(2a + 2b - 2\alpha)$$

$$= 5(a + b - \alpha)$$

Hence symmetric

$$a R b \Rightarrow 2a + 3b = 5\alpha$$

$$b R c \Rightarrow 2b + 3c = 5\beta$$

$$\text{Now } 2a + 5b + 3c = 5(\alpha + \beta)$$

$$\Rightarrow 2a + 5b + 3c = 5(\alpha + \beta)$$

$$\Rightarrow 2a + 3c = 5(\alpha + \beta - b)$$

$$\Rightarrow a R c$$

Hence relation is equivalence relation.

Question38

The minimum number of elements that must be added to the relation $R = \{(a, b), (b, c)\}$ on the set $\{a, b, c\}$ so that it becomes symmetric and transitive is:
[30-Jan-2023 Shift 1]

Options:

- A. 4
- B. 7

C. 5

D. 3

Answer: B

Solution:

Solution:

For Symmetric $(a, b), (b, c) \in R$

$\Rightarrow (b, a), (c, b) \in R$

For Transitive $(a, b), (b, c) \in R$

$\Rightarrow (a, c) \in R$

Now

1. Symmetric

$\therefore (a, c) \in R \Rightarrow (c, a) \in R$

2. Transitive

$\therefore (a, b), (b, a) \in R$

$\Rightarrow (a, a) \in R \& (b, c), (c, b) \in R$

$\Rightarrow (b, b) \& (c, c) \in R$

\therefore Elements to be added

$\left\{ \begin{array}{l} (b, a) \quad (c, b) \quad (a, c) \quad (c, a) \\ , (a, a) \quad (b, b) \quad (c, c) \end{array} \right\}$

Number of elements to be added = 7

Question 39

Let R be a relation on $N \times N$ defined by $(a, b)R(c, d)$ if and only if $ad(b - c) = bc(a - d)$. Then R is
[31-Jan-2023 Shift 1]

Options:

A. symmetric but neither reflexive nor transitive

B. transitive but neither reflexive nor symmetric

C. reflexive and symmetric but not transitive

D. symmetric and transitive but not reflexive

Answer: A

Solution:

Solution:

$(a, b)R(c, d) \Rightarrow ad(b - c) = bc(a - d)$

Symmetric:

$(c, d)R(a, b) \Rightarrow cb(d - a) = da(c - b) \Rightarrow$

Symmetric

Reflexive:

$(a, b)R(a, b) \Rightarrow ab(b - a) \neq ba(a - b) \Rightarrow$

Not reflexive

Transitive: $(2, 3)R(3, 2)$ and $(3, 2)R(5, 30)$ but

$((2, 3), (5, 30)) \notin R \Rightarrow$ Not transitive

Question40

Among the relations

$$S = \left\{ (a, b) : a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0 \right\}$$

And $T = \{(a, b) : a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\}$,
[31-Jan-2023 Shift 2]

Options:

- A. S is transitive but T is not
- B. T is symmetric but S is not
- C. Neither S nor T is transitive
- D. Both S and T are symmetric

Answer: B

Solution:

Solution:

For relation $T = a^2 - b^2 = -I$

Then, (b, a) on relation R

$$\Rightarrow b^2 - a^2 = -I$$

$\therefore T$ is symmetric

$$S = \left\{ (a, b) : a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0 \right\}$$

$$2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2, \Rightarrow \frac{b}{a} < \frac{-1}{2}$$

If $(b, a) \in S$ then

$$2 + \frac{b}{a} \text{ not necessarily positive}$$

$\therefore S$ is not symmetric

Question41

Let R be a relation on \mathbb{R} , given by $R = \{(a, b) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\}$. Then R is
[1-Feb-2023 Shift 1]

Options:

- A. Reflexive but neither symmetric nor transitive
- B. Reflexive and transitive but not symmetric
- C. Reflexive and symmetric but not transitive
- D. An equivalence relation

Answer: A

Solution:

Solution:

Check for reflexivity:

As $3(a-a) + \sqrt{7} = \sqrt{7}$ which belongs to relation so relation is reflexive

Check for symmetric:

Take $a = \frac{\sqrt{7}}{3}$, $b = 0$

Now $(a, b) \in R$ but $(b, a) \notin R$

As $3(b-a) + \sqrt{7} = 0$ which is rational so relation is not symmetric.

Check for Transitivity:

Take (a, b) as $\left(\frac{\sqrt{7}}{3}, 1\right)$

& (b, c) as $\left(1, \frac{2\sqrt{7}}{3}\right)$

So now $(a, b) \in R$ & $(b, c) \in R$ but $(a, c) \notin R$ which means relation is not transitive

Question 42

Let $P(S)$ denote the power set of $S = \{1, 2, 3, \dots, 10\}$. Define the relations R_1 and R_2 on $P(S)$ as $A R_1 B$ if $(A \cap B^c) \cup (B \cap A^c) = \emptyset$ and $A R_2 B$ if $A \cup B^c = B \cup A^c$, $\forall A, B \in P(S)$. Then :
[1-Feb-2023 Shift 2]

Options:

- A. both R_1 and R_2 are equivalence relations
- B. only R_1 is an equivalence relation
- C. only R_2 is an equivalence relation
- D. both R_1 and R_2 are not equivalence relations

Answer: A

Solution:**Solution:**

$S = \{1, 2, 3, \dots, 10\}$

$P(S)$ = power set of S

$A R_1 B \Rightarrow (A \cap \vec{B}) \cup (\vec{A} \cap B) = \emptyset$

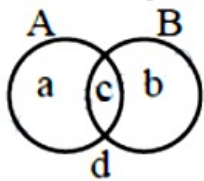
R_1 is reflexive, symmetric

For transitive

$(A \cap \vec{B}) \cup (\vec{A} \cap B) = \emptyset$; $\{a\} = \emptyset = \{b\} \Rightarrow A = B$

$(B \cap \vec{C}) \cup (\vec{B} \cap C) = \emptyset \therefore B = C$

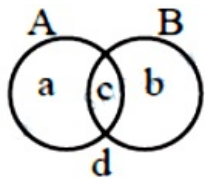
$\therefore A = C$ equivalence.



$R_2 \equiv A \cup \vec{B} = \vec{A} \cup B$

$R_2 \rightarrow$ Reflexive, symmetric

for transitive



$$A \cup \vec{B} = \vec{A} \cup B \Rightarrow \{a, c, d\} = \{b, c, d\}$$

$$\{a\} = \{b\} \therefore A = B$$

$$B \cup \vec{C} = \vec{B} \cup C \Rightarrow B = C$$

$$\therefore A = C \quad \therefore A \cup \vec{C} = \vec{A} \cup C \therefore \text{Equivalence}$$

Question43

The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has:
[24-Jan-2023 Shift 1]

Options:

A. exactly two solutions in $(-\infty, \infty)$

B. no solution

C. a unique solution in $(-\infty, 1)$

D. a unique solution in $(-\infty, \infty)$

Answer: D

Solution:

Solution:

$$x^2 - 4x + [x] + 3 = x[x]$$

$$\Rightarrow x^2 - 4x + 3 = x[x] - [x]$$

$$(x-1)(x-3) = [x] \cdot (x-1)$$

$$\Rightarrow x = 1 \text{ or } x-3 = [x]$$

$$\Rightarrow x - [x] = 3$$

$$\{x\} = 3 \text{ (Not Possible)}$$

Only one solution $x = 1$ in $(-\infty, \infty)$

Question44

Let $f(x)$ be a function such that $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^n f(k) = 3279$, then the value of n is
[24-Jan-2023 Shift 2]

Options:

A. 6

B. 8

C. 7

D. 9

Answer: C

Solution:

Solution:

$$f(x + y) = f(x) \cdot f(y) \quad \forall x, y \in \mathbb{N}, f(1) = 3$$

$$f(2) = f^2(1) = 3^2$$

$$f(3) = f(1)f(2) = 3^3$$

$$f(4) = 3^4$$

$$f(k) = 3^k$$

$$\sum_{k=1}^n f(k) = 3279$$

$$f(1) + f(2) + f(3) + \dots + f(k) = 3279$$

$$3 + 3^2 + 3^3 + \dots + 3^k = 3279$$

$$\frac{3(3^k - 1)}{3 - 1} = 3279$$

$$\frac{3^k - 1}{2} = 1093$$

$$3^k - 1 = 2186$$

$$3^k = 2187$$

$$k = 7$$

Question45

If $f(x) = \frac{2^{2x}}{2^{2x} + 2}$, $x \in \mathbb{R}$ then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \dots + f\left(\frac{2022}{2023}\right)$ equal to [24-Jan-2023 Shift 2]

Options:

A. 2011

B. 1010

C. 2010

D. 1011

Answer: D

Solution:

Solution:

$$f(x) = \frac{4^x}{4^x + 2}$$

$$f(x) + f(1 - x) = \frac{4^x}{4^x + 2} + \frac{4^{1-x}}{4^{1-x} + 2}$$

$$= \frac{4^x}{4^x + 2} + \frac{4}{4 + 2(4^x)}$$

$$= \frac{4^x}{4^x + 2} + \frac{2}{2 + 4^x}$$

$$= 1$$

$$\Rightarrow f(x) + f(1 - x) = 1$$

$$\text{Now } f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{3}{2023}\right) + \dots +$$

$$\dots + f\left(1 - \frac{3}{2023}\right) + f\left(1 - \frac{2}{2023}\right) + f\left(1 - \frac{1}{2023}\right)$$

Now sum of terms equidistant from beginning and end is 1

$$\text{Sum} = 1 + 1 + 1 + \dots + 1 \quad (1011 \text{ times})$$

Question46

For some $a, b, c \in \mathbb{N}$, let $f(x) = ax - 3$ and $g(x) = x^b + c$, $x \in \mathbb{R}$. If $(f \circ g)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$ then $(f \circ g)(ac) + (g \circ f)(b)$ is equal to _____.
[25-Jan-2023 Shift 1]

Answer: 2039

Solution:

Solution:

Let $f \circ g(x) = h(x)$

$$\Rightarrow h^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$$

$$\Rightarrow h(x) = f \circ g(x) = 2x^3 + 7$$

$$f \circ g(x) = a(x^b + c) - 3$$

$$\Rightarrow a = 2, b = 3, c = 5$$

$$\Rightarrow f \circ g(ac) = f \circ g(10) = 2007$$

$$g(f(x)) = (2x - 3)^3 + 5.$$

$$\Rightarrow g \circ f(b) = g \circ f(3) = 32$$

$$\Rightarrow \text{sum} = 2039$$

Question47

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m , such that the range of f is $[0, 2]$. Then the value of m is _____.
[25-Jan-2023 Shift 2]

Options:

A. 5

B. 3

C. 2

D. 4

Answer: A

Solution:

Solution:

Since,

$$-\sqrt{2} \leq \sin x - \cos x \leq \sqrt{2}$$

$$\begin{aligned} \therefore -2 &\leq \sqrt{2}(\sin x - \cos x) \leq 2 \\ (\text{Assume } \sqrt{2}(\sin x - \cos x) &= k) \\ -2 &\leq k \leq 2 \quad \dots (i) \\ f(x) &= \log_{\sqrt{m}}(k + k - 2) \end{aligned}$$

$$\begin{aligned} \text{Given,} \\ 0 &\leq f(x) \leq 2 \\ 0 &\leq \log_{\sqrt{m}}(k + m - 2) \leq 2 \\ 1 &\leq k + m - 2 \leq m \\ -m + 3 &\leq k \leq 2 \dots \dots (ii) \\ \text{From eq. (i) \& (ii), we get } -m + 3 &= -2 \\ \Rightarrow m &= 5 \end{aligned}$$

Question48

The number of functions $f : \{1, 2, 3, 4\} \rightarrow \{a \in \mathbb{Z} : |a| \leq 8\}$ satisfying $f(n) + \frac{1}{n}f(n+1) = 1, \forall n \in \{1, 2, 3\}$ is
[25-Jan-2023 Shift 2]

Options:

- A. 3
- B. 4
- C. 1
- D. 2

Answer: D

Solution:

$$\begin{aligned} \text{Solution:} \\ f : \{1, 2, 3, 4\} &\rightarrow \{a \in \mathbb{Z} : |a| \leq 8\} \\ f(n) + \frac{1}{n}f(n+1) &= 1, \forall n \in \{1, 2, 3\} \\ f(n+1) &\text{ must be divisible by } n \\ f(4) &\Rightarrow -6, -3, 0, 3, 6 \\ f(3) &\Rightarrow -8, -6, -4, -2, 0, 2, 4, 6, 8 \\ f(2) &\Rightarrow -8, \dots\dots\dots, 8 \\ f(1) &\Rightarrow -8, \dots\dots\dots, 8 \\ \frac{f(4)}{3} &\text{ must be odd since } f(3) \text{ should be even therefore 2 solution possible.} \end{aligned}$$

Question49

Let $f(x) = 2x^n + \lambda, \lambda \in \mathbb{R}, n \in \mathbb{N}$, and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is
[25-Jan-2023 Shift 2]

Options:

- A. 61
- B. 60

C. 58

D. 59

Answer: B

Solution:

Solution:

$$f(x) = 2x^n + \lambda$$

$$f(4) = 133$$

$$f(5) = 255$$

$$133 = 2 \times 4^n + \lambda \dots (1)$$

$$255 = 2 \times 5^n + \lambda \dots (2)$$

$$(2) - (1)$$

$$122 = 2(5^n - 4^n)$$

$$\Rightarrow 5^n - 4^n = 61$$

$$\therefore n = 3 \text{ and } \lambda = 5$$

$$\text{Now, } f(3) - f(2) = 2(3^3 - 2^3) = 38$$

Number of Divisors is 1, 2, 19, 38; & their sum is 60

Question 50

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then

[29-Jan-2023 Shift 1]

Options:

A. $f(x)$ is many-one in $(-\infty, -1)$

B. $f(x)$ is many-one in $(1, \infty)$

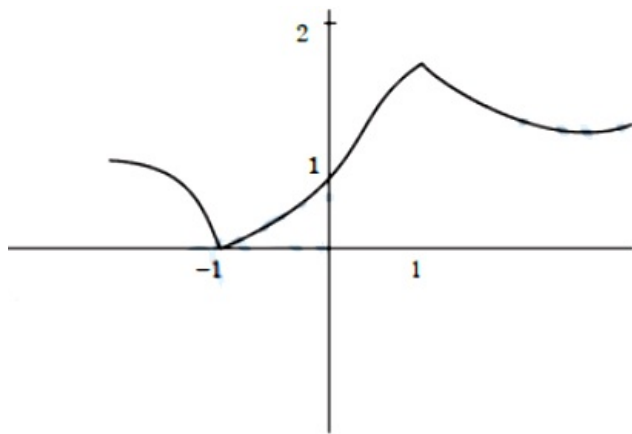
C. $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$

D. $f(x)$ is one-one in $(-\infty, \infty)$

Answer: C

Solution:

Solution:



$$f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$$

$$f(x) = 1 + \frac{2}{x + \frac{1}{x}}$$

Question51

The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_e x} - (2x+3)}$, $x \in \mathbb{R}$ is
[29-Jan-2023 Shift 1]

Options:

- A. $\mathbb{R} - \{1-3\}$
- B. $(2, \infty) - \{3\}$
- C. $(-1, \infty) - \{3\}$
- D. $\mathbb{R} - \{3\}$

Answer: B

Solution:

Solution:

$x-2 > 0 \Rightarrow x > 2$
 $x+1 > 0 \Rightarrow x > -1$
 $x+1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$
 Denominator
 $x^2 - 2x - 3 \neq 0$
 $(x-3)(x+1) \neq 0$
 $x \neq -1, 3$
 So Ans $(2, \infty) - \{3\}$

Question52

Consider a function $f : \mathbb{N} \rightarrow \mathbb{R}$, satisfying
 $f(1) + 2f(2) + 3f(3) + \dots + xf(x) = x(x+1)f(x)$; $x \geq 2$ with $f(1) = 1$. Then
 $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to

[29-Jan-2023 Shift 2]

Options:

- A. 8200
- B. 8000
- C. 8400
- D. 8100

Answer: D

Solution:

Solution:

Given for $x \geq 2$

$$f(1) + 2f(2) + \dots + xf(x) = x(x+1)f(x)$$

replace x by $x+1$

$$\Rightarrow x(x+1)f(x) + (x+1)f(x+1)$$

$$= (x+1)(x+2)f(x+1)$$

$$\Rightarrow \frac{x}{f(x+1)} + \frac{1}{f(x)} = \frac{(x+2)}{f(x)}$$

$$\Rightarrow xf(x) = (x+1)f(x+1) = \frac{1}{2}, x \geq 2$$

$$f(2) = \frac{1}{4}, f(3) = \frac{1}{6}$$

$$\text{Now } f(2022) = \frac{1}{4044}$$

$$f(2028) = \frac{1}{4056}$$

$$\text{So, } \frac{1}{f(2022)} + \frac{1}{f(2028)} = 4044 + 4056 = 8100$$

Question53

Suppose $f : \mathbb{R} \rightarrow (0, \infty)$ be a differentiable function such that

$5f(x+y) = f(x) \cdot f(y), \forall x, y \in \mathbb{R}$. If $f(3) = 320$, then $\sum_{n=0}^5 f(n)$ is equal to:

[30-Jan-2023 Shift 1]

Options:

- A. 6875
- B. 6575
- C. 6825
- D. 6528

Answer: C

Solution:

Solution:

Option (3)

$$5f(x+y) = f(x) \cdot f(y)$$

$$5f(0) = f(0)^2 \Rightarrow f(0) = 5$$

$$5f(x+1) = f(x) \cdot f(1)$$

$$\Rightarrow \frac{f(x+1)}{f(x)} = \frac{f(1)}{5}$$

$$\Rightarrow \frac{f(1)}{f(0)} \cdot \frac{f(2)}{f(1)} \cdot \frac{f(3)}{f(2)} = \left(\frac{f(1)}{5} \right)^3$$

$$\Rightarrow \frac{320}{5} = \frac{(f(1))^3}{5^3} \Rightarrow f(1) = 20$$

$$\therefore 5f(x+1) = 20 \cdot f(x) \Rightarrow f(x+1) = 4f(x)$$

$$\sum_{n=0}^5 f(n) = 5 + 5.4 + 5.4^2 + 5.4^3 + 5.4^4 + 5.4^5$$

$$= \frac{5[4^6 - 1]}{3} = 6825$$

Question54

Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the number of one functions $f : S \rightarrow P(S)$, where $P(S)$ denote the power set of S , such that $f(n) \subset f(m)$ where $n < m$ is _____.

[30-Jan-2023 Shift 1]

Answer: 3240

Solution:

Solution:

Let $S = \{1, 2, 3, 4, 5, 6\}$, then the number of one-one functions, $f : S \rightarrow P(S)$, where $P(S)$ denotes the power set of S , such that $f(n) \subset f(m)$ where $n < m$ is

$$n(S) = 6$$

$$P(S) = \left\{ \begin{array}{ccccccc} \phi & \{1\} & \dots & \{6\} & \{1, 2\} & \dots & \\ \{5, 6\} & \dots & \{1, 2, 3, 4, 5, 6\} & & & & \end{array} \right\}$$

– 64 elements

case – 1

$f(6) = S$ i.e. 1 option,

$f(5) =$ any 5 element subset A of S i.e. 6 options,

$f(4) =$ any 4 element subset B of A i.e. 5 options,

$f(3) =$ any 3 element subset C of B i.e. 4 options,

$f(2) =$ any 2 element subset D of C i.e. 3 options,

$f(1) =$ any 1 element subset E of D or empty subset i.e. 3 options,

Total functions = 1080

Case - 2

$f(6) =$ any 5 element subset A of S i.e. 6 options,

$f(5) =$ any 4 element subset B of A i.e. 5 options,

$f(4) =$ any 3 element subset C of B i.e. 4 options,

$f(3) =$ any 2 element subset D of C i.e. 3 options,

$f(2) =$ any 1 element subset E of D i.e. 2 options,

$f(1) =$ empty subset i.e. 1 option

Total functions = 720

Case – 3

$f(6) = S$

$f(5) =$ any 4 element subset A of S i.e. 15 options,

$f(4) =$ any 3 element subset B of A i.e. 4 options,

$f(3) =$ any 2 element subset C of B i.e. 3 options,

$f(2) =$ any 1 element subset D of C i.e. 2 options,

$f(1) =$ empty subset i.e. 1 option

Total functions = 360

Case – 4

$f(6) = S$
 $f(5) =$ any 5 element subset A of S i.e. 6 options,
 $f(4) =$ any 3 element subset B of A i.e. 10 options,
 $f(3) =$ any 2 element subset C of B i.e. 3 options,
 $f(2) =$ any 1 element subset D of C i.e. 2 options,
 $f(1) =$ empty subset i.e. 1 option
 Total functions = 360
 Case -5
 $f(6) = S$
 $f(5) =$ any 5 element subset A of S i.e. 6 options,
 $f(4) =$ any 4 element subset B of A i.e. 5 options,
 $f(3) =$ any 2 element subset C of B i.e. 6 options,
 $f(2) =$ any 1 element subset D of C i.e. 2 options,
 $f(1) =$ empty subset i.e. 1 option
 Total functions = 360
 Case - 6
 $f(6) = S$
 $f(5) =$ any 5 element subset A of S i.e. 6 options,
 $f(4) =$ any 4 element subset B of A i.e. 5 options,
 $f(3) =$ any 3 element subset C of B i.e. 4 options,
 $f(2) =$ any 1 element subset D of C i.e. 3 options,
 $f(1) =$ empty subset i.e. 1 option
 Total functions = 360
 \therefore Number of such functions = 3240

Question55

Let $f^1(x) = \frac{3x+2}{2x+3}$, $x \in \mathbb{R} - \left\{ \frac{-3}{2} \right\}$

For $n \geq 2$, define $f^n(x) = f^1 \circ f^{n-1}(x)$.

