Long Answer Questions

Q. 1. (i) Derive the mirror formula. What is the corresponding formula for a thin lens?

(ii) Draw a ray diagram to show the image formation by a concave mirror when the object is kept between its focus and the pole. Using this diagram, derive the magnification formula for the image formed. [CBSE Delhi 2011]

Ans. (i) Mirror Formula: M_1M_2 is a concave mirror having pole P, focus F and centre of curvature C.

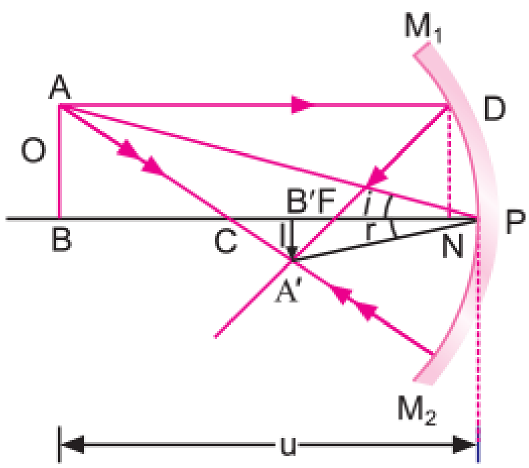
An object AB is placed in front of mirror with point B on the principal axis. The image formed by mirror is $A'B'$. The perpendicular dropped from point of incidence D on principal axis is DN

In ΔABC and $\Delta A'B'C$

$\angle ABC = \angle A'B'C$ (each equal to 90°)

$\angle ACB = \angle A'CB'$ (opposite angles)

Both triangles are similar.



$\therefore \frac{AB}{A'B'} = \frac{BC}{BC'}$ ...(*i*)

Now in ΔDNF and $A'\ B'\ F$

$\angle DNF = \angle A'B'F$ (each equal to 90°)

$\angle DFN = \angle A'FB'$ (opposite angles)

$\therefore$ Both triangles are similar

$\frac{DN}{A'B'} = \frac{FN}{BF'}$ or $\frac{AB}{A'B'} = \frac{FN}{BF'}$ $(\because AB = DN)$

...(*ii*)

Comparing (*i*) and (*ii*), we get

$\frac{BC}{BC'} = \frac{FN}{BF'}$ ...(*iii*)

If aperture of mirror is very small, the point N will be very near to P, so $FN = FP$

$\therefore \frac{BC}{BC'} = \frac{FP}{B'F}$ or $\frac{PB - PC}{PC - PB'} = \frac{FB}{PB' - PF}$ ...(*iv*)

By sign convention

Distance of object from mirror $PB = -u$

Distance of image from mirror $PB' = -v$

Focal length of mirror $PF = -f$

Radius of curvature of mirror $PC = -R = -2f$

Substituting these values in (*iv*), we get

$$\frac{-u - (-2f)}{-2f - (-v)} = \frac{-f}{-v - (-f)}$$

$$\frac{-u + 2f}{-2f + v} = \frac{-f}{-v + f}$$

$\Rightarrow 2f^2 - vf = -uf + uv + 2f^2 - 2fv$ or $fv + uf = uv$

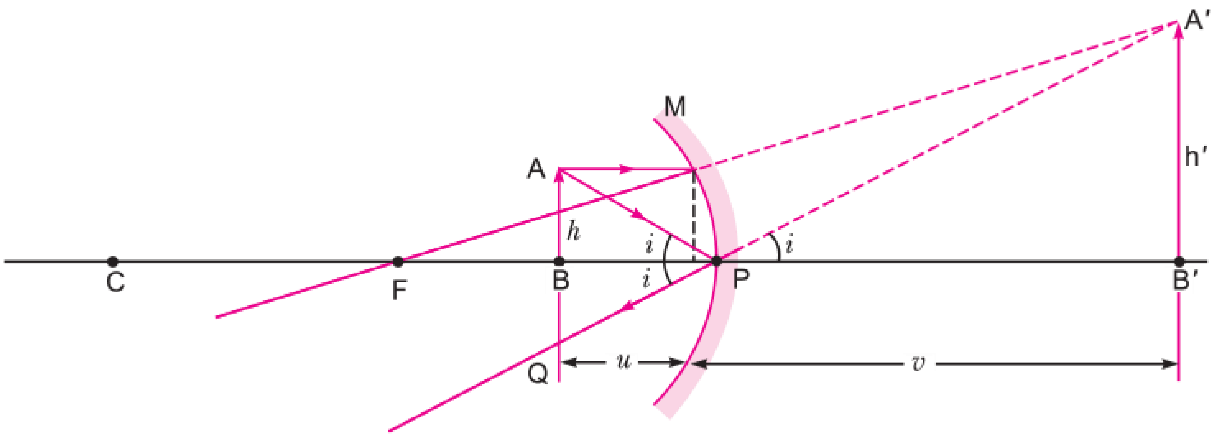
Dividing both sides by uvf we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

The corresponding formula for thin lens is

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

(ii) Ray Diagram: The ray diagram of image formation for an object between focus (F) and pole (P) of a concave mirror is shown in fig.



$$\text{Magnification: } m = \frac{\text{Size of image } (A'B')}{\text{Size of object } (AB)}$$

From fig. $APB = \angle BPQ = i$

Also, $\angle BPQ = \angle A' PB' = i$

$$\text{In } \Delta \text{ APB}, \tan i = \frac{AB}{BP} \qquad ...(i)$$

$$\text{In } \Delta \text{ AP}' B', \tan i = \frac{A'B'}{BP'} \qquad ...(ii)$$

From (i) and (ii)

$$\frac{AB}{BP} = \frac{A'B'}{BP'}$$

$$\Rightarrow \quad \text{Magnification, } m = \frac{A'B'}{AB} = \frac{BP'}{BP}$$

$$\text{or} \quad m = \frac{v}{-u} \text{ or } m = -\frac{v}{u}$$

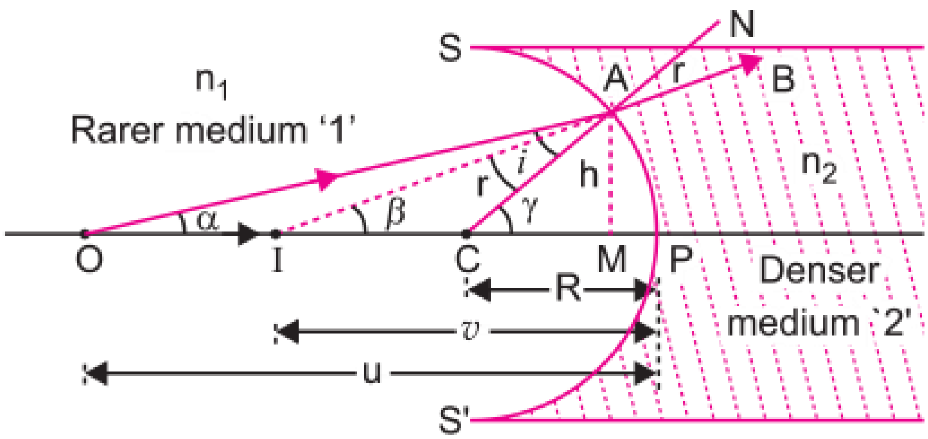
Q. 2. With the help of a ray diagram, show the formation of image of a point object due to refraction of light at a spherical surface separating two media of refractive indices n1 and n_2 ($n_2 > n_1$) respectively. Using this diagram, derive the relation

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_1 - n_2}{R}$$

Write the sign conventions used. What happens to the focal length of convex lens when it is immersed in water?

Ans. Formula for Refraction at Spherical Surface

Concave Spherical Surface : Let *SPS′* be a spherical refracting surface, which separates media ‘1’ and ‘2’. Medium ‘1’ is rarer and medium ‘2’ is denser. The refractive indices of media ‘1’ and ‘2’ are n_1 and $n_2$respectively ($n_1 < n_2$) Let *P* be the pole and *C* the centre of curvature and *PC* the principal axis of spherical refracting surface.



