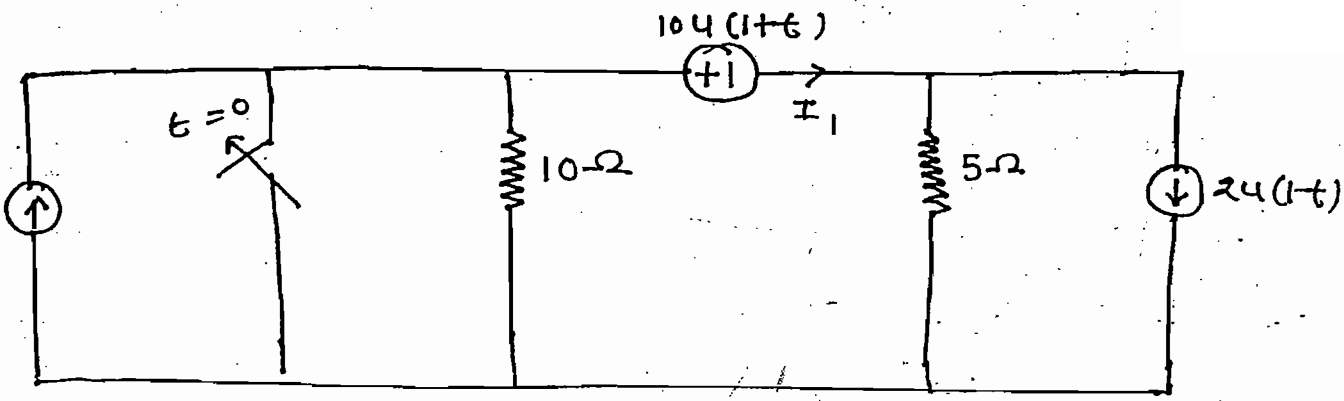
Lecture-11

Ques:- Find I_1 at $t = -2$ seconds:



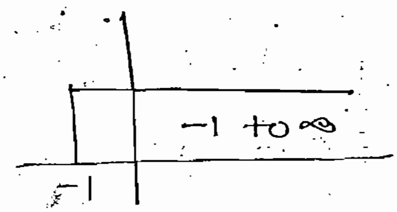
Soln:-

DC $\longrightarrow$ $-\infty$ to ∞

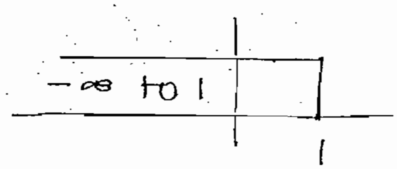
$u(t)$ $\longrightarrow$ 0 to ∞

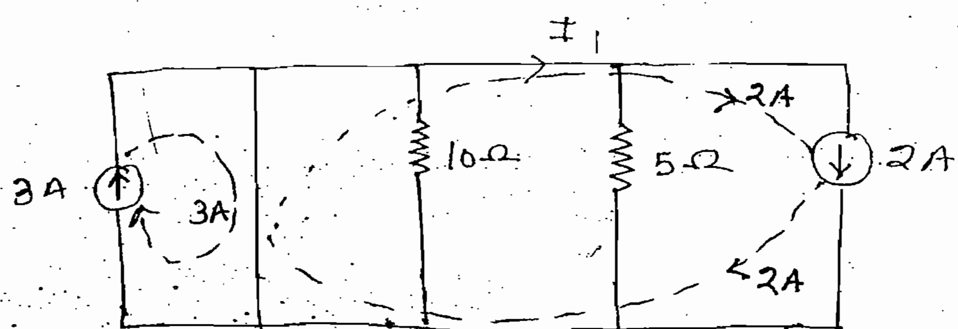
$u(-t)$ $\longrightarrow$ $-\infty$ to 0