If $f^5(x) = \frac{ax+b}{bx+a}$, $\gcd(a, b) = 1$, then $a + b$ is equal to _____.

[30-Jan-2023 Shift 1]

Answer: 3125

Solution:

Solution:

$$\begin{aligned}
 f^1(x) &= \frac{3x+2}{2x+3} \\
 \Rightarrow f^2(x) &= \frac{13x+12}{12x+13} \\
 \Rightarrow f^3(x) &= \frac{63x+62}{62x+63} \\
 \therefore f^5(x) &= \frac{1563x+1562}{1562x+1563} \\
 a+b &= 3125
 \end{aligned}$$

Question56

The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is

[30-Jan-2023 Shift 2]

Options:

A. $[\sqrt{5}, \sqrt{10}]$

B. $[2\sqrt{2}, \sqrt{11}]$

C. $[\sqrt{5}, \sqrt{13}]$

D. $[\sqrt{2}, \sqrt{7}]$

Answer: A

Solution:

Solution:

$$y^2 = 3 - x + 2 + x + 2\sqrt{(3-x)(2+x)}$$

$$= 5 + 2\sqrt{6+x-x^2}$$

$$y^2 = 5 + 2\sqrt{\frac{25}{4} - \left(x - \frac{1}{2}\right)^2}$$

$$y_{\max} = \sqrt{5+5} = \sqrt{10}$$

$$y_{\min} = \sqrt{5}$$

Question57

Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f : A \rightarrow A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to _____.

[30-Jan-2023 Shift 2]

Answer: 432

Solution:

Solution:

$$f(1) = 1; f(9) = f(3) \times f(3)$$

$$\text{i.e., } f(3) = 1 \text{ or } 3$$

$$\text{Total function} = 1 \times 6 \times 2 \times 6 \times 6 \times 1 = 432$$

Question58

If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2, 6)$, then its range is

[31-Jan-2023 Shift 1]

Options:

A. $\left[\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

B. $\left(\frac{5}{26}, \frac{2}{5} \right]$

C. $\left(\frac{5}{37}, \frac{2}{5} \right] - \left\{ \frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53} \right\}$

D. $\left(\frac{5}{37}, \frac{2}{5} \right]$

Answer: D

Solution:

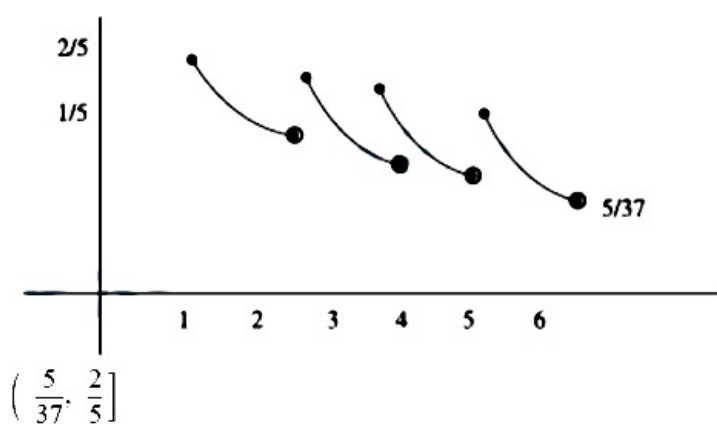
Solution:

$$f(x) = \frac{2}{1+x^2} \quad x \in [2, 3)$$

$$f(x) = \frac{3}{1+x^2} \quad x \in [3, 4)$$

$$f(x) = \frac{4}{1+x^2} \quad x \in [4, 5)$$

$$f(x) = \frac{5}{1+x^2} \quad x \in [5, 6)$$



Question59

The absolute minimum value, of the function $f(x) = x^2 - x + 1 + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1, 2]$, is :
[31-Jan-2023 Shift 2]

Options:

A. $\frac{3}{4}$

B. $\frac{3}{2}$

C. $\frac{1}{4}$

D. $\frac{5}{4}$

Answer: A

Solution:

Solution:

$$f(x) = |x^2 - x + 1| + [x^2 - x + 1]; x \in [-1, 2]$$

$$\text{Let } g(x) = x^2 - x + 1$$

$$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\therefore |x^2 - x + 1| \text{ and } [x^2 - x + 1]$$

Both have minimum value at $x = 1/2$

$$\Rightarrow \text{Minimum } f(x) = \frac{3}{4} + 0$$

$$= \frac{3}{4}$$

Question60

Let $f : \mathbb{R} - \{2, 6\} \rightarrow \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$.

Then range of f is

[31-Jan-2023 Shift 2]

Options:

A. $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

B. $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$

C. $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

D. $\left(-\infty, -\frac{21}{4}\right] \cup [1, \infty)$

Answer: A

Solution:

Solution:

$$\text{Let } y = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

By cross multiplying

$$yx^2 - 8xy + 12y - x^2 - 2x - 1 = 0$$

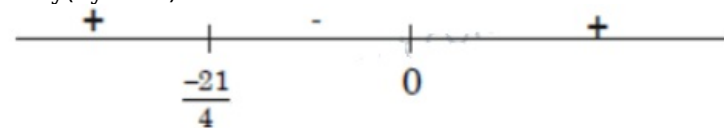
$$x^2(y - 1) - x(8y + 2) + (12y - 1) = 0$$

Case 1, $y \neq 1$

$$D \geq 0$$

$$\Rightarrow (8y + 2)^2 - 4(y - 1)(12y - 1) \geq 0$$

$$\Rightarrow y(4y + 21) \geq 0$$



$$y \in \left(-\infty, -\frac{21}{4}\right] \cup [0, \infty) - \{1\}$$

Case 2, $y = 1$

$$x^2 + 2x + 1 = x^2 - 8x + 12$$

$$10x = 11$$

$$x = \frac{11}{10} \text{ So, } y \text{ can be } 1$$

$$\text{Hence } y \in \left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$$

Question61

Let $f : \mathbb{R} - \{0, 1\} \rightarrow \mathbb{R}$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$.

Then $f(2)$ is equal to :

[1-Feb-2023 Shift 2]

Options:

A. $\frac{9}{2}$

B. $\frac{9}{4}$

C. $\frac{7}{4}$

D. $\frac{7}{3}$

Answer: B

Solution:

Solution:

$$f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$$

$$x = 2 \Rightarrow f(2) + f(-1) = 3$$

$$x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0 \dots (2)$$

$$x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2} \dots (3)$$

$$(1) + (3) - (2) \Rightarrow 2f(2) = \frac{9}{2}$$

$$\therefore f(2) = \frac{9}{4}$$

Question62

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x - 1$ and $g : \mathbb{R} - \{1, -1\} \rightarrow \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2 - 1}$. Then the function $fo g$ is :

[26-Jun-2022-Shift-2]

Options:

A. one-one but not onto

B. onto but not one-one

C. both one-one and onto

D. neither one-one nor onto

Answer: D

Solution:

$f : R \rightarrow R$ defined as

$$f(x) = x - 1 \text{ and } g : R \rightarrow \{1, -1\} \rightarrow R, g(x) = \frac{x^2}{x^2 - 1}$$

$$\text{Now } f \circ g(x) = \frac{x^2}{x^2 - 1} - 1 = \frac{1}{x^2 - 1}$$

$$\therefore \text{Domain of } f \circ g(x) = R - \{-1, 1\}$$

$$\text{And range of } f \circ g(x) = (-\infty, -1] \cup (0, \infty)$$

$$\text{Now, } \frac{d}{dx}(f \circ g(x)) = \frac{-1}{x^2 - 1} \cdot 2x = \frac{2x}{1 - x^2}$$

$$\therefore \frac{d}{dx}(f \circ g(x)) > 0 \text{ for } \frac{2x}{(1-x)(1+x)} > 0$$

$$\Rightarrow \frac{x}{(x-1)(x+1)} < 0$$

$$\therefore x \in (-\infty, -1) \cup (0, 1)$$

$$\text{and } \frac{d}{dx}(f \circ g(x)) < 0 \text{ for } x \in (-1, 0) \cup (1, \infty)$$

$\therefore f \circ g(x)$ is neither one-one nor onto.

Question63

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{2e^{2x}}{e^{2x} + e}$.

Then

$$f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right)$$

is equal to _____

[27-Jun-2022-Shift-1]

Answer: 99

Solution:

Solution:

Given,

$$f(x) = \frac{2e^{2x}}{e^{2x} + e}$$

$$\therefore f(1-x) = \frac{2e^{2(1-x)}}{e^{2(1-x)} + e}$$

$$= \frac{2 \cdot \frac{e^2}{e^{2x}}}{\frac{e^2}{e^{2x}} + e}$$

$$= \frac{2e^2}{e^2 + e^{2x} \cdot e}$$

$$= \frac{2e^2}{e(e + e^{2x})}$$

$$= \frac{2e}{e + e^{2x}}$$

$$\therefore f(x) + f(1-x) = \frac{2e^{2x}}{e^{2x} + e} + \frac{2e}{e^{2x} + e}$$

$$= \frac{2(e^{2x} + e)}{e^{2x} + e}$$

$$= 2 \dots \dots (1)$$

Now,

$$f\left(\frac{1}{100}\right) + f\left(\frac{99}{100}\right)$$

$$= f\left(\frac{1}{100}\right) + f\left(1 - \frac{1}{100}\right)$$

$$= 2 \text{ [as } f(x) + f(1-x) = 2 \text{]}$$

$$f\left(\frac{2}{100}\right) + f\left(1 - \frac{2}{100}\right) = 2$$

⋮

$$f\left(\frac{49}{100}\right) + f\left(1 - \frac{49}{100}\right) = 2$$

$$\therefore \text{Total sum} = 49 \times 2$$

$$\text{Remaining term} = f\left(\frac{50}{100}\right) = f\left(\frac{1}{2}\right)$$

Put $x = \frac{1}{2}$ in equation (1), we get

$$f\left(\frac{1}{2}\right) + f\left(1 - \frac{1}{2}\right) = 2$$

$$\Rightarrow 2f\left(\frac{1}{2}\right) = 2$$

$$\Rightarrow f\left(\frac{1}{2}\right) = 1$$

$$\therefore \text{Sum} = 49 \times 2 + 1 = 99$$

Question64

Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Define $f : S \rightarrow S$ as

$$f(n) = \begin{cases} 2n & , \text{ if } n = 1, 2, 3, 4, 5 \\ 2n - 11 & , \text{ if } n = 6, 7, 8, 9, 10, \end{cases}$$

Let $g : S \rightarrow S$ be a function such that $g(n) = \begin{cases} n + 1 & , \text{ if } n \text{ is odd} \\ n - 1 & , \text{ if } n \text{ is even} \end{cases}$

Then $g(10)g(1) + g(2) + g(3) + g(4) + g(5)$ is equal to
[27-Jun-2022-Shift-2]

Answer: 190

Solution:

Solution:

$$\therefore f(n) = \begin{cases} 2n & n = 1, 2, 3, 4, 5 \\ 2n - 11 & n = 6, 7, 8, 9, 10 \end{cases}.$$

$$\therefore f(1) = 2, f(2) = 4, \dots, f(5) = 10 \\ \text{and } f(6) = 1, f(7) = 3, f(8) = 5, \dots, f(10) = 9$$

$$\text{Now, } f(g(n)) = \begin{cases} n + 1 & \text{if } n \text{ is odd} \\ n - 1 & \text{if } n \text{ is even} \end{cases}.$$

$$\begin{aligned} f(g(10)) &= 9 \Rightarrow g(10) = 10 \\ f(g(1)) &= 2 \Rightarrow g(1) = 1 \\ \therefore f(g(2)) &= 1 \Rightarrow g(2) = 6 \\ \therefore f(g(3)) &= 4 \Rightarrow g(3) = 2 \\ f(g(4)) &= 3 \Rightarrow g(4) = 7 \\ f(g(5)) &= 6 \Rightarrow g(5) = 3 \\ \therefore g(10)g(1) + g(2) + g(3) + g(4) + g(5) &= 190 \end{aligned}$$

Question65

Let a function $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined by

$$f(n) = \begin{cases} 2n & n = 2, 4, 6, 8, \dots \\ n - 1 & n = 3, 7, 11, 15, \dots \\ \frac{n+1}{2} & n = 1, 5, 9, 13, \dots \end{cases}$$

then, f is

[28-Jun-2022-Shift-1]

Options:

- A. one-one but not onto
- B. onto but not one-one
- C. neither one-one nor onto

D. one-one and onto

Answer: D

Solution:

Solution:

When $n = 1, 5, 9, 13$ then $\frac{n+1}{2}$ will give all odd numbers.

When $n = 3, 7, 11, 15, \dots$

$n - 1$ will be even but not divisible by 4

When $n = 2, 4, 6, 8, \dots$

Then $2n$ will give all multiples of 4

So range will be \mathbb{N} .

And no two values of n give same y , so function is one-one and onto.

Question66

The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5\}$ satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

[28-Jun-2022-Shift-2]

Options:

A. $\frac{1}{24}$

B. $\frac{1}{40}$

C. $\frac{1}{30}$

D. $\frac{1}{20}$

Answer: D

Solution:

Solution:

Number of one-one function from $\{a, b, c, d\}$ to set $\{1, 2, 3, 4, 5\}$ is ${}^5P_4 = 120$ (s).

The required possible set of value $(f(a), f(b), f(c), f(d))$ such that $f(a) + 2f(b) - f(c) = f(d)$ are

$(5, 3, 2, 1), (5, 1, 2, 3), (4, 1, 3, 5), (3, 1, 4, 5), (5, 4, 3, 2)$ and $(3, 4, 5, 2)$

$\therefore n(E) = 6$

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)} = \frac{6}{120} = \frac{1}{20}$$

Question67

Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \rightarrow S : f \text{ is onto and } f(a, b) = f(b, a) \geq a \forall (a, b) \in S \times S\}$ is ____

[28-Jun-2022-Shift-2]

Answer: 37

Solution:

Solution:

There are 16 ordered pairs in $S \times S$. We write all these ordered pairs in 4 sets as follows.

$$A = \{(1, 1)\}$$

$$B = \{(1, 4), (2, 4), (3, 4), (4, 4), (4, 3), (4, 2), (4, 1)\}$$

$$C = \{(1, 3), (2, 3), (3, 3), (3, 2), (3, 1)\}$$

$$D = \{(1, 2), (2, 2), (2, 1)\}$$

All elements of set B have image 4 and only element of A has image 1 .

All elements of set C have image 3 or 4 and all elements of set D have image 2 or 3 or 4 .

We will solve this question in two cases.

Case I: When no element of set C has image 3 .

Number of onto functions = 2 (when elements of set D have images 2 or 3)

Case II: When atleast one element of set C has image 3 .

Number of onto functions = $(2^3 - 1)(1 + 2 + 2) = 35$

Total number of functions = 37

Question68

The domain of the function $\cos^{-1} \left(\frac{2\sin^{-1} \left(\frac{1}{4x^2 - 1} \right)}{\pi} \right)$ is :
[29-Jun-2022-Shift-1]

Options:

A. $\mathbb{R} - \left\{ -\frac{1}{2}, \frac{1}{2} \right\}$

B. $(-\infty, -1] \cup [1, \infty) \cup \{0\}$

C. $\left(-\infty, \frac{-1}{2} \right) \cup \left(\frac{1}{2}, \infty \right) \cup \{0\}$

D. $\left(-\infty, \frac{-1}{\sqrt{2}} \right] \cup \left[\frac{1}{\sqrt{2}}, \infty \right) \cup \{0\}$

Answer: D

Solution:

Solution:

$$-1 \leq \frac{2\sin^{-1} \left(\frac{1}{4x^2 - 1} \right)}{\pi} \leq 1$$

$$\Rightarrow -\frac{\pi}{2} \leq \sin^{-1} \left(\frac{1}{4x^2 - 1} \right) \leq \frac{\pi}{2}$$

$$\Rightarrow -1 \leq \frac{1}{4x^2 - 1} \leq 1$$

$$\therefore \frac{1}{4x^2 - 1} + 1 \geq 0$$

$$\Rightarrow \frac{1 + 4x^2 - 1}{4x^2 - 1} \geq 0$$

$$\Rightarrow \frac{4x^2}{4x^2 - 1} \geq 0$$

$$\Rightarrow \frac{4x^2}{(2x+1)(2x-1)} \geq 0 \dots (1)$$

$$\therefore x \in \left(-\alpha, -\frac{1}{2}\right) \cup \{0\} \cup \left(\frac{1}{2}, \alpha\right)$$

$$\text{And } \frac{1}{4x^2 - 1} - 1 \leq 0$$

$$\Rightarrow \frac{1 - 4x^2 + 1}{4x^2 - 1} \leq 0$$

$$\Rightarrow \frac{2 - 4x^2}{4x^2 - 1} \leq 0$$

$$\Rightarrow \frac{2x^2 - 1}{4x^2 - 1} \geq 0$$

$$\Rightarrow \frac{(\sqrt{2}x+1)(\sqrt{2}x-1)}{(2x+1)(2x-1)} \geq 0$$

$$x \in \left(-\alpha, -\frac{1}{\sqrt{2}}\right) \cup \left(-\frac{1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{\sqrt{2}}, \alpha\right)$$

From (3) and (4), we get

$$\therefore x \in \left[-\alpha, -\frac{1}{\sqrt{2}}\right) \cup \left[\frac{1}{\sqrt{2}}, \alpha\right) \cup \{0\}$$

Question69

Let $c, k \in \mathbb{R}$. If $f(x) = (c+1)x^2 + (1-c^2)x + 2k$ and $f(x+y) = f(x) + f(y) - xy$, for all $x, y \in \mathbb{R}$, then the value of $|2(f(1) + f(2) + f(3) + \dots + f(20))|$ is equal to____
[29-Jun-2022-Shift-1]

Answer: 3395

Solution:

Solution:

$f(x)$ is polynomial

Put $y = 1/x$ in given functional equation we get

$$f\left(x + \frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) - 1$$

$$\Rightarrow (c+1)\left(x + \frac{1}{x}\right)^2 + (1-c^2)\left(x + \frac{1}{x}\right) + 2K$$

$$= (c+1)x^2 + (1-c^2)x + 2K + (c+1)\frac{1}{x^2} + (1-c^2)\frac{1}{x} + 2K - 1$$

$$\Rightarrow 2(c+1) = 2K - 1 \dots (1)$$

and put $x = y = 0$ we get

$$f(0) = 2 + f(0) - 0 \Rightarrow f(0) = 0 \Rightarrow k = 0$$

$$\therefore k = 0 \text{ and } 2c = -3 \Rightarrow c = -3/2$$

$$f(x) = -\frac{x^2}{2} - \frac{5x}{4} = \frac{1}{4}(5x + 2x^2)$$

$$\left| 2 \sum_{i=1}^{20} f(i) \right| = \left| \frac{-2}{4} \left(\frac{5 \cdot 20 \cdot 21}{2} + \frac{2 \cdot 20 \cdot 21 \cdot 41}{6} \right) \right|$$

$$= \left| \frac{-1}{2} (2730 + 5740) \right|$$

$$= \left| -\frac{6790}{2} \right| = 3395.$$

Question70

Let $f(x)$ and $g(x)$ be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$ and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is _____
[29-Jun-2022-Shift-2]

Answer: 18

Solution:

Solution:

$$f(g(x)) = 8x^2 - 2x.$$

$$g(f(x)) = 4x^2 + 6x + 1.$$

$$\text{So, } g(x) = 2x - 1$$

$$\&f(x) = 2x^2 + 3x + 1$$

$$f(2) = 8 + 6 + 1 = 15$$

$$\text{Ans. 18}$$

Question71

The domain of the function

$$f(x) = \frac{\cos^{-1}\left(\frac{x^2 - 5x + 6}{x^2 - 9}\right)}{\log_e(x^2 - 3x + 2)} \text{ is :}$$

[24-Jun-2022-Shift-1]

Options:

A. $(-\infty, 1) \cup (2, \infty)$

B. $(2, \infty)$

C. $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$

D. $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{3, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Answer: D

Solution:

Solution:

$$-1 \leq \frac{x^2 - 5x + 6}{x^2 - 9} \leq 1 \text{ and } x^2 - 3x + 2 > 0, \neq 1$$

$$\frac{(x-3)(2x+1)}{x^2-9} \geq 0 \mid \frac{5(x-3)}{x^2-9} \geq 0$$

The solution to this inequality is

$$x \in \left[\frac{-1}{2}, \infty \right) - \{3\}$$

for $x^2 - 3x + 2 > 0$ and $\neq 1$

$$x \in (-\infty, 1) \cup (2, \infty) - \left\{ \frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2} \right\}$$

Combining the two solution sets (taking intersection)

$$x \in \left[-\frac{1}{2}, 1 \right) \cup (2, \infty) - \left\{ \frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2} \right\}$$

Question72

The number of one-one functions $f : \{a, b, c, d\} \rightarrow \{0, 1, 2, \dots, 10\}$ such that $2f(a) - f(b) + 3f(c) + f(d) = 0$ is [24-Jun-2022-Shift-1]

Answer: 31

Solution:

$$-1 \leq \frac{x^2 - 5x + 6}{x^2 - 9} \leq 1 \text{ and } x^2 - 3x + 2 > 0, \neq 1$$

$$\frac{(x-3)(2x+1)}{x^2-9} \geq 0 \mid \frac{5(x-3)}{x^2-9} \geq 0$$

The solution to this inequality is

$$x \in \left[\frac{-1}{2}, \infty \right) - \{3\}$$

for $x^2 - 3x + 2 > 0$ and $\neq 1$

$$x \in (-\infty, 1) \cup (2, \infty) - \left\{ \frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2} \right\}$$

Combining the two solution sets (taking intersection)

$$x \in \left[-\frac{1}{2}, 1 \right) \cup (2, \infty) - \left\{ \frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2} \right\}$$

$f(d)$ can't be 9 and 10 as if $f(d) = 9$ or 10 then $f(b) = 2 + 9 = 11$ or $f(b) = 2 + 10 = 12$, which is not possible as here any function's maximum value can be 10 .

