Type 13: Controllability and Observability

For Concept, refer to Control Systems K-Notes, State Variable Analysis

Sample Problem 13:

The second order dynamic system $\frac{dx}{dt} = PX + Qu$, Y=RX has the matrices P, Q and R as follows:

$$P = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}, \ Q = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ R=[0 \ \ 1]$$

The system has the following controllability and observability properties:
(A) Controllable and observable
(B) Not controllable but observable
(C) Controllable but not observable
(D) Not controllable and not observable

Solution: (C) is correct option

Controllability Matrix:

$C = [Q \ \ PQ]$

$$Q = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \ PQ = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

det (C) = 3 × 0 – 1 = – 1 ≠ 0 hence controllable

Observability Matrix

$$O = \begin{bmatrix} R \\ RP \end{bmatrix}$$

$$R=[0 \ \ 1] \ : RP = [0 \ \ 1]\begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix} = Q\,[0 \ \ -3]$$

$$O = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}$$

det (O) = 0 not observable

Unsolved Problems:

Q.1 A state variable model of a system is given by

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix}=\begin{bmatrix}1 & 1\\ -2 & -1\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u,$$

$$y=[1\quad 0]\begin{bmatrix}X_1\\ X_2\end{bmatrix}$$

The system is

(A) Controllable and observable

(B) Controllable but unobservable

(C) Observable and uncontrollable

(D) Uncontrollable and unobservable

Q.2 Consider the system $\frac{dx}{dt}=AX+Bu$ with $A=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ and $B=\begin{bmatrix}p\\ q\end{bmatrix}$ where p and q are arbitrary real numbers. Which of the following statements about the controllability of the system is true?

(A) The system is completely state controllable for any nonzero values of p and q.

(B) Only p=0 and q=0 result in controllability.

(C) The system is uncontrollable for all values of p and q.

(D) We cannot conclude about controllability from the given data.

Q.3 The state variable description of an LTI system is given by

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\end{bmatrix}=\begin{bmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u,\quad y=[1\quad 0\quad 0]\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix}$$

where y is the output and u is the input. The system is controllable for

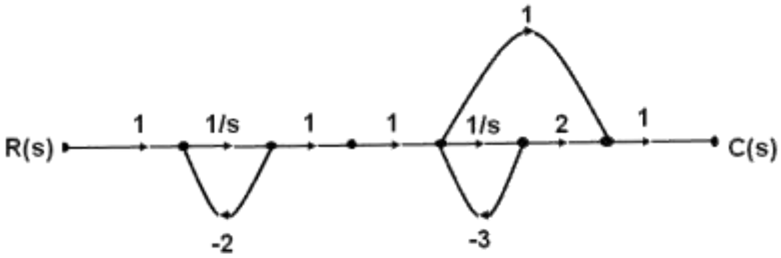
(A) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$

(B) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

(C) $a_1 = 0, a_2 \neq 0, a_3 = 0$

(D) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

Q.4 The state diagram of a system is shown in the given figure:



The system is

(A) Controllable and observable

(B) Controllable but not observable

(C) Not Controllable but observable

(D) Neither Controllable nor observable

Q.5 Consider the system defined by $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u, \quad y = [C_1 \;\; C_2 \;\; C_3] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$

except for an obvious solution choice $C_1=C_2=C_3=0$, find an example of a set of C_1,C_2,C_3 that will make the system unobservable

(A) C=[1 1 1] (B) C=[0 1 1] (C) C=[0 1 0] (D) C=[1 1 0]