O is a point-object on the principal axis. An incident ray *OA*, after refraction at *A* on the spherical surface bends towards the normal *CAN* and moves along *AB*. Another incident ray *OP* falls on the surface normally and hence passes undeviated after refraction. These two rays, when produced backward meet at point *I* on principal axis. Thus I is the virtual image of *O*.

Let angle of incidence of ray *OA* be *i* and angle of refraction be *r i.e*.

$\angle OAC = i$ and $\angle NAB = r$

Let $\angle AOP = \alpha$, $\angle AIP = \beta$ and $\angle ACP = \gamma$

In triangle *OAC* $\quad \gamma = \alpha + i$ or $\quad i = \gamma - \alpha \qquad ...(i)$

In triangle *AIC*, $\quad \gamma = \beta + r$ or $\quad r = \gamma - \beta \qquad ...(ii)$

$$\text{From Snell's law } \frac{\sin i}{\sin r} = \frac{n_2}{n_1} \qquad ...(iii)$$

If point A is very near to P, then angles i, r, α, β, γ will be very small, therefore sin i = i and sin r = r

Substituting values of i and r from (i) and (ii) we get

$$\frac{\gamma - \alpha}{\gamma - \beta} = \frac{n_2}{n_1} \quad or \quad n_1(\gamma - \alpha) = n_2(\gamma - \beta) \qquad ...(iv)$$

The length of perpendicular *AM* dropped from *A* on the principal axis is *h i.e. AM* = *h*. As angles α, *β* and *γ* are very small, therefore

$\tan \alpha = \alpha$, $\tan \beta = \beta$, $\tan \gamma = \gamma$

Substituting these values in equation (*iv*)

n_1 (tan γ - tan α) = n_2 (tan γ - tan β) ...(*v*)

As point *A* is very close to *P*, point *M* is coincident with *P*

$$\tan\ \alpha = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{AM}}{\text{MO}} = \frac{h}{\text{PO}}$$

$$\tan\beta = \frac{\text{AM}}{\text{MI}} = \frac{h}{\text{PI}}, \qquad \tan\gamma = \frac{\text{AM}}{\text{MC}} = \frac{h}{\text{PC}}$$

Substituting this value in (v), we get

$$n_1\left(\frac{h}{\text{PC}} - \frac{h}{\text{PO}}\right) = n_2\left(\frac{h}{\text{PC}} - \frac{h}{\text{PI}}\right)$$

$$\text{or} \qquad \frac{n_1}{\text{PC}} - \frac{n_1}{\text{PO}} = \frac{n_2}{\text{PC}} - \frac{n_2}{\text{PI}}$$

Let *u*, *v* and *R* be the distances of object *O*, image *I* and centre of curvature *C* from pole *P*. By sign convention *PO*, *PI* and *PC* are negative, *i.e*. *u* = – *PO*, *v* = – *PI* and *R* = – *PC*

Substituting these values in (*vi*), we get

$$\frac{n_1}{(-R)} - \frac{n_1}{(-u)} = \frac{n_2}{(-R)} - \frac{n_2}{(-v)} \quad \text{or} \quad \frac{n_1}{R} - \frac{n_1}{u} = \frac{n_2}{R} - \frac{n_2}{v}$$

$$\text{or} \qquad \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Sign Conventions:

(i) All the distances are measured from optical centre (P) of the lens.

(ii) Distances measured in the direction of incident ray of light are taken positive and vice-versa.

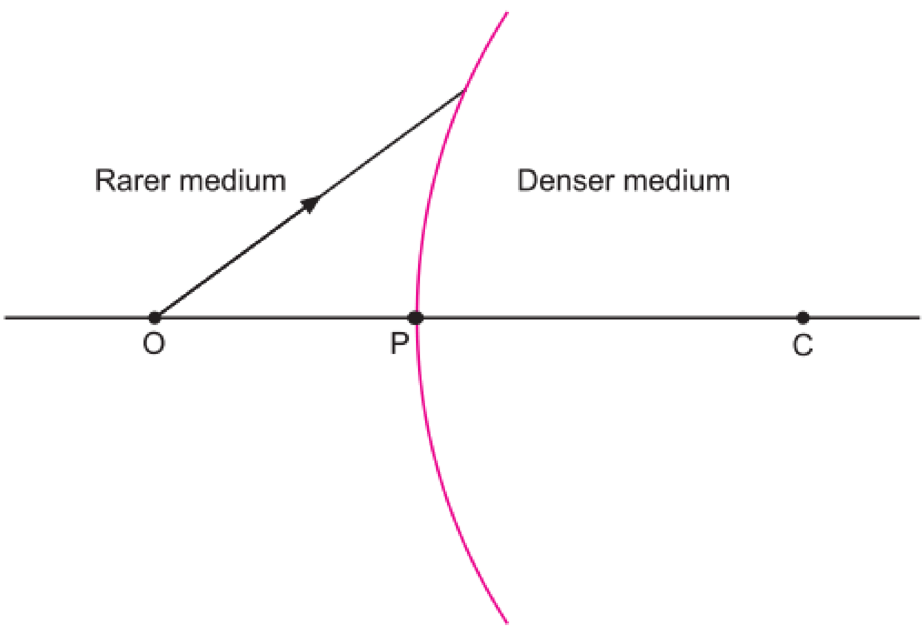
As we know

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

When convex lens is immersed in water, refractive index n decreases and hence focal length will increase i.e., the **focal length of a convex lens increases when it is immersed in water.**

Q. 3. A spherical surface of radius of curvature R, separates a rarer and a denser medium as shown in the figure. Complete the path of the incident ray of light, showing the formation of a real image. Hence derive the relation connecting object distance 'u', image distance 'v', radius of curvature R and the refractive indices n_1 and n_2 of two media.

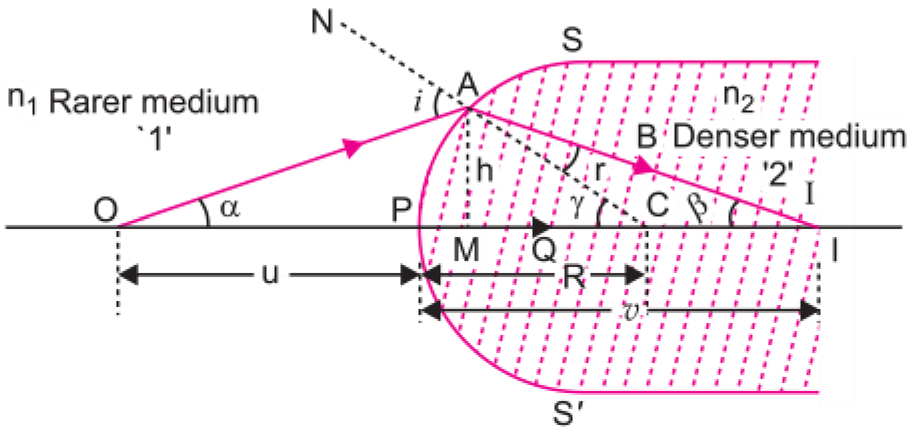
Briefly explain, how the focal length of a convex lens changes, with increase in wavelength of incident light.
[CBSE Delhi 2014; Central 2016; (F) 2017; Sample Paper 2016]



Ans. Relation of object and image distances of a convex spherical surface: Let *SPS′* be the convex spherical refracting surface, separating the two media of refractive indices n_1 and n_2 respectively ($n_1 < n_2$) *i.e*. medium '1' is rarer and medium '2' is denser. Let *P* be the pole, *C* the centre of curvature and *PC* the principal axis of convex refracting surface. *O* is a distant point object on the principal axis. The ray *OA* starting from *O* is incident on point *A* of the spherical surface, *CAN* is normal at point *A* of the surface. Due to going from rarer to denser medium the ray *OA* deviates along the normal *CAN* and is refracted along the direction *AB*. The another ray *OP* starting from *O* is incident normally on the spherical surface and passes undeviated after refraction along *PQ*. Both the rays *AB* and *PQ* meet at point *I* on the principal axis, *i.e.*, *I* is the real image of point object *O*.