∴ Total possible functions when $f(c) = 0$ and $f(a) = 1$ are $= 7$

(2) When $f(c) = 0$ and $f(a) = 2$ then

$$3 \times 0 + 2 \times 2 + f(d) = f(b)$$

$$\Rightarrow 4 + f(d) = f(b)$$

∴ possible value of $f(d) = 1, 3, 4, 5, 6$

∴ Total possible functions in this case $= 5$

(3) When $f(c) = 0$ and $f(a) = 3$ then

$$3 \times 0 + 2 \times 3 + f(d) = f(b)$$

$$\Rightarrow 6 + f(d) = f(b)$$

∴ Possible value of $f(d) = 1, 2, 4$

∴ Total possible functions in this case $= 3$

(4) When $f(c) = 0$ and $f(a) = 4$ then

$$3 \times 0 + 2 \times 4 + f(d) = f(b)$$

$$\Rightarrow 8 + f(d) = f(b)$$

∴ Possible value of $f(d) = 1, 2$

∴ Total possible functions in this case $= 2$

(5) When $f(c) = 0$ and $f(a) = 5$ then

$$3 \times 0 + 2 \times 5 + f(d) = f(b)$$

$$\Rightarrow 10 + f(d) = f(b)$$

Possible value of $f(d)$ can be 0 but $f(c)$ is already zero. So, no value to $f(d)$ can satisfy.

∴ No function is possible in this case.

∴ Total possible functions when $f(c)=0$ and $f(a)=1,2,3$ and 4 are $=7+5+3+2=17$

Case II:

(1) When $f(c)=1$ and $f(a)=0$ then

$$3 \times 1 + 2 \times 0 + f(d) = f(b)$$

$$\Rightarrow 3 + f(d) = f(b)$$

∴ Possible value of $f(d)=2,3,4,5,6,7$

∴ Total possible functions in this case $=6$

(2) When $f(c)=1$ and $f(a)=2$ then

$$3 \times 1 + 2 \times 2 + f(d) = f(b)$$

$$\Rightarrow 7 + f(d) = f(b)$$

\therefore Possible value of $f(d) = 0, 3$

\therefore Total possible functions in this case = 2

(3) When $f(c) = 1$ and $f(a) = 3$ then

$$3 \times 1 + 2 \times 3 + f(d) = f(b)$$

$$\Rightarrow 9 + f(d) = f(b)$$

\therefore Possible value of $f(d) = 0$

\therefore Total possible functions in this case = 1

\therefore Total possible functions when $f(c) = 1$ and $f(a) = 0, 2$ and 3 are $= 6 + 2 + 1 = 9$

Case III:

(1) When $f(c) = 2$ and $f(a) = 0$ then

$$3 \times 2 + 2 \times 0 + f(d) = f(b)$$

$$\Rightarrow 6 + f(d) = f(b)$$

\therefore Possible values of $f(d) = 1, 3, 4$

\therefore Total possible functions in this case = 3

(2) When $f(c) = 2$ and $f(a) = 1$ then,

$$3 \times 2 + 2 \times 1 + f(d) = f(b)$$

$$\Rightarrow 8 + f(d) = f(b)$$

\therefore Possible values of $f(d) = 0$

\therefore Total possible function in this case = 1

\therefore Total possible functions when $f(c) = 2$ and $f(a) = 0, 1$ are $= 3 + 1 = 4$

Case IV:

(1) When $f(c) = 3$ and $f(a) = 0$ then

$$3 \times 3 + 2 \times 0 + f(d) = f(b)$$

$$\Rightarrow 9 + f(d) = f(b)$$

\therefore Possible values of $f(d) = 1$

\therefore Total one-one functions from four cases

$$= 17 + 9 + 4 + 1 = 31$$

Question 73

Let R_1 and R_2 be relations on the set $\{1, 2, \dots, 50\}$ such that

$R_1 = \{(p, p^n) : p \text{ is a prime and } n \geq 0 \text{ is an integer}\}$ and

$R_2 = \{(p, p^n) : p \text{ is a prime and } n = 0 \text{ or } 1\}$

Then, the number of elements in $R_1 - R_2$ is ____

[28-Jun-2022-Shift-1]

Answer: 8

Solution:

$$R_1 - R_2 = \{(2, 2^2), (2, 2^3), (2, 2^4), (2, 2^5), (3, 3^2), (3, 3^3), (5, 5^2), (7, 7^2)\}$$

So number of elements = 8

Question74

Let $R_1 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \leq 13\}$ and $R_2 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \neq 13\}$. Then on \mathbb{N} :
[28-Jun-2022-Shift-2]

Options:

- A. Both R_1 and R_2 are equivalence relations
- B. Neither R_1 nor R_2 is an equivalence relation
- C. R_1 is an equivalence relation but R_2 is not
- D. R_2 is an equivalence relation but R_1 is not

Answer: B

Solution:

Solution:

$R_1 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \leq 13\}$ and

$R_2 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \neq 13\}$

In R_1 : $\because |2 - 11| = 9 \leq 13$

$\therefore (2, 11) \in R_1$ and $(11, 19) \in R_1$ but $(2, 19) \notin R_1$

$\therefore R_1$ is not transitive

Hence R_1 is not equivalence

In R_2 : $(13, 3) \in R_2$ and $(3, 26) \in R_2$ but $(13, 26) \notin R_2 (\because |13 - 26| = 13)$

$\therefore R_2$ is not transitive

Hence R_2 is not equivalence.

Question75

The probability that a relation R from $\{x, y\}$ to $\{x, y\}$ is both symmetric and transitive, is equal to
[29-Jun-2022-Shift-2]

Options:

A. $\frac{5}{16}$

B. $\frac{9}{16}$

C. $\frac{11}{16}$

D. $\frac{13}{16}$

Answer: A

Solution:

Solution:

Total no. of relations = $2^{2 \times 2} = 16$

Fav. relation = $\emptyset, \{(x, x)\}, \{(y, y)\}, \{(x, x)(y, y)\}$

$\{(x, x), (y, y), (x, y)(y, x)\}$

$$\text{Prob.} = \frac{5}{16}$$

Question76

The number of bijective functions

$f : \{1, 3, 5, 7, \dots, 99\} \rightarrow \{2, 4, 6, 8, \dots, 100\}$, such that

$f(3) \geq f(9) \geq f(15) \geq f(21) \geq \dots \geq f(99)$, is

[25-Jul-2022-Shift-2]

Options:

A. ${}^{50}P_{17}$

B. ${}^{50}P_{33}$

C. $33! \times 17!$

D. $\frac{50!}{2}$

Answer: B

Solution:

Solution:

As function is one-one and onto, out of 50 elements of domain set 17 elements are following restriction

$f(3) > f(9) > f(15) \dots > f(99)$

So number of ways = ${}^{50}C_{17} \cdot 17!$

$$= {}^{50}P_{33}$$

Question77

Let $f(x)$ be a quadratic polynomial with leading coefficient 1 such that

$f(0) = p$, $p \neq 0$, and $f(1) = \frac{1}{3}$. If the equations $f(x) = 0$ and

$f \circ f \circ f \circ f(x) = 0$ have a common real root, then $f(-3)$ is equal to___

[25-Jul-2022-Shift-2]

Answer: 25

Solution:

Solution:

Let $f(x) = (x - \alpha)(x - \beta)$

It is given that $f(0) = p \Rightarrow \alpha\beta = p$

and $f(1) = \frac{1}{3} \Rightarrow (1 - \alpha)(1 - \beta) = \frac{1}{3}$

Now, let us assume that, α is the common root of $f(x) = 0$ and $f(f(f(f(x)))) = 0$

$f(f(f(f(x)))) = 0$

$\Rightarrow f(f(f(0))) = 0$

$\Rightarrow f(f(p)) = 0$

So, $f(p)$ is either α or β .

$(p - \alpha)(p - \beta) = \alpha$

$(\alpha\beta - \alpha)(\alpha\beta - \beta) = \alpha \Rightarrow (\beta - 1)(\alpha - 1)\beta = 1 (\because \alpha \neq 0)$

So, $\beta = 3$

$(1 - \alpha)(1 - 3) = \frac{1}{3}$

$\alpha = \frac{7}{6}$

$f(x) = \left(x - \frac{7}{6}\right)(x - 3)$

$f(-3) = \left(-3 - \frac{7}{6}\right)(3 - 3) = 25$

Question 78

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to:
[26-Jul-2022-Shift-1]

Options:

A. 4

B. 10

C. 11

D. 16

Answer: B

Solution:

Solution:

$f(3x) - f(x) = x \dots \dots (1)$

$x \rightarrow \frac{x}{3}$

$f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3} \dots \dots (2)$

Again $x \rightarrow \frac{x}{3}$

$f\left(\frac{x}{3}\right) - f\left(\frac{x}{9}\right) = \frac{x}{3^2} \dots \dots$

Similarly

$$f\left(\frac{x}{3^{n-2}}\right) - f\left(\frac{x}{3^{n-1}}\right) = \frac{x}{3^{n-1}} \dots (n)$$

Adding all these and applying $n \rightarrow \infty$

$$\lim_{n \rightarrow \infty} \left(f(3x) - f\left(\frac{x}{3^{n-1}}\right) \right) = x \left(1 + \frac{1}{3} + \frac{1}{3^2} + \dots \right)$$

$$f(3x) - f(0) = \frac{3x}{2}$$

$$\text{Putting } x = \frac{8}{3}$$

$$f(8) - f(0) = 4$$

$$\Rightarrow f(0) = 3$$

$$\text{Putting } x = \frac{14}{3}$$

$$f(14) - 3 = 7 \Rightarrow f(14) = 10$$

Question 79

The domain of the function

$f(x) = \sin^{-1}[2x^2 - 3] + \log_2 \left(\log_{\frac{1}{2}}(x^2 - 5x + 5) \right)$, where $[t]$ is the greatest

integer function, is :

[27-Jul-2022-Shift-2]

Options:

A. $\left(-\sqrt{\frac{5}{2}}, \frac{5-\sqrt{5}}{2} \right)$

B. $\left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2} \right)$

C. $\left(1, \frac{5-\sqrt{5}}{2} \right)$

D. $\left[1, \frac{5+\sqrt{5}}{2} \right)$

Answer: C

Solution:

Solution:

$$-1 \leq 2x^2 - 3 < 2$$

$$\text{or } 2 \leq 2x^2 < 5$$

$$\text{or } 1 \leq x^2 < \frac{5}{2}$$

$$x \in \left(-\sqrt{\frac{5}{2}}, -1 \right] \cup \left[1, \sqrt{\frac{5}{2}} \right)$$

$$\log_{\frac{1}{2}}(x^2 - 5x + 5) > 0$$

$$0 < x^2 - 5x + 5 < 1$$

$$x^2 - 5x + 5 > 0 \text{ \& } x^2 - 5x + 4 < 0$$

$$x \in \left(-\infty, \frac{5-\sqrt{5}}{2} \right) \cup \left(\frac{5+\sqrt{5}}{2}, \infty \right)$$

$$\&x \in (-\infty, 1) \cup (4, \infty)$$

Taking intersection

$$x \in \left(1, \frac{5-\sqrt{5}}{2} \right)$$

Question80

The number of functions f , from the set $A = \{x \in \mathbb{N} : x^2 - 10x + 9 \leq 0\}$ to the set $B = \{n^2 : n \in \mathbb{N}\}$ such that $f(x) \leq (x - 3)^2 + 1$, for every $x \in A$, is _____.

[27-Jul-2022-Shift-2]

Answer: 1440

Solution:

Solution:

$$A = \{x \in \mathbb{N} : x^2 - 10x + 9 \leq 0\}$$

$$= \{1, 2, 3, \dots, 9\}$$

$$B = \{1, 4, 9, 16, \dots\}$$

$$f(x) \leq (x - 3)^2 + 1$$

$$f(1) \leq 5, f(2) \leq 2, \dots, f(9) \leq 37$$

$x = 1$ has 2 choices

$x = 2$ has 1 choice

$x = 3$ has 1 choice

$x = 4$ has 1 choice

$x = 5$ has 2 choices

$x = 6$ has 3 choices

$x = 7$ has 4 choices

$x = 8$ has 5 choices

$x = 9$ has 6 choices

$$\therefore \text{Total functions} = 2 \times 1 \times 1 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 1440$$

Question81

Considering only the principal values of the inverse trigonometric

functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2 - 4x + 2}{x^2 + 3}\right)$ is :

[28-Jul-2022-Shift-1]

Options:

A. $\left(-\infty, \frac{1}{4}\right]$

B. $\left[-\frac{1}{4}, \infty\right)$

C. $(-1/3, \infty)$

D. $\left(-\infty, \frac{1}{3}\right]$

Answer: B

Solution:

Solution:

$$-1 \leq \frac{x^2 - 4x + 2}{x^2 + 3} \leq 1$$

$$\Rightarrow -x^2 - 3 \leq x^2 - 4x + 2 \leq x^2 + 3$$

$$\Rightarrow 2x^2 - 4x + 5 \geq 0 \quad -4x \leq 1$$

$$x \in \mathbb{R} \text{ and } x \geq -\frac{1}{4}$$

$$\text{So domain is } \left[-\frac{1}{4}, \infty \right)$$

Question 82

Let α , β and γ be three positive real numbers. Let

$f(x) = \alpha x^5 + \beta x^3 + \gamma x$, $x \in \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be such that $g(f(x)) = x$ for all $x \in \mathbb{R}$. If $a_1, a_2, a_3, \dots, a_n$ be in arithmetic progression with mean zero,

then the value of $f \left(g \left(\frac{1}{n} \sum_{i=1}^n f(a_i) \right) \right)$ is equal to:

[28-Jul-2022-Shift-1]

Options:

A. 0

B. 3

C. 9

D. 27

Answer: A

Solution:

Solution:

$$f \left(g \left(\frac{1}{n} \sum_{i=1}^n f(a_i) \right) \right)$$

$$\frac{a_1 + a_2 + a_3 + \dots + a_n}{n} = 0$$

\therefore First and last term, second and second last and so on are equal in magnitude but opposite in sign.

$$f(x) = \alpha x^5 + \beta x^3 + \gamma x$$

$$\begin{aligned} \sum_{i=1}^n f(a_i) &= \alpha(a_1^5 + a_2^5 + a_3^5 + \dots + a_n^5) + \beta(a_1^3 + a_2^3 + \dots + a_n^3) + \gamma(a_1 + a_2 + \dots + a_n) \\ &= 0\alpha + 0\beta + 0\gamma \\ &= 0 \end{aligned}$$

$$\therefore f \left(g \left(\frac{1}{n} \sum_{i=1}^n f(a_i) \right) \right) = \frac{1}{n} \sum_{i=1}^n f(a_i) = 0$$

Question 83

The number of elements in the set

$$S = \left\{ x \in \mathbb{R} : 2 \cos \left(\frac{x^2 + x}{6} \right) = 4^x + 4^{-x} \right\} \text{ is:}$$

[29-Jul-2022-Shift-2]

Options:

- A. 1
- B. 3
- C. 0
- D. infinite

Answer: A

Solution:

Solution:

$$2 \cos \left(\frac{x^2 + x}{6} \right) = 4^x + 4^{-x}$$

L.H.S ≤ 2 . & R.H.S. ≥ 2

Hence L.H.S = 2 & R.H.S = 2

$$2 \cos \left(\frac{x^2 + x}{6} \right) = 2 \quad 4^x + 4^{-x} = 2$$

Check $x = 0$ Possible hence only one solution.

Question 84

The domain of the function $f(x) = \sin^{-1} \left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \right)$ is:

[29-Jul-2022-Shift-2]

Options:

- A. $[1, \infty)$
- B. $[-1, 2]$
- C. $[-1, \infty)$
- D. $(-\infty, 2]$

Answer: C

Solution:

Solution:

$$f(x) = \sin^{-1} \left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \right)$$

$$-1 \leq \frac{x^2 - 3x + 2}{x^2 + 2x + 7} \leq 1$$

$$\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \leq 1$$

$$x^2 - 3x + 2 \leq x^2 + 2x + 7$$

$$5x \geq -5$$

$$x \geq -1$$

$$\text{And } \frac{x^2 - 3x + 2}{x^2 + 2x + 7} \geq -1$$

$$x^2 - 3x + 2 \geq -x^2 - 2x - 7$$

$$2x^2 - x + 9 \geq 0$$

$x \in \mathbb{R}$
(i) \cap (ii)
Domain $\in [-1, \infty)$

Question 85

The total number of functions,
 $f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4, 5, 6\}$ such that $f(1) + f(2) = f(3)$, is equal to
:
[25-Jul-2022-Shift-1]

Options:

- A. 60
- B. 90
- C. 108
- D. 126

Answer: B

Solution:

Solution:

Given, $f(1) + f(2) = f(3)$

It means $f(1)$, $f(2)$ and $f(3)$ are dependent on each other. But there is no condition on $f(4)$, so $f(4)$ can be $f(4) = 1, 2, 3, 4, 5, 6$.

For $f(1)$, $f(2)$ and we have to find how many functions possible which will satisfy the condition $f(1) + f(2) = f(3)$

Case 1:

When $f(3) = 2$ then possible values of $f(1)$ and $f(2)$ which satisfy $f(1) + f(2) = f(3)$ is $f(1) = 1$ and $f(2) = 1$.