$u(1+t)$



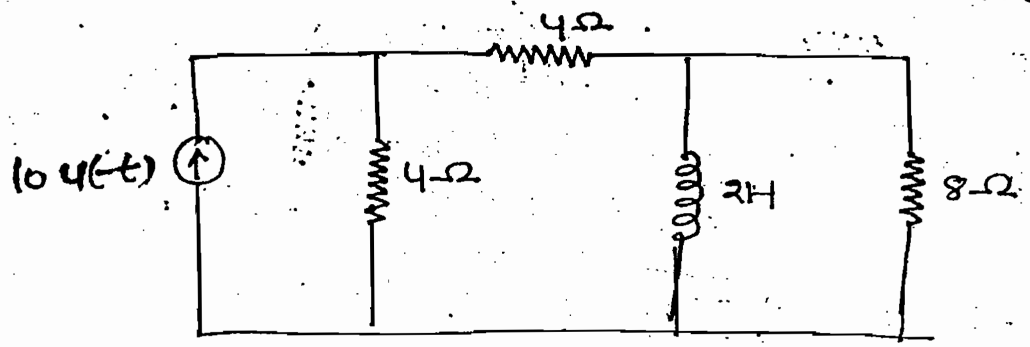
$u(1-t)$





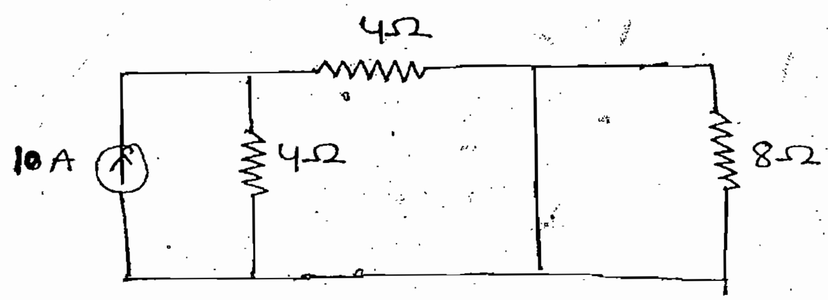
$I_1 = 2A$

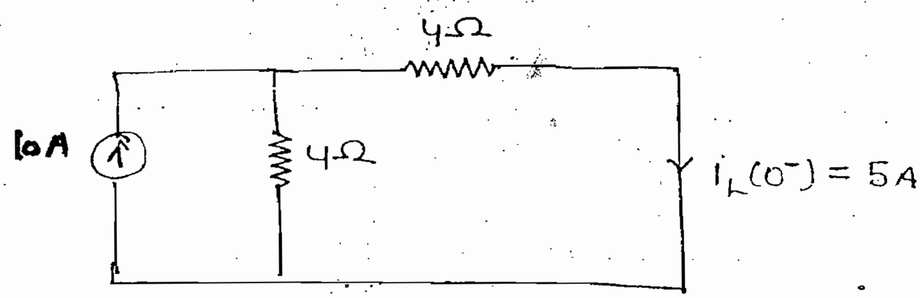
Ques:- Find current response in the inductor for $t > 0$



