Sample Question Paper - 41
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Solve for x : $x^2 - (2b - 1)x + (b^2 - b - 20) = 0$

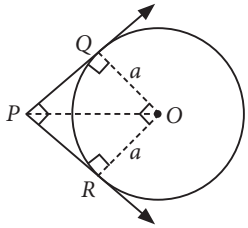
2. The angles of a quadrilateral are in A.P. whose common difference is 10°. Find the angles.

OR

The sum of the 2nd and the 7th term of an A.P. is 30. If its 15th term is 1 less than twice its 8th term, then find the A.P.

3. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)\, x + (c^2 + d^2) = 0$ has no real roots.

4. If angle between two tangents drawn from a point P to a circle of radius 'a' and centre O is 90°, then find the length of OP.



5. The length of a cold storage is double its breadth. Its height is 3 metres. The areas of its four walls (including doors) is 108 m^2. Find its volume.

OR

The radii of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recut into a solid right circular cylinder of height $10\frac{2}{3}$ cm. Find the diameter of the base of the cylinder.

6. If the mode of the given data is 340, find the missing frequency x for the following data :

Classes	0-100	100-200	200-300	300-400	400-500	500-600
Frequency	8	12	x	20	14	7

SECTION - B

7. The following table gives the literacy rate (in %) in 40 cities. Find the mean literacy rate.

Literacy rate (in %)	45-55	55-65	65-75	75-85	85-95
Number of cities	4	11	12	9	4

8. The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water.

OR

From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30°. A flagstaff is fixed at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (Take $\sqrt{3} = 1.73$)

9. In an apple orchard, the number of apples on 80 trees are as follows :

Number of apples	40-60	60-80	80-100	100-120	120-140	140-160	160-180
Number of trees	12	11	14	16	13	9	5

Find the median of the above data.

10. Construct a right triangle ABC with $AB = 6$ cm, $BC = 8$ cm and $\angle B = 90°$. Draw BD, the perpendicular from B on AC. Draw the circle through B, C and D and construct the tangents from A to this circle.

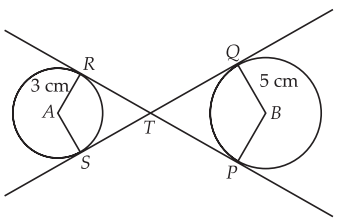
SECTION - C

11. A conical vessel of radius 12 cm and height 16 cm is completely filled with water. A sphere is lowered into the water and its size is such that, when it touches the sides, it is just immersed. What fraction of the water overflows?

12. AB and CD are two parallel chords of a circle such that $AB = 10$ cm and $CD = 24$ cm. The chords are on opposite sides of the centre and the distance between them is 17 cm. Find the radius of the circle.

OR

In the fig, RTP and STQ are common tangents to the two circles with centres A and B. The radii of the two circles are 3 cm and 5 cm respectively. If $ST : TQ = 1 : 3$ and $RT = 4$ cm. Find the length of QT and AB.



Case Study - 1

13. Amit is preparing for his upcoming semester exam. For this, he has to practice the chapter of Quadratic Equations. So he started with factorization method. Let two linear factors of $ax^2 + bx + c$ be $(px + q)$ and $(rx + s)$.

$\therefore \quad ax^2 + bx + c = (px + q)(rx + s) = prx^2 + (ps + qr)x + qs.$

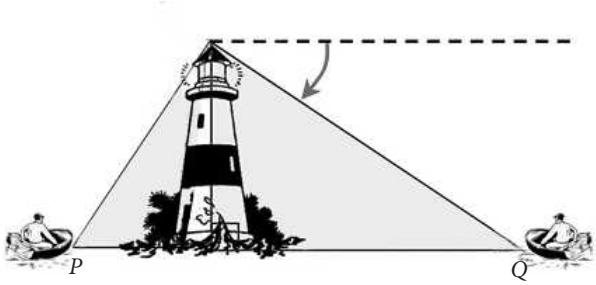
Now, factorize each of the following quadratic equations and find the roots.

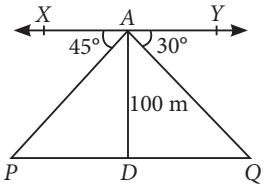
(i) $6x^2 + x - 2 = 0$

(ii) $x^2 - 28x + 160 = 0$

Case Study - 2

14. A boy is standing on the top of light house. He observed that boat P and boat Q are approaching to light house from opposite directions. He finds that angle of depression of boat P is 45° and angle of depression of boat Q is 30°. He also knows that height of the light house is 100 m.





Based on the above information, answer the following questions.

(i) Find the length of PD.

