16. Surface Area and Volume

Exercise 16.1

1. Question

A cuboid is 12 cm long, 9 cm wide and 5 cm high. Calculate the total surface area of the cuboid.

Answer

Length = l = 12 cm

Breadth = b = 9 cm

Height = h = 5 cm

Total surface area of cuboid = 2 × (lb + bh + lh)

= 2 × (12×9 + 9×5 + 12×5)

= 2 × (108 + 45 + 60)

= 2 × 213

= 426 cm^2

Therefore, total surface area of the cuboid is 426 cm^2

2. Question

Three cubes with edges 8 cm, 6 cm and 1 cm respectively are melted together and formed into a single cube. Find the total surface area of the new cube.

Answer

To find the total surface area of the new cube formed we first need to find its edge

Volume of cube = $(edge)^3$

Volume of cube with edge 8 cm = 8^3 = 512 cm^3

Volume of cube with edge 6 cm = 6^3 = 216 cm^3

Volume of cube with edge 1 cm = 1^3 = 1 cm^3

Let the edge of new cube formed be ‘a’

Volume of new cube = a^3

As the three cubes are melted and the new cube is formed

Volume of new cube = volume of all the three cubes

$\Rightarrow a^3 = 512 + 216 + 1$

$\Rightarrow a^3 = 729$

$\Rightarrow a = 9$

Surface area of cube = $6 \times (\text{edge})^2$

$\Rightarrow$ surface area of new cube = $6 \times a^2$

$= 6 \times 9^2$

$= 6 \times 81$

$= 486 \text{ cm}^2$

Therefore, surface area of new cube is 486 cm^2

3. Question

The dimensions of a box are 50 cm × 36 cm × 25 cm. How many square cm of cloth is needed to make a cover for the box?

Answer

The box is like the shape of a cuboid

Length = l = 50 cm

Breadth = b = 36 cm

Height = h = 25 cm

Total surface area of cuboid = 2 × (lb + bh + lh)

$\Rightarrow$ total surface area of box = $2 \times (50 \times 36 + 36 \times 25 + 50 \times 25)$

$= 2 \times (1800 + 900 + 1250)$

$= 2 \times 3950$

$= 7900 \text{ cm}^2$

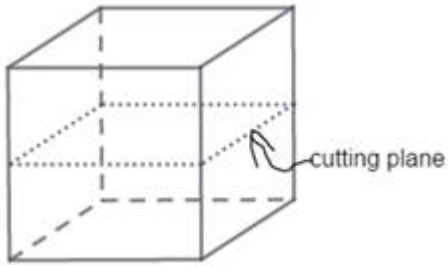
Amount of cloth required to make cover = total surface area of box

Therefore, 7900 cm^2 cloth will be required to make cover for the box

4. Question

The area of each face of a cube is 100 cm^2. If the cube is cut into two equal parts by a plane parallel to the base then find the total surface area of each equal part.

Answer



A cube has 6 surfaces

We will get two cuboids after cutting

The cutting plane will pass through 4 surfaces halving their areas

So, each part will be having 4 surfaces with area 50 cm^2

each part will have remaining 2 surfaces with area 100 cm^2

total surface area of each part = 100 + 100 + 50 + 50 + 50 + 50

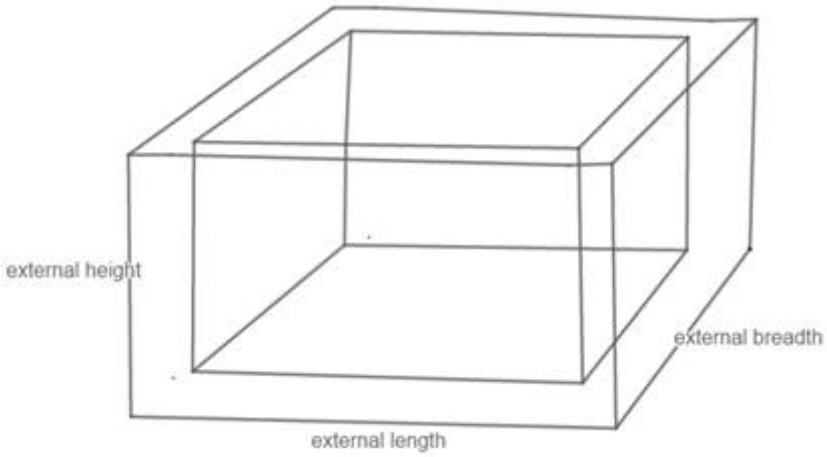
= 400 cm^2

Therefore, total surface area of each part is 400 cm^2

5. Question

An open box is made of wood 3 cm thick. Its external length, breadth and height are 146 cm, 116 cm and 83 cm respectively. Find the cost of painting the inner surface at Rs 2 per 1000 sq. cm.

Answer



the box is as shown in figure

The wood is 3 cm thick

And given length, breadth and height are external which means including the wood thickness

So, to find the inner surface area we need to subtract the wooden thickness from external length, breadth and height

Thus length, breadth and height of inner surface area are as follows

In the length and breadth, we need to subtract 3 from both the sides which means we have to subtract 6 but in the height as the box is open from one end so we need to subtract only 3cm base thickness as seen in the figure

Length = 146 – 6 = 140 cm

Breadth = 116 – 6 = 110 cm

Height = 83 – 3 = 80 cm

The shape of the box is a cuboid and it is open box, therefore, we should subtract the top rectangular surface area from the total inner surface area

Total surface area of cuboid = 2 × (lb + bh + lh) – lb

⇒ total surface area of box = 2 × (140×110 + 110×80 + 140×80) – 140×110

= 2 × (15400 + 8800 + 11200) – 15400

= 2 × 35400 – 15400

= 70800 – 15400

= 55400 cm^2

Cost of painting 1000 cm^2 is Rs 2

Thus, cost of painting 55400 cm^2 = $\frac{55400}{1000} \times 2$

= 55.4 × 2

= 110.8 Rs

Therefore, cost of painting the inner surface = 110.8 Rs

6. Question

The sum of length, breadth and height of a cuboid is 19 cm. If length of the diagonal is 11 cm then calculate the total surface area of the cuboid.

Answer

Let the length, breadth and height be l, b and h respectively

Given:

l + b + h = 19 cm

length of diagonal = 11 cm

length of diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

$\Rightarrow \sqrt{l^2 + b^2 + h^2}$ = 11 cm

Squaring both sides we get

$\Rightarrow l^2 + b^2 + h^2 = 121$

Using identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$\Rightarrow (l + b + h)^2 = l^2 + b^2 + h^2 + 2lb + 2bh + 2hl$

Substituting given values

$\Rightarrow 19^2 = 121 + 2 \times (lb + bh + hl)$

$\Rightarrow 361 = 121 + 2 \times (lb + bh + hl)$

$\Rightarrow 2 \times (lb + bh + hl) = 361 - 121$

$\Rightarrow 2 \times (lb + bh + hl) = 240\ cm^2$

Therefore, total surface area of cuboid is 240 cm^2

7. Question

A room whose floor is a square of side 6 m contains 180 cubic metre of air. Find the height of the room.

Answer

Length of room= l = 6 m

Breadth of room = b = 6 m

Let height of room be h

Room contains 180 cubic metre of air which means volume of room is 180 m^3

Room is in the shape of a cuboid

Volume of cuboid = lbh

$\Rightarrow 180 = 6 \times 6 \times h$

$\Rightarrow 180 = 36h$

$\Rightarrow h = \frac{180}{36}$

$\Rightarrow h = 5$ m

Height of room is 5 m

8. Question

Find the number of bricks, each measuring 22 cm × 10 cm × 7 cm required to construct a wall 44 m long, 1.5 m high and 85 cm thick.

Answer

Length of brick = 22 cm

Breadth of brick = 10 cm

Height of brick = 7 cm

Brick is in the shape of a cuboid

Volume of cuboid = length × breadth × height

⇒ volume of 1 brick = 22 × 10 × 7

= 1540 cm^3

Let n bricks be required to make the wall

Length of wall = 44 m = 4400 cm

Breadth of wall = 85 cm

Height of wall = 1.5m = 150 cm

Volume of wall = 4400 × 85 × 150

= 56100000 cm^3

The wall is completely made from bricks therefore

Volume of n bricks = volume of wall

⇒ n × volume of one brick = 56100000

⇒ n × 1540 = 56100000

$$\Rightarrow n = \frac{56100000}{1540}$$

$$\Rightarrow n = \frac{5610000}{154}$$

Dividing numerator and denominator by 22, we get

$$\Rightarrow n = \frac{255000}{7}$$

⇒ n = 36428.57 ~ 36429

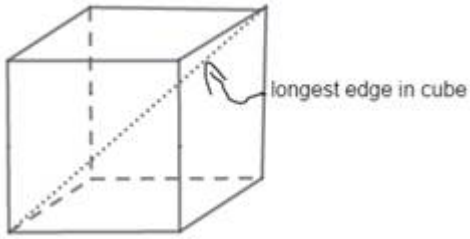
Number of brick required = 36429

9. Question

Find the length of the longest rod that can be placed in a room 10 m long, 8 m wide and 6 m high.