Let *i* be the angle of incidence of ray *OA* and *r* the angle of refraction in the denser medium *i.e.*, $\angle OAN = i$ and $\angle CAI = r$. Let $\angle AOP = a$, $\angle AIP = \beta$ and $\angle ACP = \gamma$

In triangle *OAC*, $i = \gamma + \alpha$



In triangle *AIC*, $\gamma = \beta + r$ or $r = \gamma - \beta$ ...(*ii*)

From Snell's law $\frac{\sin\ i}{\sin\ r} = \frac{n_2}{n_1}$ ...(*iii*)

If point *A* is very close to *P*, then angles *i*, *r*, α, β and γ will be very small, therefore

$\sin i = i$ and $\sin r = r$

From equation (*iii*),

$\frac{i}{r} = \frac{n_2}{n_1}$

Substituting values of *i* and *r* from (*i*) and (*ii*), we get

$\frac{\gamma+\alpha}{\gamma-\beta} = \frac{n_2}{n_1}$ or $n_1\,(\gamma+\alpha) = n_2\,(\gamma-\beta)$ ...(*iv*)

Let *h* be the height of perpendicular drawn from *A* on principal axis *i.e.* $AM = h$. As α, β and γ are very small angles.

$\tan\alpha = \alpha$, $\tan\beta = \beta$ and $\tan\gamma = \gamma$

Substituting these values in (*iv*)

$$n_1\, t(\tan \gamma + \tan \alpha) = n_2\, (\tan \gamma - \tan \beta) \qquad ...(v)$$

As point A is very close to point P, point M is coincident with P.

From figure
$$\tan \alpha = \frac{AM}{OM} = \frac{h}{OP}$$

$$\tan \beta = \frac{AM}{MI} = \frac{h}{PI}$$

$$\tan \gamma = \frac{AM}{MC} = \frac{h}{PC}$$

Substituting these values in (v), we get

$$n_1\left(\frac{h}{PC} + \frac{h}{OP}\right) = n_2\left(\frac{h}{PC} - \frac{h}{PI}\right)$$

or
$$n_1\left(\frac{1}{PC} + \frac{1}{OP}\right) = n_2\left(\frac{1}{PC} - \frac{1}{PI}\right) \qquad ...(vi)$$

If the distances of object O, image I, centre of curvature C from the pole be u, v and R respectively, then by sign convention PO is negative while PC and PI are positive. Thus,

$u = -PO$, $\quad v = +PI$, $\quad R = +PC$

Substituting these values in (*vi*), we get

$$n_1\left(\frac{1}{R}-\frac{1}{u}\right)=n_2\left(\frac{1}{R}-\frac{1}{v}\right)$$

or $$\frac{n_1}{R}-\frac{n_1}{u}=\frac{n_2}{R}-\frac{n_2}{v}$$

$\therefore$ $$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

The focal length of a convex lens is given by

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

According to Cauchy's formula

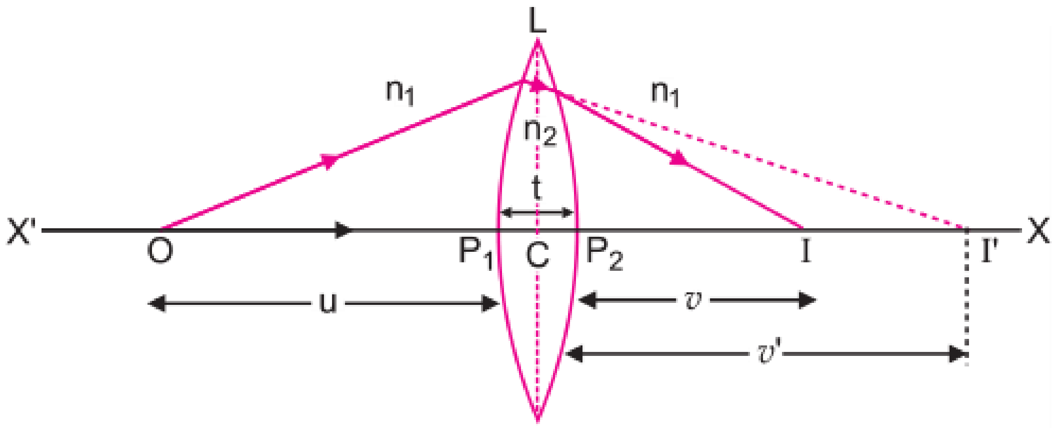
$$n=a+\frac{b}{\lambda^2}+\frac{c}{\lambda^4}+\ldots$$

Then n varies inversely as λ.

When wavelength increases, the refractive index n decreases; **so focal length of lens increases with increase of wavelength.**

Q. 4. Draw a ray diagram for formation of image of a point object by a thin double convex lens having radii of curvature R_1 and R_2. Hence, derive lens maker's formula for a double convex lens. State the assumptions made and sign convention used.
[CBSE (F) 2009, 2013, (Central) 2016]

Ans. Lens Maker's Formula: Suppose L is a thin lens. The refractive index of the material of lens is n_2 and it is placed in a medium of refractive index n_1. The optical centre of lens is C and $X'X$ is principal axis. The radii of curvature of the surfaces of the lens are R1 and R2 and their poles are P1 and P2 The thickness of lens is t, which is very small. O is a point object on the principal axis of the lens. The distance of O from pole P1 is u. The first refracting surface forms the image of O at I' at a distance v' from P_1 From the refraction formula at spherical surface



$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \qquad \ldots(i)$$

The image *I′* acts as a virtual object for second surface and after refraction at second surface, the final image is formed at *I*. The distance of *I* from pole P_2 of second surface is *v*. The distance of virtual object (*I ′*) from pole P_2 is ($v' - t$).

For refraction at second surface, the ray is going from second medium (refractive index n_2) to first medium (refractive index n_1), therefore from refraction formula at spherical surface

$$\frac{n_1}{v} - \frac{n_2}{(v' - t)} = \frac{n_1 - n_2}{R_2} \qquad \ldots(ii)$$

For a thin lens *t* is negligible as compared to *v*′ therefore from (*ii*)

$$\frac{n_1}{v} - \frac{n_2}{(v')} = -\frac{n_2 - n_1}{R_2} \qquad \ldots(iii)$$

Adding equations (*i*) and (*iii*), we get

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

i.e. $$\frac{1}{v} - \frac{1}{u} = ({}_1n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(iv)$$

where ${}_1n_2 = \frac{n_2}{n_1}$ is refractive index of second medium (*i.e.*, medium of lens) with respect to first medium.

If the object O is at infinity, the image will be formed at second focus *i.e.*

if $u = \infty$, $v = f_2 = f$

Therefore from equation (iv)

$$\frac{1}{f} - \frac{1}{\infty} = ({}_1n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

i.e. $$\frac{1}{f} = ({}_1n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ...(v)$$

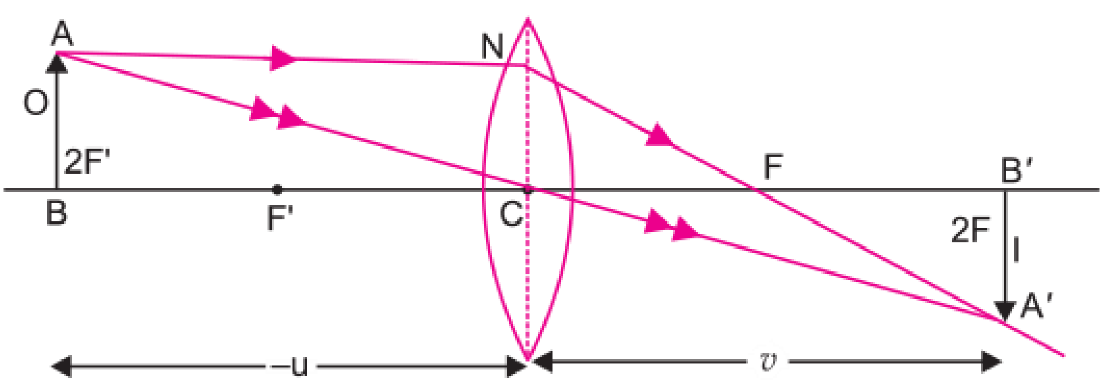
This formula is called Lens-Maker's formula.

