Chapter 3

Natural Convection and Mass Transfer

CHAPTER HIGHLIGHTS

- *Natural Convection or Free Convection*
- *Grashof Number*
- *Rayleigh Number*
- *Free Convection Correlations*
- *For Vertical Plates or Cylinders*
- *Horizontal Cylinders*
- *Horizontal Square or Circular Plates*
- *Spheres*
- *Wires*
- *Rectangular Blocks*

Natural Convection or Free Convection

When the air comes in contact with the hot surface, its molecules in the immediate vicinity of the surface receive heat from the hot surface. It causes the temperature of the molecules, its specific heat and its specific volume (i.e., air become lighter) to rise. The density of air decrease due to increase in temperature of air adjacent to the surface so this hot air rises up under the action of buoyancy forces. The place of the hot air is taken up by heavier molecules (cold air having higher density) which will also rise in similar way on receiving energy from hot surface. In this way natural motion of fluid molecules will be working. It can be said that transfer of heat from solid surface to the fluid due to density variation in the fluid causes by the temperature gradient between the surface and the fluid is called heat transfer by natural free convection coefficient of volumetric expansion at constant pressure.

$$\beta = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{P=c}$$

Negative sign indicate density decrease with increase in temperature.

For ideal gas, $PV = mRT$

$$\rho = \frac{P}{RT}$$

$$\frac{\partial \rho}{\partial T} = \frac{-1}{T_2}\cdot\frac{P}{R}$$

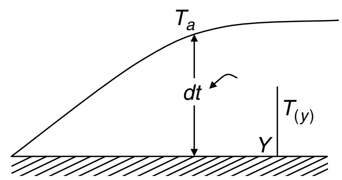
$$\beta = \frac{1}{T}\text{(per Kelvin)}$$

Important Equations

Consider an air flow over a flat plate maintained at surface temperature T_s and air temperature T_a If ($T_s > T_a$), thin heat transfer taken place from plate to air. There will be developed a thermal boundary layer. If the thermal boundary layer in ∂T, at the surface of the plate, since there is no fluid motion and heat transfer can occur only through conduction then heat flux is as

$$\frac{Q}{A} = hx(T_{1_s} - T_{1_a}) = K_f \left.\frac{dT}{dy}\right| y = 0)$$

Here, T is temperature of fluid at y distance measured in normal direction from the plate. In a thermal boundary layer region some conditions are applied as – if T represents the temperature distribution depending upon y then,



(a) At $y = 0$, $T = T_s$ (b) At $y = 0$, $\frac{\partial^2 T}{\partial y^2} = 0$

(c) At $y = \delta t$, $T = T_a$ (d) At $y = \delta_t$, $\frac{\partial T}{\partial y} = 0$

As above δ_t temperature remains same.
For free or natural convection following dimensionless numbers are used.

Grashof Number (G_r)

It represents the product of buoyant and inertia forces to square of viscous forces. It helps in determining the nature of flow (laminar or turbulent) as Reynolds number determines in forced convection.

The value of G_r depend upon the shape position of body defined by characteristic length L

$$G_r = \text{Inertia force} \times \frac{\text{Buoyancy force}}{(\text{Viscous force})_2}$$

$$= \frac{(\rho V^2 L^2)(\rho\beta g \Delta T L^3)}{(\mu V L)^2}$$

$$= \frac{\rho^2 \beta g \Delta T L^3}{\mu^2}$$

$$= \frac{\beta g \Delta\theta L^3}{v^2}, \left(v = \frac{\mu}{\rho}\right)$$

The role played by Grashoff's number in natural (free) convection is identical to the role played by Reynolds No. in forced convection. Just as Reynolds No. decides whether the fluid flow is laminar or turbulent, Grashoff number decides in the case of natural convection, whether the flow is laminar or turbulent.

When a fluid flows over a hot surface the convection process is complex, it involves both natural and forced convection. The relative importance of the mode, forced convection or natural convection is defined by the coefficient $\left(\frac{G_N}{R_N{}^2}\right).98\left(\frac{G_N}{R_N{}^2}\right) \le 1$ natural convection effects are negligible and the mode of heat transfer is to be taken as forced. If $\left(\frac{G_N}{R_N{}^2}\right) \ge 1$, forced convection effects are negligible and the mode of heat transfer may be taken as natural.

If $\left(\frac{G_N}{R_N{}^2}\right) = 1$ it is to be considered that both the effects are significant.

For natural convection, 'N_u' (Nusselt Number) is given as $N_u = f(G_N, P_N)^n$ where $N_u = \frac{hL}{K}$. From this equation the convectional heat transfer coefficient 'h' can be evaluated

$$G_r = g\frac{\beta(T_s - T_\alpha)L_c^3}{v^2}$$

Where g = Acceleration due to gravity
β = coefficient of volumetric expansion per °K
$\beta = \frac{1}{T}$ for (ideal gases)
T_s = Temperature of hot surface
T_α = Temperature of the fluid sufficiently away from the surfaces (free stream temperature)
L_c = Characteristic length m
υ = Kinematic Viscosity in m^2/s

For liquids β can be calculated if variation of density with temperature at constant pressure is

$$\beta = \frac{1}{T}$$

known for gases

$$= g\beta\rho\frac{u^2}{\mu^2}(T_\omega - T_\alpha)\frac{\rho}{u^2}L^2$$

$$G_r = g\beta\frac{(T_\omega - T_\alpha)}{v^2}L^3$$

$$\left[\frac{\rho u^2}{\frac{\mu u}{L}}\right]\left[\frac{1}{\left(\frac{\mu u}{L}\right)}g\beta\rho(T_\omega - T_\alpha)\right]$$

$$\rightarrow \frac{m}{s^2}\frac{1}{K}\cdot\frac{ks^2}{m^4}\;m^3 = 1 \times L$$

So the group is dimension–less viscous force

$$\tau = \mu\frac{du}{dy} \rightarrow \mu\frac{u}{y}$$

Inertia force is proportional to $P_N{}^2 = P_u{}^2$
Buoyant forces are proportional to $d\rho$. dH. g the group $\frac{g\beta(T_\omega - T_\alpha)}{v^2}L^3$ is rearranged as

$$= \left[\frac{\text{Inertia force}}{\text{Viscous force}}\right]\left[\frac{\text{Buoyant force}}{\text{Viscous force}}\right] \text{as } \rho\cdot\beta$$

$(T_\omega - T_\alpha) = dp$
$L = dH$

So the expression $g\ \beta\ \rho\ (T_\omega - T_\alpha)\ L$ represents buoyant force.