Answer

The longest edge of a cuboid is the body diagonal



So the length of longest rod will be same as the length of body diagonal

Length of room = l = 10 m

Breadth of room = b = 8 m

Height of room = h = 6 m

Length of body diagonal = $\sqrt{l^2 + b^2 + h^2}$

⇒ length of rod = $\sqrt{10^2 + 8^2 + 6^2}$

= $\sqrt{100 + 64 + 36}$

= $\sqrt{100 \times 2}$

Therefore, length of longest rod = $10\sqrt{2}$ m

10. Question

The volume of a cube is 512 cubic m. Find its side.

Answer

Volume = 512 m^3

Let a be the side of cube

Volume of cube = a^3

⇒ a^3 = 512

⇒ a = $\sqrt[3]{512}$

⇒ a = 8 m

11. Question

Find the number of bricks, each measuring 20 cm × 10 cm × 7.5 cm required to construct a wall 5 m long, 30 cm wide and 3 m high.

Answer

Length of brick = 20 cm

Breadth of brick = 10 cm

Height of brick = 7.5 cm

Brick is in the shape of a cuboid

Volume of cuboid = length × breadth × height

⇒ volume of 1 brick = 20 × 10 × 7.5

= 1500 cm^3

Let n bricks be required to make the wall

Length of wall = 5 m = 500 cm

Breadth of wall = 30 cm

Height of wall = 3 m = 300 cm

Volume of wall = 500 × 30 × 300

= 4500000 cm^3

The wall is completely made from bricks therefore

Volume of n bricks = volume of wall

⇒ n × volume of one brick = 56100000

⇒ n × 1500 = 4500000

$\Rightarrow n = \frac{4500000}{1500}$

$\Rightarrow n = \frac{45000}{15}$

⇒ n = 3000

Number of brick required = 3000

12. Question

The dimensions of a cuboid are in the ratio 5 : 3 : 2. If total surface area of the cuboid is 558 cm^2, then find the length of edges of the cuboid.

Answer

Let

Length of cuboid = l = 5x

Breadth of cuboid = b = 3x

Height of cuboid = h = 2x

Such that the ratio is 5:3:2 as mentioned in question where x is any positive number

Total surface area of cuboid = 558 cm^2

Total surface area of cuboid = 2 × (lb + bh + hl)

$\Rightarrow 558 = 2 \times [(5x)(3x) + (3x)(2x) + (2x)(5x)]$

$\Rightarrow \frac{558}{2} = 15x^2 + 6x^2 + 10x^2$

$\Rightarrow 279 = 31x^2$

$\Rightarrow x^2 = \frac{279}{31}$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

x is length therefore x cannot be negative therefore x = 3

therefore,

length of cuboid = 5x = 5 × 3 = 15 cm

breadth of cuboid = 3x = 3 × 3 = 9 cm

height of cuboid = 2x = 2 × 3 = 6 cm

length of edges of cuboid are 15 cm, 9 cm and 6 cm

Exercise 16.2

1. Question

The diameter of a cylinder is 14 cm and its height is 15 cm. Calculate the total surface area and volume of the cylinder.

Answer

Diameter of cylinder = d = 14 cm

$\Rightarrow$ radius of cylinder = $r = \frac{d}{2} = \frac{14}{2} = 7$ cm

Height of cylinder = h = 15 cm

Total surface area will be the sum of curved surface area and the area of two circles flat surfaces

Total surface area of cylinder = $2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 15 + 2 \times \frac{22}{7} \times 7^2$

$= 30 \times 22 + 2 \times 22 \times 7$

$= 660 + 308$

$= 968\ cm^2$

Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 7^2 \times 15$

$= 22 \times 7 \times 15$

$= 22 \times 105$

$= 2310\ cm^3$

Therefore, total surface area of cylinder is 968 cm^2 and volume is 2310 cm^3

2. Question

The height of a right circular cylinder is 7 cm and radius of its base is 3 cm. Find the curved surface area, total surface area and volume of the cylinder.

Answer

Height of cylinder = h = 7 cm

Radius of base of cylinder = r = 3 cm

Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times 3 \times 7$

$= 6 \times 22$

$= 132\ cm^2$

Total surface area will be the sum of the curved surface area and the area of two circles flat surfaces

Total surface area of cylinder = $132 + 2\pi r^2$

$= 132 + 2 \times 3.14 \times 3^2$

$= 132 + 6.28 \times 9$

= 132 + 56.52

= 188.52 cm^2

Volume of cylinder = $\pi r^2 h$

= $\frac{22}{7} \times 3^2 \times 7$

= 9 × 22

= 198 cm^3

Therefore, curved surface area of cylinder is 132 cm^2, total surface area is 188.52 cm^2 and volume is 198 cm^3

3. Question

The area of circular base of a cylinder is 154 cm^2 and its height is 21 cm. Find the volume and curved surface area of the cylinder.

Answer

Area of circular base = 154 cm^2

Height = h = 21 cm

Let r be radius of circular base

Area = πr^2

$\Rightarrow 154 = \frac{22}{7} \times r^2$

$\Rightarrow \frac{154 \times 7}{22} = r^2$

$\Rightarrow r^2 = 7 \times 7 = 49$

$\Rightarrow r = \pm 7$

Radius cannot be negative hence r = 7

Volume of cylinder = $\pi r^2 h$

= $\frac{22}{7} \times 7^2 \times 21$

= 7 × 21 × 22

= 3234 cm^3

Curved surface area of cylinder = $2\pi rh$

= $2 \times \frac{22}{7} \times 7 \times 21$

$= 14 \times 22 \times 3$

$= 924\ cm^2$

Therefore, volume of cylinder is 3234 cm^3 and curved surface area is 924 cm^2

4. Question

The radii of two right circular cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 4. Calculate the ratio of their curved surface areas and the ratio of their volumes.

Answer

Let the radii of two circles r_1 and r_2

$\Rightarrow \frac{r_1}{r_2} = \frac{2}{3}$

Let the heights of two cylinders h_1 and h_2

$\Rightarrow \frac{h_1}{h_2} = \frac{5}{4}$

Curved surface area of cylinder = $2\pi rh$

Ratio of their surface areas = $\frac{2\pi r_1 h_1}{2\pi r_2 h_2}$

$= \frac{r_1}{r_2} \times \frac{h_1}{h_2}$

$= \frac{2}{3} \times \frac{5}{4}$

$= \frac{5}{6}$

Volume of cylinder = $\pi r^2 h$

Ratio of their volumes = $\frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$

$= \frac{r_1^2}{r_2^2} \times \frac{h_1}{h_2}$

$= \frac{2^2}{3^2} \times \frac{5}{4}$

$= \frac{4}{9} \times \frac{5}{4}$

$= \frac{5}{9}$

Therefore, ratio of their curved surface areas is 5:6 and the ratio of their volumes is 5:9

5. Question

A solid cylinder has total surface area of 462 m^2. Its curved surface area is one–third of its total surface area. Find the volume of the cylinder.

Answer

Total surface area of cylinder = 462 m^2

Curved surface area of cylinder = $\frac{1}{3}$ × Total surface area of cylinder

= $\frac{1}{3}$ × 462

= 154 m^2

Curved surface area of cylinder = 154 m^2

We know

Total surface area of cylinder = curved surface area + area of circles at top and bottom of cylinder

⇒ 462 = 154 + area of circles at top and bottom of cylinder

⇒ area of circles at top and bottom of cylinder = 462 – 154 = 308 m^2

Let r be the radius of to and bottom circle of cylinder

area of circles at top and bottom of cylinder = $2\pi r^2$

⇒ $2\pi r^2 = 308$

⇒ $2 \times \frac{22}{7} \times r^2 = 308$

⇒ $r^2 = \frac{308 \times 7}{2 \times 22}$

⇒ $r^2 = 7 \times 7$

⇒ $r = \pm 7$

r is radius and radius cannot be negative hence r = 7 m

now to find volume we need to find one more parameter about the cylinder which is the height

let us assume the height to be h

curved surface area of cylinder = $2\pi rh$

$\Rightarrow 2\pi rh = 154$

$\Rightarrow 2 \times \frac{22}{7} \times 7 \times h = 154$

$\Rightarrow h = \frac{154}{22 \times 2}$

$\Rightarrow h = \frac{7}{2} m$

volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 7^2 \times \frac{7}{2}$

$= 11 \times 7 \times 7$

$= 539\ m^3$

Therefore, volume of cylinder is 539 m^3

6. Question

The lateral curved surface area of a cylinder is 660 cm^2 and its height is 15 cm. Find the volume of the cylinder.