(ii) Find the length of DQ.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. We have, $x^2 - (2b - 1)x + (b^2 - b - 20) = 0$
Discriminant, $D = (2b - 1)^2 - 4(1)(b^2 - b - 20)$
$= 4b^2 + 1 - 4b - 4b^2 + 4b + 80 = 81$
Using quadratic formula,

$$x = \frac{(2b-1) \pm \sqrt{81}}{2(1)} = \frac{2b-1 \pm 9}{2}$$

$$\Rightarrow \quad x = \frac{(2b-1)+9}{2} \text{ or } x = \frac{(2b-1)-9}{2}$$

$$\Rightarrow \quad x = \frac{2b+8}{2} = b+4 \quad \text{or} \quad x = \frac{2b-10}{2} = b-5$$

2. Let the four angles of a quadrilateral are n, $(n + 10°)$, $(n + 20°)$ and $(n + 30°)$.
$\because$ Sum of all the angles of a quadrilateral = 360°
$\Rightarrow n + (n + 10°) + (n + 20°) + (n + 30°) = 360°$
$\Rightarrow 4n + 60° = 360° \Rightarrow n = 300°/4 = 75°$
$\therefore$ Angles are 75°, 85°, 95° and 105°.

OR

Let a be the first term and d be the common difference of the A.P. Now, according to the question, $a_2 + a_7 = 30$
$\Rightarrow a + d + a + 6d = 30 \Rightarrow 2a + 7d = 30$ ...(i)
Given, $a_{15} = 2a_8 - 1$
$\Rightarrow a + 14d = 2(a + 7d) - 1 \Rightarrow a + 14d = 2a + 14d - 1$
$\Rightarrow a = 1$ ...(ii)
Substituting (ii) in (i), we get
$2 + 7d = 30 \Rightarrow 7d = 28 \Rightarrow d = 4$
Hence, the A.P. is formed as 1, 5, 9, ...

3. We have, $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$
Discriminant, $D = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$
$= 4(a^2c^2 + b^2d^2 + 2acbd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$
$= 4(a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2)$
$= 4(2abcd - a^2d^2 - b^2c^2) = -4(ad - bc)^2 < 0 \quad [\because ad \neq bc]$
Thus, given equation has no real roots.

4. In given figure, PQ and PR be the tangents.
Since $\angle P = 90°$
$\therefore$ Using angle sum property in quad $OQPR$,
$\angle QOR = 90°$
Also, $OR = OQ = a$
$\therefore$ $PQOR$ is a square
$\Rightarrow OP = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$

5. Let the length, breadth and height of the cold storage be l, b and h metres respectively. Then $l = 2b$ (given) and $h = 3$ m.
Now, area of the four walls = 108 m^2
$\Rightarrow 2(l + b)h = 108$
$\Rightarrow 2(2b + b) \times 3 = 108$
$\Rightarrow 18b = 108 \Rightarrow b = 6$
$\Rightarrow l = 2 \times 6 = 12$ m
Hence, volume of the cold storage
$= l \times b \times h = 12 \times 6 \times 3 = 216$ m^3.

OR

Let the radius of the base of the cylinder be r cm. Then,
Volume of the metallic solid cylinder of height $10\frac{2}{3}$ cm = Volume of the metal in the spherical shell

$$\Rightarrow \quad \pi \times r^2 \times \frac{32}{3} = \frac{4}{3}\pi(5^3 - 3^3)$$

$$\Rightarrow \quad \frac{32}{3}r^2 = \frac{4}{3}(125 - 27) \quad \Rightarrow \quad r^2 = \frac{3}{32} \times \frac{4}{3} \times 98$$

$$\Rightarrow \quad r^2 = \frac{49}{4} \Rightarrow r = \frac{7}{2} \text{ cm}$$

Hence, diameter of the base of the cylinder = 7 cm

6. Here, mode = 340 which lies in the interval 300-400.
$\therefore$ Modal class = 300-400
Now, Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$\Rightarrow \quad 340 = 300 + \left(\frac{20 - x}{2 \times 20 - x - 14}\right) \times 100$$

$$\Rightarrow \quad 340 - 300 = \left(\frac{20 - x}{26 - x}\right) \times 100$$

$\Rightarrow 6x = 96 \Rightarrow x = 16$

7. The frequency distribution table from the given data can be drawn as :

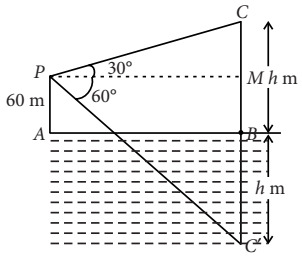
Literacy rate	Class-marks (x_i)	Frequency (f_i)	$u_i = \frac{x_i - A}{h}$	f_iu_i
45-55	50	4	–2	–8
55-65	60	11	–1	–11
65-75	70	12	0	0
75-85	80	9	1	9
85-95	90	4	2	8
Total		$\Sigma f_i = 40$		$\Sigma f_iu_i = -2$

Let assumed mean (A) = 70

$$\therefore \quad \text{Mean } (\bar{X}) = A + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 70 + \left(\frac{-2}{40}\right) \times 10 = 70 - \frac{20}{40} = 69.5$$

8. Let AB be the surface of the lake and C be the position of cloud and C' be its reflection or shadow in the lake. Also, let height of cloud is h m.