If first medium is air and refractive index of material of lens be n, then ${}_1n_2 = n$, therefore the modified equation (v) may be written as

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Q. 5. Draw a ray diagram to show the formation of real image of the same size as that of the object placed in front of a converging lens. Using this ray diagram establish the relation between u, v and f for this lens.

Ans. Thin Lens Formula : Suppose an object *AB* of finite size is placed normally on the principal axis of a thin convex lens (fig.). A ray *AP* starting from *A* parallel to the principal axis, after refraction through the lens, passes through the second focus *F*. Another ray *AC* directed towards the optical centre *C* of the lens, goes straight undeviated. Both the rays meet at *A′* Thus *A′* is the real image of *A*. The perpendicular *A′ B′* dropped from *A′* on the principal axis is the whole image of *AB*.



Let distance of object *AB* from lens = *u*

Distance of image *A'B′* from lens = *v*

Focal length of lens = *f*. We can see that

Triangles *ABC* and *A′B′C′* are similar

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \qquad ...(i)$$

Similarly triangles *PCF* and *A′B′F* are similar

$$\frac{PC}{A'B'} = \frac{CF}{FB'}$$

But $PC = AB$

$$\frac{AB}{A'B'} = \frac{CF}{FB'} \qquad ...(ii)$$

From (i) and (ii), we get $\frac{CB}{CB'} = \frac{CF}{FB'} \qquad ...(iii)$

From sign convention, $CB = -u$, $CB' = v$, $CF = f$

and $FB' = CB' - CF = v - f$

Substituting this value in (iii), we get, $-\frac{u}{v} = \frac{f}{v-f}$

or $-u\left(v - f\right) = vf$ or $-uv + uf = uf$

Dividing throughout by uvf, we get $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad ...(iv)$

Q. 6. Derive the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a thin concave lens, using the necessary ray diagram.

Ans. The formation of image by a concave lens 'L' is shown in fig. AB is object and A′ B′ is the image. Triangles ABO and A′ B′ O are similar

$$\frac{AB}{A'B'} = \frac{OB}{OB'} \qquad \dots(i)$$

Also triangles NOF and $A'B'F$ are similar

$$\frac{NO}{A'B'} = \frac{OF}{FB'}$$

But $NO = AB$

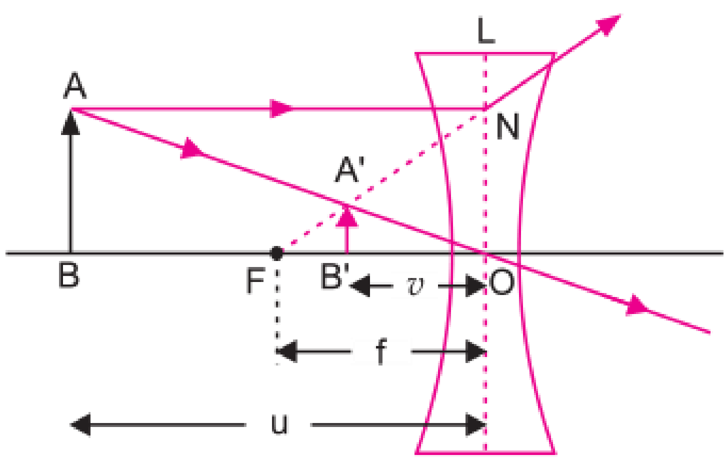
$$\frac{AB}{A'B'} = \frac{OF}{FB'} \qquad \dots(ii)$$

Comparing equation (i) and (ii)

$$\frac{OB}{OB'} = \frac{OF}{FB'} \Rightarrow \frac{OB}{OB'} = \frac{OF}{OF - OB'}$$

Using sign conventions of coordinate geometry

$OB = -u$, $OB' = -v$, $OF = -f$



$$\frac{-u}{-v} = \frac{-f}{-f+v} \quad \Rightarrow \quad \text{uf} - \text{uv} = \text{vf}$$

$$\Rightarrow \qquad uv = uf - uf$$

Dividing throughout by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This is the required lens formula.

Q. 7. Define power of a lens. Write its units. Deduce the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ for two thin lenses kept in contact coaxially. [CBSE (F) 2012]

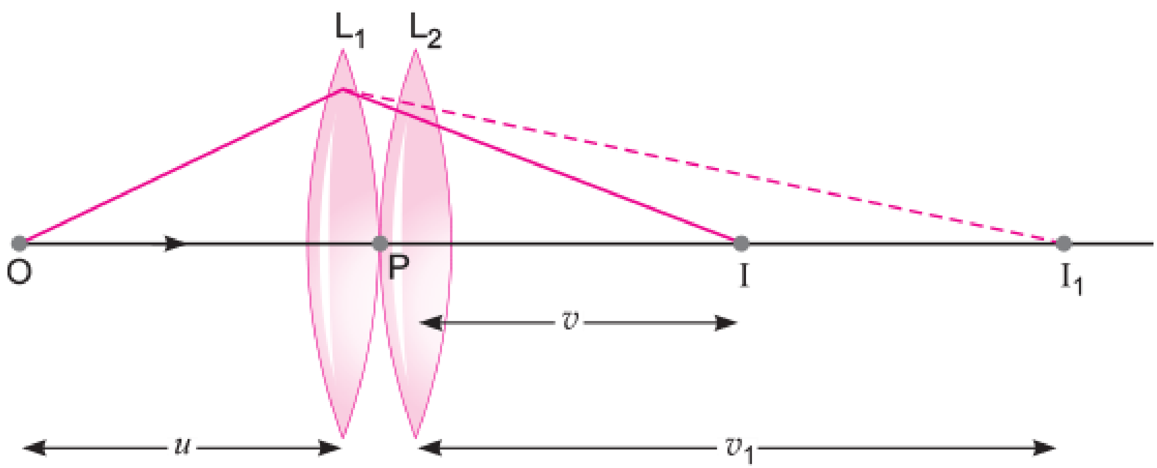
Ans. Power of lens: It is the reciprocal of focal length of a lens.

$p = \frac{1}{f}$ (f is in metre)

Unit of power of a lens is Diopter.

An object is placed at point O. The lens A produces an image at I_1 which serves as a virtual object for lens B which produces final image at I.

Given, the lenses are thin. The optical centres (P) of the lenses L_1 and L_2 is co-incident.



For lens L_1, we have

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \quad ...(i)$$

For lens L_2, we have $\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$ $...(ii)$

Adding equations (i) and (ii),

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad ...(iii)$$

If two lenses are considered as equivalent to a single lens of focal length f, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad ...(iv)$$

From equation (iii) and equation (iv), we can write

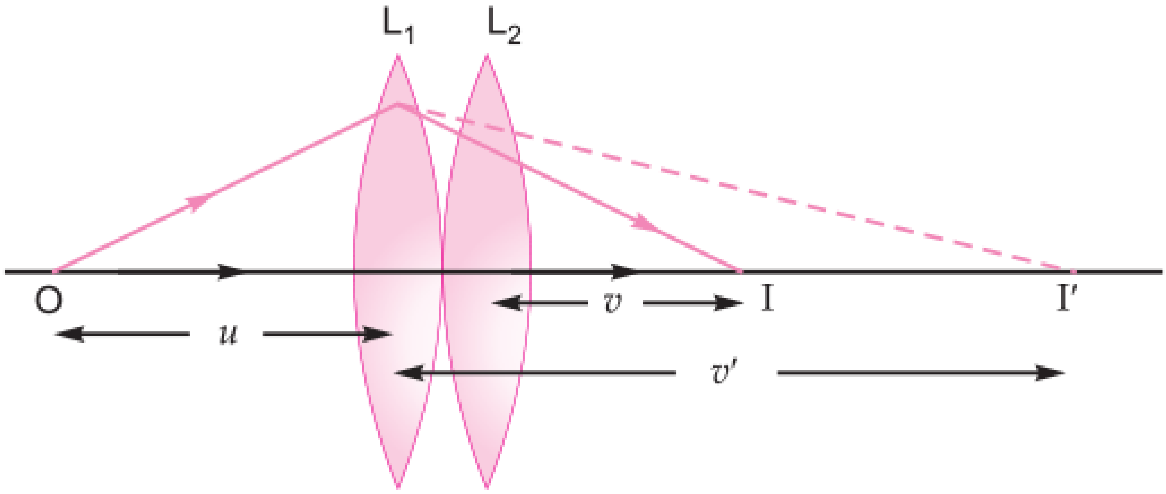
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Q. 8. Two thin convex lenses L_1 and L_2 of focal lengths f_1 and f_2 respectively, are placed coaxially in contact. An object is placed at a point beyond the focus of lens L_1. Draw a ray diagram to show the image formation by the combination and hence derive the expression for the focal length of the combined system. [CBSE Panchkula 2015]