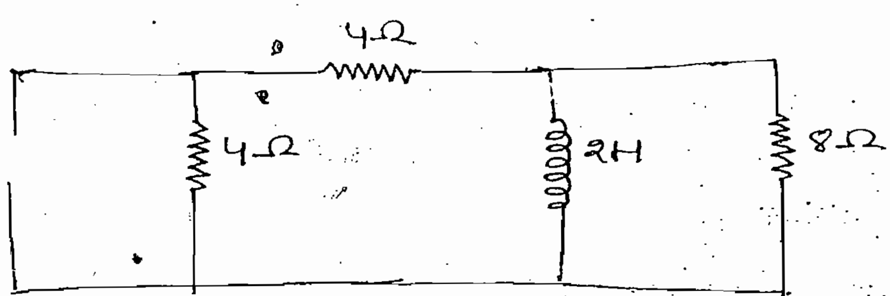
Soln:- $t = 0^-$

$u(-t) \longrightarrow -\infty$ to 0

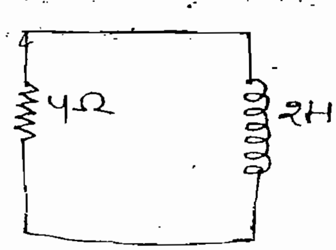




$t > 0$



$$i(t) = I_0 e^{-R/L\, t}$$

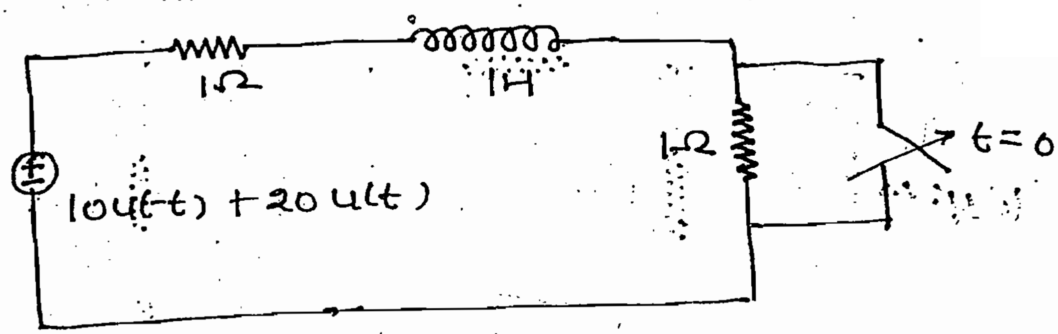


$$i(t) = 5 e^{-2t}$$, Ans

$I_0 = i_L(0^-) = 5A$

9p

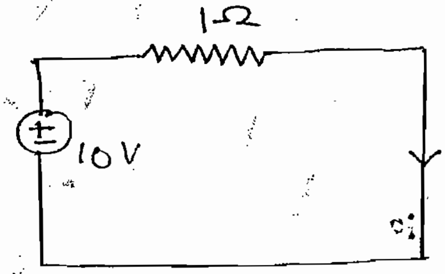
Ques:- Find current in the inductor for $t > 0$



Soln:- $i(t) = [i(0^+) - i(\infty)] e^{-\frac{R}{L}t} + i(\infty)$

$t = 0^-$

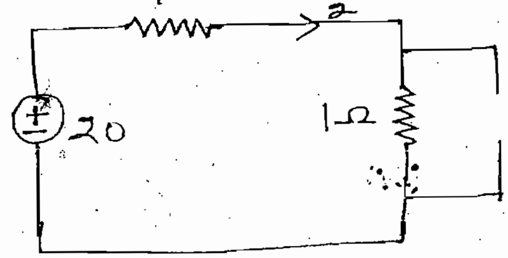
$i(0^+) = i(0^-) = 10A$



$i(0^+) = i(0^-)$
$= 10A.$

$t = \infty$

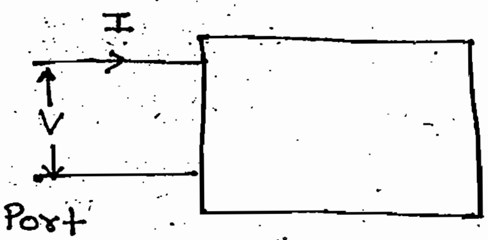
$\frac{20}{2} = 10A = i(\infty)$



$i(t) = [10 - 10]. e^{-R/L\, t} + 10 = 10$ Ans.

TWO PORT NETWORK

→ A pair of terminals at which signal may enter or leave from the N/w is called as port.



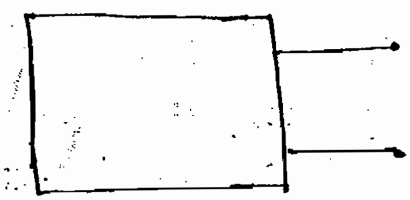
Port ↙ A pair of terminals

→ When the N/w is having one pair of terminals then the N/w is called as single port N/w

eg:- Motor, Generator

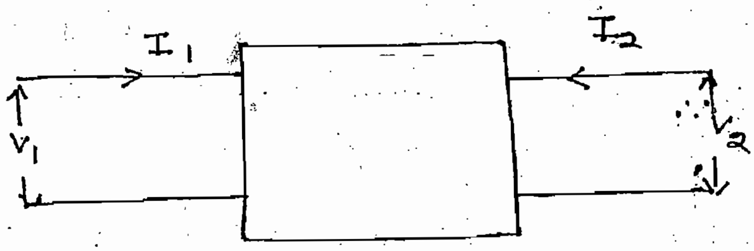
Single Port

eg:- Motor



eg:- Generator

eg:- Transformer



Classification of Parameters:-

(I) z - Parameter (O.C Parameter)
(II) Y - Parameter (S.C Parameter)
(III) h - Parameter (Hybrid Parameter)
(IV) g - Parameter (Inverse hybrid Parameter)
(V) ABCD - Parameter (Transmission line parameter)
(VI) abcd - Parameter (Inverse transmission line parameter)