Rayleigh Number(R_a)

It is the product of Grashof number and Prandtl number
$R_a = G_r P_r$

$$= \frac{\rho^2 \beta g \Delta T L^3}{\mu^2} \times \frac{\mu C p}{K} = \frac{g\beta L^3 \Delta T}{v \cdot \alpha}$$

If free or natural convection then

$10^4 < R_a < 10^9$ laminar flow

$R_a > 10^9$ – turbulent flow

Free Convection Correlations

For Vertical Plates or Cylinders

$$N_{u_a} = \frac{h_a L}{K} = 0.53(G_r P_r)^{\frac{1}{4}} \text{ for } G_r P_r < 10^5$$

$$N_{u_a} = \frac{h_a L}{K} = 0.56(G_r P_r)^{\frac{1}{4}} \text{ for } 10^5 < G_r P_r < 10^8$$

$$N_{u_a} = \frac{h_a L}{K} = 0.13(G_r P_r)^{\frac{1}{3}} \text{ for } 10^8 < G_r P_r < 10^{12}$$

Characteristic length is the height of the plate or cylinder (L).

Horizontal Cylinders

$$N_{u_a} = \frac{h_a d}{K} = 1.1(G_r P_r)^{\frac{1}{6}}$$

for $\frac{1}{10} < G_r P_r < 10^4$

$$N_{u_a} = \frac{h_a d}{K} = 0.53(G_r P_r)^{\frac{1}{4}} \text{ for } 10^4 < G_r P_r < 10^9$$

$N_{u_a} = \frac{h_a d}{K} = 0.13(G_r P_r)^{\frac{1}{3}}$ For $10^9 < G_r P_r < 10^{12}$ the characteristic length is the diameter of pipe.

Horizontal square or circular plates. For horizontal hot surface facing up ward or cold surface facing down ward

$$N_{u_a} = \frac{h_a L}{K} = 0.71(G_r P_r)^{\frac{1}{4}} \text{ for } 10^3 < G_r P_r < 10^9$$

$$N_{u_a} = \frac{h_a L}{K} = 0.17(G_r P_r)^{\frac{1}{3}} \text{ For } G_r P_r > 10^9$$

For horizontal hot surface facing down ward or cold surface facing up ward

$$N_{u_a} = \frac{h_a L}{K} = 0.35(G_r P_r)^{\frac{1}{4}} \text{ for } 10^3 < G_r P_r < 10^9$$

$$N_{u_a} = \frac{h_a L}{K} = 0.08(G_r P_r)^{\frac{1}{3}} \text{ for } G_r P_r > 10^9$$

Characteristic length is the side of the plate.

The above equations can also be used for circular plates taking characteristic length as the diameter of the plate. If plate is rectangle, the greater side should be taken as characteristic length.

Spheres

The equations used for horizontal cylinders are also used for spheres when $G_r > 10^3$ and taking radius of the sphere as characteristic length in N_u and G_r.

Free convection heat transfer from sphere to air is also as given below.

$N_u = 2 + 0.43\,(G_r P_r)^{1/4}$

Wires

For laminal and turbulent flow over wires following equation is suggested

$$h = 2\left(\frac{\Delta L}{d}\right)^{\frac{1}{4}} \text{ when } 10^4 < G_r P_r < 10^9$$

$$h = 10.07(\Delta T)^{\frac{1}{3}}$$

Rectangular Blocks

Equation used for horizontal cylinders can also be used for rectangular blocks provided, the characteristic length L is calculated, by using the following formula $\frac{1}{L_e} = \frac{1}{L_v} + \frac{1}{L_h}$

L_v = vertical dimension

L_h = higher horizontal dimension

The following formulae are used for solving the problems

1. $\frac{u}{u_x} = \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2$
2. $u_{\max} = \frac{4}{27} u_x (\text{at } x = x)$
3. $U_m = \frac{27}{48} u_{\max} (\text{at } y = x)$
4. $U_a = 5.17\nu\left(P_r + \frac{20}{21}\right)^{\frac{-1}{2}}$

   $\left(\frac{\beta g \theta m}{\nu^2}\right)^{\frac{1}{2}} (X)^{\frac{1}{2}}$ where $\theta_m = (T_\omega - T_\infty)$
5. $\frac{\delta}{x} = 3.93\left[\frac{P_r + \frac{20}{21}}{P_r^2 (G_r)_x}\right]^{\frac{1}{4}}$
6. m = mass flow through a particular section of boundary at $x = x = \frac{\rho}{12}(\delta u_x)$ is boundary layer thickness at $x = x$.
7. δ_m mass flow between two sections

   $\frac{\rho}{12} = [u_x \delta |_{x=x_2} - ux\delta |_{x=x_1}]$

   Where $x_2 > x_1$
8. Mass flow through the complete boundary.

$$m_1 = 1.7\rho v\left[\frac{G_r\ell}{P_r^{\,2}\left(P_r+\frac{20}{21}\right)}\right]^{\frac{1}{4}}$$