Ans. O – object

I′ → Image formed by L_1/virtual object for L_2

I → Image formed by L_2



Lens formula, $\frac{1}{-u} + \frac{1}{v} = \frac{1}{f}$

For lens L_1 $\quad \frac{1}{-u} + \frac{1}{v'} = \frac{1}{f_1}$ ...(*i*)

For lens L_2 $\quad \frac{1}{-v'} + \frac{1}{v} = \frac{1}{f_2}$ ...(*ii*)

Adding (i) and (ii), we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \Rightarrow \quad f = \frac{f_1 f_2}{f_1 + f_2}$$

Q. 9. (a) Draw the labelled ray diagram for the formation of image by a compound microscope. Derive an expression for its total magnification (or magnifying power), when the final image is formed at the near point. [CBSE Delhi 2009, 2010, 2013]

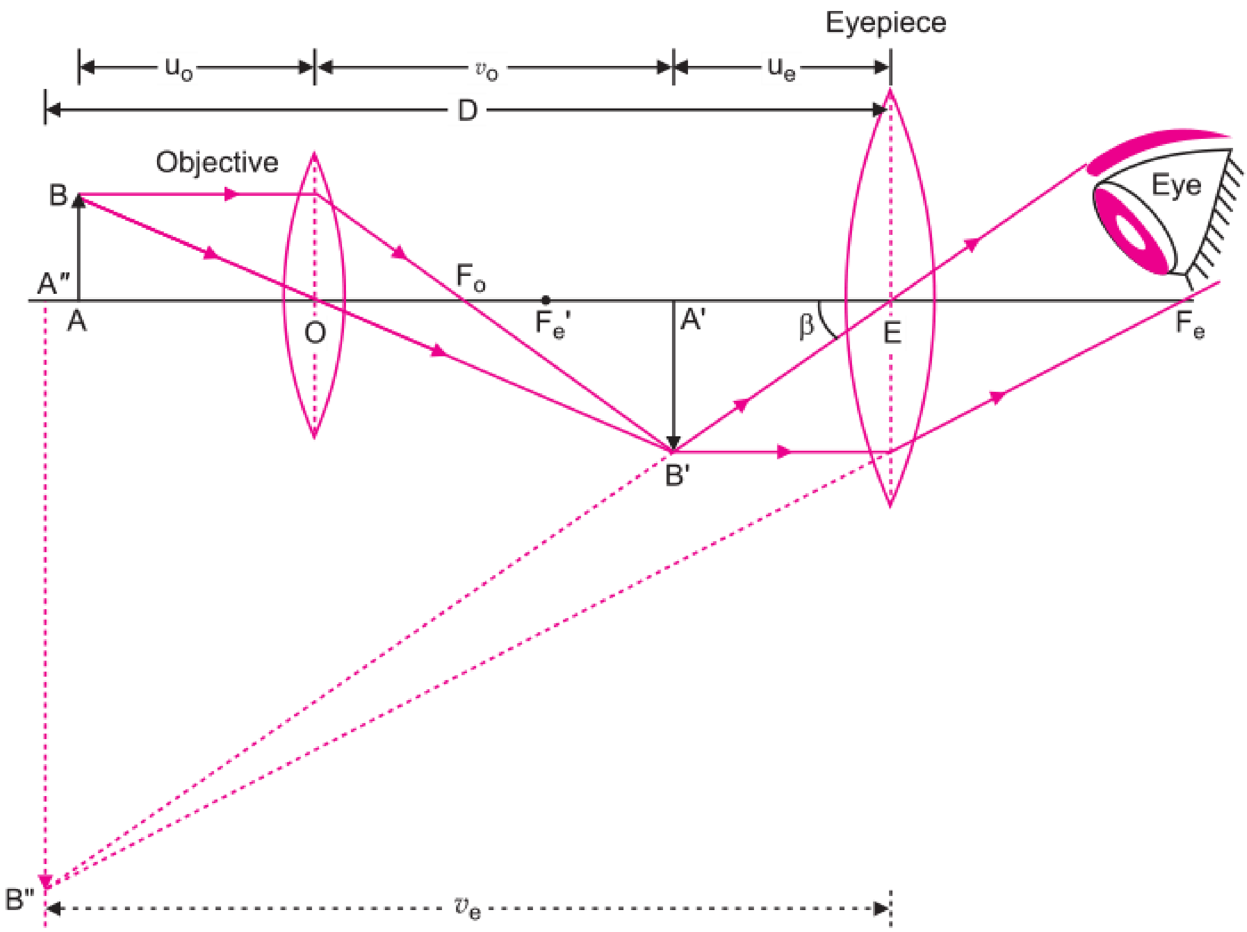
(b) Why both objective and eyepiece of a compound microscope must have short focal lengths?

Draw a ray diagram showing the image formation by a compound microscope. Hence obtain expression for total magnification when the image is formed at infinity. [CBSE Delhi 2013]

Ans. (a) Compound Microscope : It consists of a long cylindrical tube, containing at one end a convex lens of small aperture and small focal length. This is called the objective lens (O). At the other end of the tube another co-axial smaller and wide tube is fitted, which carries a convex lens (E) at its outer end. This lens is towards the eye and is called the eye-piece. The focal length and aperture of eyepiece are somewhat larger than those of objective lens.

Cross-wires are mounted at a definite distance before the eyepiece. The entire tube can be moved forward and backward by the rack and pinion arrangement.

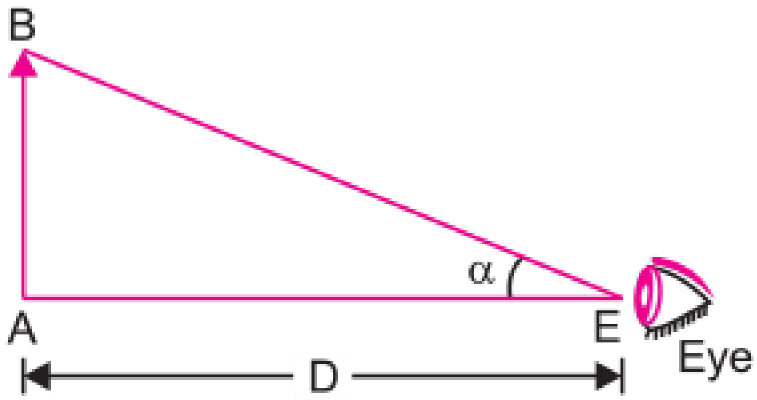
Adjustment: First of all the eyepiece is displaced backward and forward to focus it on cross-wires. Now the object is placed just in front of the objective lens and the entire tube is moved by rack and pinion arrangement until there is no parallax between image of object and cross wire. In this position the image of the object appears quite distinct.



Working : Suppose a small object *AB* is placed slightly away from the first focus F_0' of the objective lens. The objective lens forms the real, inverted and magnified image *A′ B′* which acts as an object for eyepiece. The eyepiece is so adjusted that the image *A′ B′* lies between the first focus F_e' and the eyepiece *E*. The eyepiece forms its image *A″ B″* which is virtual, erect and magnified. Thus the final image *A″ B″* formed by the microscope is inverted and magnified and its position is outside the objective and eyepiece towards objective lens.