Answer

Height = 15 cm

lateral curved surface area of a cylinder = 660 sq. cm

let r be the radius

lateral curved surface area of a cylinder = $2\pi rh$

$\Rightarrow 2\pi rh = 660$

$\Rightarrow 2 \times \frac{22}{7} \times r \times 15 = 660$

$\Rightarrow 30 \times r = \frac{660 \times 7}{22}$

$\Rightarrow 30 \times r = 30 \times 7$

$\Rightarrow r = 7$ cm

Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 7^2 \times 15$

$= 22 \times 7 \times 15$

$= 2310\ cm^3$

Therefore, volume is 2310 cm^3

7. Question

The volume of a cylinder is 30π cm^3 and the area of it base is 6π cm^2. Find the height of the cylinder.

Answer

Let the radius of base of cylinder be r

Area of base = 6π cm^2

Area of base = πr^2

$\Rightarrow \pi r^2 = 6\pi$

$\Rightarrow r^2 = 6$

Volume of cylinder = 30π cm^3

Let h be the height of cylinder

Volume of cylinder = $\pi r^2 h$

$\Rightarrow \pi r^2 h = 30\pi$

$\Rightarrow 6h = 30$

$\Rightarrow h = 5$ cm

Height of cylinder is 5 cm

8. Question

The volume and curved surface area of a cylinder are 1650 cm^3 and 660 cm^2 respectively. Find the radius and the height of the cylinder.

Answer

Let r be the radius of base and h be the height of cylinder

Curved surface area of cylinder = 660 cm^2

Curved surface area = $2\pi rh$

$\Rightarrow 2\pi rh = 660$

$\Rightarrow 2 \times \frac{22}{7} \times r \times h = 660$

$\Rightarrow r \times h = \frac{660 \times 7}{22 \times 2}$

$\Rightarrow r \times h = \frac{30 \times 7}{2}$

$\Rightarrow r \times h = 15 \times 7$

$\Rightarrow r \times h = 105$

$\Rightarrow rh = 105$

Volume of cylinder = 1650 cm^3

Volume of cylinder = $\pi r^2 h$

$\Rightarrow \pi r^2 h = 1650$

$\Rightarrow \frac{22}{7} \times r \times r \times h = 1650$

Using r × h = 105 we get

$\Rightarrow \frac{22}{7} \times r \times 105 = 1650$

$\Rightarrow 22 \times 15 \times r = 1650$

$\Rightarrow r = \frac{1650}{22 \times 15}$

$\Rightarrow r = \frac{110}{22}$

$\Rightarrow r = 5$ cm

Substitute r = 5 in r × h = 105 to get h

$\Rightarrow 5 \times h = 105$

$\Rightarrow h = \frac{105}{5}$

$\Rightarrow h = 21$ cm

Therefore, radius is 5 cm and height is 21 cm

9. Question

The height and radius of a cylinder are 7.5 cm and 3.5 cm respectively. Find the ratio between the total surface area and curved surface area of the cylinder

Answer

Let r be the radius and h be the height of cylinder

h = 7.5 cm = $\frac{15}{2}$ cm

$r = 3.5 \text{ cm} = \frac{7}{2} \text{ cm}$

total surface area = $2\pi rh + 2\pi r^2$

curved surface area = $2\pi rh$

ratio of total surface area to curved surface area = $\frac{\text{total surface area}}{\text{curved surface area}}$

$\Rightarrow \text{ratio} = \frac{2\pi rh+2\pi r^2}{2\pi rh}$

$\Rightarrow \text{ratio} = \frac{h+r}{h}$

$\Rightarrow \text{ratio} = 1 + \frac{r}{h}$

$\Rightarrow \text{ratio} = 1 + \frac{\frac{7}{2}}{\frac{15}{2}}$

$\Rightarrow \text{ratio} = 1 + \frac{7}{15}$

$\Rightarrow \text{ratio} = \frac{15+7}{15}$

$\Rightarrow \text{ratio} = \frac{22}{15}$

Therefore, ratio between the total surface area and curved surface area of the cylinder is 22:15

10. Question

A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Answer

The volume of earth removed by digging for cylindrical well is the same volume used to make platform which is cuboidal in shape

Therefore, volume of well dug = volume of platform formed

Parameters of well

Diameter = d = 7 m

$\Rightarrow \text{radius} = r = \frac{d}{2} = \frac{7}{2} \text{ m}$

The well is 20 m deep so we can say that its height is 20 m

Height = h = 20 m

Volume of cylinder = $\pi r^2 h$

$\Rightarrow$ volume of well dug = $\frac{22}{7} \times \frac{7^2}{2^2} \times 20$

$= \frac{22}{7} \times \frac{49}{4} \times 20$

$= 22 \times 7 \times 5$

$= 770\ m^3$

Parameters for platform

Length = l = 22 m

Breadth = b = 14 m

Let height be h

Volume of cuboid = l × b × h

$\Rightarrow$ Volume of platform = 22 × 14 × h

$= 308h\ m^3$

As volume of well dug = volume of platform formed

$\Rightarrow 770 = 308h$

$\Rightarrow h = \frac{770}{308}$

Divide numerator and denominator by 7 we get

$\Rightarrow h = \frac{110}{44}$

Divide numerator and denominator by 22 we get

$\Rightarrow h = \frac{5}{2} = 2.5$

Therefore, height of platform is 2.5 m

11. Question

30800 cm^3 of water can be filled in a cylindrical container. The inner radius of the container is 14 cm. Find the inner curved surface area of the container.

Answer

Volume of water filled in cylinder = volume of cylindrical container

Radius = r = 14 cm

Let Height = h

Volume of cylindrical container = 30800

Volume of cylinder = $\pi r^2 h$

$\Rightarrow \pi r^2 h = 30800$

$\Rightarrow \frac{22}{7} \times 14^2 \times h = 30800$

$\Rightarrow \frac{22}{7} \times 14 \times 14 \times h = 30800$

$\Rightarrow 28 \times 22 \times h = 30800$

$\Rightarrow h = \frac{30800}{28 \times 22}$

$\Rightarrow h = \frac{1400}{28}$

Divide numerator and denominator by 14

$\Rightarrow h = \frac{100}{2} = 50$ cm

Curved surface area of cylinder = $2\pi rh$

$\Rightarrow$ inner curved surface area = $2 \times \frac{22}{7} \times 14 \times 50$

$= 100 \times 22 \times 2$

$= 4400\ cm^2$

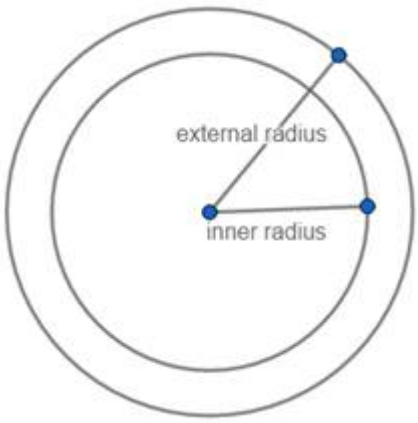
Therefore, inner curved surface area of the container is $4400\ cm^2$

12. Question

A hollow cylinder, open at both ends has a thickness of 2 cm. If the inner diameter is 14 cm and its height is 26 cm then calculate the total surface of the hollow cylinder.

Answer

Cross section of cylinder i.e. top/bottom of cylinder will look like as shown



Inner diameter of cylinder = 14 cm

$\Rightarrow$ Inner radius = $r_i = \frac{14}{2} = 7$ cm

As the thickness is 2 cm it can be seen from figure that external radius = inner radius + 2

$\Rightarrow$ external radius = $r_e = 7 + 2 = 9$ cm

Height of cylinder = h = 26 cm

Curved surface area of cylinder = $2\pi rh$

$\Rightarrow$ external curved surface area of cylinder = $2\pi r_e h$

$\Rightarrow$ inner curved surface area of cylinder = $2\pi r_i h$

As seen the figure the area between the external circle and inner circle will also be counted in total surface area

That area is given by subtracting inner circle area from external circle area

Let us call that area as ring area

$\Rightarrow$ ring area = $\pi r_e^2 - \pi r_i^2$

$= \pi \times (r_e^2 - r_i^2)$

Using identity $a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow$ ring area = $\pi \times (r_e + r_i) \times (r_e - r_i)$

We have two such ring areas at the top and bottom of cylinder

So total ring area = $2 \times \pi \times (r_e + r_i) \times (r_e - r_i)$

Now,

Total surface area of hollow cylinder = external curved surface area of cylinder + inner curved surface area of cylinder + total ring area

$\Rightarrow$ Total surface area of hollow cylinder = $2\pi r_e h + 2\pi r_i h + 2 \times \pi \times (r_e + r_i) \times (r_e - r_i)$

$\Rightarrow$ Total surface area of hollow cylinder = $2\pi h(r_e + r_i) + 2\pi(r_e + r_i)(r_e - r_i)$

$\Rightarrow$ Total surface area of hollow cylinder = $2\pi(r_e + r_i)[h + r_e - r_i]$

Substituting values

$\Rightarrow$ Total surface area of hollow cylinder = $2 \times \frac{22}{7} \times (9 + 7)[26 + 9 - 7]$

$= 2 \times \frac{22}{7} \times 16 \times 28$

$= 32 \times 22 \times 4$

$= 2816$

Therefore, total surface of the hollow cylinder is 2816 cm^2

13. Question

A hollow cylinder, open at both ends has a height of 20 cm. The inner and outer diameters are 26 cm and 30 cm respectively. Find the volume of this hollow cylinder.