9. $$N_{u_x} = \frac{h_x x}{K} = 0.508\left[\frac{P_r^{\,2}G_{r_x}}{P_r+\frac{20}{21}}\right]^{\frac{1}{4}}$$

10. $$N_{u_a} = \frac{h_a L}{K} = 0.667\left[\frac{(P_r)^2 G_r\ell}{P_r+\frac{20}{21}}\right]^{\frac{1}{4}}$$

11. $$N_{u_a} = \frac{h_a L}{K} = 0.667\left[\frac{P_r^2 G_r\ell\cos\alpha}{P_r+\frac{20}{21}}\right]^{\frac{1}{4}}$$

(For inclined plate)

12. $$N_{u_a} = \frac{h_a L}{K} = 0.0246\left[\frac{(P_r)^{1.17} g_r L}{1+0.495(P_o)^{\frac{2}{3}}}\right]^{\frac{9}{4}}$$

(For turbulent flow)

13. Acceleration of rotating blade = $R_m\omega^2$.

14. $\beta = \frac{1}{T}$ (For ideal gases only)

$$\beta = \left(\frac{\rho_c-\rho_h}{\rho_h}\right)\times\frac{1}{\Delta T} \text{ for fluids where}$$

$\Delta T = (T_h - T_c)$.

Solved Examples

Example 1: A vertical cylinder 3 m high and 30 cm in diameter is maintained at a temperature of 100°C in atmospheric environment of 20°C. Calculate the heat lost by free convection from cylinder.

Proportion of air at mean temperature of 60°C

$\rho = 1.06$ Kg/m^3

C_P= 1008J/kg-K

$K = 0.028$ W/mK

$\mu = 20 \times 10^{-6}$ kg/ms

$\nu = 18.97 \times 10^{-6}$ m^2/s

(A) 1378 W (B) 1268.9 W

(C) 1420 W (D) 1523 W

Solution:

$$\beta = \frac{1}{60+273} = \frac{1}{333}\,°\text{K}$$

$$P_r = \frac{\mu C_p}{K} = (20\times10^{-6})\times\frac{1008}{0.028} = 0.71$$

$$G_r = \frac{\beta g L^3(\Delta\theta)}{\nu^2}$$

$$\frac{1}{333}\times\frac{9.81\times3^3\times80}{(18.97\times10^{-6})^2} = 0.177\times10^{12}$$

$G_rP_r = 0.177 \times 10^{12} \times 0.71$

$= 1.2567 \times 10^{11}$

Now using the equation

$$N_u = 0.12(G_rP_r)^{\frac{1}{3}}$$

$= 0.12(1.256 \times 10^{11})^{1/3}$

$= 601$

$$h_a = \frac{601\times0.028}{3} = 5.61 \text{ W/m}^2\text{K}$$

$Q = Ah(\Delta T) = \pi \times 0.30 \times 3 \times 80 \times 5.61$

$= 1268.9$ W

Example 2: Determine the surface coefficient of convection of the inside surface of the tube when the saturated steam at 200°C flows through extra heavy 20 cm diameter pipe at a velocity of 3000 m/min.

(A) 628.7 W/m^{2}K (B) 348.7 W/m^{2}K

(C) 583.8 W/m^{2}K (D) 835.7 W/m^{2}K

Properties of steam at 200°C;.

$\rho = 7.56$ kg/m^3

C_p= 2800 J/Kg-K

$K = 0.0305$ W/m^{2}K

$\mu = 15.79 \times 10^{-6}$ m^2/s

$\nu = 2.03 \times 10^{-6}$ m^2/s.

Solution:

$$P_r = \frac{\mu C_p}{K} = [15.79\times10^{-6}]\times\frac{2800}{0.03880}$$

$= 1.14$

$$V = \frac{3000}{60} = 50 \text{ m/sec}$$

$$R_e = \frac{LV}{\nu} = \frac{0.2\times50}{2.03\times10^{-6}}$$

$= 4.93 \times 10^6$

Using the following equations for average heat transfer coefficient

$N_u = 0.023R_e^{0.8}P_r^{0.4}$

$= 0.023(4.93 \times 10^6)^{0.8} \times (1.14)^{0.4}$

$= 5480$

$$h_a = \frac{5480\times0.0305}{0.2}$$

$= 835.7$ W/m^{2}K.

Example 3: A hot plate kept vertical in air develops a boundary layer by free convection. The thickness of the boundary layer at a distance of 0.4 m was 19 mm. (a) Estimate the location where it will be 22.5 mm. (b) If the gas was carbon dioxide with $v = 12.5 \times 10^{-6}$ m²/s. As compared to air with $v = 23.13 \times 10^{-6}$ m²/s. Determine the boundary layer thickness at 0.4 m. Assume that the Prandtl number is nearly the same and all other conditions remain unchanged.

Solution: For a given situation δx is proportional to $xG_r^{-0.25}$ or $x^{+0.25}$ (It is assumed that laminar conditions prevail in both cases)

$$\frac{22.5}{19} = \frac{x^{0.25}}{0.4^{0.25}} = x = 0.787 \text{ m}$$

δx is proportional to x

$G_r^{-0.25}$ as x is the same and other conditions remain unchanged.

δx is proportional to $\frac{1}{(v^2)^{0.25}}$ or $\frac{1}{v^{0.5}}$

$$\frac{\delta}{19} = \left(\frac{23.13\times10^{-6}}{12.6\times10^{-6}}\right)^{0.5}$$

$$\delta = 25.74 \text{ mm.}$$

Direction for questions 4 and 5: An athlete lies still on the ground in cool air at 24°C. His body temperature is 96.8°F Approximating his body to be a cylinder 0.3 m dia and 2 m long.