Magnifying power of a microscope is defined as the ratio of angle (β) subtended by final image on the eye to the angle (*a*) subtended by the object on eye, when the object is placed at the least distance of distinct vision, *i.e.*,

Magnifying power M = $\frac{\beta}{\alpha}$ ...(i)



As object is very small, angles α and β are very small and so tan α = α and tan β = β By definition the object *AB* is placed at the least distance of distinct vision.

$\alpha = \tan\alpha = \frac{AB}{EA}$

By sign convention $EA = -D$, $\therefore$ $a = \frac{AB}{-D}$

and from figure $\beta = \tan\ \beta = \frac{A'B'}{EA'}$

If u_e is distance of image $A'B'$ from eye-piece E, then by sign convention, $EA' = -u_e$

and so, $\beta = \frac{A'B'}{(-u_e)}$

Hence magnifying power $M = \frac{\beta}{\alpha} = \frac{A'B'/(-u_e)}{AB\,(-D)} = \frac{A'B'}{AB} \cdot \frac{D}{u_e}$

By sign conventions, magnification of objective lens

$\frac{A'B'}{AB} = \frac{v_0}{(-u_0)}$

$M = -\frac{v_0}{u_0} \cdot \frac{D}{u_e}$ ...(ii)

Using lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for eye-lens, (*i.e.*, using $f = fe$, v_e, $u = -u_e$), we get

$\frac{1}{f_e} = \frac{1}{-v_e} - \frac{1}{(-u_e)}$ or $\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{v_e}$

Magnifying power $M = -\frac{v_0}{u_0} D\left(\frac{1}{f_e} + \frac{1}{v_e}\right)$

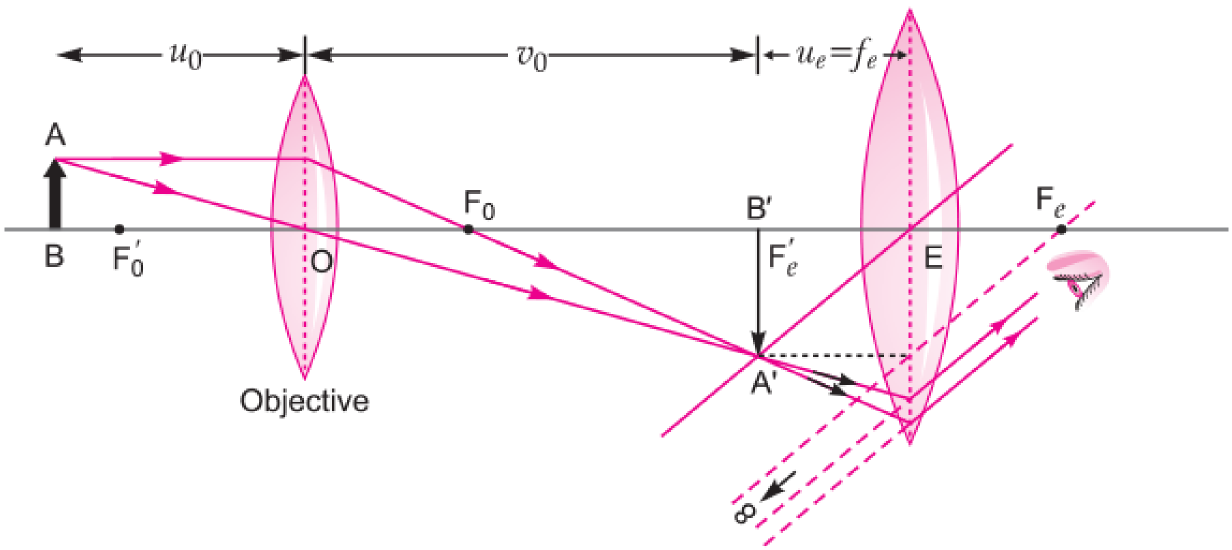
or $\quad M = -\frac{v_0}{u_0}\left(\frac{D}{f_e} + \frac{D}{v_e}\right)$

When final image is formed at the distance of distinct vision, $v_e = D$

Magnification, $\quad M = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

For greater magnification of a compound microscope, f_e should be small. As $f_0 < f_e$, so f0 is small. Hence, for greater magnification both f0 and f_e should be small with f_0 to be smaller of the two.

(b) If image A′B′ is exactly at the focus of the eyepiece, then image A'B' is formed at infinity.



If the object AB is very close to the focus of the objective lens of focal length fo, then magnification Mo by the objective lens

If the object AB is very close to the focus of the objective lens of focal length fo, then magnification Mo by the objective lens

$$M_e = \frac{L}{f_0}$$

where L is tube length (or distance between lenses L_o and L_e)

Magnification M_e by the eyepiece

$$M_e = \frac{D}{f_e}$$

where D = Least distance of distinct vision

Total magnification, $m = M_o M_e = \left(\frac{L}{f_o}\right)\left(\frac{D}{f_e}\right)$

Q. 10. Answer the following question : [CBSE (F) 2015]

(i) Draw a ray diagram showing image formation in a compound microscope. Define the term 'limit of resolution' and name the factors on which it depends. How is it related to resolving power of a microscope?

(ii) Suggest two ways by which the resolving power of a microscope can be increased.

(iii) "A telescope resolves whereas a microscope magnifies." Justify this statement.

Ans. (i) For diagram Refer to Q. 9. above

Limit of resolution

The minimum linear or angular separation between two points at which they can be just separately seen or resolved by an optical instrument. It depends on

(i) Wavelength of light used
(ii) Medium between object and objective lens

Resolving power of microscope is the reciprocal of its limit of resolution.

(ii) Resolving power of compound microscope can be increased by

(i) Decreasing wavelength
(ii) Increasing refractive index of the medium between object and objective of the microscope

(iii) A telescope produces an angularly magnified image of the far object and thereby enables us to resolve them

A microscope magnifies small objects which are near to our eye.

Q. 11. Explain with the help of a labelled ray diagram, how is image formed in an astronomical telescope. Derive an expression for its magnifying power.
[CBSE (F) 2014]

OR

Draw a ray diagram showing the image formation of a distant object by a refracting telescope. Define its magnifying power and write the two important factors considered to increase the magnifying power.

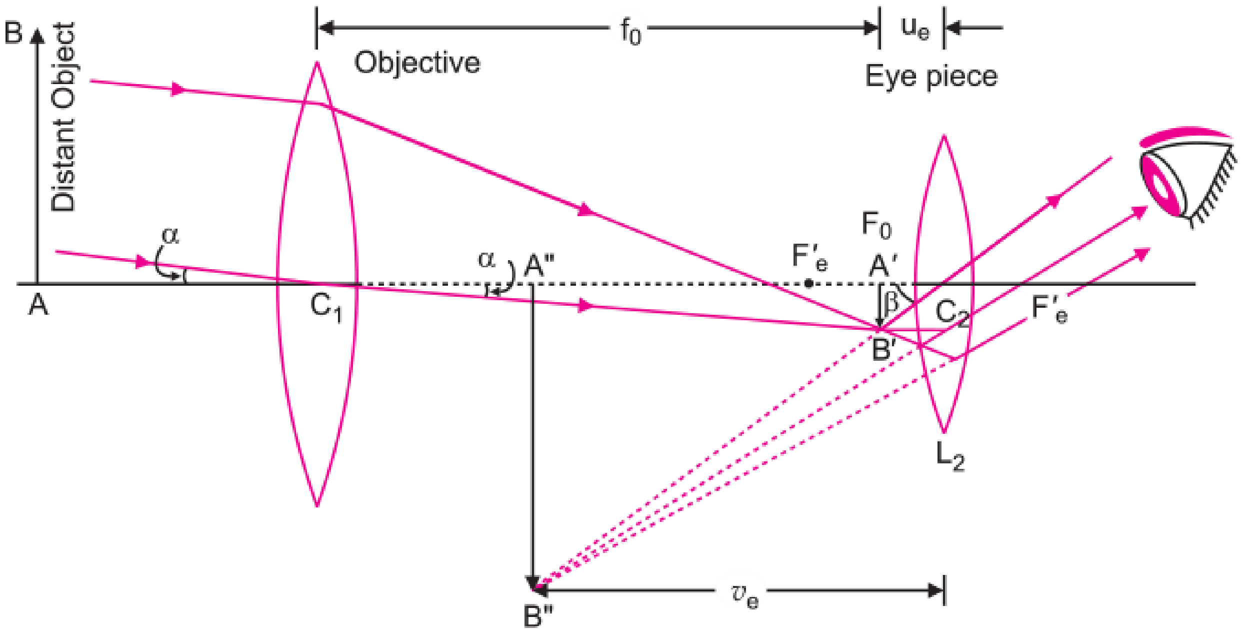
Describe briefly the two main limitations and explain how far these can be minimised in a reflecting telescope. [CBSE (F) 2015]