(I) **Z-Parameter :-**

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—(I)} \longrightarrow KVL$$

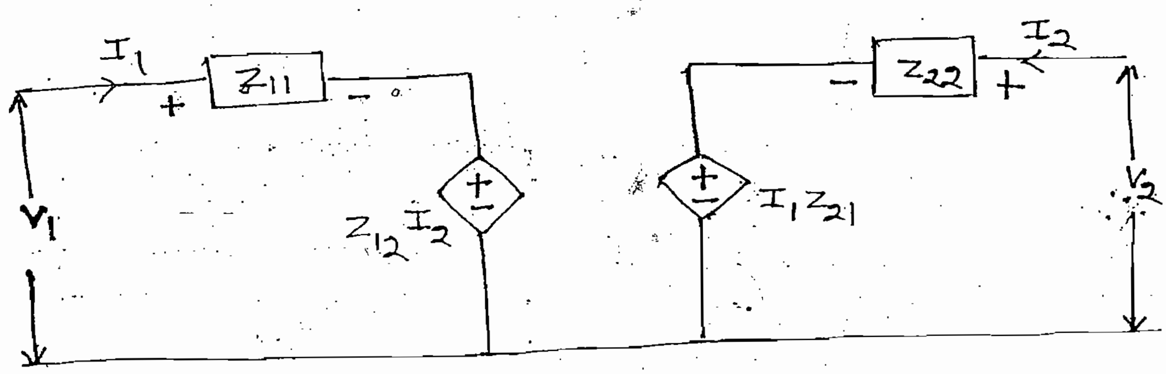
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—(II)} \longrightarrow KVL$$

$V_1, V_2 \longrightarrow$ Dependent Variables

Source $\rightarrow I_1, I_2 \longrightarrow$ Independent Variables

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$



$Z_{11} \Rightarrow$ Open ckt i/p impedance / Driving point i/p impedance

Z_{21} = Forward transfer impedance

Z_{12} = Reverse transfer impedance

Z_{22} = Open ckt o/p impedance / Driving point o/p impedance

(II) **Y-Parameter :-**

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—(I)} \longrightarrow KCL$$

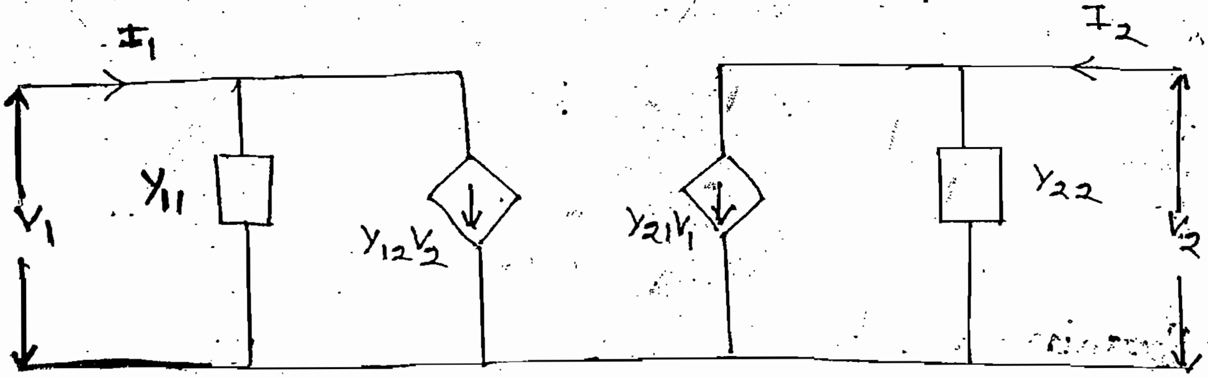
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—(II)} \longrightarrow KCL$$

$I_1, I_2 \longrightarrow$ Dependent Variables

$V_1, V_2 \longrightarrow$ Independent Variables
(source)

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



Y_{11} = S.C i/p admittance / Driving point i/p admittan[ce]

Y_{21} = forward transfer admittance

Y_{12} = Reverse transfer admittance

Y_{22} = S.C o/p admittance / Driving point o/p admittance

h-Parameter:-

$$V_1 = h_{11}I_1 + h_{12}V_2 \text{ —— (I)} \longrightarrow \text{KVL}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \text{ —— (II)} \longrightarrow \text{KCL}$$

h_{11} (Ω) $= \left.\frac{V_1}{I_1}\right|_{V_2=0}$ $\qquad \left(h_{11} \neq Z_{11} \ , \ h_{11} = \frac{1}{Y_{11}} \right)$

h_{21} (unitless) $= \left.\frac{I_2}{I_1}\right|_{V_2=0}$ $\qquad \left(h_{21} = \frac{Y_{21}}{Y_{11}} = \frac{I_2/\cancel{V_1}}{I_1/\cancel{V_1}} \right)$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \left(h_{12} = \frac{Z_{12}}{Z_{22}} \right)$$