Considering free convections over horizontal cylinder, the film temperature is 60/2 = 30°C. The property values are ρ = 1.165 kg/m³, ν =16 × 10⁻⁶m²/s, $P_r = 0.701$, $K = 26.75 \times 10^{-3}$ W/mk. Use $N_u = 0.36 + .518\,(G_rP_r)^{.025}/\,[(1 + 0.559/P_r)^{9/16}]^{4/9}$.

Example 4: Determine the heat loss from the body

(A) 58.5 W (B) 60 W
(C) 70 W (D) 78 W

Solution:

$$G_r = \frac{9.81}{303} \times \frac{12\times0.3^3}{(16\times10^{-6})^2} = 4.098\times10^7$$

$$G_rP_r = 2.872 \times 10^7$$

A suitable correlation is $N_u = 0.36 + \frac{0.518(G_rP_r)^{0.25}}{\left[1+\left(\frac{0.559}{P_r}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} = 29$

$$N_u = \frac{hd}{K}$$

$$h = 2.586 \text{ W/m}^2\text{K}$$

Heat loss rate = $\pi \times 0.3 \times 2 \times 2.586(12)$
= 58.5 W

Example 5: The heat loss when he runs 400 m distance in 55 sec the other surrounding conditions remaining the same. Neglect end losses.

$P_r = 0.701$, $K = 26.75 \times 103$ W/mK.

$N_u = 0.266\, R_e^{.805} \times P_r^{1/3}$

(A) 258.5 W (B) 262.8 W
(C) 279.1 W (D) 285.9 W

Solution: If the athlete runs then it is forced convection

$$U_\infty = \frac{400}{55} \text{ m/s}$$

$$R_e = \frac{0.3\times400}{55\times16\times16^{-6}} = 1.36\times10^5$$

$$N_u = 0.266\, R_e^{0.805} P_r^{\frac{1}{3}} = 3206$$

$$h = 285.86 \text{ W/m}^2\text{K.}$$

Example 6: Water is heated in a tank using horrrizontal pipes of diameter 50 mm with a wall temperature of 60°C maintained by steam condensing on the inside tubes. Water in the tank is at 20°C. Calculate the value of convection coefficient if the water is stagnant

(A) 948 W/m² (B) 793.8 W/m²
(C) 838 W/m² (D) 738 W/m²

Solution:

This is a case convection over a horizontal pipe.

The film temperature is (60 + 20)/2 = 40°C

The property values are

$\rho = 995$, $v = 0.657 \times 10^{-6}$m²/s

$P_r = 4.34$, $K = 0.628$

$\beta = 0.41 \times 10^{-3}$

$G_r = 9.81 \times 0.41 \times 10^{-3}(60 - 20) \times 0.05^3/(0.657 \times 10^{-6})^2$
$= 4.66 \times 10^7$

$N_u = 0.53(G_rP_r)^{0.25}$
$= 0.53(4.66 \times 10^7 \times 4.34)^{0.25}$

$N_u = 63.2$

$h = 793.8$ W/m²K.

Example 7: Banks of plates are used to dissipate the heat from a transformer. The plates are 0.6 m high and 0.18 m wide. The plate surface is at 80°C and the air is at 40°C. Determine the distance between plates, so that the boundary layers do not interfere. Also calculate the number of plates requires dissipating 2 kW. Use $N_u = \frac{0.508P_r^{0.5}G_r^{0.25}}{(0.952+P_r)^{0.25}}$

(A) 35, 30 mm (B) 40, 45 mm
(C) 50, 28 mm (D) 60, 39 mm

Solution:

For boundary layers not to interfere, the distance between plates should be greater than twice the boundary layer thickness.

The film temperature is (40 + 80)/2 = 60ºC

The property values are $\rho = 1.06$,

$v = 18.97 \times 10^{-6}$ m/s, $P_r = 0.696$, $K = 28.96 \times 10^3$ W/mK

$$G_r = \frac{9.81 \times 1 \times 40 \times 0.6^3}{333 \times (18.97 \times 10^{-6})^2}$$

$$= 0.707 \times 10^9 \therefore \text{ laminar}$$

$$\delta x = 3.93 \times P_r^{-0.05} (0.952 + P_r)^{0.25}$$

$$\times G_{rx}^{-0.25} = 0.01794 \text{ m}$$

Distance between plates $2 = \delta x = 39$ mm or say sum for safe operation.

$$N_{u_x} = \frac{508 P_r^{0.5} G_r^{0.25}}{(0.952 + P_r)^{0.25}} = 374.04$$

$h_x = 2.94$ W/m^{2}K

$h = 3.93$ W/m^{2}K

$A = 2 \times N \times 0.18 \times 0.6 \times 3.93(80 - 40)$

$N = 59$ plates or say 60.

Example 8: A vertical plate is maintained at 40°C in 20°C still air. Determine the height at which the boundary layer will turn turbulent if turbulence set in at $G_rP_r = 10^9$.

The property values of air should be evaluated at T_f, (40 + 20)/2 = 30°C

$v = 16 \times 10^{-6}$ m^2/s $P_r = 0.701$,

$\rho = 1.165$ kg/m^3

(A) 0.75 m (B) 0.912 m
(C) 0.402 m (D) 0.818 m

Solution:

Flow become turbulent

where $G_rP_r = 10^9$

$$10^9 = 9.81 \times \frac{1}{303} \times x^3 \frac{(40-20)}{(16 \times 10^{-6})^2} \times 0.701$$

Solving $x = 0.8262$ m

The flow turns turbulent at this height.

Example 9: The problem being the same as in above no (3.8) but fluid is water.

The property values at 30°C are

$v = 0.8315 \times 10^{-6}$ m^2/s

$P_r = 5.68, - = 0.31 \times 10^{-3}$ K

Solution: Turbulence sets in at $G_rP_r = 10^9$

$$10^9 = 9.81 \times 0.31 \times 10^{-3} \times \frac{(40-20)x^3}{(0.8315 \times 10^{-6})^2} \times 0.568x$$

$= 0.271$ m.