And $f(4)$ can be $= 1, 2, 3, 4, 5, 6$

\therefore Total possible functions $= 1 \times 6 = 6$

Case 2 :

When $f(3) = 3$ then possible values

(1) $f(1) = 1$ and $f(2) = 2$

(2) $f(1) = 2$ and $f(2) = 1$

And $f(4)$ can be $= 1, 2, 3, 4, 5, 6$.

\therefore Total functions $= 2 \times 6 = 12$

Case 3 :

When $f(3) = 4$ then

(1) $f(1) = 1$ and $f(2) = 3$

(2) $f(1) = 2$ and $f(2) = 2$

(3) $f(1) = 3$ and $f(2) = 1$

And $f(4)$ can be $= 1, 2, 3, 4, 5, 6$

\therefore Total functions $= 3 \times 6 = 18$

Case 4 :

When $f(3) = 5$ then

(1) $f(1) = 1$ and $f(4) = 4$

(2) $f(1) = 2$ and $f(4) = 3$

(3) $f(1) = 3$ and $f(4) = 2$

(4) $f(1) = 4$ and $f(4) = 1$

And $f(4)$ can be $= 1, 2, 3, 4, 5$ and 6

\therefore Total functions $= 4 \times 6 = 24$

Case 5 :

When $f(3) = 6$ then

(1) $f(1) = 1$ and $f(2) = 5$

(2) $f(1) = 2$ and $f(2) = 4$

(3) $f(1) = 3$ and $f(2) = 3$

(4) $f(1) = 4$ and $f(2) = 2$

(5) $f(1) = 5$ and $f(2) = 1$

And $f(4)$ can be $= 1, 2, 3, 4, 5$ and 6

\therefore Total possible functions = $5 \times 6 = 30$

\therefore Total functions from those 5 cases we get = $6 + 12 + 18 + 24 + 30 = 90$

Question 86

Let $f : \mathbb{N} \rightarrow \mathbb{R}$ be a function such that $f(x + y) = 2f(x)f(y)$ for natural numbers x and y . If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1)$$

holds, is :

[25-Jun-2022-Shift-1]

Options:

A. 2

B. 3

C. 4

D. 6

Answer: C

Solution:

Solution:

Given,

$$f(x+y) = 2f(x)f(y)$$

$$\text{and } f(1) = 2$$

For $x = 1$ and $y = 1$,

$$f(1+1) = 2f(1)f(1)$$

$$\Rightarrow f(2) = 2(f(1))^2 = 2(2)^2 = 2^3$$

For $x = 1, y = 2$

$$f(1+2) = 2f(1)f(2)$$

$$\Rightarrow f(3) = 2 \cdot 2 \cdot 2^3 = 2^5$$

For $x = 1, y = 3$

$$f(1+3) = 2f(1)f(3)$$

$$\Rightarrow f(4) = 2 \cdot 2 \cdot 2^5 = 2^7$$

For $x = 1, y = 4$

$$f(1+4) = 2f(1)f(4)$$

$$\Rightarrow f(5) = 2 \cdot 2 \cdot 2^7 = 2^9 \dots$$

Also given

$$\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1)$$

$$\Rightarrow f(\alpha+1) + f(\alpha+2) + f(\alpha+3) + \dots + f(\alpha+10) = \frac{512}{3}(2^{20} - 1)$$

$$\Rightarrow f(\alpha+1) + f(\alpha+2) + f(\alpha+3) + \dots + f(\alpha+10) = \frac{2^9((2^2)^{10} - 1)}{2^2 - 1}$$

This represent a G.P with first term $= 2^9$ and common ratio $= 2^2$

$$\therefore \text{First term} = f(\alpha+1) = 2^9 \dots (2)$$

$$\text{From equation (1), } f(5) = 2^9$$

\therefore From (1) and (2), we get

$$f(\alpha+1) = 2^9 = f(5)$$

$$\Rightarrow f(\alpha+1) = f(5)$$

$$\Rightarrow f(\alpha+1) = f(4+1)$$

Comparing both sides we get, $\alpha = 4$

Question87

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \left(2 \left(1 - \frac{x^{25}}{2} \right) (2 + x^{25}) \right)^{\frac{1}{50}}$. If the function $g(x) = f(f(f(x))) + f(f(x))$, then the greatest integer less than or equal to $g(1)$ is ____
[25-Jun-2022-Shift-1]

Answer: 2

Solution:

Given,

$$f(x) = \left(2 \left(1 - \frac{x^{25}}{2} \right) (2 + x^{25}) \right)^{\frac{1}{50}}$$

$$\text{and } g(x) = f(f(f(x))) + f(f(x))$$

$$\therefore g(1) = f(f(f(1))) + f(f(1))$$

$$\text{Now, } f(1) = \left(2 \left(1 - \frac{1^{25}}{2} \right) (2 + 1^{25}) \right)^{\frac{1}{50}}$$

$$= \left(2 \left(1 - \frac{1}{2} \right) (2 + 1) \right)^{\frac{1}{50}}$$

$$= (3)^{\frac{1}{50}}$$

$$\therefore f(f(1)) = f\left(3^{\frac{1}{50}}\right)$$

$$= \left(2 \left(1 - \frac{\left(3^{\frac{1}{50}}\right)^{25}}{2}\right) \left(2 + \left(3^{\frac{1}{50}}\right)^{25}\right)\right)^{\frac{1}{50}}$$

$$= \left(2 \left(1 - \frac{3^{\frac{1}{2}}}{2}\right) \left(2 + 3^{\frac{1}{2}}\right)\right)^{\frac{1}{50}}$$

$$= \left(2 \times \left(\frac{2 - \sqrt{3}}{2}\right) (2 + \sqrt{3})\right)^{\frac{1}{50}}$$

$$= [(2 - \sqrt{3})(2 + \sqrt{3})]^{\frac{1}{50}}$$

$$= (4 - 3)^{\frac{1}{50}}$$

$$= 1^{\frac{1}{50}} = 1$$

$$\text{Now, } f(f(f(1))) = f(1) = 3^{\frac{1}{50}}$$

$$\therefore g(1) = f(f(f(1))) + f(f(1))$$

$$= 3^{\frac{1}{50}} + 1$$

Now, greatest integer less than or equal to $g(1)$

$$= [g(1)]$$

$$= \left[3^{\frac{1}{50}} + 1\right]$$

$$= \left[3^{\frac{1}{50}}\right] + [1]$$

$$= [1.02] + 1$$

$$= 1 + 1 = 2$$

Question88

Let $f(x) = \frac{x-1}{x+1}$, $x \in \mathbb{R} - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^n(x))$ for all $n \in \mathbb{N}$, then $f^6(6) + f^7(7)$ is equal to :
[26-Jun-2022-Shift-1]

Options:

A. $\frac{7}{6}$

B. $-\frac{3}{2}$

C. $\frac{7}{12}$

D. $-\frac{11}{12}$

Answer: B

Solution:

Given,

$$f(x) = \frac{x-1}{x+1}$$

Also given,

$$f^{n+1}(x) = f(f^n(x)) \dots (1)$$

\therefore For $n = 1$

$$f^{1+1}(x) = f(f^1(x))$$

$$\Rightarrow f^2(x) = f(f(x))$$

$$= f\left(\frac{x-1}{x+1}\right)$$

$$= \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$$

$$= \frac{\frac{x-1-x-1}{x+1}}{\frac{x-1+x+1}{x+1}}$$

$$= \frac{-2}{2x} = -\frac{1}{x}$$

From equation (1), when $n = 2$ $f^{2+1}(x) = f(f^2(x))$

$$\Rightarrow f^3(x) = f(f^2(x))$$

$$= f\left(-\frac{1}{x}\right)$$

$$= \frac{-\frac{1}{x} - 1}{-\frac{1}{x} + 1}$$

$$= \frac{\frac{-1-x}{x}}{\frac{-1+x}{x}}$$

$$= \frac{-1-x}{-1+x} = \frac{-(x+1)}{x-1}$$

Similarly,

$$f^4(x) = f(f^3(x))$$

$$= f\left(\frac{-x+1}{x-1}\right)$$

$$= \frac{\frac{-(x+1)}{x-1} - 1}{\frac{-(x+1)}{x-1} + 1}$$

$$= \frac{\frac{x-1-x+1}{x-1}}{\frac{-x-1+x-1}{x-1}}$$

$$= \frac{-2x}{-2} = x$$

$$\therefore f^5(x) = f(f^4(x))$$

$$= f(x)$$

$$= \frac{x-1}{x+1}$$

$$f^6(x) = f(f^5(x))$$

$$= f\left(\frac{x-1}{x+1}\right)$$

$$= -\frac{1}{x} \text{ (Already calculated earlier)}$$

$$f^7(x) = f(f^6(x))$$

$$= f\left(-\frac{1}{x}\right)$$

$$= \frac{-\frac{1}{x} - 1}{-\frac{1}{x} + 1}$$

$$= \frac{-(x+1)}{x-1}$$

$$\therefore f^6(6) = -\frac{1}{6}$$

$$\text{and } f^7(7) = \frac{-(7+1)}{7-1} = -\frac{8}{6}$$

$$\text{So, } f^6(6) + f^7(7)$$

$$= -\frac{1}{6} - \frac{8}{6}$$

$$= -\frac{3}{2}$$

Question 89

The range of the function,

$$f(x) = \log_{\sqrt{5}} \left(3 + \cos \left(\frac{3\pi}{4} + x \right) + \cos \left(\frac{\pi}{4} + x \right) + \cos \left(\frac{\pi}{4} - x \right) - \cos \left(\frac{3\pi}{4} - x \right) \right)$$

is

[2021, 01 Sep. Shift-II]

Options:

A. $(0, \sqrt{5})$

B. $[-2, 2]$

C. $\left[\frac{1}{\sqrt{5}}, \sqrt{5} \right]$

D. $[0, 2]$

Answer: D

Solution:

Solution:

$$\begin{aligned}f(x) &= \log_{\sqrt{5}} \left(3 + \cos \left(\frac{3\pi}{4} + x \right) + \cos \left(\frac{\pi}{4} + x \right) \right. \\&\quad \left. + \cos \left(\frac{\pi}{4} - x \right) - \cos \left(\frac{3\pi}{4} - x \right) \right) \\&= \log_{\sqrt{5}} (3 - \sqrt{2} \sin x + \sqrt{2} \cos x) \\&\because -2 \leq -\sqrt{2} \sin x + \sqrt{2} \cos x \leq 2 \\&\Rightarrow 1 \leq 3 - \sqrt{2} \sin x + \sqrt{2} \cos x \leq 5 \\&\Rightarrow \log_{\sqrt{5}} 1 \leq \log_{\sqrt{5}} (3 - \sqrt{2} \sin x + \sqrt{2} \cos x) \\&\Rightarrow 0 \leq f(x) \leq 2 \\&\Rightarrow f(x) \in [0, 2]\end{aligned}$$

Question90

The domain of the function $f(x) = \sin^{-1} \left(\frac{3x^2 + x - 1}{(x-1)^2} \right) + \cos^{-1} \left(\frac{x-1}{x+1} \right)$ is [2021, 31 Aug. Shift-II]

Options:

A. $\left[0, \frac{1}{4} \right]$

B. $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2} \right]$

C. $\left[\frac{1}{4}, \frac{1}{2} \right] \cup \{0\}$

D. $\left[0, \frac{1}{2} \right]$

Answer: C

Solution:

$$f(x) = \sin^{-1} \left(\frac{3x^2 + x - 1}{(x-1)^2} \right) + \cos^{-1} \left(\frac{x-1}{x+1} \right)$$

$$-1 \leq \frac{x-1}{x+1} \leq 1 \Rightarrow -1 - 1 \leq \frac{x-1}{x+1} - 1 \leq 1 - 1$$

$$\Rightarrow -2 \leq \frac{-2}{x+1} \leq 0 \Rightarrow 0 \leq \frac{1}{x+1} \leq 1$$

$$\Rightarrow 1 \leq x+1 < \infty$$

$$\Rightarrow 0 \leq x < \infty$$

$$\Rightarrow x \in [0, \infty)$$

$$\text{and } -1 \leq \frac{3x^2 + x - 1}{(x-1)^2} \leq 1$$

$$\Rightarrow -(x-1)^2 \leq 3x^2 + x - 1 \leq (x-1)^2, x \neq 1$$

$$\Rightarrow -(x^2 - 2x + 1) \leq 3x^2 + x - 1$$

$$\text{and } 3x^2 + x - 1 \leq x^2 - 2x + 1$$

$$\Rightarrow 4x^2 - x \geq 0$$

$$\text{and } 2x^2 + 3x - 2 \leq 0$$

$$\Rightarrow x(4x - 1) \geq 0$$

$$\text{and } (x+2)(2x-1) \leq 0$$

$$\Rightarrow x \in (-\infty, 0] \cup \left[\frac{1}{4}, \infty \right)$$

$$\text{and } x \in \left[-2, \frac{1}{2} \right]$$

$$\Rightarrow x \in (-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2} \right]$$

Domain of f in Eq. (i) \cap Eq. (ii)

$$\therefore x \in \{0\} \cup \left[\frac{1}{4}, \frac{1}{2} \right]$$

Question91

Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be a function such that $f(m+n) = f(m) + f(n)$ for every $m, n \in \mathbb{N}$. If $f(6) = 18$, then $f(2) \cdot f(3)$ is equal to

[2021, 31 Aug. Shift-11]

Options:

- A. 6
- B. 54
- C. 18
- D. 36

Answer: B

Solution:

$$f(m+n) = f(m) + f(n), m, n \in \mathbb{N}$$

$$\therefore f(3+3) = f(3) + f(3)$$

$$\Rightarrow f(6) = 2f(3) = 18 \quad [\because f(6) = 18]$$

$$\text{Also } f(3) = f(2+1) = f(2) + f(1)$$

$$= f(1+1) + f(1)$$

$$f(3) = f(1) + f(1) + f(1)$$

$$\Rightarrow 9 = 3f(1) \Rightarrow f(1) = 3$$

$$\therefore f(2) = f(1+1) = f(1) + f(1) = 3 + 3 = 6$$

$$\text{Hence, } f(2) \cdot f(3) = 6 \cdot 9 = 54$$

Question92

The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is
[2021, 26 Aug. Shift-II]

Options:

A. $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$

B. $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

C. $\left(-\frac{1}{2}, \infty\right) - \{0\}$

D. $\left[-\frac{1}{2}, \infty\right) - \{0\}$

Answer: D

Solution:

$$f(x) = \operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right) \quad \left| \frac{1+x}{x} \right| \geq 1$$

Clearly, $x \neq 0$

$$|1+x| \geq |x|$$

$$1+x^2+2x \geq x^2$$

$$2x+1 \geq 0$$

$$x \geq -\frac{1}{2}$$

So,

$$x \in \left[-\frac{1}{2}, \infty\right) - \{0\}$$

Question93

Which of the following is not correct for relation R on the set of real numbers ?

[2021, 31 Aug. Shift-1]

Options:

- A. $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric.
- B. $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive.
- C. $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric.
- D. $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive and symmetric.

Answer: B

Solution:

Solution:

According to the question, let's consider option (b) (2, 3) and (3, 4) satisfy $0 < |x - y| \leq 1$ but (2, 4) does not satisfy it.

Question94

Let N be the set of natural numbers and a relation R on N be defined by $R = \{ (x, y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0 \}$. Then the relation R is [2021, 27 July Shift-11]

Options:

- A. symmetric but neither reflexive nor transitive.
- B. reflexive but neither symmetric nor transitive.
- C. reflexive and symmetric, but not transitive.
- D. an equivalence relation.

Answer: B

Solution:

Solution:

Given, relation R on N is defined by

$$R = \{(x, y) \in N \times N : x^3 - 3x^2 - xy^2 + 3y^3 = 0\}$$

$$x^3 - 3x^2y - xy^2 + 3y^3 = 0$$

$$\Rightarrow x^3 - xy^2 - 3x^2y + 3y^3 = 0$$

$$\Rightarrow x(x^2 - y^2) - 3y(x^2 - y^2) = 0$$

$$\Rightarrow (x - 3y)(x^2 - y^2) = 0$$

$$\Rightarrow (x - 3y)(x - y)(x + y) = 0$$

$$\text{Now, } x - x = 0$$

$$\Rightarrow x = x, \forall (x, x) \in N \times N$$

So, R is a reflexive relation.

But not symmetric and transitive relation because,

(3, 1) satisfies but (1, 3) does not. Also, (3, 1) and

(1, -1) satisfies but (3, -1) does not.

Hence, relation R is reflexive but neither symmetric nor transitive.

Question95

Define a relation R over a class of $n \times n$ real matrices A and B as "ARB, if there exists a non-singular matrix P such that $PAP^{-1} = B'$. Then which of the following is true ?
[2021, 18 March Shift-II]

Options:

A. R is symmetric, transitive but not reflexive.

B. R is reflexive, symmetric but not transitive.

C. R is an equivalence relation.

D. R is reflexive, transitive but not symmetric.

Answer: C

Solution:**Solution:**

For reflexive relation, $\forall (A, A) \in R$ for matrix P.

$$\Rightarrow A = PAP^{-1} \text{ is true for } P = 1$$

So, R is reflexive relation.

For symmetric relation,

Let $(A, B) \in R$ for matrix P.

$$\Rightarrow A = PBP^{-1} \text{ After pre-multiply by } P^{-1} \text{ and post-multiply by } P_1$$

we get

$$P^{-1}AP = B$$

So, $(B, A) \in R$ for matrix P^{-1} .

So, R is a symmetric relation.

For transitive relation,

Let ARB and BRC

$$\text{So, } A = PBP^{-1} \text{ and } B = PCP^{-1}$$

$$\text{Now, } A = P(PCP^{-1})P^{-1}$$

$$\Rightarrow A = (P)^2C(P^{-1})^2 \Rightarrow A = (P)^2 \cdot C \cdot (P^2)^{-1}$$

$$\therefore (A, C) \in R \text{ for matrix } P^2.$$

$\therefore R$ is transitive relation.

Hence, R is an equivalence relation.

Question96

Let $A = \{2, 3, 4, 5, \dots, 30\}$ and $' \sim '$ be an equivalence relation on $A \times A$, defined by $(a, b) \sim (c, d)$, if and only if $ad = bc$. Then, the number of ordered pairs, which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to
[2021, 16 March Shift-II]

Options:

- A. 5
- B. 6
- C. 8
- D. 7

Answer: D

Solution:

Solution:

$$A = \{2, 3, 4, 5, \dots, 30\}$$

$$a = bc$$

$$\therefore (a, b)R(4, 3)$$

$$\Rightarrow 3a = 4b$$

$$\Rightarrow a = \left(\frac{4}{3}\right)b$$

b must be a multiple of 3, b can be
(3, 6, 9, ... 30).

Also, a must be less than or equal to 30.

$$(a, b) = (4, 3), (8, 6), (12, 9), (16, 12), (20, 15)$$

$$(24, 18), (28, 21)$$

\Rightarrow 7 ordered pairs

Question97

Let $R = \{(P, O) \mid P \text{ and } Q \text{ are at the same distance from the origin}\}$ be a relation, then the equivalence class of $(1, -1)$ is the set
[2021, 26 Feb. Shift-1]

Options:

- A. $S = \{(x, y) \mid x^2 + y^2 = 4\}$
- B. $S = \{(x, y) \mid x^2 + y^2 = 1\}$
- C. $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$
- D. $S = \{(x, y) \mid x^2 + y^2 = 2\}$

Answer: D

Solution:

Solution:

Let $P(a, b)$ and $Q(c, d)$ are any two points.