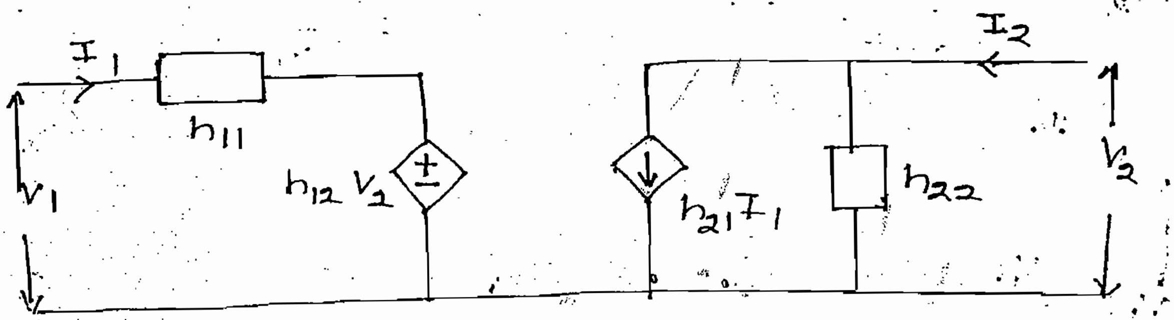
$\downarrow$

Unitless

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \quad \left(h_{22} \neq Y_{22} \ , \ h_{22} = \frac{1}{Z_{22}} \right)$$

$\downarrow$

(mho)



g-Parameters :-

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad \text{--- (I)}$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad \text{--- (II)}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \quad \left(g_{11} \neq Y_{11} \ , \ g_{11} = \frac{1}{Z_{11}} \right)$$

$\downarrow$

mho

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \quad \left(g_{21} = \frac{Z_{21}}{Z_{11}} \right)$$

$\downarrow$

Unitless

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \quad \left(g_{12} = \frac{Y_{12}}{Y_{22}} \right)$$

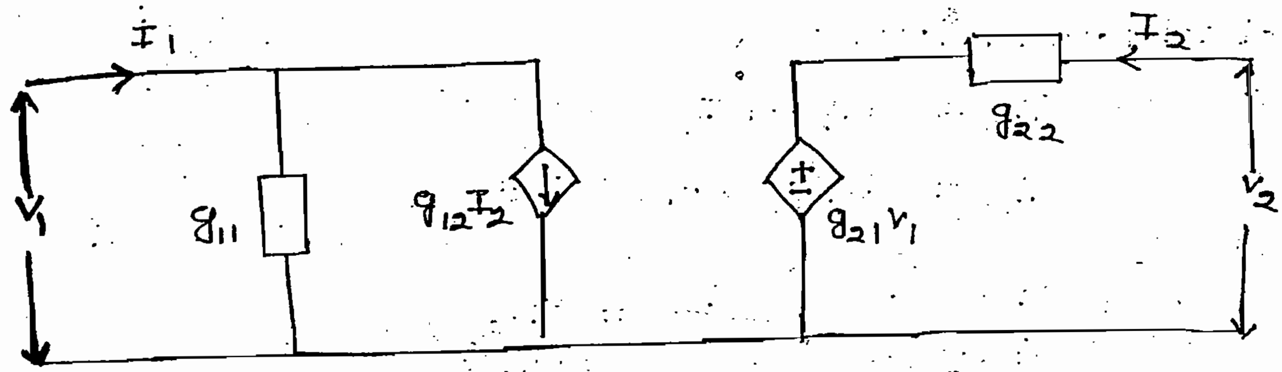
$\downarrow$

(unitless)

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \quad \left(g_{22} \neq Z_{22} \ , \ g_{22} = \frac{1}{Y_{22}} \right)$$

$\downarrow$

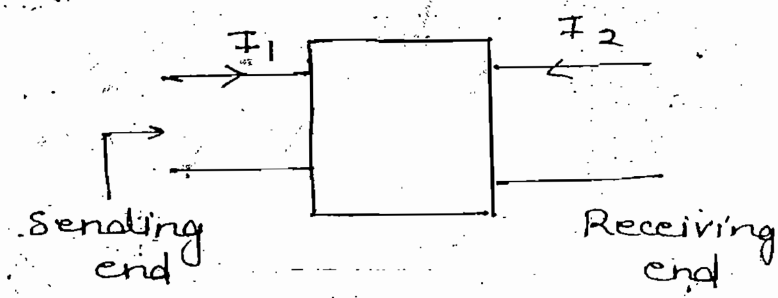
Ω



ABCD Parameters:-

$$V_1 = AV_2 - BI_2 \quad \text{——— (I)}$$

$$I_1 = CV_2 - DI_2 \quad \text{——— (II)}$$



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad \left(A = \frac{Z_{11}}{Z_{21}}\right)$$