Answer

Height of cylinder = h = 20 cm

Inner diameter = 26 cm

$\Rightarrow$ Inner radius = $r_i = \frac{26}{2} = 13$ cm

Inner diameter = 30 cm

$\Rightarrow$ Inner radius = $r_o = \frac{30}{2} = 15$ cm

Volume of hollow cylinder = $\pi(r_i^2 - r_o^2)h$

$\Rightarrow$ volume of hollow cylinder = $\frac{22}{7} \times (15^2 - 13^2) \times 20$

$= \frac{22}{7} \times (225 - 169) \times 20$

$= \frac{22}{7} \times 56 \times 20$

$= 22 \times 8 \times 20$

$= 3520\ cm^3$

Therefore, volume of cylinder is 3520 cm^3

Exercise 16.3

1. Question

The height of a cone is 28 cm and radius of its base is 21 cm. Find its curved surface area, total surface area and its volume.

Answer

Height = h = 28 cm

Radius of base = r = 21 cm

Curved surface area of cone = πrl

Where l = $\sqrt{r^2 + h^2}$

⇒ l = $\sqrt{21^2 + 28^2}$

⇒ l = $\sqrt{441 + 784}$

⇒ l = $\sqrt{1225}$

⇒ l = 35 cm

Therefore,

Curved surface area = $\frac{22}{7}$ × 21 × 35

= 22 × 21 × 5

= 2310 cm^2

Total surface area of cone = curved surface area + area of base

= 2310 + πr^2

= 2310 + $\frac{22}{7}$ × 21 × 21

= 2310 + 22 × 3 × 21

= 2310 + 1386

= 3696 cm^2

Volume of cone = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7}$ × 21 × 21 × 28

= 22 × 21 × 28

= 12936 cm^3

Therefore, curved surface area is 2310 cm^2, total surface area is 3696 cm^2 and volume is 12936 cm^3

2. Question

The volume of a right circular cone is 1232 cm^3 and its height is 24 cm. Find the slant height of the cone.

Answer

Height of cone = h = 24 cm

Volume of cone = 1232 cm^3

Let r be the radius of base of cone

Volume of cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232$

$\Rightarrow \frac{22}{7} \times r^2 \times 8 = 1232$

$\Rightarrow \frac{22}{7} \times r^2 = \frac{1232}{8}$

$\Rightarrow \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{154 \times 7}{22}$

$\Rightarrow r^2 = 7 \times 7$

$\Rightarrow r = 7$

Slant height l = $\sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{7^2 + 24^2}$

$\Rightarrow l = \sqrt{49 + 576}$

$\Rightarrow l = \sqrt{625}$

$\Rightarrow l = 25$ cm

Therefore, slant height of cone = 25 cm

3. Question

The diameter of the base of a cone is 14 m and its slant height is 25 m. Find the total surface area of the cone.

Answer

Diameter of base = 14 m

$\Rightarrow$ radius of base = r = $\frac{14}{2}$ = 7 m

Let h be the height of cone

Slant height = l = 25 m

Total surface area of cone = curved surface area of cone + base area of cone

$\Rightarrow$ total surface area = $\pi rl + \pi r^2$

$= \pi r(l + r)$

$= \frac{22}{7} \times 7 \times (25 + 7)$

$= 22 \times 32$

$= 704\ m^2$

Therefore, total surface area of the cone is 704 m^2

4. Question

Radius of the base of a cone is 14 cm and its slant height is 50 cm. Find the curved surface area and the total surface area of the cone.

Answer

Radius of base of cone = r = 14 cm

Slant height = l = 50 cm

Curved surface area of cone = πrl

$= \frac{22}{7} \times 14 \times 50$

$= 22 \times 2 \times 50$

$= 2200\ cm^2$

Total surface area of cone = curved surface area of cone + base area of cone

$\Rightarrow$ total surface area = $4400 + \pi r^2$

$= 4400 + \frac{22}{7} \times 14 \times 14$

$= 2200 + 22 \times 2 \times 14$

$= 2816\ cm^2$

Therefore, curved surface area is 2200 cm^2 and total surface area is 2816 cm^2

5. Question

The height of a right circular cone is 8 cm and the radius of its base is 6 cm. Find its volume.

Answer

Height of cone = 8 cm

Radius of base = r = 6 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$

$= \frac{22}{7} \times 2 \times 48$

$= \frac{22 \times 96}{7}$

$= \frac{2112}{7}$

$= 301.714 \text{ cm}^3$

Therefore, volume is 301.714 cm^3

6. Question

The curved surface area of a cone is 1884.4 m^2 and its slant height is 12 m. On the basis of this information, calculate the radius of the cone.

Answer

Curved surface area of cone = 1884.4 m^2

Slant height = l = 12 m

Let r be the radius of base of cone

Curved surface area of cone = πrl

$\Rightarrow 1884.4 = \frac{22}{7} \times r \times 12$

$\Rightarrow 1884.4 = \frac{22}{7} \times r \times 12$

$\Rightarrow r = \frac{1884.4 \times 7}{22 \times 12}$

$\Rightarrow r = \frac{13190.8}{264}$

$\Rightarrow r = 49.96 \sim 50 \text{ m}$

Therefore, radius of base of cone is 50 m

7. Question

The area of the base of a right circular cone is 154 cm^2. If its slant height is 25 cm then find the height of the cone.

Answer

Area of base of cone = 154 cm^2

Let r be the radius of cone

Area of base of cone = πr^2

$\Rightarrow \pi r^2 = 154$

$\Rightarrow \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{154 \times 7}{22}$

$\Rightarrow r^2 = 7 \times 7$

$\Rightarrow r = 7$ cm

Slant height = 25 cm

Let h be the height of cone

Slant height = $\sqrt{r^2 + h^2}$

$\Rightarrow 25 = \sqrt{7^2 + h^2}$

Squaring both sides we get

$\Rightarrow 625 = 7^2 + h^2$

$\Rightarrow 625 = 49 + h^2$

$\Rightarrow h^2 = 625 - 49$

$\Rightarrow h^2 = 576$

$\Rightarrow h = \pm 24$

Height cannot be negative therefore h = 24 cm

Height of cone is 24 cm

8. Question

The diameters of two cones are equal and their slant height are in the ratio 5 : 4. If the curved surface area of the smaller cone is 400 cm^2 then find the curved surface area of the bigger cone.

Answer

As the diameters of the cones are equal, their radius is also equal

Let the radius of both cones be r

Let the slant height of bigger cone be l_1 and that of smaller cone be l_2

$l_1 : l_2 = 5:4$

$\Rightarrow \frac{l_1}{l_2} = \frac{5}{4}$

Curved surface area of smaller cone = 400 cm^2

Curved surface area of cone = πrl

Curved surface area of bigger cone = πrl_1

Curved surface area of smaller cone = πrl_2

Consider ratio of curved surface area of bigger cone to smaller cone

$\Rightarrow \frac{\text{curved surface area of bigger cone}}{\text{Curved surface area of smaller cone}} = \frac{\pi rl_1}{\pi rl_2}$

$\Rightarrow \frac{\text{curved surface area of bigger cone}}{400} = \frac{l_1}{l_2}$

$\Rightarrow \frac{\text{curved surface area of bigger cone}}{400} = \frac{5}{4}$

$\Rightarrow$ Curved surface area of bigger cone = $\frac{5 \times 400}{4}$

$\Rightarrow$ Curved surface area of bigger cone = 5 × 100

$\Rightarrow$ Curved surface area of bigger cone = 500 cm^2

Therefore, curved surface area of bigger cone is 500 cm^2

9. Question

The slant height and radius of a cone are in the ratio 7 : 4. If its curved surface area is 792 cm^2, find its radius.

Answer

Let the slant height of cone be l and radius of base of cone be r

Ratio of slant height to radius = 7:4

$\Rightarrow \frac{l}{r} = \frac{7}{4}$

$\Rightarrow l = \frac{7}{4} \times r$

Curved surface area of cone = 792 cm^2

$\Rightarrow \pi rl = 792$

put $l = \frac{7}{4} \times r$

$\Rightarrow \frac{22}{7} \times r \times \frac{7}{4} \times r = 792$

$\Rightarrow \frac{22}{4} \times r^2 = 792$

$\Rightarrow r^2 = \frac{792 \times 4}{22}$

$\Rightarrow r^2 = 36 \times 4$

$\Rightarrow r^2 = 144$

$\Rightarrow r^2 = \sqrt{144}$

$\Rightarrow r = \pm 12$

r is radius and radius cannot be negative thus r = 12 cm

10. Question

The circumference of the base of a 9 m high cone is 44 m. Find the volume of air present in it.