Exercises

Practice Problems I

Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. A vertical cylinder 2 m high and 20 cm in diameter is maintained at a temperature of 100°C in an atmospheric environment of 20°C. The heat lost by free convection from this cylinder is (Properties of air at mean temperature of 60°C)
   Use the heat transfer equation $N_u = 0.12\ (G_rP_r)^{1/3}$
   $\rho = 1.06$ kg/m^3, $C_p = 1008$ j/kg-K,
   $K = 0.028$ W/m – K, $\mu = 20 \times 10^{-6}$kg/m.s
   $\nu = 18.97 \times 10^{-6}$m^2/s
   (A) 566 W (B) 620 W
   (C) 732 W (D) 800 W
2. Hot air at 300°C flows through 30 cm OD horizontal pipe which is exposed to atmosphere at 20°C. The heat lost to the atmosphere per meter length of pipe by natural convection from outside surface is.
   Use $N_u = 0.1(G_rP_r)^{1/3}$
   [Properties of air at mean temperature of 160°C
   $\rho = 0.815$ kg/m^2; $C_p = 1.019$ kJ/kg
   $\nu = 30.09 \times 10^{-6}$ m^2/sec, $K = 0.035$ W/mK.
   $\mu = 24.24 \times 10^{-6}$ kg/m/sec]
   (A) 1680 W/m (B) 1496 W/m
   (C) 1552 W/m (D) 1340 W/m
3. The heat is given up by convection from a vertical surface 3m high and 2 m wide to atmospheric air at rest which is 40°C colder than the surface. Ratio when compared with the of heat that would be given up by a surface 2 m high and 3 m wide and 40°C warmer than surrounding air is. (All other conditions are being same.)
   Heat transfer equation is $C(G_rP_r)$
   (A) 1:1.1 (B) 1.2
   (C) 1.5 (D) 1.8
4. A cylinder 8 cm in diameter and 15 cm high having a surface temperature of 50°C is placed vertically in water at 15°C. Find the heat transfer from the cylinder per hour. Properties of water at mean temperature of 32.5°C. $\rho = 995.7$ kg/m^3, $Cp = 4.18$ kJ/kgK
   $k = 0.615$ W/mk, $\mu = 0.798 \times 10^{-3}$ kg/ms
   $\nu = 0.805 \times 10^{-6}$ m^2/s, $\beta = 4.15 \times 10^{-4}$
   $K = 0.615$ W/mk

   $$N_{u_x} = \frac{h_x}{K}$$

   $$h_x = 0.0288\, R_e = \left(\frac{v}{\nu}\right)^{\frac{4}{5}} P_r^{\frac{1}{3}}$$

   (A) 1110 W (B) 998 W
   (C) 746 W (D) 830 W

5. Considering the body of a man as a cylinder of 30 cm diameter and 160 cm height, if the temperature of the body is to be maintained at 97.7°F. Find the amount of heat generated in the body of man per hour.

Temperature of surrounding is 13.5°C.

Properties of air at the mean temperature of 25°C

The heat transfer equation is

$N_u = 0.12\ (G_r P_r)^{1/3}$

$\rho = 1.6$ kg/m^3

$C_p = 1.006$ kJ/kg-K

$K = 0.0256$ W/mK

$\mu = 1.82 \times 10^{-5}$ kg/ms

$\nu = 15.46 \times 10^{-6}$m^2/sec

(A) 150 W (B) 140 W
(C) 138 W (D) 151 W

6. Air is flowing with 150 km/h on a plate which is maintained at 100°C. If the temperature of the air is 20°C, the heat lost per hour from the plate assuming the plate is 50 cm long along the flow and 30 cm wide is [given $N_{u_x} = 0.0288\ (R_{e_x})^{4/5} \cdot (P_r)^{1/3}$]

(Properties of air at mean temperature of 60°C

$\rho = 1.06$ kg/m^3, $C_p = 1.008$ kJ/kgK

$K = 0.0285$/kcal/m hr°C,

$\mu = 20.03 \times 10^{-6}$ kg/m^{-s}

$\nu = 18.97 \times 10^{-6}$ m^2/sec)

(A) 2188 W (B) 1323 W
(C) 1528 W (D) 1492.6 W

7. Water flows inside a tube 50 mm in diameter and 3 meter long at a velocity of 0.8 m/sec. The heat transfer coefficient, if the mean water temperature is 50°C and wall temperature is 70°C is

[Properties of water at mean temperature of 60°C

Given $N_u = 0.023\ R_e^{0.8}(P_r)^{0.4}$

$\rho = 933.2$ kg/m^3; $C_p = 4148$ J/kg-K

$K = 0.65$ W/m-K, $\mu = 0.467 \times 10^{-3}$kg/ms

$\nu = 0.478 \times 10^{-6}$ m^2/sec]

(A) 5201 W/m^{2}K (B) 4120 W/m^{2}K
(C) 3812 W/m^{2}K (D) 4017 W/m^{2}K

8. The mean water temperature is 50°C and wall temperature is 70°C for water flowing through a 50 mm diameter tube. Find total quantity of heat transferred per hour using the following equation for finding average heat transfer coefficient

$$N_u = 021(R_{e_f})^{0.8}(P_{r_f})^{0.42}\left(\frac{P_{r_f}}{P_{r_w}}\right)^{0.25}$$

Where P_{r_f} = Prandtl number at mean temperature of water = 3.6 at 50°C

P_{r_w} = Prandtl number of water at wall temperature = 2.60 at 70°C

R_{e_f} = Reynold number at mean temperature of water

(A) 10425 W (B) 41380 W
(C) 35600 W (D) 52130 W

9. Determine the surface coefficient due to convection inside the tube surface when 6000 kg of water per hour at an average temperature of 50°C is being heated per hour in a clean smooth horizontal tube 5 cm in diameter. Assume the difference between steam and film temperature will be small.

