CBSE Test Paper 02

CH-04 Principle of Mathematical Induction

1. The greatest positive integer, which divides $(n+1)(n+2)(n+3)\ldots\ldots\ldots\ldots\ldots\ldots(n+r)\forall n \in W$, is
   a. n+r
   b. r
   c. $(r+1)!$
   d. r !
2. If P (n) = 2+4+6+....................+2n , n $\in$ N , then P (k) = k (k + 1) + 2 $\Rightarrow$ P (k + 1) = (k + 1) (k +2) + 2 for all k $\in$ N . So we can conclude that P (n) = n (n + 1) +2 for
   a. n $>$ 2
   b. all n $\in$ N
   c. nothing can be said
   d. n $>$ 1
3. If n is a +ve integer, then $3.5^{2n+1} + 2^{3n+1}$ is divisible by
   a. 64
   b. 24
   c. none of these
   d. 17
4. The nth terms of the series 4+14+30+52+80+114+..... is =
   a. $2n^2 + 2n$
   b. $3n^2 + n$

c. 5n-1

d. $2n^2 + 2$

5. The statement P (n) : "$(n + 3)^2 > 2^{n+3}$ " is true for :

a. all n $\geq$ 2

b. no n $\in$ N ,

c. all n $\geq$ 3

d. all n .

6. Fill in the blanks:

n^3 - 7n + 3 is divisible by ________, for all natural numbers n.

7. Fill in the blanks:

The two basic process of reasoning are ________ and ________.

8. Prove by the principle of mathematical induction that for all n $\in$ N: $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6} n(n + 1)(2n + 1)$

9. Prove the following by using the principle of mathematical induction for all $n \in N$: $1^1 + 3^2 + 5^2 + \ldots + (2n - 1)^2 = \frac{n(2n-1)(2n+1)}{3}$

10. Prove that $(1 + x)^n \geq (1 + nx)$, for all natural number n, where x > -1.

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Solution

1. (d) r !

   Explanation: If n = 0 the given expression becomes 1.2.3.4........r = r! Also when n = 1 one more extra term will be there in the product 2.3.4........(r+1) which is also divisible by r!.

2. (c) nothing can be said

   Explanation: Because the statement is incomplete without the conclusion/ RHS
3. (d) 17

   Explanation: When n = 1 the value is 391 which is divisible by 17.
4. (b) $3n^2 + n$

   Explanation: When n = 1 we get 3. When n = 2 we get 12+2= 14...

5. (b) no n $\in$ N ,

   Explanation: When n = 1 we get $16 > 16$, which is false. when n = 2 we get $25 > 32$,which is false as well. As n = 3,4,5....the inequalty does not hold correct.

6. 3

7. deduction, induction

8. Let P(n) be the statement given by

   P (n): $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6}$ n(n + 1) (2n + 1)

   We have,

   P(1) : $1^2 = \frac{1}{6}$ (1) (1 + 1) (2 $\times$ 1 + 1)

   $\Rightarrow$ 1 = 1

   So, P(1) is true

   Let P(m) be true. Then,

   $1^2 + 2^2 + 3^2 + \ldots + m^2 = \frac{1}{6}$ m (m + 1)(2m + 1) ....(i)

   We wish to show that P(m + 1) is true. For this we have to show that,

$1^2 + 2^2 + 3^2 + \ldots + m^2 + (m + 1)^2 = \frac{1}{6}$ (m +1) {(m +1) +1} {2(m + 1) + 1}

Now, $1^2 + 2^2 + 3^2 + \ldots + m^2 + (m + 1)^2$

= $\{1^2 + 2^2 + 3^2 + \ldots + m^2\} + (m + 1)^2 = \frac{1}{6}$ m(m + 1)(2m + 1) + $(m + 1)^2$ ...[using (i)]

= $\frac{1}{6}$ (m + 1) [m (2m + 1) + 6 (m + 1)}

= $\frac{1}{6}$ (m +1) $\{2m^2 + 7m + 6\}$

= $\frac{1}{6}$ (m + 1) (m + 2) (2m + 3) = $\frac{1}{6}$ (m + 1) {(m + 1) + 1} {2(m + 1) + 1}

So, P(m + 1) is true

Thus, P(m) is true $\Rightarrow$ P(m + 1) is true

Hence, by the principle of mathematical induction, the given result is true for all n $\in$ N.

9. Let P(n)

$1^1 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

For n = 1

$P(1) = (2 \times 1 - 1) = \frac{1(2\times1-1)(2\times1+1)}{3} \Rightarrow 1 = \frac{1\times1\times3}{3}$

$\therefore$ P(1) is true

Let P(n) be true for n = k.

$\therefore P(k) = 1^2 + 3^2 + 5^2 + \ldots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$ .... (i)

For P(k+1)

R.H.S. $= \frac{(k+1)(2k+1)(2k+3)}{3}$

L.H.S. $= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$ [Using (i)]

$= (2k+1)\left[\frac{k(2k-1)}{3} + (2k+1)\right] = (2k+1)\left[\frac{2k^2-k+6k+3}{3}\right]$

$= \frac{(2k+1)(2k^2+5k+3)}{3} = \frac{(2k+1)(k+1)(2k+3)}{3}$

$= \frac{(k+1)(2k+1)(2k+3)}{3}$

$\therefore$ P(k + 1) is true

Thus P(k) is true $\Rightarrow$ P (k + 1) is true

Hence by principle of mathematical induction, P(n) is true for all $n \in N$.

10. **Step I** Let P(n) be the given statement. Then,

$P(n) : (1+x)^n \geq (1+nx)$, for x > - 1

Step II For n = 1, we have $(1+x)$ = $(1+x)$

Thus, $P(n)$ is true when n = 1

Step III For n = k, assume that P(k) is true, i.e.,

$P(k) : (1+x)^k \geq (1+kx)$ for $x > -1$ ...(i)

Step IV For n = k + 1, we have to show that P(k + 1) is true for x > - 1, whenever P(k) is true

Consider the identity

$(1+x)^{k+1} = (1+x)^k(1+x)$ ...(ii)

Given that, $x > -1$, so $(1+x) > 0$

Therefore, by using $(1 + x)^k \geq (1 + kx)$, we get

$(1+x)^{k+1} \geq (1+kx)(1+x)$

i.e., $(1+x)^{k+1} \geq (1+x+kx+kx^2)$ ...(iii)

Here, k is a natural number and $x^2 \geq 0$, which implies $kx^2 \geq 0$

Therefore, $(1+x+kx+kx^2) \geq (1+x+kx)$ and so we obtain

$(1+x)^{k+1} \geq (1+x+kx)$ [using Eq. (iii)]

or $(1+x)^{k+1} \geq [1+(1+k)x]$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the Principle of Mathematical Induction, P(n) is true for all natural numbers.