Given, $OP = OQ$

$$\text{i.e. } \sqrt{a^2 + b^2} = \sqrt{c^2 + d^2}$$

Squaring on both sides,

$$R = \{(a, b), (c, d) : a^2 + b^2 = c^2 + d^2\}$$

$R(x, y), S(1, -1), OR = OS$ (equivalence class)

$$\text{This gives } OR = \sqrt{x^2 + y^2} \text{ and } OS = \sqrt{2}$$

$$1 \Rightarrow \sqrt{x^2 + y^2} = \sqrt{2}$$

$$\Rightarrow x^2 + y^2 = 2 \quad (\text{Squaring on both sides})$$

$$\therefore S = \{(x, y) : x^2 + y^2 = 2\}$$

Question 98

Let $\{S = 1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible functions $f : S \rightarrow S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to
[2021, 27 July Shift-I]

Answer: 490

Solution:

Solution:

$$S = \{1, 2, 3, 4, 5, 6, 7\}$$

$$f : S \rightarrow S$$

$$f(m \cdot n) = f(m)f(n)$$

$$m, n \in S \Rightarrow m, n \in S$$

$$\text{If } mn \in S \Rightarrow mn \leq 7$$

$$\text{So, } (1 \cdot 1, 1 \cdot 2, \dots, 1 \cdot 7) \leq 7$$

$$(2 \cdot 2, 2 \cdot 3) \leq 7$$

$$\text{When } m = 1, f(n) = f(1) \cdot f(n) \Rightarrow f(1) = 1$$

$$\text{When } m = n = 2,$$

$$f(4) = f(2)f(2) = \begin{cases} f(2) = 1 \Rightarrow f(4) = 1 & \text{or} \\ f(2) = 2 \Rightarrow f(4) = 4. \end{cases}$$

$$\text{When, } m = 2, n = 3$$

$$f(6) = f(2)f(3) \begin{cases} \text{When, } f(2) = 1 \\ f(3) = 1 \text{ to } 7 \\ \text{When, } f(2) = 2 \\ f(3) = 1 \text{ or } 2 \text{ or } 3. \end{cases}$$

And $f(5), f(7)$ can take any value (1-7) [$\because f(5) = f(1) \cdot f(5) \leq 7$ and $f(7) = f(1) \cdot f(7) \leq 7$] The possible combination is

$$1) f(1) = 1 \quad f(1) = 1$$

$$f(2) = 1 \quad f(2) = 2$$

$$f(3) = (1-7) \quad f(3) = (1-3)$$

$$f(4) = 1 \quad f(4) = 4$$

$$f(5) = (1-7) \quad f(5) = (1-7)$$

$$f(6) = f(3) \quad f(6) = f(3)$$

$$f(7) = (1-7) \quad f(7) = (1-7)$$

$$\text{So, total} = (1 \times 1 \times 7 \times 1 \times 7 \times 1 \times 7)$$

$$+ (1 \times 1 \times 3 \times 1 \times 7 \times 1 \times 7)$$

$$= 490$$

Question99

If $[x]$ be the greatest integer less than or equal to x , then $\sum_{n=8}^{100} \left[\frac{(-1)^n n}{2} \right]$ is equal to
[25 July 2021, Shift-III]

Options:

- A. 0
- B. 4
- C. -2
- D. 2

Answer: D

Solution:

Solution:

We have,

$$\sum_{n=8}^{100} \left[\frac{(-1)^n n}{2} \right] (\because [x] \text{ is the greatest integer function})$$

Substitute the values of n

$$\begin{aligned} &= [4] + [-4.5] + [5] + [-5.5] \\ &\quad + \dots + [-49.5] + [50] \\ &= 4 - 5 + 5 - 6 + \dots - 50 + 50 \\ &= 4 \end{aligned}$$

Question100

If the domain of the function $f(x) = \frac{\cos^{-1} \sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1} \left(\frac{2x - 1}{2} \right)}}$ is the interval (α, β) , then $\alpha + \beta$ is equal to
[2021, 22 July Shift-II]

Options:

- A. $\frac{3}{2}$
- B. 2
- C. $\frac{1}{2}$
- D. 1

Answer: A

Solution:

Solution:

$$f(x) = \frac{\cos^{-1} \sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1} \left(\frac{2x-1}{2} \right)}}$$

$$\Rightarrow x \in \mathbb{R}, \quad x(x-1) \leq 0$$

$$x^2 - x + 1 \geq 0 \text{ and } x^2 - x + 1 \leq 1$$

$$0 \leq x \leq 1 \quad \dots\dots\dots (i) \Rightarrow 0 < \sin^{-1} \left(\frac{2x-1}{2} \right) < \frac{\pi}{2}$$

$$\Rightarrow 0 < \frac{2x-1}{2} < 1$$

$$\Rightarrow \frac{1}{2} < x < \frac{3}{2} \quad \dots\dots\dots (ii)$$

$$(A) \cap (B) = x \in \left(\frac{1}{2}, 1 \right]$$

$$\therefore \alpha + \beta = \frac{3}{2}$$

Question 101

Let $[x]$ denote the greatest integer $\leq x$, where $x \in \mathbb{R}$. If the domain of the real valued function

$$f(x) = \sqrt{\frac{[x] - 2}{[x] - 3}}$$
 is $(-\infty, a) \cup [b, c)$

$\cup [u, \infty)$, $a < b < c$, then the value of $a + b + c$ is
[2021, 20 July Shift-I]

Options:

- A. 8
- B. 1
- C. -2
- D. -3

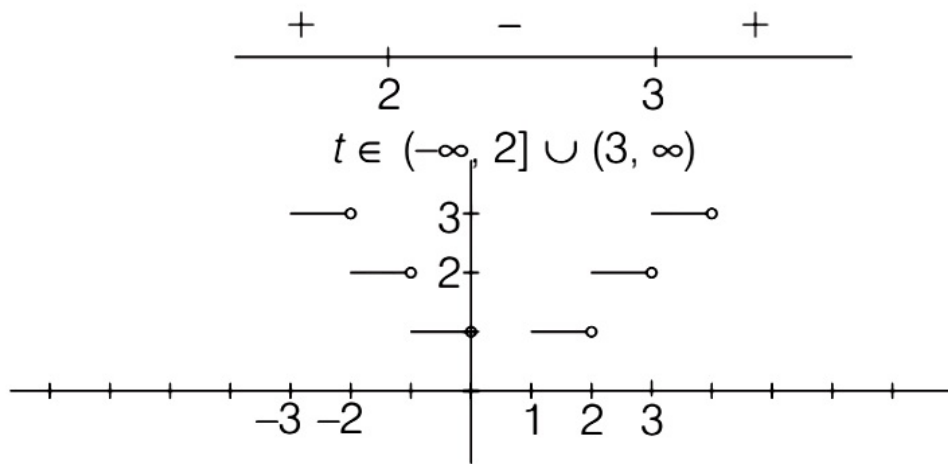
Answer: C

Solution:

Solution:

$$f(x) = \sqrt{\frac{[x] - 2}{[x] - 3}} \geq 0$$

$$\text{Let } [x] = t$$



$| [x] | = 3 \Rightarrow x \in [-3, -2) \cup [3, 4)$
 Domain of $x = [-\infty, -3) \cup [-2, 3) \cup [4, \infty)$
 $a = -3$
 $b = -2$
 $c = 3$
 $\therefore a + b + c = -3 + (-2) + 3 = -2$

Question102

The real valued function $f(x) = \frac{\operatorname{cosec}^{-1}x}{\sqrt{x} - [x]}$, where $[x]$ denotes the greatest integer less than or equal to x , is defined for all x belonging to [2021, 18 March Shift-I]

Options:

- A. all reals except integers
- B. all non-integers except the interval $[-1, 1]$
- C. all integers except 0, $-1, 1$
- D. all reals except the interval $[-1, 1]$

Answer: B

Solution:

Solution:

$$\text{Given, } f(x) = \frac{\operatorname{cosec}^{-1}x}{\sqrt{x} - [x]}$$

$$\Rightarrow f(x) = \frac{\operatorname{cosec}^{-1}x}{\sqrt{\{x\}}}$$

For $f(x)$ to be defined,

$$\begin{cases} |x| \geq 1 \\ \{x\} > 0. \end{cases} \Rightarrow \begin{cases} x \leq -1 \text{ or } x \geq 1 \\ x \neq 1 \text{ integers} \end{cases}$$

i.e. $x \in (-\infty, -1] \cup [1, \infty) - \{ \text{integers} \}$

i.e. all non-integers except the interval $[-1, 1]$
(here, -1 and 1 are included in except case, because of -1 and 1 are integers).

Question103

If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions?

$f + g$, $f - g$, f / g , g / f , $g - f$, where

$$(f \pm g)(x) = f(x) \pm g(x), (f / g)(x) = \frac{f(x)}{g(x)}$$

[2021, 18 March, Shift-1]

Options:

A. $0 \leq x \leq 1$

B. $0 \leq x < 1$

C. $0 < x < 1$

D. $0 < x \leq 1$

Answer: C

Solution:

Solution:

Given, $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$

\therefore Domain of $f(x) = D_1$ is $x \geq 0$

i.e. $D_1 : x \in (0, \infty)$

and domain of $g(x) = D_2$ is $1 - x \geq 0 \Rightarrow x \leq 1$

i.e. $D_2 : x \in (-\infty, 1]$

As, we know that, the domain of $f + g$, $f - g$, $g - f$ will be $D_1 \cap D_2$ as well as the domain for $\frac{f}{g}$ is $D_1 \cap D_2$ except all those value(s) of x , such that $g(x) = 0$.

Similarly, for $\frac{g}{f}$ is $D_1 \cap D_2$ but $f(x) \neq 0$.

Hence, common domain for $(f + g)$, $(f - g)$, $\left(\frac{f}{g}\right)$, $\left(\frac{g}{f}\right)$ and $(g - f)$ will be $0 < x < 1$

Question104

A function $f(x)$ is given by $f(x) = \frac{5^x}{5^x + 5}$, then the sum of the series

$$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \dots + f\left(\frac{39}{20}\right)$$

is equal to

[2021, 25 Feb. Shift-II]

Options:

A. $\frac{29}{2}$

B. $\frac{49}{2}$

C. $\frac{39}{2}$

D. $\frac{19}{2}$

Answer: C

Solution:

Solution:

Given, $f(x) = \frac{5^x}{5^x + 5}$, then,

$$f(2-x) = \frac{5^{2-x}}{5^{2-x} + 5}$$

$$= \frac{5}{5^x + 5}$$

This gives, $f(x) + f(2-x) = \frac{5^x + 5}{5^x + 5} = 1 \Rightarrow f\left(\frac{1}{20}\right) + f\left(2 - \frac{1}{20}\right) = f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right) = 1$

Similarly,

cf $\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right) = 1$ and so on,

$\therefore f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + \dots + f\left(\frac{38}{20}\right) + f\left(\frac{39}{20}\right)$

$= 1 + 1 + \dots + 1 + f\left(\frac{20}{20}\right)$

$= 19 + f(1) = 19 + \frac{1}{2} = \frac{39}{2}$

Question105

If $a + \alpha = 1$, $b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$, $x \neq 0$, then the value

of expression $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$ is

[2021, 24 Feb. Shift-II]

Answer: 2

Solution:

Solution:

Given, $a + \alpha = 1$

$b + \beta = 2$

$\therefore a \cdot f(x) + \alpha \cdot f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x} \dots\dots\dots (i)$

Replace x by $\frac{1}{x}$,

$af\left(\frac{1}{x}\right) + af(x) = \frac{b}{x} + \beta x$

Adding Eqs. (i) and (ii), we get

$$\&(a + \alpha) \left[f(x) + f\left(\frac{1}{x}\right) \right] = \left(x + \frac{1}{x}\right)(b + \beta) \dots\dots\dots (ii)$$

$$\Rightarrow \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{b + \beta}{a + \alpha} = \frac{2}{1} = 2$$

Question 106

Let $f(x) = \sin^{-1}x$ and

$$g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$$

If $g(2) = \lim_{x \rightarrow 2} g(x)$, then the domain of the function $f \circ g$ is

[2021, 26 Feb. Shift-II]

Options:

A. $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

B. $(-\infty, -2] \cup [-1, \infty)$

C. $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

D. $(-\infty, -1] \cup [2, \infty)$

Answer: C

Solution:

Solution:

Given, $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$, $f(x) = \sin^{-1}x$

$$f(g(x)) = \sin^{-1}(g(x))$$

$$f \circ g(x) = \sin^{-1}\left(\frac{x^2 - x - 2}{2x^2 - x - 6}\right)$$

For the domain of $f \circ g(x)$,

$$|g(x)| \leq 1$$

[\because Domain of $f(x)$ is $[-1, 1]$]

$$\Rightarrow \left| \frac{x^2 - x - 2}{2x^2 - x - 6} \right| \leq 1$$

$$\Rightarrow \left| \frac{(x+1)(x-2)}{(2x+3)(x-2)} \right| \leq 1$$

$$\Rightarrow \left| \frac{x+1}{2x+3} \right| \leq 1$$

$$\Rightarrow -1 \leq \frac{x+1}{2x+3} \leq 1$$

$$\Rightarrow \left(\frac{x+1}{2x+3} \right)^2 \leq 1$$

$$\Rightarrow (x+1)^2 \leq (2x+3)^2$$

$$\Rightarrow 3x^2 + 10x + 8 \geq 0$$

$$\Rightarrow (3x + y)(x + 2) \geq 0$$

This implies,

$$x \in (-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$$

Question 107

Let $g : \mathbb{N} \rightarrow \mathbb{N}$ be defined as

$$g(3n + 1) = 3n + 2$$

$$g(3n + 2) = 3n + 3,$$

$$g(3n + 3) = 3n + 1, \text{ for all } n \geq 0.$$

Then which of the following statements is true?

[2021, 25 July Shift-1]

Options:

A. There exists an onto function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $f \circ g = f$

B. There exists a one-one function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $f \circ g = f$

C. $g \circ g \circ g = g$

D. There exists a function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $g \circ f = f$

Answer: A

Solution:

$$g(3n + 1) = 3n + 2$$

$$g(3n + 2) = 3n + 3$$

$$g(3n + 3) = 3n + 1, \text{ for all } n \geq 0$$

$$g : \mathbb{N} \rightarrow \mathbb{N}$$

$$g(1) = 2, g(4) = 5, g(7) = 8$$

$$g(2) = 3, g(5) = 6, g(8) = 9$$

$$g(3) = 1, g(6) = 4, g(9) = 7$$

$$\Rightarrow f[g(1)] = f(1)$$

$$\Rightarrow f(2) = f(1)$$

Clearly, it is not a one - one function.

$$\text{Now, } f[g(2)] = f(2)$$

$$f(3) = f(2)$$

$$\text{And, } f[g(3)] = f(3)$$

$$f(1) = f(3)$$

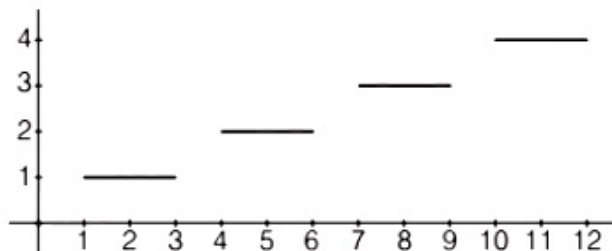
$$\text{Similarly, } f[g(4)] = f(4)$$

$$f(5) = f(4)$$

And, so on

$$\text{If } f(1) = f(2) = f(3) = 1$$

$$f(4) = f(5) = f(6) = 2$$



Now, there can be a possibility such that

So, $f(x)$ can be onto function.

$$\text{When } f(1) = f(2) = f(3) = 1$$

$$f(4) = f(5) = f(6) = 2$$

and so on.

Question108

Consider function $f : A \rightarrow B$ and $g : B \rightarrow C$ ($A, B, C \subseteq \text{eqR}$) such that $(g \circ f)^{-1}$ exists, then
[2021, 25 July Shift-II]

Options:

- A. f and g both are one-one
- B. f and g both are onto
- C. f is one-one and g is onto
- D. f is onto and g is one-one

Answer: C

Solution:

Solution:

Given functions, $f : A \rightarrow B$ and $g : B \rightarrow C$ ($A, B, C \subseteq \text{eqR}$)
 $\therefore (g \circ f)^{-1}$ exists $\Rightarrow g \circ f$ is a bijective function.
 $\Rightarrow 'f'$ must be 'one-one' and ' g ' must be 'onto' function.

Question109

Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then, the number of bijective functions $F : A \rightarrow A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to
[2021, 22 July Shift-III]

Answer: 720

Solution:

Solution:

$f(1) + f(2) = 3 - f(3)$
 $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$
 $f : A \rightarrow A$
So, $f(1) + f(2) + f(3) = 3$
 $0 + 1 + 2 = 3$ is the only possibility.
So, $f(0)$ can be either 0 or 1 or 2 .
Similarly, $f(1)$ and $f(2)$ can be 0,1 and 2 .
and $\{3, 4, 5, 6, 7\} \rightarrow \{3, 4, 5, 6, 7\}$

They have $5!$ choices.
And $\{0, 1, 2\}$

They have $3!$ choices.
Number of bijective functions
 $= 3! \times 5! = 720$

Question110

Let $f : \mathbb{R} - \left\{ \frac{\alpha}{6} \right\} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{5x+3}{6x-\alpha}$$

Then, the value of α for which $(f \circ f)(x) = x$, for all $x \in \mathbb{R} - \left\{ \frac{\alpha}{6} \right\}$ is
[2021, 20 July Shift-II]

Options:

A. No such α exists

B. 5

C. 8

D. 6

Answer: B

Solution:

Solution:

$$f(x) = \frac{5x+3}{6x-\alpha}$$

$$\text{Now, } f \circ f(x) = f\left(\frac{5x+3}{6x-\alpha}\right)$$

$$= \frac{5\left(\frac{5x+3}{6x-\alpha}\right) + 3}{6\left(\frac{5x+3}{6x-\alpha}\right) - \alpha}$$

$$= \frac{5(5x+3) + 3(6x-\alpha)}{6(5x+3) - \alpha(6x-\alpha)}$$

$$= \frac{5(5x+3) + 3(6x-\alpha)}{6(5x+3) - \alpha(6x-\alpha)}$$

Given, $f \circ f(x) = x$

$$\Rightarrow \frac{5(5x+3) + 3(6x-\alpha)}{6(5x+3) - \alpha(6x-\alpha)} = x$$

$$\Rightarrow 25x + 15 + 18x - 3\alpha$$

$$= 30x^2 + 18x - 6\alpha x^2 + \alpha^2 x$$

$$\Rightarrow x^2(30 - 6\alpha) - x(\alpha^2 - 25) + 3\alpha - 15 = 0$$

Comparing coefficients,

$$30 - 6\alpha = 0$$

$$\Rightarrow 6\alpha = 30$$

$$\Rightarrow \alpha = 5$$

Question111

Let $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which

$$f^{-1}(x) + g^{-1}(x) = \frac{13}{2} \text{ is equal to}$$

[2021, 18 March Shift-II]

Options:

- A. 7
- B. 2
- C. 5
- D. 3

Answer: C

Solution:

Solution:

$$\text{Given, } f(x) = \frac{x-2}{x-3}$$

$$g(x) = 2x - 3$$

$$\text{Let } y = f(x) = \frac{x-2}{x-3}$$

$$\Rightarrow xy - 3y = x - 2 \Rightarrow xy - x = 3y - 2$$

$$\Rightarrow x(y - 1) = 3y - 2$$

$$\Rightarrow x = \frac{3y-2}{y-1}$$

$$\Rightarrow f^{-1}(y) = \frac{3y-2}{y-1}$$

$$\Rightarrow f^{-1}(x) = \frac{3x-2}{x-1}$$

$$\text{Similarly, } g^{-1}(x) = \frac{x+3}{2}$$

$$\text{Given, } f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$$

$$\Rightarrow \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$$

$$\Rightarrow x^2 + 8x - 7 = 13(x - 1)$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow (x-2)(x-3) = 0$$

$$\Rightarrow x = 2, 3$$

$$\therefore \text{Sum} = 2 + 3 = 5$$

Question112

**The inverse of $y = 5^{\log x}$ is
[2021, 17 March Shift-I]**

Options:

A. $x = 5^{\log y}$

B. $x = y^{\log 5}$

C. $x = y^{\frac{1}{\log 5}}$

D. $x = 5^{\frac{1}{\log y}}$

Answer: C

Solution:

Solution:

$$y = 5^{\log x}$$

Taking log on both sides,

$$\Rightarrow \frac{\log y}{\log 5} = \log x \cdot \log 5$$

$$\Rightarrow \frac{1}{\log 5} = \frac{\log x}{\log y}$$

$$\frac{1}{\log 5} = \log_y x$$

$$x = y^{\frac{1}{\log 5}}$$

Question 113

Let $A = \{1, 2, 3, \dots, 10\}$ and $f : A \rightarrow A$ be defined as defined as

$$f(x) = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x & \text{if } x \text{ is even} \end{cases}.$$

Then, the number of possible functions $g : A \rightarrow A$, such that $g \circ f = f$ is [2021, 26 Feb. Shift-II]

Options:

A. 10^5

B. ${}^{10}C_5$

C. 5^5

D. $5!$

Answer: A

Solution:

Solution:

$$f(x) = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x & \text{if } x \text{ is even} \end{cases}.$$

Given, $g : A \rightarrow A$ such that,

$$g(f(x)) = f(x)$$

When x is even, then

$$g(x) = x$$

When x is odd, then

$$g(x+1) = x+1$$

This implies,

$$g(x) = x \text{ if } x \text{ is even.}$$

\Rightarrow If x is odd, then $g(x)$ can take any value in set A .