Properties of water at 50°C.

Use the relation $N_u = 0.023\ (R_e)^{0.5}\ (P_r)^{0.4}$

$\rho = 988.1$ kg/m^3, $K = 0.557$ W/mK

$C_p = 4182$ J/kgK, $\mu = 0.55 \times 10^{-3}$kg/m^{-s}

$\nu = 0.556 \times 10^{-6}$m^2/sec

(A) 355 W/m^{2}K (B) 379 W/m^{2}K
(C) 126 W/m^{2}K (D) 411 W/m^{2}K

10. A triangular plate of equal sides of 0.6 m is maintained horizontally in air at 20°C. The plate temperature is 80°C. The hotter side of the plate faces down. Determine the value of convection coefficient. Property values at 50°C $\nu = 17.95 \times 10^{-6}$ m^2/s, $P_r = 0.698$, $K = 28.26 \times 10^{-3}$ W/mK use $N_u = 0.27 \times (G_r P_r)^{0.25}$

(A) 4.8 W/m^{2}K (B) 3.5 W/m^{2}K
(C) 5.1 W/m^{2}K (D) 2.8 W/m^{2}K

11. A fluid is flowing through a rectangular tube of width 'a' and depth 'b'. The equivalent length to be taken for heat calculation (L_e) is

(A) $a + b$ (B) $2\ (a + b)$
(C) $\dfrac{2ab}{a+b}$ (D) $\dfrac{4(a,\ b)}{a+b}$

12. Peclet number is defined as

(A) $(R_e G_r)$ (B) $(R_e P_r)$
(C) $(G_r P_r)$ (D) $R_e^{\ m}\ P_r^{\ n}$

Direction for questions 13 and 17: A plate with dimension 100 cm × 50 cm × 1 cm is placed horizontally. The temperature of the top surface is 100°c. Air is flowing over the plate at a velocity of 2 m/s. Air temperature is 20°C. Conductivity of plate material is 20 W/mK. Air is flowing parallel to the 100 cm side.

The proportion of air at mean temperature 20°C is

$\rho = 1.06$ kg/m^3, $C_p = 1008$ J/kgK,

$k = 0.0285$ W/mK, $\nu = 18.97 \times 10^{-6}$m^2/s,

$P_r = 0.71$

Use the formula $N_u = 0.332\ R_e^{0.5}\ P_r^{0.33}$

13. Nusslet number is

(A) 107 (B) 84
(C) 96 (D) 116

14. The average value of the convection heat transfer coefficient is

(A) 5.48 W/m^2K (B) 8.76 W/m^2K
(C) 12.32 W/m^2K (D) 10.7 W/m^2K

15. The amount of heat lost from the plate is
(A) 254.3 W (B) 219.2 W
(C) 248.1 W (D) 301.2 W

16. The amount of heat conducted through the plate is
(A) 312.8 W (B) 296.5 W
(C) 219.2 W (D) 212.3 W

17. The bottom surface temperature of the plate under steady state condition is
(A) 100.12 °C (B) 100.22 °C
(C) 101.31 °C (D) 101.92 °C

Direction for questions 18, 19, 20: A hot plate is kept vertical. Height of the plate is 25 cm and width is 50 cm. The temperature of the plate is maintained at 110°C on both sides. The ambient temperature is 30°C. The properties of air at 70°C is
$\rho = 1.03$ kJ/m³, $\nu = 20.02 \times 10^{-6}$ m²/s, $C_p = 1005$ J/kgK, $K = 2.6 \times 10^{-2}$ W/mK. Use the convection heat transfer equation average Nusselt $N_u = 0.56\ (G_rP_r)^{1/3}$

18. The value of (G_rP_r) is
(A) 3.2×10^7 (B) 4.52×10^7
(C) 5.45×10^7 (D) 7.1×10^7

19. The average heat transfer coefficient is
(A) 14.68 W/m^2K (B) 17.32 W/m^2K
(C) 22.98 W/m^2K (D) 24.12 W/m^2K

20. The heat lost from both sides
(A) 386 W (B) 423 W
(C) 482 W (D) 520 W

Practice Problems 2

Direction for questions 1 to 20: Select the correct alternative from the given choices.

Direction for questions 1 to 4: A pipe kept horizontal has a diameter 30 cm. It is maintained at a temperature of 240°C in a room where the ambient temperature is 30° c. The properties of air at 135°C are

$K = 0.034$ W/mK. $\nu = 26.25 \times 10^{-6}$ m²/s, $P_r = 0.70$. The heat transfer equation is $N_u = 0.53\ (G_rP_r)^{0.25}$

1. The value of the product (G_rP_r) is
(A) 1.38×10^8 (B) 1.42×10^8
(C) 1.69×10^8 (D) 1.84×10^8

2. The convective heat transfer coefficient is
(A) 7.8 W/m^2K (B) 6.5 W/m^2K
(C) 5.6 W/m^2K (D) 5.2 W/m^2K

3. The heat transfer per unit length of the pipe is
(A) 1412.5 W (B) 1393.47 W
(C) 1376.4 W (D) 1286.5 W

4. Stanton number is given by
(A) $\frac{N_u}{P_rG_r}$ (B) $\frac{P_r}{G_rN_u}$
(C) $\frac{N_u}{R_eP_r}$ (D) $\frac{G_r}{P_rN_u}$

Direction for questions 5 to 10: Water is heated in a vessel by dipping a hot plate of constant temperature 120°C. The water temperature is 20°C. The size of the plate is 40 cm × 30 cm Properties of water at 70°C are