Answer

Height of cone = h = 9 m

Circumference of base of cone = 44 m

Let r be the radius of base of cone

Circumference of base of cone = $2\pi r$

$\Rightarrow 2\pi r = 44$

$\Rightarrow \pi r = 22$

$\Rightarrow \frac{22}{7} \times r = 22$

$\Rightarrow r = \frac{22 \times 7}{22}$

$\Rightarrow r = 7$ m

Volume of air present inside the cone = volume of cone

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 9$

$= 22 \times 7 \times 3$

$= 462\ m^3$

Therefore, volume of air inside cone is 462 m^3

11. Question

A conical vessel whose internal radius is 10 cm and height 18 cm is full of water. The water is emptied into a cylindrical vessel of internal radius 5 cm. Find the height to which the water rises.

Answer

Radius of base of conical vessel = r = 10 cm

Height of conical vessel = h = 18 cm

Volume of water in conical vessel = volume of cone

Volume of cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow$ Volume of water in conical vessel = $\frac{1}{3} \times \pi \times 10^2 \times 18$

$= \pi \times 100 \times 6$

$= 600\pi$

Now this volume of water is transferred in cylindrical vessel whose radius is 5 cm

Let h_1 be the height up to which the water rises in the cylindrical vessel

Volume of water in cylindrical vessel = $\pi(\text{radius})^2 h$

$= \pi \times 5^2 \times h$

$= 25h\pi$

As water is transferred from conical vessel to cylindrical vessel volume of water is same

$\Rightarrow$ Volume of water in conical vessel = Volume of water in cylindrical vessel

$\Rightarrow 600\pi = 25h\pi$

$\Rightarrow 25h = 600$

$\Rightarrow h = \frac{600}{25}$

$\Rightarrow h = 24$ cm

Therefore, the height to which the water rises is 24 cm

12. Question

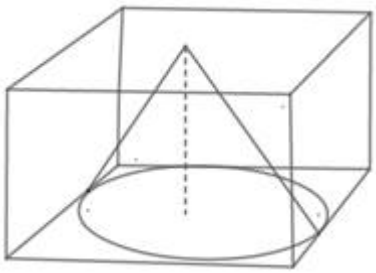
Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 14 cm.

Answer

Volume of cone = $\frac{1}{3}\pi r^2 h$

For volume to be largest the radius of base of cone 'r' and height of cone 'h' should be maximum

In a cube whose edge is 14 cm



As it can be seen from the figure that the maximum height of cone can be 14 cm and also the maximum diameter can be 14 cm

Radius = r = $\frac{14}{2}$ = 7 cm

Height = h = 14 cm

⇒ Volume of cone = $\frac{1}{3} \times \frac{22}{7} \times 7^2 \times 14$

= $\frac{1}{3} \times 22 \times 49 \times 2$

= $\frac{44 \times 49}{3}$

= $\frac{2156}{3}$

= 718.67 cm^2

Therefore, volume of the largest right circular cone that can be cut out of the cube is 718.67 cm^2

13. Question

The base radius and the height of cone are 7 cm and 24 cm respectively. Find the slant height, curved surface area, total surface area and its volume.

Answer

Base radius of cone = r = 7 cm

Height of cone = h = 24 cm

Slant height = l = $\sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{7^2 + 24^2}$

$\Rightarrow l = \sqrt{49 + 576}$

$\Rightarrow l = \sqrt{625}$

$\Rightarrow l = 25$ cm

Curved surface area = πrl

$= \frac{22}{7} \times 7 \times 25$

$= 22 \times 25$

$= 550\ cm^2$

Total surface area of cone = curved surface area + area of base

$= \pi rl + \pi r^2$

$= \pi r(l + r)$

$= \frac{22}{7} \times 7 \times (25 + 7)$

$= 22 \times 32$

$= 704\ cm^2$

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24$

$= 22 \times 7 \times 8$

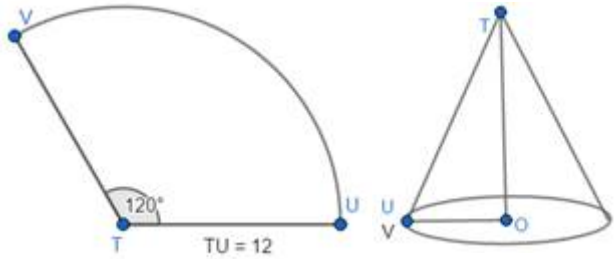
$= 1232\ cm^3$

Therefore, slant height of cone is 25 cm, curved surface area is 550 cm^2, total surface area is 704 cm^2 and volume is 1232 cm^3

14. Question

The radius of a sector of a circle is 12 cm and its angle is 120°. Its straight edges are joined together to form a cone. Find the volume of this cone.

Answer



The sector is as shown in the figure with TU = 12 as radius and ∠VTU = 120°

Now edges VT and UT are joined to form the cone as shown and hence points U and V become the same

For the cone formed TU or TV is the slant height of cone

TU = l = 12 cm

To find volume of cone we need to find base radius of cone and height of cone

Radius of base of cone as seen in figure is OV or OU let us assume that to be r_c

To find base radius of cone we can use the fact that the cone is made by folding the sector which means the curved surface area of cone will be same as area of sector thus we will find those areas and equate them

Area of sector = $\frac{\pi r^2 \theta}{360}$

r is the radius of sector which is TU = 12 cm and θ is the angle which is ∠VTU = 120°

⇒ area of sector = $\frac{\pi (TU)^2 \times \angle VTU}{360}$

= $\frac{\pi \times 12^2 \times 120}{360}$

= $\frac{\pi \times 144}{3}$

= 48π

Curved surface area of cone = $\pi r_c l$

= $\pi \times r_c \times 12$

= $12\pi r_c$

Therefore,

⇒ $48\pi = 12\pi r_c$

⇒ $r_c = 4$ cm

Now we have found the base radius of cone we need to find one more parameter which is the height to find volume

Let the height of the cone be h which is TO in the diagram

using the formula

$l = \sqrt{r_c^2 + h^2}$

$\Rightarrow 12 = \sqrt{4^2 + h^2}$

Squaring both the sides

$\Rightarrow 144 = 16 + h^2$

$\Rightarrow h^2 = 144 - 16$

$\Rightarrow h^2 = 128$

$\Rightarrow h^2 = 64 \times 2$

$\Rightarrow h = 8\sqrt{2}$ cm

Volume of cone $= \frac{1}{3}\pi r_c^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4^2 \times 8\sqrt{2}$

$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 8\sqrt{2}$

$= \frac{22 \times 128\sqrt{2}}{21}$

$= \frac{2816\sqrt{2}}{21}$

$= 134.095 \times \sqrt{2}$

$= 134.095 \times 1.414$

$= 189.61$ cm^3

Therefore, volume of cone is 189.61 cm^3

Exercise 16.4

1. Question

Find the surface area and volume of a sphere whose radius is 1.4 cm.

Answer

Radius of sphere = r = 1.4 cm

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 1.4 \times 1.4$

$= 5.6 \times 22 \times 0.2$

$= 24.64\ cm^2$

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$

$= \frac{4}{3} \times 22 \times 0.2 \times 1.4 \times 1.4$

$= \frac{4}{3} \times 8.624$

$= 11.49 \sim 11.5\ cm^3$

Therefore, surface area of sphere is 24.64 cm^2 and volume of sphere is 11.5 cm^3

2. Question

The surface area of a sphere is 616 cm^2. Find its volume.

Answer

Surface area of sphere = 616 cm^2

Let the radius of sphere be r

Surface area of sphere = $4\pi r^2$

$\Rightarrow 4\pi r^2 = 616$

$\Rightarrow \pi r^2 = \frac{616}{4}$

$\Rightarrow \pi r^2 = 154$

$\Rightarrow \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{154 \times 7}{22}$

$\Rightarrow r^2 = 7 \times 7$

$\Rightarrow r = 7\ cm$

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7^3$

$= \frac{4}{3} \times 22 \times 7^2$

$= \frac{4 \times 1078}{3}$

$= \frac{4312}{3}$

$= 1437.33 \text{ cm}^3$

Therefore, volume of sphere is 1437.33 cm^3

3. Question

The radius of a hemisphere is 4.5 cm. Find its surface area and its volume.