So, number of $g(x) = 10^5$

Question 114

Let $f, g : \mathbb{N} \rightarrow \mathbb{N}$, such that $f(n+1) = f(n) + f(1) \forall n \in \mathbb{N}$ and g be any

**arbitrary function. Which of the following statements is not true?
[2021, 25 Feb. Shift-1]**

Options:

- A. if $f \circ g$ is one-one, then g is one-one.
- B. if f is onto, then $f(n) = n, \forall n \in \mathbb{N}$.
- C. f is one-one.
- D. if g is onto, then $f \circ g$ is one-one.

Answer: D

Solution:

Solution:

Given, $f(n+1) = f(n) + f(1), \forall n \in \mathbb{N}$

$\Rightarrow f(n+1) - f(n) = f(1)$

It is an AP with common difference $= f(1)$

Also, general term

$11 = T_n = f(1) + (n-1)f(1) = nf(1)$

$\Rightarrow f(n) = nf(1)$

Clearly, $f(n)$ is one-one.

For $f \circ g$ to be one-one, g must be one-one.

For f to be onto, $f(n)$ should take all the values of natural numbers.

As, $f(x)$ is increasing, $f(1) = 1$

$\Rightarrow f(n) = n$

If g is many-one, then $f \circ g$ is many one.

So, if g is onto, then $f \circ g$ is one-one.

Question 115

**Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$. Then,
[2021, 25 Feb. Shift-II]**

Options:

- A. $2y = 91x$
- B. $2y = 273x$
- C. $y = 91x$
- D. $y = 273x$

Answer: A

Solution:

$x = \{ f : A \rightarrow B, f \text{ is one - one } \}$

$y = \{ g : A \rightarrow A \times B, g \text{ is one one } \}$

Number of elements in $A = 3$ i.e. $|A| = 3$
 Similarly, $|B| = 5$
 Then, $|A \times B| = |A| \times |B| = 3 \times 5 = 15$
 Now, number of one-one function from A to B will be

$${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$$

 $\therefore x = 60$
 Now, number of one-one function from A
 to $A \times B$ will be $= {}^{15}P_3 = \frac{15!}{(15-3)!} = \frac{15!}{12!}$
 $= 15 \times 14 \times 13 = 2730$
 $\therefore y = 2730$ l c $\therefore y = 2730$
 Thus, $2 \times (2730) = 91 \times (60)$
 $\Rightarrow 2y = 91x$

Question 116

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 2x - 1$ and $g : \mathbb{R} - \{1\} \rightarrow \mathbb{R}$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$. Then the composition function $f(g(x))$ is :
 24 Feb 2021 Shift 1

Options:

- A. onto but not one-one
- B. both one-one and onto
- C. one-one but not onto
- D. neither one-one nor onto

Answer: C

Solution:

Solution:

$$f(g(x)) = 2g(x) - 1 = 2\left(\frac{2x - 1}{2(x - 1)}\right) - 1 = \frac{x}{x - 1} = 1 + \frac{1}{x - 1}$$

Range of $f(g(x)) = \mathbb{R} - \{1\}$

Range of $f(g(x))$ is not onto

& $f(g(x))$ is one-one

So, $f(g(x))$ is one-one but not onto.

Question 117

Let R_1 and R_2 be two relations defined as follows :

$$R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\} \text{ and } R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\},$$

where \mathbb{Q} is the set of all rational numbers. Then :

[Sep. 03, 2020 (II)]

Options:

- A. Neither R_1 nor R_2 is transitive.

B. R_2 is transitive but R_1 is not transitive.

C. R_1 is transitive but R_2 is not transitive.

D. R_1 and R_2 are both transitive.

Answer: A

Solution:

Solution:

(a) For R_1 let $a = 1 + \sqrt{2}$, $b = 1 - \sqrt{2}$, $c = 8^{1/4}$

$$aR_1b \Rightarrow a^2 + b^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in \mathbb{Q}$$

$$bR_1c \Rightarrow b^2 + c^2 = (1 - \sqrt{2})^2 + (8^{1/4})^2 = 3 \in \mathbb{Q}$$

$$aR_1c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (8^{1/4})^2 = 3 + 4\sqrt{2} \notin \mathbb{Q}$$

$\therefore R_1$ is not transitive.

For R_2 let $a = 1 + \sqrt{2}$, $b = \sqrt{2}$, $c = 1 - \sqrt{2}$

$$aR_2b \Rightarrow a^2 + b^2 = (1 + \sqrt{2})^2 + (\sqrt{2})^2 = 5 + 2\sqrt{2} \notin \mathbb{Q}$$

$$bR_2c \Rightarrow b^2 + c^2 = (\sqrt{2})^2 + (1 - \sqrt{2})^2 = 5 - 2\sqrt{2} \notin \mathbb{Q}$$

$$aR_2c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in \mathbb{Q}$$

$\therefore R_2$ is not transitive.

Question 118

The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$.

Then a is equal to :

[Sep. 02, 2020 (I)]

Options:

A. $\frac{\sqrt{17}}{2}$

B. $\frac{\sqrt{17}-1}{2}$

C. $\frac{1+\sqrt{17}}{2}$

D. $\frac{\sqrt{17}}{2} + 1$

Answer: C

Solution:

Solution:

$$\therefore f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$$

$$\therefore -1 \leq \frac{|x|+5}{x^2+1} \leq 1$$

$$\Rightarrow |x| + 5 \leq x^2 + 1$$

$$[\because x^2 + 1 \neq 0]$$

$$\Rightarrow x^2 - |x| - 4 \geq 0$$

$$\Rightarrow \left(|x| - \frac{1 - \sqrt{17}}{2} \right) \left(|x| - \frac{1 + \sqrt{17}}{2} \right) \geq 0$$

$$\Rightarrow x \in \left(-\infty, -\frac{1 + \sqrt{17}}{2} \right) \cup \left[\frac{1 + \sqrt{17}}{2}, \infty \right)$$

$$\therefore a = \frac{1 + \sqrt{17}}{2}$$

Question119

If $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers \mathbb{Z} , then the domain of R^{-1} is :
[Sep. 02, 2020 (I)]

Options:

- A. $\{-2, -1, 1, 2\}$
- B. $\{0, 1\}$
- C. $\{-2, -1, 0, 1, 2\}$
- D. $\{-1, 0, 1\}$

Answer: D

Solution:

Solution:

Since, $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$
 $\therefore R = \{(1, 1), (2, 1), (1, -1), (0, 1), (1, 0)\}$
 $\Rightarrow D_{R^{-1}} = \{-1, 0, 1\}$

Question120

Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x , $[x]^2 + 2[x + 2] - 7 = 0$ has :
[Sep. 04, 2020 (I)]

Options:

- A. exactly two solutions
- B. exactly four integral solutions
- C. no integral solution
- D. infinitely many solutions

Answer: D

Solution:

Solution:

The given equation $[x]^2 + 2[x] + 4 - 7 = 0$
 $\Rightarrow [x]^2 + 2[x] - 3 = 0$
 $\Rightarrow [x]^2 + 3[x] - [x] - 3 = 0$
 $\Rightarrow ([x] + 3)([x] - 1) = 0 \Rightarrow [x] = 1 \text{ or } -3 \Rightarrow x \in [-3, -2) \cup [1, 2)$
 \therefore The equation has infinitely many solutions.

Question121

Let $f(x)$ be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in:
[Sep. 02, 2020 (II)]

Options:

- A. $(-1, 0)$
- B. $(1, 3)$
- C. $(-3, -1)$
- D. $(0, 1)$

Answer: A

Solution:

Solution:

Let $f(x) = ax^2 + bx + c$
 Given : $f(-1) + f(2) = 0$
 $a - b + c + 4a + 2b + c = 0$
 $\Rightarrow 5a + b + 2c = 0 \dots\dots(i)$
 and $f(3) = 0 \Rightarrow 9a + 3b + c = 0 \dots\dots(ii)$
 From equations (i) and (ii),
 $\frac{a}{1-6} = \frac{b}{18-5} = \frac{c}{15-9} \Rightarrow \frac{a}{-5} = \frac{b}{13} = \frac{c}{6}$
 Product of roots, $\alpha\beta = \frac{c}{a} = \frac{-6}{5}$ and $\alpha = 3$
 $\Rightarrow \beta = \frac{-2}{5} \in (-1, 0)$

Question122

Let $f(1, 3) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$ where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is:
[Jan. 8, 2020 (II)]

Options:

- A. $\left(\frac{2}{5}, \frac{3}{5}\right) \cup \left(\frac{3}{4}, \frac{4}{5}\right)$
- B. $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right)$
- C. $\left(\frac{2}{5}, \frac{4}{5}\right)$

D. $\left(\frac{3}{5}, \frac{4}{5}\right)$

Answer: B

Solution:

Solution:

$$f(x) = \begin{cases} \frac{x}{x^2 + 1} & x \in (1, 2) \\ \frac{2x}{x^2 + 1} & x \in [2, 3). \end{cases}$$

$$f'(x) = \begin{cases} \frac{1 - x^2}{1 + x^2} & x \in (1, 2) \\ \frac{1 - 2x^2}{1 + x^2} & x \in [2, 3). \end{cases}$$

$\therefore f(x)$ is a decreasing function

$$\therefore y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{6}{10}, \frac{4}{5}\right]$$

$$\Rightarrow y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$$

Question123

Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f : A \rightarrow B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$ is _____.
[NA Sep. 05,2020 (II)]

Answer: 19

Solution:

Solution:

The desired functions will contain either one element or two elements in its codomain of which '2' always belongs to $f(A)$.

\therefore The set B can be $\{2\}, \{1, 2\}, \{2, 3\}, \{2, 4\}$

$$\text{Total number of functions} = 1 + (2^3 - 2)3 = 19$$

Question124

The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1, 1)$, is _____.

[Jan. 8, 2020 (I)]

Options:

A. $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

B. $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

C. $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

D. $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

Answer: A

Solution:

Solution:

$$y = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$$

$$\frac{1+y}{1-y} = \frac{8^{2x}}{8^{-2x}} \Rightarrow 8^{4x} = \frac{1+y}{1-y}$$

$$\Rightarrow 4x = \log_8\left(\frac{1+y}{1-y}\right)$$

$$\Rightarrow x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$$

$$\therefore f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$$

Question125

If $g(x) = x^2 + x - 1$ and $(g \circ f)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to:
[Jan. 7, 2020 (I)]

Options:

A. $\frac{3}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $-\frac{3}{2}$

Answer: B

Solution:

Solution:

$$(g \circ f)(x) = g(f(x)) = f^2(x) + f(x) - 1$$

$$g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10 \cdot \frac{5}{4} + 5 = -\frac{5}{4}$$

$$[\because g(f(x)) = 4x^2 - 10x + 5]$$

$$g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$-\frac{5}{4} = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$$

$$\left(f\left(\frac{5}{4}\right) + \frac{1}{2}\right)^2 = 0$$

$$f\left(\frac{5}{4}\right) = -\frac{1}{2}$$

Question126

For a suitably chosen real constant a , let a function, $f : \mathbb{R} - \{-a\} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{a-x}{a+x}$. Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, $(f \circ f)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to:

[Sep. 06, 2020 (II)]

Options:

A. $\frac{1}{3}$

B. $-\frac{1}{3}$

C. -3

D. 3

Answer: D

Solution:

Solution:

$$f(f(x)) = \frac{a - \left(\frac{a-x}{a+x}\right)}{a + \left(\frac{a-x}{a+x}\right)} = x$$

$$\Rightarrow \frac{a-ax}{1+x} = f(x) \Rightarrow \frac{a(1-x)}{1+x} = \frac{a-x}{a+x} \Rightarrow a = 1$$

$$\therefore f(x) = \frac{1-x}{1+x} \Rightarrow f\left(-\frac{1}{2}\right) = 3$$

Question127

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in \mathbb{R}$. Then the range of f is :

[Jan. 11, 2019 (I)]

Options:

A. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

B. $\mathbb{R} - [-1, 1]$

C. $\mathbb{R} - \left[-\frac{1}{2}, \frac{1}{2}\right]$

D. $(-1, 1) - \{0\}$

Answer: A

Solution:

Solution:

$$f(x) = \frac{x}{1+x^2}, x \in \mathbb{R}$$

$$\text{Let, } y = \frac{x}{1+x^2}$$

$$\Rightarrow yx^2 - x + y = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2}$$

$$\Rightarrow 1 - 4y^2 \geq 0$$

$$\Rightarrow 1 \geq 4y^2$$

$$\Rightarrow |y| \leq \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

$$\Rightarrow \text{The range of } f \text{ is } \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Question 128

The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is:

[April. 09, 2019 (II)]

Options:

A. $(-1, 0) \cup (1, 2) \cup (3, \infty)$

B. $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

C. $(-1, 0) \cup (1, 2) \cup (2, \infty)$

D. $(1, 2) \cup (2, \infty)$

Answer: C

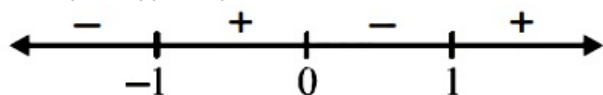
Solution:

Solution:

To determine domain, denominator $\neq 0$ and $x^3 - x > 0$

i.e., $4 - x^2 \neq 0 \Rightarrow x \neq \pm 2$ (1)

and $x(x-1)(x+1) > 0$



$x \in (-1, 0) \cup (1, \infty)$ (2)

Hence domain is intersection of (1)&(2).

i.e., $x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

Question129

If $f(x) = \log_e \left(\frac{1-x}{1+x} \right)$, $|x| < 1$, then $f \left(\frac{2x}{1+x^2} \right)$ is equal to

[April 8, 2019 (I)]

Options:

A. $2f(x)$

B. $2f(x^2)$

C. $(f(x))^2$

D. $-2f(x)$

Answer: A

Solution:

Solution:

$$f(x) = \log \left(\frac{1-x}{1+x} \right), |x| < 1$$

$$f \left(\frac{2x}{1+x^2} \right) = \log \left(\frac{1 - \frac{2x}{1+x^2}}{1 + \frac{2x}{1+x^2}} \right)$$

$$= \log \left(\frac{1+x^2-2x}{1+x^2+2x} \right) = \log \left(\frac{1-x}{1+x} \right)^2$$

$$= 2 \log \left(\frac{1-x}{1+x} \right) = 2f(x)$$

Question130

Let $f(x) = a^x (a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y) + f_1(x-y)$ equals:

[April. 08, 2019 (II)]

Options:

A. $2f_1(x)f_1(y)$

B. $2f_1(x+y)f_1(x-y)$

C. $2f_1(x)f_2(y)$

D. $2f_1(x+y)f_2(x-y)$

Answer: A

Solution:

Solution:

Given function can be written as

$$f(x) = a^x = \left(\frac{a^x + a^{-x}}{2} \right) + \left(\frac{a^x - a^{-x}}{2} \right)$$

where $f_1(x) = \frac{a^x + a^{-x}}{2}$ is even function

$f_2(x) = \frac{a^x - a^{-x}}{2}$ is odd function

$$\Rightarrow f_1(x+y) + f_1(x-y)$$

$$= \left(\frac{a^{x+y} + a^{-x-y}}{2} \right) + \left(\frac{a^{x-y} + a^{-x+y}}{2} \right)$$

$$= \frac{1}{2}[a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})]$$

$$= \frac{(a^x + a^{-x})(a^y + a^{-y})}{2} = 2f_1(x) \cdot f_1(y)$$

Question131

Let a function $f : (0, \infty) \rightarrow (0, \infty)$ be defined by $f(x) = \left| 1 - \frac{1}{x} \right|$. Then f is :
[Jan. 11, 2019 (II)]

Options:

- A. not injective but it is surjective
- B. injective only
- C. neither injective nor surjective
- D. (Bonus)

Answer: D

Solution:**Solution:**

$$f : (0, \infty) \rightarrow (0, \infty)$$

$$f(x) = \left| 1 - \frac{1}{x} \right| \text{ is not a function}$$

$\because f(1) = 0$ and $1 \in \text{domain}$ but $0 \notin \text{co-domain}$

Hence, $f(x)$ is not a function.

Question132

The number of functions f from $\{1, 2, 3, \dots, 20\}$ onto $\{1, 2, 3, \dots, 20\}$ such that $f(k)$ is a multiple of 3, whenever k is a multiple of 4 is :
[Jan. 11, 2019 (II)]

Options:

- A. $6^5 \times (15)!$
- B. $5! \times 6!$
- C. $(15)! \times 6!$

D. $5^6 \times 15$

Answer: C

Solution:

Solution:

Domain and codomain = $\{1, 2, 3, \dots, 20\}$.

There are five multiple of 4 as 4, 8, 12, 16 and 20.

and there are 6 multiple of 3 as 3, 6, 9, 12, 15, 18.

Since, when ever k is multiple of 4 then $f(k)$ is multiple of 3 then total number of arrangement = ${}^6C_5 \times 5! = 6!$

Remaining 15 elements can be arranged in $15!$ ways.

Since, for every input, there is an output

\Rightarrow function $f(k)$ is onto

\therefore Total number of arrangement = $15! \cdot 6!$

Question 133

Let N be the set of natural numbers and two functions f and g be defined

$$\text{as } f, g : N \rightarrow N \text{ such that } f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

and $g(n) = n - (-1)^n$. Then $f \circ g$ is:

[Jan. 10, 2019 (II)]

Options:

A. onto but not one-one.

B. one-one but not onto.

C. both one-one and onto.

D. neither one-one nor onto.

Answer: A

Solution:

Solution:

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

$$g(n) = \begin{cases} 2, & n = 1 \\ 1, & n = 2 \\ 4, & n = 3 \\ 3, & n = 4 \\ 6, & n = 5 \\ 5, & n = 6 \end{cases}$$

Then,

$$f(g(n)) = \begin{cases} \frac{g(n)+1}{2}, & \text{if } g(n) \text{ is odd} \\ \frac{g(n)}{2}, & \text{if } g(n) \text{ is even} \end{cases}$$

$$f(g(n)) = \begin{cases} 1, & n = 1 \\ 1, & n = 2 \\ 2, & n = 3 \\ 2, & n = 4 \\ 3, & n = 5 \\ 3, & n = 6 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \end{cases}$$

\Rightarrow fog is onto but not one - one

Question134

Let $A = \{x \in \mathbb{R} : x \text{ is not a positive integer}\}$. Define a function $f : A \rightarrow \mathbb{R}$ as $f(x) = \frac{2x}{x-1}$, then f is:
[Jan. 09, 2019 (II)]

Options:

- A. not injective
- B. neither injective nor surjective
- C. surjective but not injective
- D. injective but not surjective

Answer: D

Solution:

Solution:

As $A = \{x \in \mathbb{R} : x \text{ is not a positive integer}\}$

A function $f : A \rightarrow \mathbb{R}$ given by $f(x) = \frac{2x}{x-1}$

$$f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$$

So, f is one-one.

As $f(x) \neq 2$ for any $x \in A \Rightarrow f$ is not onto.

Hence f is injective but not surjective.