Here, $PM = AB$ and $BM = AP = 60$ m

In ΔPCM, $\tan 30° = \frac{CM}{PM} = \frac{BC - BM}{AB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h - 60}{AB}$

$\Rightarrow \quad AB = (h - 60)\sqrt{3}$ ...(i)

In $\Delta PMC'$, $\tan 60° = \frac{C'M}{PM} = \frac{BC' + BM}{AB}$

$\Rightarrow \quad \sqrt{3} = \frac{h + 60}{AB} \Rightarrow \quad AB = \frac{h + 60}{\sqrt{3}}$ ...(ii)

From (i) and (ii), we have

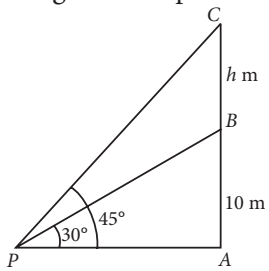
$(h - 60)\sqrt{3} = \frac{h + 60}{\sqrt{3}} \Rightarrow 3h - 180 = h + 60$

$\Rightarrow 2h = 240 \Rightarrow h = 120$

Thus, height of the cloud from the surface of water is 120 m.

OR

Let AB be the building and BC be the flagstaff of height h m. AP is the distance of the building from the point P.



In right ΔPAB,

$\tan 30° = \frac{AB}{AP}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{AP}$

$\Rightarrow \quad AP = 10\sqrt{3}$ m

$\Rightarrow AP = 10 \times 1.73$ m $= 17.3$ m ...(i)

So, the distance of the building from the point P is 17.3 m.

In right ΔPAC, $\tan 45° = \frac{AC}{AP}$

$\Rightarrow \quad 1 = \frac{10 + h}{17.3}$ (Using (i))

$\Rightarrow 10 + h = 17.3$

$\Rightarrow h = 17.3 - 10 \Rightarrow h = 7.3$

Thus, the length of the flagstaff is 7.3 m.

9. The frequency distribution table from the given data can be drawn as :

Class	**Frequency (f_i)**	**Cumulative frequency (*c.f.*)**
40-60	12	12
60-80	11	23
80-100	14	37
100-120	16	53
120-140	13	66
140-160	9	75
160-180	5	80
Total	80	

Clearly, $\frac{N}{2} = \frac{80}{2} = 40$ lies in the class interval 100-120.

So, 100-120 is the median class.

$\therefore \quad l = 100$, c.f. $= 37$, $f = 16$, $h = 20$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$

$$= 100 + \left(\frac{40 - 37}{16}\right) \times 20$$

$$= 100 + \frac{60}{16} = 100 + 3.75 = 103.75$$

$\therefore$ Median = 103.75

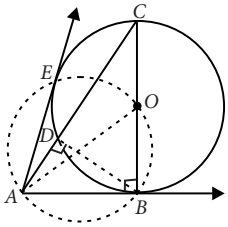
10. Steps of construction :

Step-I : Draw ΔABC and perpendicular BD from B on AC.

Step-II : Draw a circle with BC as diameter. This circle will pass through D.

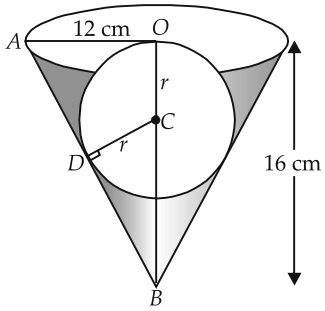
Step-III : Let O be the mid-point of BC. Join AO.

Step-IV : Draw a circle with AO as diameter. This circle cuts the circle drawn in step 2 at B and E.



Step-V : Join AE. AE and AB are desired tangents drawn from A to the circle passing through B, C and D.