Answer

Hemisphere is half of a sphere

So the volume and area will be halved

But for total surface area there would be one more area which is the flat circle of a hemisphere

Radius of hemisphere = r = 4.5 cm = $\frac{9}{2}$

Surface area of hemisphere = $3\pi r^2$

$= 3 \times \frac{22}{7} \times \frac{9^2}{2^2}$

$= 3 \times \frac{11}{7} \times \frac{81}{2}$

$= \frac{3 \times 891}{14}$

$= \frac{2673}{14}$

$= 190.92 \text{ cm}^2$

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{9 \times 9 \times 9}{2 \times 2 \times 2}$

$= \frac{11 \times 3 \times 9 \times 9}{7 \times 2}$

$= \frac{2673}{14}$

$= 190.92\ cm^3$

Therefore, surface area is $190.92\ cm^2$ and volume is $190.92\ cm^3$

4. Question

Volume of a sphere is $38808\ cm^3$. Find its surface area.

Answer

Volume of sphere = $38808\ cm^3$

Let r be the radius of sphere

Volume of sphere = $\frac{4}{3}\pi r^3$

$\Rightarrow \frac{4}{3}\pi r^3 = 38808$

$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = 38808$

$\Rightarrow r^3 = \frac{38808 \times 7 \times 3}{4 \times 22}$

$\Rightarrow r^3 = \frac{1764 \times 21}{4}$

$\Rightarrow r^3 = 441 \times 21$

$\Rightarrow r^3 = 9261$

$\Rightarrow r = 21\ cm$

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 21 \times 21$

$= 4 \times 22 \times 3 \times 21$

$= 5544\ cm^2$

Therefore, surface area is $5544\ cm^2$

5. Question

A cylinder made out of lead has radius 4 cm and height 10 cm. It is melted to form lead shots of radius 2 cm. Find the number of lead shots.

Answer

Radius of each lead shot = $r_s = 2\ cm$

Radius of cylinder = $r_c = 4\ cm$

Height of cylinder = h = 10 cm

volume of sphere = $\frac{4}{3}\pi r_s^3$

Volume of cylinder= $\pi r_c^2 h$

The lead shots area made by melting the cylinder which means same melted volume is used to make the lead shots thus the volume remains the same

Let 'n' be the number of lead shots formed

i.e. volume of n lead shots = volume of cylinder

$\Rightarrow n \times \frac{4}{3}\pi r_s^3 = \pi r_c^2 h$

$\Rightarrow 4 \times r_s^3 \times n = 3 \times r_c^2 \times h$

Substituting values

$\Rightarrow 4 \times 2^3 \times n = 3 \times 4^2 \times 10$

$\Rightarrow 4 \times 8 \times n = 3 \times 16 \times 10$

$\Rightarrow 4 \times n = 3 \times 2 \times 10$

$\Rightarrow 4 \times n = 3 \times 20$

$\Rightarrow n = 3 \times 5$

$\Rightarrow n = 15$

Number of lead shots made = 15

6. Question

A hollow spherical shell is 2 cm thick. If its outer radius is 8 cm, then find the volume of the metal to make such a shell.

Answer

Outer radius = r_1 = 8 cm

The shell is 2 cm thick therefore we must subtract 2 cm from outer radius to get the inner radius

Inner radius = r_2 = 8 – 2 = 6 cm

Volume of metal required to make shell = volume of shell

Volume of shell we can get by subtracting the volume of hollow part of sphere from the complete volume of sphere with 8 cm radius

$\Rightarrow$ Volume of shell = $\frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_2^3$

$= \frac{4}{3}\pi \times (r_1^3 - r_2^3)$

Substituting values

$\Rightarrow$ Volume of shell $= \frac{4}{3} \times \frac{22}{7} \times (8^3 - 6^3)$

$= \frac{4}{3} \times \frac{22}{7} \times (512 - 216)$

$= \frac{4}{3} \times \frac{22}{7} \times 296$

$= \frac{88 \times 296}{21}$

$= 1240.38 \text{ cm}^3$

Therefore, volume of the metal to make the shell is 1240.38 cm^3

7. Question

How many cones of radius 3 cm and height 6 cm can be made by melting a metallic sphere of radius 9 cm.

Answer

base radius of cone= $r_c = 3$ cm

Height of cone = $h = 6$ cm

Radius of sphere = $r_s = 9$ cm

volume of cone = $\frac{1}{3}\pi r_c^2 h$

Volume of sphere = $\frac{4}{3}\pi r_s^3$

Let n be the number of cones made

As the sphere is melted and then the cones are made from the same amount of melted sphere therefore the volume remains same

i.e. volume of n cones made = volume of sphere

$\Rightarrow n \times \frac{1}{3}\pi r_c^2 h = \frac{4}{3}\pi r_s^3$

$\Rightarrow n r_c^2 h = 4 r_s^3$

Substituting values

$\Rightarrow n \times 3^2 \times 6 = 4 \times 9^3$

$\Rightarrow n \times 6 = 4 \times 9 \times 9$

$\Rightarrow n = 6 \times 9$

$\Rightarrow n = 54$

Number of cones formed = 54

8. Question

Eight spheres of equal radii are obtained by melting a metallic sphere of radius 10 cm. Find the surface area of each sphere thus obtained.

Answer

Let the radius of small spheres formed be r_2

Radius of sphere which is melted = r_1 = 10 cm

As the sphere is melted and then the small spheres are made from the same amount of melted sphere, therefore, the volume remains same

8 small spheres are made

Volume of sphere = $\frac{4}{3}\pi r^3$

$\Rightarrow 8 \times \frac{4}{3}\pi {r_2}^3 = \frac{4}{3}\pi {r_1}^3$

$\Rightarrow 8{r_2}^3 = {r_1}^3$

Substituting values

$\Rightarrow 8 \times {r_2}^3 = 10^3$

$\Rightarrow 8 \times {r_2}^3 = 1000$

$\Rightarrow {r_2}^3 = \frac{1000}{8}$

$\Rightarrow r_2 = \frac{10}{2} = 5$ cm

Surface area of sphere = $4\pi r^2$

$\Rightarrow$ surface area of small spheres made = $4\pi {r_2}^2$

$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

$= 100\pi$ cm^2

Therefore, surface area of each sphere obtained is 100π cm^2

9. Question

If surface area of a sphere is 5544 cm^2, then find the volume of the sphere.

Answer

Surface area of sphere = 5544 cm^2

Let r be the radius of sphere

$\Rightarrow 4\pi r^2 = 5544$

$\Rightarrow \pi r^2 = 1386$

$\Rightarrow \frac{22}{7} \times r^2 = 1386$

$\Rightarrow r^2 = 63 \times 7$

$\Rightarrow r^2 = 441$

$\Rightarrow r = \pm 21$

r is radius and radius cannot be negative thus r = 21

volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= 4 \times 22 \times 3 \times 7 \times 21$

= 38808 cm^3

Therefore, volume of the sphere is 38808 cm^3

10. Question

How many spherical lead shots each 4.2 cm in diameter can be obtained from a rectangular solid of lead with dimensions 66 cm, 42 cm and 21 cm.

Answer

Diameter of spherical lead shot = 4.2 cm

$\Rightarrow$ radius of spherical lead shots = r = $\frac{4.2}{2}$ = 2.1 cm

Length of solid lead = l = 66 cm

Breadth = b = 42 cm

Height = h = 21 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of rectangular lead = lbh

As the spherical lead shots are obtained from rectangular lead volume remains same

Let n spherical lead shots are made

$\Rightarrow n \times \frac{4}{3}\pi r^3 = lbh$

Substituting values

$\Rightarrow n \times \frac{4}{3} \times \frac{22}{7} \times 2.1^3 = 66 \times 42 \times 21$

$\Rightarrow n \times \frac{4}{3} \times \frac{1}{7} \times 2.1^2 = 3 \times 42 \times 10$

$\Rightarrow n \times \frac{4}{3} \times \frac{1}{7} \times 2.1 = 3 \times 20 \times 10$

$\Rightarrow n \times \frac{4}{3} \times 0.3 = 600$

$\Rightarrow n \times 4 \times 0.1 = 600$

$\Rightarrow n \times 4 \times \frac{1}{10} = 600$

$\Rightarrow n \times 4 = 6000$

$\Rightarrow n = 1500$

Therefore, 1500 spherical lead shots are formed

11. Question

A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?

Answer

Diameter of sphere = 6 cm

Radius of sphere = $r_s = \frac{6}{2} = 3$ cm

The diameter of cylindrical vessel = 12 cm

Radius of cylindrical vessel = $r_c = \frac{12}{2} = 6$ cm

The volume of sphere submerged in water is equal to the volume of water rise

Let h be the rise of water level

it is given that the sphere is submerged completely

thus, volume of sphere = volume of water rise

volume of sphere = $\frac{4}{3}\pi r_s^3$

volume of cylinder = $\pi r_c^2 h$

$\Rightarrow \frac{4}{3}\pi r_s^3 = \pi r_c^2 h$

$\Rightarrow 4r_s^3 = 3r_c^2 h$

$\Rightarrow 4 \times 3^3 = 3 \times 6^2 \times h$

$\Rightarrow 4 \times 9 \times 3 = 3 \times 36 \times h$

$\Rightarrow 4 \times 3 = 3 \times 4 \times h$

$\Rightarrow 3 = 3 \times h$

$\Rightarrow h = 1$ cm

Therefore, rise in water level in the cylindrical vessel is 1 cm

12. Question

A hemispherical bowl of internal radius 9 cm contains liquid. This liquid is to be filled in cylindrical bottles of diameter 3 cm and height 4 cm. How many bottles are required to empty the bowl?