Question135

For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$

If $\phi(x) = ((h \circ f) \circ g)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to

[April 12, 2019 (I)]

Options:

A. $\tan \frac{\pi}{12}$

B. $\tan \frac{11\pi}{12}$

C. $\tan \frac{7\pi}{12}$

D. $\tan \frac{5\pi}{12}$

Answer: B

Solution:

Solution:

$$\because \phi(x) = ((\text{hof}) \circ g)(x)$$

$$\because \phi\left(\frac{\pi}{3}\right) = h\left(f\left(g\left(\frac{\pi}{3}\right)\right)\right) = h(f(\sqrt{3})) = h(3^{1/4})$$

$$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} = -\frac{1}{2}(1 + 3 - 2\sqrt{3}) = \sqrt{3} - 2 = -(-\sqrt{3} + 2)$$

$$= -\tan 15^\circ = \tan(180^\circ - 15^\circ) = \tan\left(\pi - \frac{\pi}{12}\right) = \tan \frac{11\pi}{12}$$

Question 136

Let $f(x) = x^2$, $x \in \mathbb{R}$. For any $A \subseteq \mathbb{R}$, define $g(A) = \{x \in \mathbb{R} : f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true?
[April 10, 2019 (I)]

Options:

A. $g(f(S)) \neq S$

B. $f(g(S)) = S$

C. $g(f(S)) = g(S)$

D. $f(g(S)) \neq f(S)$

Answer: C

Solution:

Solution:

$$f(x) = x^2; x \in \mathbb{R}$$

$$g(A) = \{x \in \mathbb{R} : f(x) \in A\} \quad S = [0, 4]$$

$$g(S) = \{x \in \mathbb{R} : f(x) \in S\}$$

$$= \{x \in \mathbb{R} : 0 \leq x^2 \leq 4\} = \{x \in \mathbb{R} : -2 \leq x \leq 2\}$$

$$\because g(S) \neq S \therefore f(g(S)) \neq f(S)$$

$$g(f(S)) = \{x \in \mathbb{R} : f(x) \in f(S)\}$$

$$= \{x \in \mathbb{R} : x^2 \in S^2\} = \{x \in \mathbb{R} : 0 \leq x^2 \leq 16\}$$

$$= \{x \in \mathbb{R} : -4 \leq x \leq 4\}$$

$$\therefore g(f(S)) \neq g(S)$$

$$\therefore g(f(S)) = g(S) \text{ is incorrect.}$$

Question137

For $x \in \mathbb{R} - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then $J(x)$ is equal to:

[Jan. 09, 2019 (I)]

Options:

A. $f_3(x)$

B. $\frac{1}{x}f_3(x)$

C. $f_2(x)$

D. $f_1(x)$

Answer: A

Solution:

Solution:

The given relation is

$$(f_2 \circ J \circ f_1)(x) = f_3(x) = \frac{1}{1-x}$$

$$\Rightarrow (f_2 \circ J)(f_1(x)) = \frac{1}{1-x}$$

$$\Rightarrow (f_2 \circ J)\left(\frac{1}{x}\right) = \frac{1}{1 - \frac{1}{x}} = \frac{\frac{1}{x}}{\frac{1}{x} - 1} \left[\because f_1(x) = \frac{1}{x} \right]$$

$$\Rightarrow (f_2 \circ J)(x) = \frac{x}{x-1} \left[\frac{1}{x} \text{ is replaced by } x \right]$$

$$\Rightarrow f_2(J(x)) = \frac{x}{x-1}$$

$$\Rightarrow 1 - J(x) = \frac{x}{x-1} \left[\because f_2(x) = 1 - x \right]$$

$$\therefore J(x) = 1 - \frac{x}{x-1} = \frac{1}{1-x} = f_3(x)$$

Question138

Let N denote the set of all natural numbers. Define two binary relations on N as $R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and

$R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then

[Online April 16, 2018]

Options:

A. Both R_1 and R_2 are transitive relations

B. Both R_1 and R_2 are symmetric relations

C. Range of R_2 is $\{1, 2, 3, 4\}$

D. Range of R_1 is $\{2, 4, 8\}$

Answer: C

Solution:

Solution:

Here, $R_1 = \{(x, y) \in \mathbb{N} \times \mathbb{N} : 2x + y = 10\}$ and

$R_2 = \{(x, y) \in \mathbb{N} \times \mathbb{N} : x + 2y = 10\}$

For R_1 ; $2x + y = 10$ and $x, y \in \mathbb{N}$

So, possible values for x and y are:

$x = 1, y = 8$ i.e. $(1, 8)$;

$x = 2, y = 6$ i.e. $(2, 6)$;

$x = 3, y = 4$ i.e. $(3, 4)$ and $x = 4, y = 2$ i.e. $(4, 2)$.

$R_1 = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$

Therefore, Range of R_1 is $\{2, 4, 6, 8\}$

R_1 is not symmetric

Also, R_1 is not transitive because $(3, 4), (4, 2) \in R_1$ but $(3, 2) \notin R_1$

Thus, options A, B and D are incorrect.

For R_2 ; $x + 2y = 10$ and $x, y \in \mathbb{N}$

So, possible values for x and y are: $x = 8, y = 1$ i.e. $(8, 1)$;

$x = 6, y = 2$ i.e. $(6, 2)$;

$x = 4, y = 3$ i.e. $(4, 3)$ and

$x = 2, y = 4$ i.e. $(2, 4)$

$R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$

Therefore, Range of R_2 is $\{1, 2, 3, 4\}$

R_2 is not symmetric and transitive.

Question 139

Consider the following two binary relations on the set

$A = \{a, b, c\} : R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\}$ and

$R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)\}$. Then

[Online April 15, 2018]

Options:

A. R_2 is symmetric but it is not transitive

B. Both R_1 and R_2 are transitive

C. Both R_1 and R_2 are not symmetric

D. R_1 is not symmetric but it is transitive

Answer: A

Solution:

Solution:

Both R_1 and R_2 are symmetric as

For any $(x, y) \in R_1$, we have

$(y, x) \in R_1$ and similarly for R_2

Now, for R_2 , $(b, a) \in R_2$, $(a, c) \in R_2$ but $(b, c) \notin R_2$

Similarly, for R_1 , $(b, c) \in R_1$, $(c, a) \in R_1$ but $(b, a) \notin R_1$

Therefore, neither R_1 nor R_2 is transitive.

Question140

Let $f : A \rightarrow B$ be a function defined as $f(x) = \frac{x-1}{x-2}$, where $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$. Then f is
[Online April 15, 2018]

Options:

A. invertible and $f^{-1}(y) = \frac{2y+1}{y-1}$

B. invertible and $f^{-1}(y) = \frac{3y-1}{y-1}$

C. no invertible

D. invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$

Answer: D

Solution:

Solution:

Suppose $y = f(x)$

$$\Rightarrow y = \frac{x-1}{x-2}$$

$$\Rightarrow yx - 2y = x - 1$$

$$\Rightarrow (y-1)x = 2y-1$$

$$\Rightarrow x = f^{-1}(y) = \frac{2y-1}{y-1}$$

As the function is invertible on the given domain and its inverse can be obtained as above.

Question141

The function $f : \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is:
[2017]

Options:

A. neither injective nor surjective

B. invertible

C. injective but not surjective

D. surjective but not injective

Answer: D

Solution:

Solution:

We have $f : \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$,

$$f(x) = \frac{x}{1+x^2} \quad \forall x \in \mathbb{R}$$

$$\Rightarrow f'(x) = \frac{(1+x^2) \cdot 1 - x \cdot 2x}{(1+x^2)^2} = \frac{-(x+1)(x-1)}{(1+x^2)^2}$$

sign of $f'(x)$

$\Rightarrow f'(x)$ changes sign in different intervals.

\therefore Not injective Now $y = \frac{x}{1+x^2}$

$$\Rightarrow y + yx^2 = x$$

$$\Rightarrow yx^2 - x + y = 0$$

$$\text{For } y \neq 0, D = 1 - 4y^2 \geq 0$$

$$\Rightarrow y \in \left[-\frac{1}{2}, \frac{1}{2}\right] - \{0\}$$

$$\text{For } y = 0 \Rightarrow x = 0$$

$$\therefore \text{Range is } \left[-\frac{1}{2}, \frac{1}{2}\right]$$

\Rightarrow Surjective but not injective

Question142

The function $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = x - 5 \left[\frac{x}{5} \right]$, where \mathbb{N} is set of natural numbers and $[x]$ denotes the greatest integer less than or equal to x , is:

[Online April 9, 2017]

Options:

A. one-one and onto.

B. one-one but not onto.

C. onto but not one-one.

D. neither one-one nor onto.

Answer: D

Solution:**Solution:**

$$\left. \begin{aligned} f(1) &= 1 - 5[1/5] = 1 \\ f(6) &= 6 - 5[6/5] = 1 \end{aligned} \right\} \rightarrow \text{Many one}$$

$$f(10) = 10 - 5(2) = 0 \text{ which is not in co-domain.}$$

Neither one-one nor onto.

Question143

Let $f(x) = 2^{10} \cdot x + 1$ and $g(x) = 3^{10} \cdot x - 1$. If $(f \circ g)(x) = x$, then x is equal to:

[Online April 8, 2017]

Options:

A. $\frac{3^{10} - 1}{3^{10} - 2^{-10}}$

B. $\frac{2^{10} - 1}{2^{10} - 3^{-10}}$

C. $\frac{1 - 3^{-10}}{2^{10} - 3^{-10}}$

D. $\frac{1 - 2^{-10}}{3^{10} - 2^{-10}}$

Answer: D

Solution:

Solution:

$$f(g(x)) = x$$

$$\Rightarrow f(3^{10}x - 1) = 2^{10}(3^{10} \cdot x - 1) + 1 = x$$

$$\Rightarrow 2^{10}(3^{10}x - 1) + 1 = x$$

$$\Rightarrow x(2^{10} - 1) = 2^{10} - 1$$

$$\Rightarrow x = \frac{2^{10} - 1}{2^{10} - 1} = \frac{1 - 2^{-10}}{1 - 2^{-10}}$$

Question 144

For $x \in \mathbb{R}$, $x \neq 0$, let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, $n = 0, 1, 2, \dots$

Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to :

[Online April 9, 2016]

Options:

A. $\frac{8}{3}$

B. $\frac{4}{3}$

C. $\frac{5}{3}$

D. $\frac{1}{3}$

Answer: C

Solution:

Solution:

$$f_1(x) = f_{0+1}(x) = f_0(f_0(x)) = \frac{1}{1 - \frac{1}{1-x}} = \frac{x-1}{x}$$

$$f_2(x) = f_{1+1}(x) = f_0(f_1(x)) = \frac{1}{1 - \frac{x-1}{x}} = x$$

$$f_3(x) = f_{2+1}(x) = f_0(f_2(x)) = f_0(x) = \frac{1}{1-x}$$

$$f_4(x) = f_{3+1}(x) = f_0(f_3(x)) = \frac{x-1}{x}$$

$$\therefore f_0 = f_3 = f_6 = \dots = \frac{1}{1-x}$$

$$f_1 = f_4 = f_7 = \dots = \frac{x-1}{x}$$

$$f_2 = f_5 = f_8 = \dots = x$$

$$f_{100}(3) = \frac{3-1}{3} = \frac{2}{3} f_1\left(\frac{2}{3}\right) = \frac{\frac{2}{3}-1}{\frac{2}{3}} = -\frac{1}{2}$$

$$f_2\left(\frac{3}{2}\right) = \frac{3}{2}$$

$$\therefore f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = \frac{5}{3}$$

Question145

Let $A = \{x_1, x_2, \dots, x_7\}$ and $B = \{y_1, y_2, y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions $f : A \rightarrow B$ that are onto, if there exist exactly three elements x in A such that $f(x) = y_2$, is equal to
(Online April 11, 2015)

Options:

A. $14 \cdot {}^7C_3$

B. $16 \cdot {}^7C_3$

C. $14 \cdot {}^7C_2$

D. $12 \cdot {}^7C_2$

Answer: A

Solution:

Solution:

Number of onto function such that exactly three elements in $x \in A$ such that $f(x) = \frac{1}{2}$ is equal to
 $= {}^7C_3 \cdot \{2^4 - 2\} = 14 \cdot {}^7C_3$

Question146

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{|x|-1}{|x|+1}$ then f is:
[Online April 19, 2014]

Options:

A. both one-one and onto

- B. one-one but not onto
- C. onto but not one-one
- D. neither one-one nor onto.

Answer: C

Solution:

Solution:

$$f(x) = \frac{|x| - 1}{|x| + 1}$$

for one-one function if $f(x_1) = f(x_2)$ then

x_1 must be equal to x_2

Let $f(x_1) = f(x_2)$

$$\frac{|x_1| - 1}{|x_1| + 1} = \frac{|x_2| - 1}{|x_2| + 1} \quad |x_1| |x_2| + |x_1| - |x_2| - 1 = |x_1| |x_2| - |x_1| + |x_2| - 1$$

$$\Rightarrow |x_1| - |x_2| = |x_2| - |x_1|$$

$$2|x_1| = 2|x_2|$$

$$|x_1| = |x_2|$$

$$x_1 = x_2, x_1 = -x_2$$

here x_1 has two values therefore function is many one function.

For onto : $f(x) = \frac{|x| - 1}{|x| + 1}$

for every value of $f(x)$ there is a value of x in domain set.

If $f(x)$ is negative then $x = 0$

for all positive value of $f(x)$, domain contain atleast one element. Hence $f(x)$ is onto function.

Question 147

Let P be the relation defined on the set of all real numbers such that

$P = \{(a, b) : \sec^2 a - \tan^2 b = 1\}$. Then P is:

[Online April 9, 2014]

Options:

- A. reflexive and symmetric but not transitive.
- B. reflexive and transitive but not symmetric.
- C. symmetric and transitive but not reflexive.
- D. an equivalence relation.

Answer: D

Solution:

Solution:

$$P = \{(a, b) : \sec^2 a - \tan^2 b = 1\}$$

For reflexive :

$$\sec^2 a - \tan^2 a = 1 \text{ (true } \forall a \text{)}$$

For symmetric :

$$\sec^2 b - \tan^2 a = 1$$

L.H.S

$$1 + \tan^2 b - (\sec^2 a - 1) = 1 + \tan^2 b - \sec^2 a + 1$$

$$= -(\sec^2 a - \tan^2 b) + 2$$

$$= -1 + 2 = 1$$

So, Relation is symmetric For transitive :

$$\text{if } \sec^2 a - \tan^2 b = 1 \text{ and } \sec^2 b - \tan^2 c = 1$$

$$\sec^2 a - \tan^2 c = (1 + \tan^2 b) - (\sec^2 b - 1)$$

$$= -\sec^2 b + \tan^2 b + 2$$

$$= -1 + 2 = 1$$

So, Relation is transitive.

Hence, Relation P is an equivalence relation

Question148

Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100} \right] n$, where $[n]$ denotes the greatest integer less than or equal to n . Then $\sum_{n=1}^{56} f(n)$ is equal to:

[Online April 19, 2014]

Options:

- A. 56
- B. 689
- C. 1287
- D. 1399

Answer: D

Solution:

Solution:

$$\text{Let } f(n) = \left[\frac{1}{3} + \frac{3n}{100} \right] n$$

where $[n]$ is greatest integer function,

$$= \left[0.33 + \frac{3n}{100} \right] n$$

For $n = 1, 2, \dots, 22$, we get $f(n) = 0$ and for $n = 23, 24, \dots, 55$, we get $f(n) = 1 \times n$ For $n = 56$, $f(n) = 2 \times n$

$$\text{So, } \sum_{n=1}^{56} f(n) = 1(23) + 1(24) + \dots + 1(55) + 2(56)$$

$$= (23 + 24 + \dots + 55) + 112$$

$$= \frac{33}{2} [46 + 32] + 112$$

$$= \frac{33}{2} (78) + 112 = 1399$$

Question149

Let f be an odd function defined on the set of real numbers such that for $x \geq 0$, $f(x) = 3 \sin x + 4 \cos x$ Then $f(x)$ at $x = -\frac{11\pi}{6}$ is equal to:

[Online April 11, 2014]

Options:

- A. $\frac{3}{2} + 2\sqrt{3}$

B. $-\frac{3}{2} + 2\sqrt{3}$

C. $\frac{3}{2} - 2\sqrt{3}$

D. $-\frac{3}{2} - 2\sqrt{3}$

Answer: C

Solution:

Solution:

Given f be an odd function

$$f(x) = 3 \sin x + 4 \cos x$$

$$\text{Now, } f\left(\frac{-11\pi}{6}\right) = 3 \sin\left(\frac{-11\pi}{6}\right) + 4 \cos\left(\frac{-11\pi}{6}\right)$$

$$f\left(\frac{-11\pi}{6}\right) = 3 \sin\left(-2\pi + \frac{\pi}{6}\right) + 4 \cos\left(-2\pi + \frac{\pi}{6}\right)$$

$$f\left(\frac{-11\pi}{6}\right) = 3 \sin\left\{-\left(2\pi - \frac{\pi}{6}\right)\right\} + 4 \cos\left\{-\left(2\pi - \frac{\pi}{6}\right)\right\}$$

$$\left\{ \begin{array}{l} \text{For odd functions} \\ \sin(-\theta) = -\sin \theta \\ \text{and } \cos(-\theta) = \cos \theta \end{array} \right\}$$

$$\therefore f\left(\frac{-11\pi}{6}\right) = -3 \sin\left(2\pi - \frac{\pi}{6}\right) - 4 \cos\left(2\pi - \frac{\pi}{6}\right)$$

$$\Rightarrow f\left(\frac{-11\pi}{6}\right) = +3 \sin\left(\frac{\pi}{6}\right) - 4 \cos\frac{\pi}{6}$$

$$\Rightarrow f\left(\frac{-11\pi}{6}\right) = 3 \times \frac{1}{2} - 4 \times \frac{\sqrt{3}}{2}$$

$$\text{or } f\left(\frac{-11\pi}{6}\right) = \frac{3}{2} - 2\sqrt{3}$$

Question150

If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to:
[2014]

Options:

A. $\frac{1}{1 + \{g(x)\}^5}$

B. $1 + \{g(x)\}^5$

C. $1 + x^5$

D. $5x^4$

Answer: B

Solution:

Solution:

Since f(x) and g(x) are inverse of each other

$$\therefore g'(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow g'(f(x)) = 1 + x^5 \left(\because f'(x) = \frac{1}{1+x^5} \right)$$

Here $x = g(y)$

$$\therefore g'(y) = 1 + [g(y)]^5$$

$$\Rightarrow g'(x) = 1 + (g(x))^5$$

Question151

Let $R = \{ (x, y) : x, y \in \mathbb{N} \text{ and } x^2 - 4xy + 3y^2 = 0 \}$, where \mathbb{N} is the set of all natural numbers. Then the relation R is:

[Online April 23, 2013]

Options:

- A. reflexive but neither symmetric nor transitive.
- B. symmetric and transitive.
- C. reflexive and symmetric,
- D. reflexive and transitive.

Answer: D

Solution:

Solution:

$$R = \{ (x, y) : x, y \in \mathbb{N} \text{ and } x^2 - 4xy + 3y^2 = 0 \}$$

$$\text{Now, } x^2 - 4xy + 3y^2 = 0$$

$$\Rightarrow (x - y)(x - 3y) = 0$$

$$\therefore x = y \text{ or } x = 3y$$

$$\therefore R = \{ (1, 1), (3, 1), (2, 2), (6, 2), (3, 3), (9, 3), \dots \}$$

Since $(1, 1), (2, 2), (3, 3), \dots$ are present in the relation, therefore R is reflexive.

Since $(3, 1)$ is an element of R but $(1, 3)$ is not the element of R , therefore R is not symmetric

Here $(3, 1) \in R$ and $(1, 1) \in R \Rightarrow (3, 1) \in R$ $(6, 2) \in R$ and $(2, 2) \in R \Rightarrow (6, 2) \in R$

For all such $(a, b) \in R$ and $(b, c) \in R$

$$\Rightarrow (a, c) \in R$$

Hence R is transitive.

Question152

Let $R = \{ (3, 3), (5, 5), (9, 9), (12, 12), (5, 12), (3, 9), (3, 12), (3, 5) \}$ be a relation on the set $A = \{3, 5, 9, 12\}$. Then, R is :

[Online April 22, 2013]

Options:

- A. reflexive, symmetric but not transitive.
- B. symmetric, transitive but not reflexive.
- C. an equivalence relation.
- D. reflexive, transitive but not symmetric.

Answer: D

Solution:

Solution:

Let $R = \{(3, 3), (5, 5), (9, 9), (12, 12), (5, 12), (3, 9), (3, 12), (3, 5)\}$ be a relation on set

$A = \{3, 5, 9, 12\}$

Clearly, every element of A is related to itself.

Therefore, it is a reflexive.

Now, R is not symmetry because 3 is related to 5 but 5 is not related to 3.

Also R is transitive relation because it satisfies the property that if aRb and bRc then aRc .

Question153

Let $A = \{1, 2, 3, 4\}$ and $R : A \rightarrow A$ be the relation defined by $R = \{(1, 1), (2, 3), (3, 4), (4, 2)\}$. The correct statement is:
[Online April 9, 2013]

Options:

- A. R does not have an inverse.
- B. R is not a one to one function.
- C. R is an onto function.
- D. R is not a function.

Answer: C

Solution:

Solution:

Domain = $\{1, 2, 3, 4\}$

Range = $\{1, 2, 3, 4\}$

\therefore Domain = Range

Hence the relation R is onto function.