11. Radius of conical vessel = R = 12 cm



Height of the conical vessel = h = 16 cm

Let the radius of sphere = r

Now, $\Delta BOA \sim \Delta BDC$,

$$\Rightarrow \quad \frac{AO}{CD} = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{12}{r} = \frac{20}{16-r} \quad \left[\text{In } \Delta AOB, AB = \sqrt{16^2 + 12^2} = 20\right]$$

$$\Rightarrow \quad 12 \times (16 - r) = 20r \Rightarrow 32r = 16 \times 12$$

$$\Rightarrow \quad r = \frac{16 \times 12}{32} \text{cm} = 6 \text{ cm}$$

Volume of water that overflows = Volume of the sphere

$$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7}(6)^3 = \frac{6336}{7} \text{cm}^3$$

Volume of water in conical vessel $= \frac{1}{3}\pi R^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (12)^2 \times 16 = \frac{16896}{7}$$

Fraction of water that over flows

$$= \frac{\text{Volume of water overflows}}{\text{Volume of water in conical vessel}}$$

$$= \frac{6336}{7} \times \frac{7}{16896} = \frac{3}{8}$$

12. Given, AB = 10 cm, CD = 24 cm

EF (distance between two chords) = 17 cm

Let $OE = x$, then $OF = EF - OE = 17 - x$

and $OC = OA = r$ [Radii of the circle]

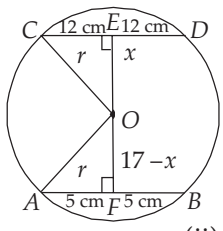
In ΔOCE, right angled at E

$$OC^2 = CE^2 + OE^2$$

$$\Rightarrow \quad r^2 = 12^2 + x^2$$

$$\Rightarrow \quad r^2 = 144 + x^2 \qquad \ldots(i)$$

In ΔOAF, right angled at F



$$OA^2 = AF^2 + OF^2$$

$$\Rightarrow \quad r^2 = 5^2 + (17 - x)^2$$

$$\Rightarrow \quad r^2 = 25 + 289 - 34x + x^2 \qquad \ldots(ii)$$

From (i) and (ii), we get

$$144 + x^2 = 25 + 289 - 34x + x^2$$

$$\Rightarrow \quad 34x = 170 \Rightarrow x = 5 \text{ cm}$$

Now, from eq (i), we have

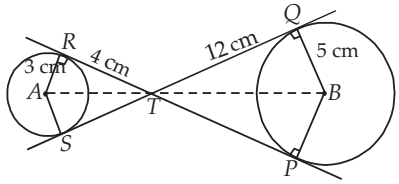
$$r^2 = 144 + 25 = 169$$

$$\Rightarrow \quad r = 13 \text{ cm}$$

Hence radius of the circle is 13 cm.

OR

We have, AR = 3 cm, BQ = 5 cm, RT = 4 cm and $ST : TQ = 1 : 3$



TR and TS are two tangents drawn from an external point T to the circle with centre A

$$\therefore \quad TR = TS \text{ and } \frac{ST}{TQ} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{TR}{TQ} = \frac{1}{3} \Rightarrow \frac{4}{TQ} = \frac{1}{3} \qquad [\because TR = 4 \text{ cm}]$$

$$\Rightarrow \quad TQ = 12 \text{ cm}$$

Now, in ΔART, $\angle R = 90°$

$$\therefore \quad AT^2 = AR^2 + RT^2$$

$$\Rightarrow \quad AT^2 = (3)^2 + (4)^2 = 9 + 16 = 25 \Rightarrow AT = 5 \text{ cm}$$

and in ΔBQT, $\angle Q = 90°$

$$\therefore \quad BT^2 = BQ^2 + TQ^2$$

$$\Rightarrow \quad BT^2 = (5)^2 + (12)^2 = 25 + 144 = 169$$

$$\Rightarrow \quad BT = 13 \text{ cm}$$

Now $AB = AT + BT$

$$\Rightarrow \quad AB = 5 + 13 = 18 \text{ cm}$$

Hence, QT = 12 cm and AB = 18 cm

13. (i) We have, $6x^2 + x - 2 = 0$

$$\Rightarrow \quad 6x^2 - 3x + 4x - 2 = 0 \Rightarrow (3x + 2)(2x - 1) = 0$$

$$\Rightarrow \quad x = \frac{1}{2}, \frac{-2}{3}$$

(ii) $x^2 - 28x - 160 = 0$

$$\Rightarrow \quad x^2 - 20x - 8x + 160 = 0$$

$$\Rightarrow \quad x(x - 20) - 8(x - 20) = 0$$

$$\Rightarrow \quad (x - 20)(x - 8) = 0$$

$$\therefore \quad x = 20 \text{ or } 8.$$

14. (i) $\angle XAP = 45°$ (Given)

$$\therefore \quad \angle APD = 45° \qquad \text{[Alternate interior angles]}$$

In ΔAPD, $\frac{AD}{DP} = \tan 45°$

$$\Rightarrow \quad \frac{100}{DP} = 1 \Rightarrow DP = 100 \text{ m}$$

(ii) In ΔAQD, $\frac{AD}{QD} = \tan 30°$

$$\Rightarrow \quad \frac{100}{QD} = \frac{1}{\sqrt{3}} \Rightarrow QD = 100\sqrt{3} \text{ m}$$