Answer

Radius of hemispherical bowl = $r_h = 9$ cm

Diameter of cylindrical bottle = 3 cm

Radius of cylindrical bottle = $r_c = \frac{3}{2}$ cm

Height of cylindrical bottle = h = 4 cm

The volume of water is transferred from hemispherical bowl to cylindrical bottles

Suppose there are n bottles then volume of water filled in n bottles will be same as volume of water in hemispherical bowl

Volume of hemisphere = $\frac{2}{3}\pi r_h^3$

Volume of cylinder = $\pi r_c^2 h$

$\Rightarrow \frac{2}{3}\pi r_h^3 = n \times \pi r_c^2 h$

$\Rightarrow 2r_h^3 = 3nr_c^2h$

Substitute values

$\Rightarrow 2 \times 9^3 = 3n \times \frac{3^2}{2^2} \times 4$

$\Rightarrow 2 \times 9^3 = 3n \times 9$

$\Rightarrow 2 \times 9^2 = 3n$

$\Rightarrow 162 = 3n$

$\Rightarrow n = 54$

Therefore, 54 bottles are required to empty the bowl

13. Question

The diameter of a sphere is 0.7 cm. By putting 3000 such spheres in a tank full of water, the tank is emptied completely. Find the volume of water that overflows.

Answer

Total volume of sphere = 3000 × volume of one sphere

diameter of sphere = 0.7 cm

radius of sphere = $r = \frac{0.7}{2} = \frac{\frac{7}{10}}{2} = \frac{7}{20}$ cm

Volume of sphere = $\frac{4}{3}\pi r^3$

$\Rightarrow$ volume of one sphere = $\frac{4}{3} \times \frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times \frac{7}{20}$

$= \frac{88}{3} \times \frac{49}{8000}$ cm^3

$= \frac{11}{3} \times \frac{49}{1000}$ cm^3

$= \frac{539}{3000}$ cm^3

3000 such spheres are submerged

Total volume of sphere = 3000 × volume of one sphere

$= 3000 \times \frac{539}{3000}$

$= 539$ cm^2

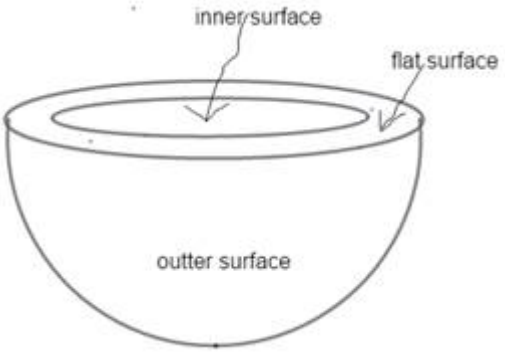
The total volume of spheres submerged in tank is the total amount of water that overflows

Therefore, 539 cm^2 volume of water overflows

14. Question

The outer and inner diameter of a hollow hemi–spherical vessel are 43 cm and 42 cm. Find the cost of colouring the vessel at 7 paise per square cm.

Answer



The hemispherical bowl will look like as shown

To find the cost required to paint we first need to calculate its total surface area

Total surface area will include the area of outter curved surface the inner curved surface and the flat surface as shown

Diameter of outer surface = 43 cm

Radius of outer surface = $r_o = \frac{43}{2}$ cm = 21.5 cm

Diameter of inner surface = 42 cm

Radius of inner surface = $r_i = \frac{42}{2}$ cm = 21 cm

Curved surface area of hemisphere = $2\pi r^2$

$\Rightarrow$ outter curved surface area = $2\pi r_o^2$

$= 2 \times \pi \times \frac{43^2}{2^2}$

$= \frac{1849}{2}\pi$ cm^2

$= 924.5\pi$ cm^2

$\Rightarrow$ inner curved surface area = $2\pi r_i^2$

$= 2 \times \pi \times 21^2$

$= 882\pi \text{ cm}^2$

The flat area can be found by subtracting the inner circle area from outter circle area

$\Rightarrow$ flat area $= \pi r_o^2 - \pi r_i^2$

$= \pi(21.5)^2 - \pi(21)^2$

$= 462.25\pi - 441\pi$

$= 21.25\pi \text{ cm}^2$

Total surface area of hemispherical vessel = outter curved surface area + inner curved surface area + flat area

$\Rightarrow$ Total surface area of hemispherical vessel $= 924.5\pi + 882\pi + 21.25\pi = 1827.75\pi \text{ cm}^2$

Now cost of colouring 1 cm^2 is 7 paise

So, cost of colouring $1827.75\pi \text{ cm}^2 = 7 \times 1827.75 \times \pi$

$= 7 \times 1827.75 \times \frac{22}{7}$

$= 1827.75 \times 22$

$= 40210.5$ paise

100 paise is 1 Rs

$\Rightarrow 40210.5 \text{ paise} = \frac{40210.5}{100} = 402.105$ Rs

Therefore, cost of colouring the vessel is 402.105 Rs

Miscellaneous Exercise 16

1. Question

Total surface area of a cube is 486 cm^2. The side of the cube is:

A. 6 cm

B. 8 cm

C. 9 cm

D. 7 cm

Answer

Let the side of cube be a

Total surface area of cube = $6a^2$

$\Rightarrow 6a^2 = 486$

$\Rightarrow a^2 = 81$

$\Rightarrow a = 9$ cm

2. Question

The length, breadth and height of a cuboid are 9 m, 2 m and 1 m respectively. The lateral surface area of the cuboid is:

A. 12 m^2

B. 11 m^2

C. 21 m^2

D. 22 m^2

Answer

Length of cuboid = l = 9 m

Breadth of cuboid = b = 2 m

Height of cuboid = h = 1 m

Lateral surface area of cuboid is surface area without the top and bottom surfaces

So here we won't consider the area l × b

Lateral surface area = 2lh + 2bh

= 2h(l + b)

= 2 × 1 × (9 + 2)

= 22 m^2

3. Question

The diameter of a sphere is 6 cm. Its volume is—

A. 16π cm^3

B. 20π cm^3

C. 36π cm^3

D. 30π cm^3

Answer

Diameter of sphere = 6 cm

Radius of sphere = r = $\frac{6}{2}$ = 3 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

= $\frac{4}{3} \times \pi \times 3^3$

= $4 \times 3^2 \times \pi$

= 36π cm^3

4. Question

The radius of base of a cylinder is 14 cm and its height is 10 cm. The curved surface area of the cylinder is—

A. 810 cm^2

B. 880 cm^2

C. 888 cm^2

D. 890 cm^2

Answer

Radius of cylinder = r = 14 cm

Height of cylinder = h = 10 cm

Curved surface area of cylinder = $2\pi rh$

= $2 \times \frac{22}{7} \times 14 \times 10$

= $2 \times 22 \times 2 \times 10$

= 880 cm^2

5. Question

The volume of a cone is 308 cm^3. Its height is 6 cm. The radius of its base is—

A. 7 cm

B. 8 cm

C. 6 cm

D. None of these

Answer

Let r be the base radius of cone

Volume of cone = 308 cm^3

Height = h = 6 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow 308 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 6$

$\Rightarrow \frac{308 \times 7}{22} = r^2 \times 2$

$\Rightarrow 14 \times 7 = r^2 \times 2$

$\Rightarrow r^2 = 7 \times 7$

$\Rightarrow r = 7$ cm

6. Question

The diameter of a solid metallic hemisphere is 42 cm. Find the cost of polishing its total surface at 20 paise per square cm.

Answer

Diameter of hemisphere = 42 cm

Radius of hemisphere = r = $\frac{42}{2}$ = 21 cm

Total surface area of hemisphere = $3\pi r^2$

$= 3 \times \frac{22}{7} \times 21^2$

$= 3 \times 22 \times 3 \times 21$

= 4158 cm^2

Cost of polishing 1 sq. cm is 20 paise

$\Rightarrow$ cost of polishing 4158 cm^2 = 4158 × 20

= 83160 paise

100 paise is 1 Rs

$\Rightarrow$ 83160 paise = $\frac{83160}{100}$ = 831.6 Rs

Therefore, cost of polishing its total surface is 831.6 Rs

7. Question

A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find the ratio of their volumes.