Question154

If $P(S)$ denotes the set of all subsets of a given set S , then the number of one-to-one functions from the set $S = \{1, 2, 3\}$ to the set $P(S)$ is
[Online May 19, 2012]

Options:

- A. 24
- B. 8
- C. 336
- D. 320

Answer: C

Solution:

Solution:

Let $S = \{1, 2, 3\} \rightarrow n(S) = 3$

Now, $P(S)$ = set of all subsets of S

total no. of subsets = $2^3 = 8$

$\therefore n[P(S)] = 8$

Now, number of one-to-one functions from $S \rightarrow P(S)$ is ${}^8P_3 = \frac{8!}{5!} = 8 \times 7 \times 6 = 336$

Question 155

If $A = \{x \in \mathbb{Z}^+ : x < 10 \text{ and } x \text{ is a multiple of 3 or 4}\}$, where \mathbb{Z}^+ is the set of positive integers, then the total number of symmetric relations on A is.

[Online May 12, 2012]

Options:

- A. 2^5
- B. 2^{15}
- C. 2^{10}
- D. 2^{20}

Answer: B

Solution:

Solution:

A relation on a set A is said to be symmetric iff $(a, b) \in A \Rightarrow (b, a) \in A, \forall a, b \in A$

Here $A = \{3, 4, 6, 8, 9\}$

Number of order pairs of $A \times A = 5 \times 5 = 25$

Divide 25 order pairs of $A \times A$ in 3 parts as follows:

Part - A : $(3, 3), (4, 4), (6, 6), (8, 8), (9, 9)$

Part - B : $(3, 4), (3, 6), (3, 8), (3, 9), (4, 6), (4, 8), (4, 9), (6, 8), (6, 9), (8, 9)$

Part - C : $(4, 3), (6, 3), (8, 3), (9, 3), (6, 4), (8, 4), (9, 4), (8, 6), (9, 6), (9, 8)$

In part - A, both components of each order pair are same.

In part - B, both components are different but not two such order pairs are present in which first component of one order pair is the second component of another order pair and vice-versa.

In part - C, only reverse of the order pairs of part - B are present i.e., if (a, b) is present in part - B, then (b, a) will be present in part - C

For example $(3, 4)$ is present in part - B and $(4, 3)$ present in part - C.

Number of order pair in A, B and C are 5, 10 and 10 respectively.

In any symmetric relation on set A , if any order pair of part - B is present then its reverse order pair of part - C will must be also present.

Hence number of symmetric relation on set A is equal to the number of all relations on a set D , which contains all the order pairs of part -A and part- B.

Now $n(D) = n(A) + n(B) = 5 + 10 = 15$

Hence number of all relations on set $D = (2)^{15}$

\Rightarrow Number of symmetric relations on set $D = (2)^{15}$

Question 156

The range of the function $f(x) = \frac{x}{1+|x|}$, $x \in \mathbb{R}$, is
[Online May 7, 2012]

Options:

- A. \mathbb{R}
- B. $(-1,1)$
- C. $\mathbb{R} - \{0\}$
- D. $[-1,1]$

Answer: B

Solution:

Solution:

$$f(x) = \frac{x}{1+|x|}, x \in \mathbb{R}$$

$$\text{If } x > 0, |x| = x \Rightarrow f(x) = \frac{x}{1+x}$$

which is not defined for $x = -1$

$$\text{If } x < 0, |x| = -x \Rightarrow f(x) = \frac{x}{1-x} \text{ which is not defined for } x = 1$$

Thus $f(x)$ defined for all values of \mathbb{R} except 1 and -1

Hence, range = $(-1, 1)$

Question157

Let A and B be non empty sets in \mathbb{R} and $f : A \rightarrow B$ is a bijective function.

Statement 1: f is an onto function.

Statement 2: There exists a function $g : B \rightarrow A$ such that $f \circ g = I_B$

[Online May 26, 2012]

Options:

- A. Statement 1 is true, Statement 2 is false.
- B. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
- C. Statement 1 is false, Statement 2 is true.
- D. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1.

Answer: D

Solution:

Solution:

Let A and B be non-empty sets in \mathbb{R} .

Let $f : A \rightarrow B$ is bijective function.

Clearly statement - 1 is true which says that f is an onto function.

Statement -2 is also true statement but it is not the correct explanation for statement-1

Question158

Let R be the set of real numbers.

Statement-1: $A = \{ (x, y) \in R \times R : y - x \text{ is an integer} \}$ is an equivalence relation on R .

Statement- 2: $B = \{ (x, y) \in R \times R : x = \alpha y \text{ for some rational number } \alpha \}$ is an equivalence relation on R .

[2011]

Options:

- A. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
- B. Statement-1 is true, Statement-2 is false.
- C. Statement-1 is false, Statement-2 is true.
- D. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Answer: A

Solution:

Solution:

$\because x - x = 0 \in I (\therefore R \text{ is reflexive})$

Let $(x, y) \in R$ as $x - y$ and $y - x \in I (\therefore R \text{ is symmetric})$

Now $x - y \in I$ and $y - z \in I \Rightarrow x - z \in I$

So, R is transitive.

Hence R is equivalence.

Similarly as $x = \alpha y$ for $\alpha = 1$. B is reflexive symmetric and transitive. Hence B is equivalence.

Both relations are equivalence but not the correct explanation.

Question159

The domain of the function $f(x) = \frac{1}{\sqrt{|x|} - x}$ is

[2011]

Options:

- A. $(0, \infty)$
- B. $(-\infty, 0)$
- C. $(-\infty, \infty) - \{0\}$
- D. $(-\infty, \infty)$

Answer: B

Solution:

Solution:

$$f(x) = \frac{1}{\sqrt{|x|} - x}, f(x) \text{ is defined if } |x| - x > 0$$

$$\Rightarrow |x| > x, \Rightarrow x < 0$$

Hence domain of $f(x)$ is $(-\infty, 0)$

Question 160

Let f be a function defined by

$$f(x) = (x - 1)^2 + 1, (x \geq 1)$$

Statement -1: The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$

Statement -2: f is a bijection and $f^{-1}(x) = 1 + \sqrt{x - 1}, x \geq 1$

[2011 RS]

Options:

A. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1.

C. Statement-1 is true, Statement-2 is false.

D. Statement-1 is false, Statement-2 is true.

Answer: A

Solution:

Solution:

Given f is a bijective function

$$\therefore f : [1, \infty) \rightarrow [1, \infty)$$

$$f(x) = (x - 1)^2 + 1, x \geq 1$$

$$\text{Let } y = (x - 1)^2 + 1 \Rightarrow (x - 1)^2 = y - 1$$

$$\Rightarrow x = 1 \pm \sqrt{y - 1} \Rightarrow f^{-1}(y) = 1 \pm \sqrt{y - 1}$$

$$\Rightarrow f^{-1}(x) = 1 + \sqrt{x - 1} \{ \because x \geq 1 \}$$

Hence statement- 2 is correct

$$\text{Now } f(x) = f^{-1}(x)$$

$$\Rightarrow f(x) = x \Rightarrow (x - 1)^2 + 1 = x$$

$$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$$

Hence statement- 1 is correct

Question 161

Consider the following relations:

$R = \{ (x, y) \mid x, y \text{ are real numbers and } x = wy \text{ for some rational number } w \};$

$S = \left\{ \left(\frac{m}{n}, \frac{p}{q} \right) \mid m, n, p, \text{ and } q \text{ are integers such that } n, q \neq 0 \text{ and } \right.$

$qm = pn \}$

. Then

[2010]

Options:

- A. Neither R nor S is an equivalence relation
- B. S is an equivalence relation but R is not an equivalence relation
- C. R and S both are equivalence relations
- D. R is an equivalence relation but S is not an equivalence relation

Answer: B

Solution:

Solution:

Let xRy .

$$\Rightarrow x = wy \Rightarrow y = \frac{x}{w}$$

$$\Rightarrow (y, x) \notin R$$

R is not symmetric

$$\text{Let } S : \frac{m}{n} S \frac{p}{q}$$

$$\Rightarrow qm = pn \Rightarrow \frac{p}{q} = \frac{m}{n}$$

$$\therefore \frac{m}{n} = \frac{m}{n} \therefore \text{reflexive}$$

$$\frac{m}{n} = \frac{p}{q} \Rightarrow \frac{p}{q} = \frac{m}{n} \therefore \text{symmetric}$$

$$\text{Let } \frac{m}{n} S \frac{p}{q}, \frac{p}{q} S \frac{r}{s}$$

$$\Rightarrow qm = pn, ps = rq$$

$$\Rightarrow \frac{p}{q} = \frac{m}{n} = \frac{r}{s}$$

$$\Rightarrow ms = rn \text{ transitive}$$

.S is an equivalence relation.

Question 162

Let $f(x) = (x + 1)^2 - 1, x \geq -1$

Statement -1: The set $\{x : f(x) = f^{-1}(x) = \{0, -1\}\}$.

Statement- 2: f is a bijection.

[2009]

Options:

- A. Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
- B. Statement-1 is true, Statement-2 is false.
- C. Statement-1 is false, Statement-2 is true.
- D. Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

Answer: D

Solution:

Solution:

Given that $f(x) = (x + 1)^2 - 1, x \geq -1$

Clearly $D_f = [-1, \infty)$ but co-domain is not given. Therefore $f(x)$ is onto.

Let $f(x_1) = f(x_2)$

$$\Rightarrow (x_1 + 1)^2 - 1 = (x_2 + 1)^2 - 1$$

$$\Rightarrow x_1 = x_2$$

$\therefore f(x)$ is one-one, hence $f(x)$ is bijection

$\because (x + 1)$ being something +ve, $\forall x > -1$

$\therefore f^{-1}(x)$ will exist. Let $(x + 1)^2 - 1 = y$

$$\Rightarrow x + 1 = \sqrt{y + 1} \text{ (+ve square root as } x + 1 \geq 0 \text{)}$$

$$\Rightarrow x = -1 + \sqrt{y + 1}$$

$$\Rightarrow f^{-1}(x) = \sqrt{x + 1} - 1$$

Then $f(x) = f^{-1}(x)$

$$\Rightarrow (x + 1)^2 - 1 = \sqrt{x + 1} - 1$$

$$\Rightarrow (x + 1)^2 = \sqrt{x + 1} \Rightarrow (x + 1)^4 = (x + 1)$$

$$\Rightarrow (x + 1)[(x + 1)^3 - 1] = 0 \Rightarrow x = -1, 0$$

\therefore The statement- 1 and statement- 2 both are true.

Question163

Let R be the real line. Consider the following subsets of the plane $R \times R$:

$$S = \{ (x, y) : y = x + 1 \text{ and } 0 < x < 2 \}$$

$$T = \{ (x, y) : x - y \text{ is an integer} \}$$

Which one of the following is true?

[2008]

Options:

- A. Neither S nor T is an equivalence relation on R
- B. Both S and T are equivalence relation on R
- C. S is an equivalence relation on R but T is not
- D. T is an equivalence relation on R but S is not

Answer: D

Solution:

Solution:

Given that

$$S = \{ (x, y) : y = x + 1 \text{ and } 0 < x < 2 \}$$

$$\because x \neq x + 1 \text{ for any } x \in (0, 2)$$

$$\Rightarrow (x, x) \notin S$$

So, S is not reflexive.

Hence, S is not an equivalence relation.

$$\text{Given } T = \{ (x, y) : x - y \text{ is an integer} \}$$

$$\because x - x = 0 \text{ is an integer, } \forall x \in R$$

So, T is reflexive.

$$\text{Let } (x, y) \in T \Rightarrow x - y \text{ is an integer then } y - x \text{ is also an integer } \Rightarrow (y, x) \in T$$

$\therefore T$ is symmetric

$$\text{If } x - y \text{ is an integer and } y - z \text{ is an integer then } (x - y) + (y - z) = x - z \text{ is also an integer.}$$

$\therefore T$ is transitive

Hence T is an equivalence relation.

Question164

Let $f : \mathbb{N} \rightarrow Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in \mathbb{N} : y = 4x + 3 \text{ for some } x \in \mathbb{N}\}$. Show that f is invertible and its inverse is [2008]

Options:

A. $g(y) = \frac{3y + 4}{3}$

B. $g(y) = 4 + \frac{y + 3}{4}$

C. $g(y) = \frac{y + 3}{4}$

D. $g(y) = \frac{y - 3}{4}$

Answer: D

Solution:

Solution:

Clearly $f(x) = 4x + 3$ is one one and onto, so it is invertible.

Let $f(x) = 4x + 3 = y$

$$\Rightarrow x = \frac{y - 3}{4}$$

$$\therefore g(y) = \frac{y - 3}{4}$$

Question 165

Let W denote the words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W \mid \text{the words } x \text{ and } y \text{ have at least one letter in common}\}$. Then R is [2006]

Options:

A. not reflexive, symmetric and transitive

B. reflexive, symmetric and not transitive

C. reflexive, symmetric and transitive

D. reflexive, not symmetric and transitive

Answer: B

Solution:

Solution:

Clearly $(x, x) \in R, \forall x \in W$

\therefore All letters are common in some word. So R is reflexive.

Let $(x, y) \in R$, then $(y, x) \in R$ as x and y have at least one letter in common. So, R is symmetric.

But R is not transitive for example

Let $x = \text{BOY}$, $y = \text{TOY}$ and $z = \text{THREE}$

then $(x, y) \in R$ (O, Y are common) and $(y, z) \in R$ (T is common) but $(x, z) \notin R$. (as no letter is common)

Question 166

A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where a is a given constant and $f(0) = 0$, $f(2a - x)$ is equal to [2005]

Options:

- A. $-f(x)$
- B. $f(x)$
- C. $f(a) + f(a - x)$
- D. $f(-x)$

Answer: A

Solution:

Solution:

Given that $f(0) = 0$ and put

$x = 0, y = 0$

$$f(0) = f^2(0) - f^2(a)$$

$$\Rightarrow f^2(a) = 0 \Rightarrow f(a) = 0$$

$$f(2a - x) = f(a - (x - a))$$

$$= f(a)f(x - a) - f(0)f(x)$$

$$= f(a)f(x - a) - f(x) = -f(x)$$

$$\Rightarrow f(2a - x) = -f(x)$$

Question 167

Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is [2005]

Options:

- A. reflexive and transitive only
- B. reflexive only
- C. an equivalence relation
- D. reflexive and symmetric only

Answer: A

Solution:

Solution:

R is reflexive and transitive only.

Here $(3, 3), (6, 6), (9, 9), (12, 12) \in R$ [So, reflexive]

$(3, 6), (6, 12), (3, 12) \in R$ [So, transitive]

$\therefore (3, 6) \in R$ but $(6, 3) \notin R$ [So, non-symmetric]

Question 168

Let $f : (-1, 1) \rightarrow B$, be a function defined by $f(x) = \tan^{-1} \frac{2x}{1-x^2}$, then f is both one – one and onto when B is the interval [2005]

Options:

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left[0, \frac{\pi}{2}\right)$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Answer: D

Solution:**Solution:**

Given $f(x) = \tan^{-1} \left(\frac{2x}{1-x^2} \right) = 2\tan^{-1}x$

for $x \in (-1, 1)$

If $x \in (-1, 1) \Rightarrow \tan^{-1}x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\Rightarrow 2\tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Clearly, range of $f(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

For f to be onto, codomain = range

\therefore Co-domain of function = $B = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Question 169

The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then [2004]

Options:

A. $f(x) = -f(-x)$

B. $f(2+x) = f(2-x)$

C. $f(x) = f(-x)$

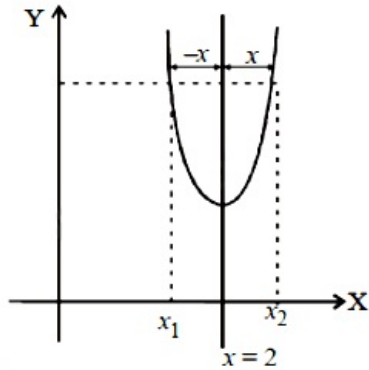
D. $f(x+2) = f(x-2)$

Answer: B

Solution:

Solution:

(b) Given that a graph symmetrical. with respect to line $x = 2$ as shown in the figure.



From the figure

$$f(x_1) = f(x_2), \text{ where } x_1 = 2 - x \text{ and } x_2 = 2 + x$$

$$\therefore f(2 - x) = f(2 + x)$$

Question170

Let $R = \{(1,3), (4, 2), (2, 4), (2,3), (3,1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is
[2004]

Options:

A. reflexive

B. transitive

C. not symmetric

D. a function

Answer: C

Solution:

Solution:

$$\because (1, 1) \notin R \Rightarrow R \text{ is not reflexive}$$

$$\because (2, 3) \in R \text{ but } (3, 2) \notin R$$

$$\therefore R \text{ is not symmetric}$$

$$\because (4, 2) \text{ and } (2, 4) \in R \text{ but } (4, 4) \notin R$$

$$\Rightarrow R \text{ is not transitive}$$

Question171

If $f : \mathbb{R} \rightarrow \mathbb{S}$, defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$, is onto, then the interval of \mathbb{S} is [2004]

Options:

- A. $[-1, 3]$
- B. $[-1, 1]$
- C. $[0, 1]$
- D. $[0, 3]$

Answer: A

Solution:

Solution:

Given that $f(x)$ is onto
 $\therefore \text{range of } f(x) = \text{codomain} = \mathbb{S}$

Now, $f(x) = \sin x - \sqrt{3} \cos x + 1$

$$= 2 \sin \left(x - \frac{\pi}{3} \right) + 1$$

we know that $-1 \leq \sin \left(x - \frac{\pi}{3} \right) \leq 1$

$$-1 \leq 2 \sin \left(x - \frac{\pi}{3} \right) + 1 \leq 3 \therefore f(x) \in [-1, 3] = \mathbb{S}$$

Question 172

Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is [2003]

Options:

- A. $(-1, 0) \cup (1, 2) \cup (2, \infty)$
- B. $(a, 2)$
- C. $(-1, 0) \cup (a, 2)$
- D. $(1, 2) \cup (2, \infty)$

Answer: A

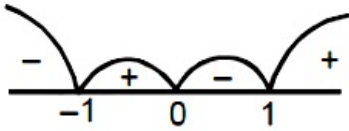
Solution:

Solution:

$$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$$

$$4 - x^2 \neq 0; x^3 - x > 0$$

$$x \neq \pm\sqrt{4} \text{ and } -1 < x < 0 \text{ or } 1 < x < \infty$$



$$\therefore D = (-1, 0) \cup (1, \infty) - \{\sqrt{4}\}$$

$$D = (-1, 0) \cup (1, 2) \cup (2, \infty)$$

Question173

If $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$ and $f(1) = 7$, then $\sum_{r=1}^n f(r)$ is [2003]

Options:

- A. $\frac{7n(n+1)}{2}$
- B. $\frac{7n}{2}$
- C. $\frac{7(n+1)}{2}$
- D. $7n + (n+1)$

Answer: A

Solution:

Solution:

$$f(x + y) = f(x) + f(y)$$

$$\therefore f(1) = 7$$

$$f(2) = f(1 + 1) = f(1) + f(1) = 14$$

$$f(3) = f(1 + 2) = f(1) + f(2) = 21$$

$$\begin{aligned} & \text{-----} \\ & \text{-----} \\ \therefore \sum_{r=1}^n f(r) &= 7(1 + 2 + 3 + \dots + n) \\ &= \frac{7n(n+1)}{2} \end{aligned}$$

Question174

A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases} \text{ is}$$

[2003]

Options:

- A. neither one -one nor onto

- B. one-one but not onto
- C. onto but not one-one
- D. one-one and onto both.

Answer: D

Solution:

Solution:

We have $f : \mathbb{N} \rightarrow \mathbb{I}$

Let x and y are two even natural numbers, and $f(x) = f(y) \Rightarrow \frac{-x}{2} = \frac{-y}{2} \Rightarrow x = y$

$\therefore f(n)$ is one-one for even natural number.

Let x and y are two odd natural numbers and $f(x) = f(y) \Rightarrow \frac{x-1}{2} = \frac{y-1}{2} \Rightarrow x = y$

$\therefore f(n)$ is one-one for odd natural number.

Hence f is one-one.

Let $y = \frac{n-1}{2} \Rightarrow 2y + 1 = n$

This shows that n is always odd number for $y \in \mathbb{I}$ (i)

and $y = \frac{-n}{2} \Rightarrow -2y = n$

This shows that n is always even number for $y \in \mathbb{I}$ (ii)

From (i) and (ii)

Range of $f = \mathbb{I} = \text{codomain}$

$\therefore f$ is onto.

Hence f is one one and onto both.
