CBSE Test Paper 02

Chapter 3 Matrices

1. The system of equations, x + y + z = 1, 3x + 6y + z = 8, αx + 2y + 3z = 1 has a unique solution for
   a. all real α
   b. α not equal to 0
   c. all integral α
   d. all rational α
2. If $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then for all natural numbers n, A_n is equal to
   a. $\begin{bmatrix} 1 & 0 \\ 1 & n \end{bmatrix}$
   b. $\begin{bmatrix} n & 0 \\ 1 & 1 \end{bmatrix}$
   c. $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$
   d. none of these
3. If A and B are square matrices of the same order and AB = 3I, then A^{-1} is equal to
   a. $3B^{-1}$
   b. 3 B
   c. $\frac{1}{3}B$
   d. none of these
4. If A and B are square matrices of the same order, then $(A + B)^2 = A^2 + 2AB + B^2$ implies
   a. none of these
   b. AB = BA
   c. AB + BA = O
   d. AB = O
5. If A and B are two matrices such that AB = BA and BA = A, then $A^2 + B^2$ =.
   a. A + B
   b. 2 BA
   c. AB
   d. 2 AB

6. If A and B are matrices of same order, then (3A - 2B)' is equal to ________.
7. If A is matrix of order m × n and B is a matrix such that AB' and B'A are both defined, then order of matrix B is ________.
8. If A is a symmetric matrix, then A^3 is a ________ matrix.
9. Write the value of x - y + z from following equation.
$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$
10. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and A^2 = kA, then write the value of k.
11. Given an example of matrix A and B such that AB = 0 but A $\neq$ 0, B $\neq$ 0.
12. Show by an example that for $A \neq 0,\ B \neq 0$, AB = 0.
13. If the matrix A is both symmetric and skew symmetric, then prove that A will be a Zero matrix.
14. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, Prove that A - A^t is a skew symmetric matrix.
15. Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.
16. Express the matrix A as the sum of a symmetric and a skew symmetric matrix, where
$$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}.$$
17. If AB = BA for any two square matrices, then prove by mathematical induction that $AB^n = BA^n$ for all $n \in N$.
18. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that
$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}.$$

Solution

1. c. all integral

   Explanation: The given system of equations has unique solution,

   if : $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 6 & 1 \\ \alpha & 2 & 3 \end{vmatrix} \neq 0 \Rightarrow 1(18-2) - 1(9-\alpha) + 1(6-6\alpha) \neq 0$

   $\Rightarrow 13 - 5\alpha \neq 0 \Rightarrow \alpha \neq \frac{13}{5}$.

   Therefore , unique solution exists for all integral values of alpha α.

2. c. $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$

   Explanation: $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$,

   A^2 = A.A = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

   A^3 = A.A.A = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$,

   ...A^n = A.A.A........A (n -times) = $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$

3. c. $\frac{1}{3}B$

   Explanation: If A and B are square matrices of the same order and AB = 3 I, then , $\frac{1}{3}AB = IA^{-1} = \frac{1}{3}B$.

4. b. AB = BA

   Explanation:

   If A and B are square matrices of same order, then, product of the matrices is not commutative.Therefore, the given result is true only when AB = BA.

5. a. A + B

   Explanation: AB = B $\Rightarrow$ (AB)A = BA

   $\Rightarrow$ A(BA) = BA $\Rightarrow$ A(A) = A,

   $\Rightarrow A^2 = A$

AB = B $\Rightarrow$ B(AB) = BB

$\Rightarrow$ (BA)B = B^2

$\Rightarrow$ AB = B^2

$\Rightarrow$ B = B^2

$\therefore$ $A^2 + B^2 = A + B$

6. 3A' - 2B'
7. m × n
8. symmetric

9. According to the question,

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Equating the corresponding elements,

$x + y + z = 9$ ...(i)

$x + z = 5$ ...(ii)

and $y + z = 7$ ...(iii)

Putting the value of x + z from Eq. (ii) in Eq. (i)

$y + 5 = 9 \Rightarrow y = 4$

On putting y = 4 in Eq. (iii), we get z = 3

Again, putting z = 3 in Eq. (ii), we get x = 2

$\therefore$ x - y + z = 2 - 4 + 3 = 1

10. According to the question, $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ ...(i)

and $A^2 = kA$ ...(ii)

Now, $A^2 = A \cdot A$

$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix}$ [multiplying row by column]

$= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow$ A^2 = 2A [from Eq. (i)]

Comparing with Eq. (ii),

k = 2

11. $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

Therefore, $AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

12. Let $A = \begin{bmatrix} 0 & -4 \\ 0 & 2 \end{bmatrix} \neq 0$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} \neq 0$

$\therefore AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ Hence proved.