Ans. Astronomical (Refracting) Telescope:

Construction: It consists of two co-axial cylindrical tubes, out of which one tube is long and wide, while the other tube is small and narrow. The narrow tube may be moved in and out of the wide tube by rack and pinion arrangement. At one end of wide tube an achromatic convex lens L1 is placed, which faces the object and is so called **objective (lens).** The focal length and aperture of this lens are kept large. The large aperture of objective is taken that it may collect sufficient light to form a bright image of a distant object. The narrow tube is towards eye and carries an achromatic convex lens L2 of small focal length and small aperture on its outer end. This is called **eye-lens** or **eyepiece.** The small aperture of eye-lens is taken so that the whole light refracted by it may reach the eye. Cross-wires are fitted at a definite distance from the eye-lens.

Due to large focal length of objective lens and small focal length of eye lens, the final image subtends a large angle at the eye and hence the object appears large. The distance between the two lenses may be arranged by displacing narrow tube in or out of wide tube by means of rack and pinion arrangement.

Adjustment : First of all the eyepiece is moved backward and forward in the narrow tube and focused on the cross-wires. Then the objective lens is directed towards the object and narrow tube is displaced in or out of wide tube until the image of object is formed on cross-wires and there is no parallax between the image and cross-wires. In this position a clear image of the object is seen. As the image is formed by refraction of light through both the lenses, this telescope is called the **refracting telescope.**



Working : Suppose *AB* is an object whose end *A* is on the axis of telescope. The objective lens (L_1) forms the image *A*' *B*' of the object *AB* at its second principal focus F_0 This image is real, inverted and diminished. This image *A*' *B*' acts as an object for the eye-piece L_2 and lies between first focus Fe and optical centre $C_2$of lens L_2. Therefore eye-piece forms its image *A*'' *B*'' which is virtual, erect and magnified.

Thus the final image *A*' *B*' of object *AB* formed by the telescope is magnified, inverted and lies between objective and eyepiece.

Magnifying Power : The magnifying power of a telescope is measured by the ratio of angle (β) subtended by final image on the eye to the angle (α) subtended by` object on the eye, *i.e.*,

Magnifying power M = $\frac{\beta}{\alpha}$

As α and β are very small angles, therefore, from figure.

The angle subtended by final image *A″ B″* on eye

β = angle subtended by image *A′ B′* on eye

$=\tan\beta = \frac{A'B'}{C_2A'}$

As the object is very far (at infinity) from the telescope, the angle subtended by object at eye is same as the angle subtended by object on objective lens.

$$\alpha = \tan\alpha = \frac{A'B'}{C_1A'}$$

$$M = \frac{\beta}{\alpha} = \frac{A'B'/C_2A'}{A'B'/C_1A'} = \frac{C_1A'}{C_2A'}$$

If the focal lengths of objective and eye-piece be f_o, and f_e , distance of image $A'B'$ from eye-piece be ue, then by sign convention

$C_1A' = +f_0$, $C_2A' = -u_e$

$$M = -\frac{f_0}{u_e} \qquad ...(i)$$

If v_e is the distance of $A''B''$ from eye-piece, then by sign convention, fe is positive, ue are ve both negative. Hence by lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we have

$$\frac{1}{f_e} = \frac{1}{-v_e} - \frac{1}{(-u_e)} \text{ or } \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{v_e}$$

Substituting this value in (i), we get

$$M = -f_0\left(\frac{1}{f_e} + \frac{1}{v_e}\right) \qquad ...(ii)$$

This is the general formula for magnifying power. In this formula only numerical values of f_0 , f_e and v_e are to be used because signs have already been used.

Length of Telescope : The distance between objective and eye-piece is called the length (0) of the telescope. Obviously

$L = L_1L_2 = C_1C_2 = f_o + u_e$ ..(iii)

Now there arise two cases:

(i) When the final image is formed at minimum distance (D) of distinct vision : then v_e = D

$$M = -f_0\left(\frac{1}{f_e} + \frac{1}{D}\right) = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right) \qquad ..(iv)$$

Length of telescope $L = f_o + u_e$

(ii) In normal adjustment position, the final image is formed at infinity : For relaxed eye, the final image is formed at infinity. In this state, the image A′ B′ formed by objective lens should be at first the principal focus of eyepiece, i.e.,

$u_e = f_o$ and $v_e = \infty$

$$\therefore \text{ Magnifying power, } M = -\ f_o\left(\frac{1}{f_e}+\frac{1}{\infty}\right) = -\frac{f_o}{f_e}$$

Length of telescope = $f_0 + f_e$.

For large magnifying power, f_o should be large and f_e should be small.

For high resolution of the telescope, diameter of the objective should be large.

Factors for increasing the magnifying power

(i) Increasing focal length of objective
(ii) Decreasing focal length of eye piece

Limitations

(i) Suffers from chromatic aberration
(ii) Suffers from spherical aberration
(iii) Small magnifying power
(iv) Small resolving power

Advantages of reflecting telescope

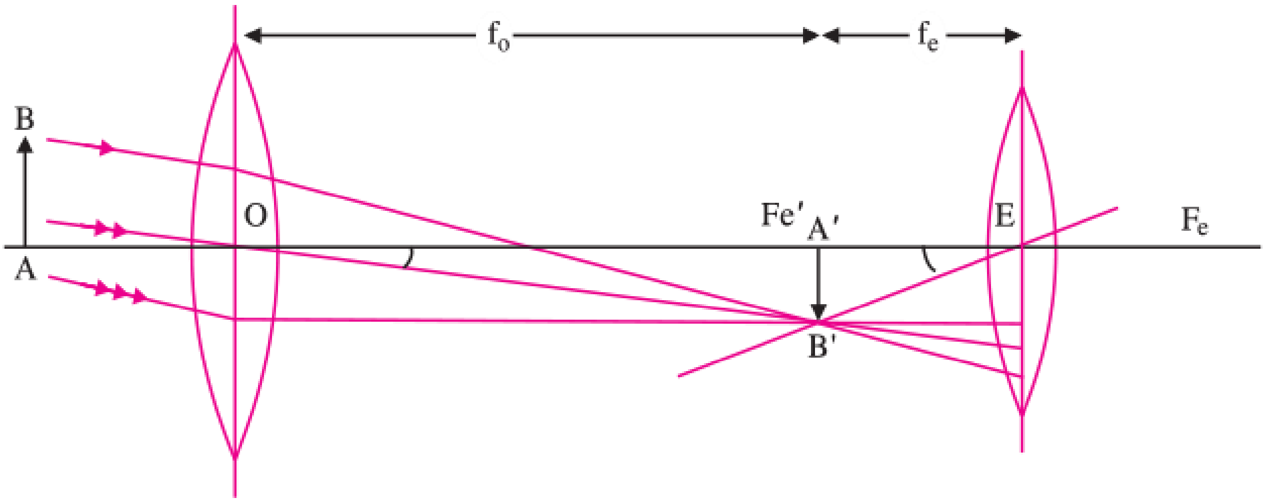
(i) No chromatic aberration, because mirror is used.
(ii) Spherical aberration can be removed by using a parabolic mirror.
(iii) Image is bright because no loss of energy due to reflection.
(iv) Large mirror can provide easier mechanical support.