Answer

The cone, hemisphere and cylinder stand on equal bases which means their base radius is same

Let that base radius be r

It is given that height = r

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$

Volume of hemisphere = $\frac{2}{3}\pi r^3$

Volume of cylinder = $\pi r^2 h = \pi r^3$

Ratio of cone to hemisphere = $\frac{\frac{1}{3}\pi r^3}{\frac{2}{3}\pi r^3} = \frac{1}{2}$

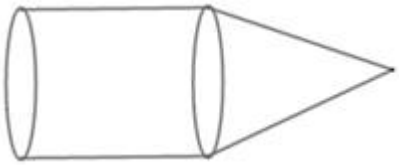
Ratio of hemisphere to cylinder = $\frac{\frac{2}{3}\pi r^3}{\pi r^3} = \frac{2}{3}$

⇒ ratio of volumes of all three solid = 1:2:3

8. Question

The left part of a solid is in the shape of a cylinder while the right part is in the shape of cone. The diameter of cylinder is 14 cm and height is 40 cm while the diameter of the conical part is 14 cm and height is 12 cm. Find the volume of the solid.

Answer



The solid will look like as shown

Diameter of cylinder = 14 cm

Radius of cylinder = r = $\frac{14}{2}$ = 7 cm

Height of cylinder = h = 40 cm

Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 7^2 \times 40$

$= 22 \times 7 \times 40$

$= 6160\ cm^3$

Diameter of base of cone = 14 cm

Radius of base of cone = $r_1 = \frac{14}{2} = 7$ cm

Height of cone = 12 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 12$

$= 22 \times 7 \times 4$

$= 616\ cm^3$

Volume of solid = Volume of cylinder + Volume of cone

$= 6160 + 616$

$= 6776\ cm^3$

Therefore, volume of solid is $6776\ cm^3$

9. Question

Find the number of cones of radius 3 cm and height 6 cm that can be made by melting a metallic sphere of radius 9 cm.

Answer

Radius of metal sphere = $r_s = 9$ cm

Radius of cone formed = $r_c = 3$ cm

Height of cone = h = 6 cm

The sphere is melted and the volume of sphere is used to make cones

Volume of sphere = $\frac{4}{3}\pi {r_s}^3$

Volume of cone = $\frac{1}{3}\pi {r_c}^2 h$

Let n cones are made then the volume of n cones is same as the volume of sphere

$\Rightarrow \frac{4}{3}\pi r_s^3 = n \times \frac{1}{3}\pi r_c^2 h$

$\Rightarrow 4r_s^3 = nr_c^2 h$

Substitute values

$\Rightarrow 4 \times 9^3 = n \times 3^2 \times 6$

$\Rightarrow 4 \times 9^2 = n \times 6$

$\Rightarrow 4 \times 9^2 = 6n$

$\Rightarrow 324 = 6n$

$\Rightarrow n = 54$

Therefore, 54 cones are made

10. Question

A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Answer

Length of tank = l = 20 m

Breadth of tank = b = 15 m

Height of tank = h = 6 m

Volume of water in tank = volume of cuboidal tank

Volume of cuboid = lbh

$\Rightarrow$ volume of cuboidal tank = 20 × 15 × 6

= 1800 m^3

1 m^3 is 1000 litres

$\Rightarrow$ 1800 m^3 = 1800 × 1000 = 1800000 litres

150 litres of water is required per head per day and the population of village is 4000

$\Rightarrow$ volume of water consumed by 4000 people per day = 4000 × 150 = 600000 litres

In one day village consumes 600000 litres of water

$\Rightarrow$ days require to consume 1800000 litres of water = $\frac{1800000}{600000} = 3$

Thus water of tank will last 3 days

11. Question

Three solid sphere of iron whose radii are 6 cm, 8 cm and 10 cm respectively are melted into a single solid sphere. Find the radius of the solid sphere.

Answer

Three spheres are melted and one big sphere is made out of the melted spheres

Thus volume of sphere formed will be same as volume of 3 melted spheres

Let the radius of spheres be $r_1 = 6$ cm, $r_2 = 8$ cm and $r_3 = 10$ cm

$\Rightarrow$ Volume of spheres melted $= \frac{4}{3}\pi {r_1}^3 + \frac{4}{3}\pi {r_2}^3 + \frac{4}{3}\pi {r_3}^3$

$= \frac{4}{3}\pi({r_1}^3 + {r_2}^3 + {r_3}^3)$

Let r be the radius of sphere formed

$\Rightarrow$ Volume of sphere formed $= \frac{4}{3}\pi r^3$

As volumes are same

$\Rightarrow$ Volume of spheres melted = Volume of sphere formed

$\Rightarrow \frac{4}{3}\pi({r_1}^3 + {r_2}^3 + {r_3}^3) = \frac{4}{3}\pi r^3$

$\Rightarrow {r_1}^3 + {r_2}^3 + {r_3}^3 = r^3$

Substituting values

$\Rightarrow 6^3 + 8^3 + 10^3 = r^3$

$\Rightarrow 216 + 512 + 1000 = r^3$

$\Rightarrow r^3 = 1728$

$\Rightarrow r = 12$ cm

Therefore, radius of sphere made is 12 cm

12. Question

A conical vessel of radius 10 cm and height 18 cm is filled with water. If the water is poured in a cylindrical vessel of radius 5 cm, find the height of water in the cylindrical vessel.

Answer

Base radius of conical vessel = r_1 = 10 cm

Height of conical vessel = h_1 =18 cm

Base radius of cylindrical vessel = r_2 = 5 cm

Let h be the height of water in the cylindrical vessel

(note that we does not need to know the height of cylindrical vessel just assume that the height will be till the water level)

As the water is transferred from conical vessel to cylindrical vessel volume is unchanged

Volume of conical vessel = $\frac{1}{3}\pi r_1^2 h_1$

Volume of cylindrical vessel = $\pi r_2^2 h$

$\Rightarrow \frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h$

$\Rightarrow r_1^2 h_1 = 3r_2^2 h$

Substituting values

$\Rightarrow 10^2 \times 18 = 3 \times 5^2 \times h$

$\Rightarrow 100 \times 6 = 25 \times h$

$\Rightarrow h = 4 \times 6$

$\Rightarrow h = 24$ cm

Therefore, height of water in the cylindrical vessel is 24 cm

13. Question

From a wax cuboid of dimensions 11 cm × 3.5 cm × 2.5 cm, a candle of diameter 2.8 cm is made. Find the length of the candle.

Answer

Length of wax cuboid = l = 11 cm

Breadth of wax cuboid = b = 3.5 cm = $\frac{7}{2}$ cm

Height of wax cuboid = h = 2.5 cm = $\frac{5}{2}$ cm

Diameter of candle = 2.8 cm

Radius of candle = r = $\frac{2.8}{2}$ = 1.4 cm

Let the length of candle be l_1

The candle is cylindrical so the length of candle will be its height

Candle is made from wax cuboid hence volume of cuboid wax is equal to volume of cylindrical candle candle

Volume of wax cuboid = lbh

Volume of cylindrical candle = $\pi r^2 l_1$

$\Rightarrow lbh = \pi r^2 l_1$

$\Rightarrow 11 \times \frac{7}{2} \times \frac{5}{2} = \frac{22}{7} \times 1.4^2 \times l_1$

$\Rightarrow \frac{7}{2} \times \frac{5}{2} = 2 \times 0.2 \times 1.4 \times l_1$

$\Rightarrow \frac{7}{2} \times \frac{5}{2} = 0.56 \times l_1$

$\Rightarrow \frac{1}{2} \times \frac{5}{2} = 0.08 \times l_1$

$\Rightarrow \frac{1}{2} \times \frac{5}{2} = \frac{8}{100} \times l_1$

$\Rightarrow l_1 = \frac{100 \times 5}{4 \times 8}$

$\Rightarrow l_1 = \frac{25 \times 5}{8}$

$\Rightarrow l_1 = \frac{125}{8}$

$\Rightarrow l_1 = 15.625$ cm

Therefore, candle is 15.625 cm long

14. Question

The diameter of a metallic sphere is 6 cm. It is melted section. If the length of the wire is 36 m, then find the radius of the wire.

Answer

Diameter of sphere = 6 cm

Radius of sphere = $r_s = \frac{6}{2} = 3$ cm

Length of wire = h = 36 m = 3600 cm

Let the radius of wire be r_w

Wire is cylindrical

Wire is formed by melting the sphere thus volume of wire is same as volume of sphere

Volume of sphere = $\frac{4}{3}\pi r_s^3$

Volume of cylindrical wire = $\pi r_w^2 h$

$\Rightarrow \frac{4}{3}\pi r_s^3 = \pi r_w^2 h$

$\Rightarrow \frac{4}{3} \times r_s^3 = r_w^2 \times h$

Substitute values

$\Rightarrow \frac{4}{3} \times 3^3 = r_w^2 \times 3600$

$\Rightarrow 4 \times 3^2 = r_w^2 \times 3600$

$\Rightarrow 36 = r_w^2 \times 3600$

$\Rightarrow r_w^2 = \frac{1}{100}$

$\Rightarrow r_w = 0.1$ cm

Therefore, radius of wire is 0.1 cm