$\rho = 977.8$ kJ/m³, $\mu = 0.4 \times 10^{-3}$ kJ/ms
$\nu = 0.415 \times 10^{-6}$ m²/s, $C_p = 4200$ j/kgK, $K = 0.66$ W/mK. $\beta = 62 \times 10^{-4}$/K. The heat transfer equation is $N_u = 0.13\ (G_rP_r)^{1/3}$

5. If the plate is immersed such that the side 40 cm stands vertical, the value of $(G_r \cdot P_r)$ is
(A) 4878×10^8 (B) 5186×10^8
(C) 5752×10^8 (D) 6111×10^8

6. The convective heat transfer coefficient is
(A) 1784.15 W/m^2K (B) 1523 W/m^2K
(C) 1487.32 W/m^2K (D) 1412.11 W/m^2K

7. The total heat loss from the plate is
(A) 53.1 (B) 60.3
(C) 42.82 (D) 58.1

8. If the plate is immersed in water with the side of length 30 cm vertical then the value of $G_r \cdot P_r$ is
(A) 1842.3×10^8 (B) 2426.7×10^8
(C) 3118.7×10^8 (D) 3286.6×10^8

9. The value of convective heat transfer coefficient is
(A) 1522.32 W/m^2K (B) 1783.8 W/m^2K
(C) 2312.42 W/m^2K (D) 2480.6 W/m^2K

10. The heat loss from the plate is
(A) 42.81 kW (B) 38.43 kW
(C) 38.29 kW (D) 51.45 kW

Direction for questions 11 and 12: A copper tube of diameter 1 cm carrying refrigerants at –30°C. The tube is surrounded by still air at 40°c. Length of the tubes is 3 m. The property of air may be taken as

$\nu = 16.6 \times 10^{-6}$m²/s, $K = 0.028$ W/mK, $P = 0.69$ convection heat transfer co-efficient $h = 1.32\left(\frac{\Delta T}{d}\right)^{\frac{1}{4}}$, if $G_rP_r < 10^9$ other wise N_u 0.17 $(G_rP_r)^{1/4}$.

11. The heat transfer coefficient is
(A) 8.12 W/m^2K (B) 8.7 W/m^2K
(C) 9.2 W/m^2K (D) 12 W/m^2K

12. The heat gain of pipe
(A) 79 W (B) 62 W
(C) 57.5 W (D) 52 W

13. If water flows through an annulus of outer diameter D and inner diameter d for heat transfer calculations, the characteristics length for flow configuration is.

(A) $D - d$ (B) $\frac{\pi(D-d)}{D+d}$

(C) $\frac{D-d}{\pi(D+d)}$ (D) $\frac{4D^2}{d^2}$

14. Nusselt number provides a measure of
(A) The convective heat transfer occurring at surface
(B) The conductive heat transfer occurring at surface
(C) The radiative heat transfer occurring at surface
(D) None of these

15. Which one of the following is true
(A) $h_{gas} = h_{liquid}$ (B) $h_{gas} < h_{liquid}$
(C) $h_{gas} < ch_{liquid}$ (D) $h_{gas} > h_{liquid}$

Direction for questions 16 and 17: An electric heater of exposed surface area 0.09 m^2 and output 600 watts is designed to operate fully sub merged in water.

16. When the water at 37°C and the surface coefficient of heat transfer is 285.3 W/m^2°K, the surface temperature of the heater will be
(A) 30.5°C (B) 60.5°C
(C) 90.5°C (D) 120.5°C

17. When the heater is mistakenly operated at 37°C in air with a surface coefficient of 8.5 W/m^2°K the surface temperature of the heater will be
(A) 321°C (B) 621°C
(C) 821°C (D) None of these

18. A well is 40 m deep and 9 m diameter and the atmosphere temperature is 25°C. The air at the top is having relative humidity of 50%. Determine the rate of diffusion of water vapour through the well $D = 2.58 \times 10^{-5}$ m^2/s (diffusion co efficience) use

$m_a = A\frac{DP}{R_a T(x_2 - x_1)} \ln \frac{P - P_b^2}{P - P_{b_1}}$ Partial pressure is saturation pressure at 25°C = 0.03169 bar

$R_a = \frac{8315}{18}$ JK gK partial

(A) 4.84×10^{-7} kg/s (B) 5.8×10^{-7} kg/s
(C) .8 kg/s (D) .07 kg/s

Direction for questions 19 and 20: Saturated steam at 110°C flows inside a copper pipe (Thermal conductivity 450 W/m°K) having an internal diameter of 10 cm and an external diameter of 12 cm. The surface resistance on the steam side is 12000W/m^2°K and that on the outside surface of the pipe is 18 W/m^2°K.

19. The total thermal resistance will be
(A) 0.07°W (B) 0.147° W
(C) 0.317° W (D) None of these

20. When it is located in space at 25°c, heat loss from the pipe will be
(A) 510 W/m (B) 620 W/m
(C) 578.2 W/m (D) 490 W/m

21. A hot plate 1 × 1.5 m is maintained at 300°C. Air at 20°C blows over the plate. If the rate of heat transfer is 8.4 kW. What is the convective heat transfer coefficient.
(A) 20 W/m°K (B) 0.02 W/m^2°K
(C) 40 Wm°K (D) 8 W/m^2°K

22. Kinematic viscosity of air at 20°C is given to be 1.6×10^{-5}m^2/sec. Its kinematic viscosity at 70°C will be approximately.
(A) 2.2×10^{-5} m^2/s (B) 1.6×10^{-5} m^2/s
(C) 1.2×10^{-5} m^2/s (D) 1×10^{-5} m^2/s

Direction for questions 23 and 24: Oil at 25°C flows at velocity of 0.1 m/s over a flat plate in a certain process. The plate is 4.5 m long and is maintained at a uniform temperature of 90°C. Take the thermo physical properties of oil at mean flow temperature $\frac{95+25}{2} = 60°C$, $\nu = 0.65 \times 10^{-4}$, $\propto = 7.2 \times 10^{-8}$.