Q. 12. (i) Draw a labelled ray diagram to obtain the real image formed by an astronomical telescope in normal adjustment position. Define its magnifying power.

(ii) You are given three lenses of power 0.5 D, 4 D and 10 D to design a telescope.

(a) Which lenses should be used as objective and eyepiece? Justify your answer.
(b) Why is the aperture of the objective preferred to be large?
[CBSE (Central) 2016]

Ans. (i)



Definition: It is the ratio of the angle (β) subtended at the eye, by the final image, to the angle (α) subtended by the object on the eye, *i.e.*, M = $\frac{\beta}{\alpha}$

(ii) (a) Objective = 5 D

Eye lens = 10 D

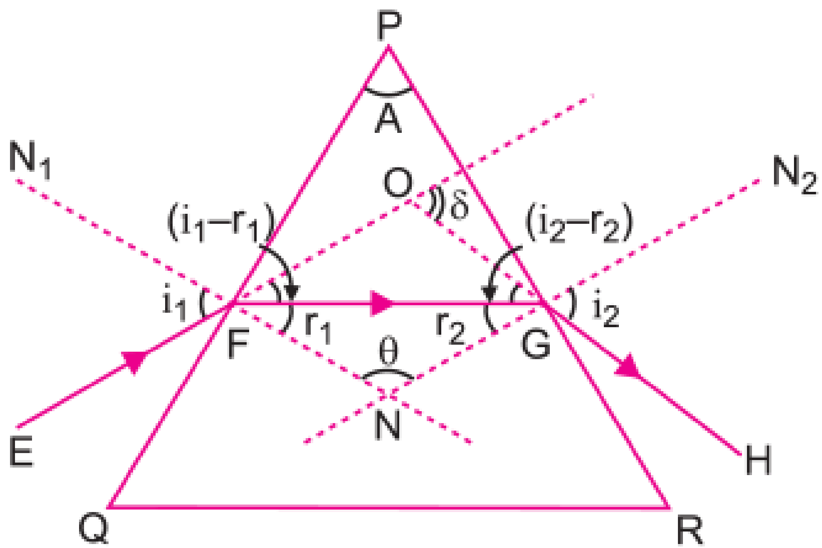
This choice would give higher magnification as

$$M = \frac{f_0}{f_e} = \frac{P_e}{P_0}$$

(b) The aperture of the objective lens is preferred to be large that it may collect sufficient light to form a brighter image of a distant object.

Q. 13. Draw a graph to show the angle of deviation δ with the variation of angle of incidence i for a monochromatic ray of light passing through a prism of refracting angle A. Deduce the relation
[CBSE Delhi 2011, 2016; (F) 2011, 2017; Sample Paper 2016]

$$n = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$



Ans. Graph of deviation in δ with variation in angle of incidence *i* : The homogeneous transparent medium (such as glass) enclosed by two plane refracting surfaces is called a prism. The angle between the refracting surfaces is called the refracting angle or angle of prism. The section cut by a plane perpendicular to the refracting surfaces is called the principal section of the prism.

Let *PQR* be the principal section of the prism. The refracting angle of the prism is *A*.

A ray of monochromatic light *EF* is incident on face *PQ* at angle of incidence i_1 The refractive index of material of prism for this ray is *n*. This ray enters from rarer to denser medium and so is deviated towards the normal *FN* and gets refracted along the direction *FG*. The angle of refraction for this face is r_1 The refracted ray *FG* becomes incident on face *PR* and is refracted away from the normal GN_2 and emerges in the direction *GH*. The angle of incidence on this face is r_2 (into prism) and angle of refraction (into air) is i_2. The incident ray *EF* and emergent ray *GH* when produced meet at *O*. The angle between these two rays is called angle of deviation 'δ'.

$\angle OFG = i_1 - r_1$ and $\angle OGF = i_2 - r_2$

In ΔFOG, δ is exterior angle

$\delta = \angle OFG + \angle OGF = (i_1 - r_1) + (i_2 - r_2)$

$= (i_1 + i_2) - (r_1 - r_2)$ ...(i)

The normals FN_1 and GN_2 on faces PQ and PR respectively, when produced meet at N. Let $\angle FNG = \theta$ In ΔFGN, $r_1 + r_2 + \theta = 180^\circ$ ...(*ii*)

In quadrilateral $PFNG$, $\angle PFN = 90^\circ$, $\angle PGN = 90^\circ$

$A + 90^\circ + \theta + 90^\circ = 360^\circ$ or $A + \theta = 180^\circ$ ...(*iii*)

Comparing (*ii*) and (*iii*), $r_1 + r_2 = A$ ...(*iv*)

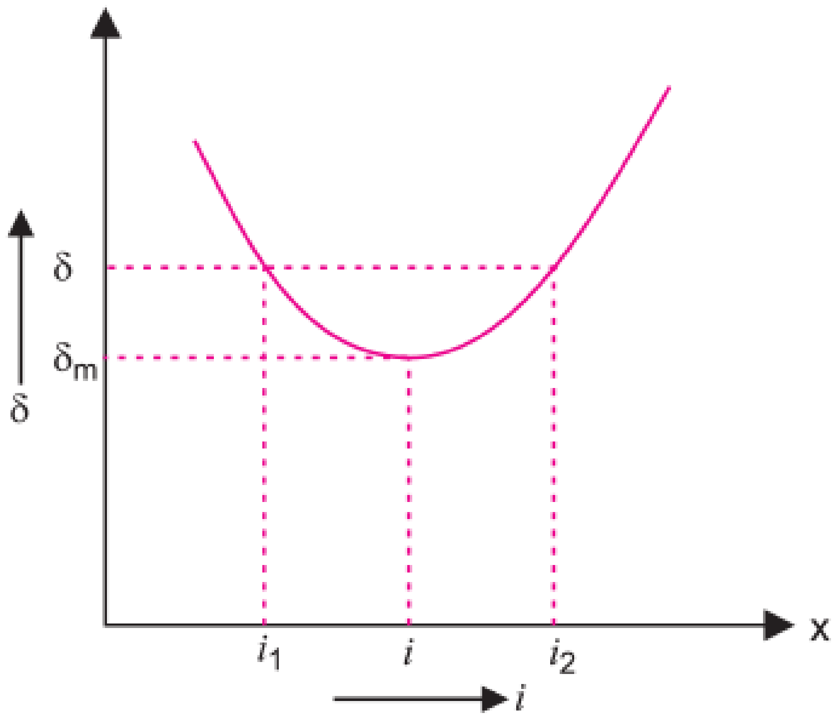
Substituting this value in (*i*), we get

$\delta = i_1 + i_2 - A$ ...(*v*)

or $i_1 + i_2 = A + \delta$ ...(*vi*)

From Snell's law $n = \dfrac{\sin i_1}{\sin r_1} = \dfrac{\sin i_2}{\sin r_2}$ ...(*vii*)

Minimum Deviation : From equation (*v*), it is clear that the angle of deviation depends upon the angle of incidence i_1 As the path of light is reversible, therefore if angle of incidence be i_2 then angle of emergence will be i_1 Thus for two angles of incidence i_1 and i_2 there will be one angle of deviation.



If we determine experimentally, the angles of deviation corresponding to different angles of incidence and then plot i (*on X*-axis) and δ (on *Y*-axis), we get a curve as shown in figure. Clearly if angle of incidence is gradually increased, from a small value, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then begins to increase. Obviously for one angle of deviation (δ) there are two angles of incidences i_1 and i_2 , but ***for one and only one particular value of angle of incidence (i), the angle of deviation is the minimum***. This minimum angle of deviation is represented by δ_m. For minimum deviation i_1 and i_2 become coincident, *i.e.*, $i_1 = i_2 = i$ (say)

So from (*vii*) $r_1 = r_2 = r$ (say)

Hence from (*iv*) and (*vi*), we get $r + r = A$ or $r = A / 2$ or

and $$i + i = A + \delta_m \text{ or } i = \frac{A+\delta_m}{2}$$

Hence from Snell's law, $$n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$