13. It is given that $A^t = A$ & $A^t = -A$

$\Rightarrow$ A = - A

$\Rightarrow$ 2A = O

$\Rightarrow$ A = O

14. Let $P = A - A^t$

$= \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ -3 & -5 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$P' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$P' = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

P' = -P

Hence $A - A^t$ is a skew symmetric matrix.

15. Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

On solving a + 4b = -7 and 2a + 5b = -8 & c + 4d=2 and 2c+5d=4

we get a = 1, b = -2, c = 2, d = 0

$X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

16. We have

$$A=\begin{bmatrix}2&4&-6\\7&3&5\\1&-2&4\end{bmatrix}\text{, then }A'=\begin{bmatrix}2&7&1\\4&3&-2\\-6&5&4\end{bmatrix}$$

Hence $\frac{A+A'}{2}=\frac{1}{2}\begin{bmatrix}4&11&-5\\11&6&3\\-5&3&8\end{bmatrix}=\begin{bmatrix}2&\frac{11}{2}&\frac{5}{2}\\\frac{11}{2}&3&\frac{3}{2}\\\frac{-5}{2}&\frac{3}{2}&4\end{bmatrix}$

And $\frac{A-A'}{2}=\frac{1}{2}\begin{bmatrix}0&-3&-7\\3&0&7\\7&-7&0\end{bmatrix}=\begin{bmatrix}0&-\frac{3}{2}&-\frac{7}{2}\\\frac{3}{2}&0&\frac{7}{2}\\\frac{7}{2}&-\frac{7}{2}&0\end{bmatrix}$

Therefore,

$$\frac{A+A'}{2}+\frac{A-A'}{2}=\begin{bmatrix}2&\frac{11}{2}&\frac{-5}{2}\\\frac{11}{2}&3&\frac{3}{2}\\\frac{-5}{2}&\frac{3}{2}&4\end{bmatrix}+\begin{bmatrix}0&\frac{-3}{2}&\frac{-7}{2}\\\frac{3}{2}&0&\frac{7}{2}\\\frac{7}{2}&\frac{-7}{2}&0\end{bmatrix}=\begin{bmatrix}2&4&-6\\7&3&5\\0&-2&4\end{bmatrix}=A$$

17. For n = 1,we have, $AB^1 = B^1A$

$\Rightarrow$ AB = BA, which is true.

Let it be true for n = m i.e $AB^m = BA^m$.........(1)

Then,for n = m + 1,

$AB^{m+1} = A(B^m B) = (AB^m)B = (B^mA)B$ [by (1)]

$= B^m(AB) = B^m(AB)$ [as AB=BA ,given]

$= (B^mB)A = B^{m+1}A$. So ,it is true for n=m+1

$\therefore$ $AB^n = B^nA$

18. L.H.S. $I+A=\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}0&-\tan\frac{\alpha}{2}\\\tan\frac{\alpha}{2}&0\end{bmatrix}=\begin{bmatrix}1&-\tan\frac{\alpha}{2}\\\tan\frac{\alpha}{2}&1\end{bmatrix}$

Now, $N-A=\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}0&-\tan\frac{\alpha}{2}\\\tan\frac{\alpha}{2}&0\end{bmatrix}=\begin{bmatrix}1&\tan\frac{\alpha}{2}\\-\tan\frac{\alpha}{2}&1\end{bmatrix}$

R.H.S. $=(I-A)\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}=\begin{bmatrix}1&\tan\frac{\alpha}{2}\\-\tan\frac{\alpha}{2}&1\end{bmatrix}\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & -\sin\alpha + \cos\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ -\cos\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \sin\alpha & \sin\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \cos\alpha \end{bmatrix}$$

$$\begin{bmatrix} \frac{\cos\alpha\cos\frac{\alpha}{2} + \sin\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{-\sin\alpha\cos\frac{\alpha}{2} + \cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \frac{-\cos\alpha\sin\frac{\alpha}{2} + \sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{\sin\alpha\sin\frac{\alpha}{2} + \cos\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{-\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \\ \frac{\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} \frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$\therefore$ L.H.S. = R.H.S. Proved.