23. The hydrodynamic boundary layer thickness is
(A) 270.4 mm (B) 250 mm
(C) 192.8 mm (D) 320 mm

24. The thermal boundary layer thickness is
(A) 27.98 mm (B) 26.46 mm
(C) 20.32 mm (D) 33.86 mm

25. A spherical heater of diameter 0.2 m with surface at 60°C is used to heat water at 20°C in a tank. Determine the value of convective heat transfer coefficient.

The film temperature is 40°C. The property values are $\rho = 995$, $\nu = 0.657 \times 10^{-6}$m^2/s,

$P_r = 4.34$

$K = 0.628$ w/mk, $\beta = 0.41 \times 10^{-3}$/K

The correlation is $N_u = 2 + 0.43(G_r P_r)^{0.25}$
(A) 461.7 W/m^{2}K (B) 520.2 W/m^{2}K
(C) 338 W/m^{2}K (D) 490.1 W/m^{2}K

26. A vertical plate 4 m high and 1 m wide is maintained at 60°C in still air at 0°C. The value of convection coefficient is (average value of temperature = (0 + 60/2) = 30°C. Property values $\rho = 1.165$, $\nu = 16 \times 10^{-6}$,

$P_r = 0.701$, $k = 0.02675$, $\beta = 1/303$) use

$N_u = 0.02\ (G_r P_r)^{0.4}$

$N_u = 021\ (G_r P_r)^{0.4}$
(A) 6.33 W/m^{2}K (B) 5.76 W/m^{2}K
(C) 731 W/m^{2}K (D) 7.21 W/m^{2}K

27. A horizontal cylinder of 0.4 m diameter at a surface temperature of 40°C is placed in air at 80°c. The film temperature is 60°C. The property values of air are $\rho = 1.06$ $\nu = 18.97 \times 10^{-6}$m^2/s $P_r = 0.696$ $k = 28.96 \times 10^{-3}$w/mk correlation applicable $N_u = 0.053\ (G_r P_r)^{1/4.}$

The heat transfer co-efficient is
(A) 4.22 W/m^{2}K (B) 7.33 W/m^{2}K
(C) 388 W/m^{2}K (D) 520 W/m^{2}K

28. A long horizontal cooling water tube of 2.5 cm OD is immersed in hot oil bath at 100°C. the tube surface is at 20°c. The value of convection coefficient is

(Film temperature at 60°c oil is on the outside

$\rho = 864$, $\nu = 83 \times 10^{-6}\text{m}^2/\text{s}$, $P_r = 1050$,

$k = 0.1407$ W/mK $\beta = 6.944 \times 10^{-4}$/K),

$$N_u = \frac{0.36 + 0.518(G_r P_r)^{\frac{1}{4}}}{\left[\left(\frac{1+0.559}{P_r}\right)\frac{9}{16}\right]^{\frac{4}{9}}}$$

(A) 90.1 W/m^{2}K (B) 102 W/m^{2}K
(C) 99.8 W/m^{2}K (D) 210 W/m^{2}K

29. A ceramic block at 480°C is 0.2 m × 0.1 with 0.1 vertical. It is exposed to air at 20°C. The heat loss to the air if the mean temperature 250°C and air properties:

$\nu = 40.61 \times 10^{-6}$ m^2/s,

$P_r = 0.677$, $K = 42.68 \times 10^{-3}$ W/mK and

$N_u = 0.52\,(G_r P_r)^{0.25}$

(A) 563.4 W (B) 820 W
(C) 870 W (D) 780.3 W

30. A square duct of 0.3 m side carrying conditioned air at 10°C passes in a room at 30°C. The heat gain for 1 m length when

$\nu = 15.06 \times 10^{-6}\text{m}^2/\text{s}$, $P_r = 0.703$,

$k = 25.93 \times 10^{-3}$w/mk, $N_u = 0.52\,(G_r P_r)^{0.25}$

(A) 78.48 W/m (B) 83 W/m
(C) 95 W/m (D) 102 W/m

Previous Years' Questions

1. Match Group A with Group B:

Group A	Group B
P. Biot number	**1.** Ratio of buoyancy to viscous force
Q. Grashof number	**2.** Ratio of inertia force to viscous force
R. Prandtl number	**3.** Ratio of momentum to thermal diffusivities
S. Reynolds number	**4.** Ratio of internal thermal resistance to boundary layer thermal resistance

(A) P – 4, Q – 1, R – 3, S – 2
(B) P – 4, Q – 3, R – 1, S – 2
(C) P – 3, Q – 2, R – 1, S – 4
(D) P – 2, Q – 1, R – 3, S – 4

2. Grashof number signifies the ratio of: **[2016]**
(A) inertia force to viscous force
(B) buoyancy force to viscous force
(C) buoyancy force to inertia force
(D) inertia force to surface tension force

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** C	**3.** A	**4.** A	**5.** B	**6.** D	**7.** D	**8.** A	**9.** C	**10.** B
11. C	**12.** B	**13.** C	**14.** A	**15.** B	**16.**	**17.** B	**18.** D	**19.** D	**20.** C

Practice Problems 2

1. A	**2.** B	**3.** D	**4.** C	**5.** C	**6.** A	**7.** C	**8.** B	**9.** B	**10.** A
11. D	**12.** A	**13.** A	**14.** A	**15.** B	**16.** B	**17.** C	**18.** A	**19.** B	**20.** C
21. A	**22.** A	**23.** A	**24.** A	**25.** A	**26.** B	**27.** A	**28.** C	**29.** D	**30.** A

Previous Years' Questions

1. A **2.** B