↓ Unitless

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad \left(C = \frac{1}{Z_{21}}\right)$$

↓ mho

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0} \qquad \left(B = -\frac{1}{Y_{21}}\right)$$

↓ Ω

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0} \qquad \left(D = -\frac{Y_{11}}{Y_{21}}\right)$$

↓ Unitless

By using eq-(I) & (II) it is not possible to develo eq ckt since both equations are developed w.r.t i/p

abcd Parameter:-

$$V_2 = aV_1 - bI_1 \quad \text{—— (I)}$$

$$I_2 = cV_1 - dI_1 \quad \text{—— (II)}$$

$$a = \left.\frac{V_2}{V_1}\right|_{I_1=0} \qquad \left(a = \frac{Z_{22}}{Z_{12}}\right)$$

$\downarrow$

Unitless

$$c = \left.\frac{I_2}{V_1}\right|_{I_1=0} \qquad \left(c = \frac{1}{Z_{12}}\right)$$

$\downarrow$

mho

$$b = \left.\frac{-V_2}{I_1}\right|_{V_1=0} \qquad \left(b = -\frac{1}{Y_{12}}\right)$$

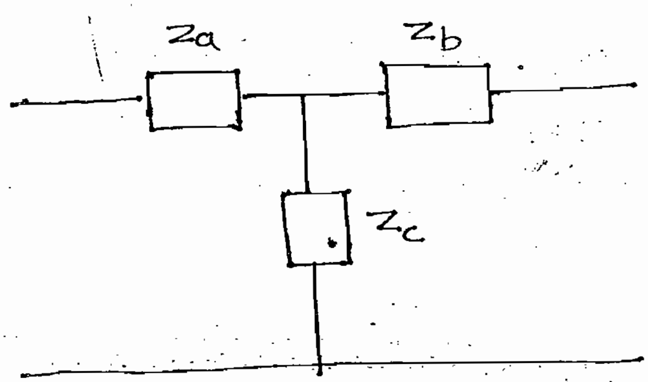
$\downarrow$

Ω

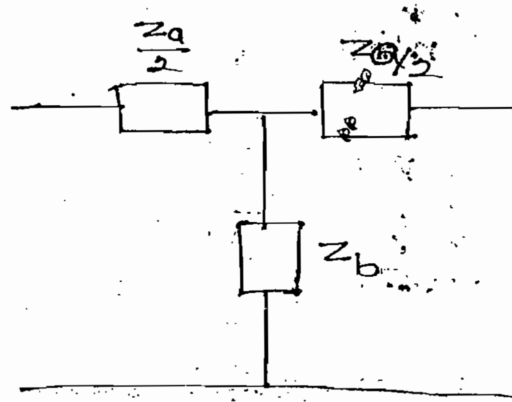
$$d = \left.-\frac{I_2}{I_1}\right|_{V_1=0} \qquad \left(d = -\frac{Y_{22}}{Y_{12}}\right)$$

$\downarrow$

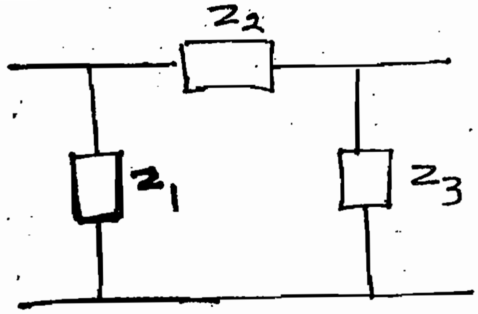
No units



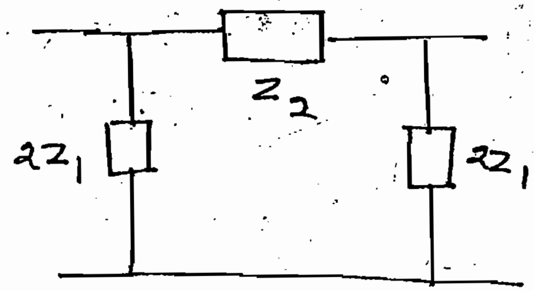
→ Unsymmetrical T-N/w



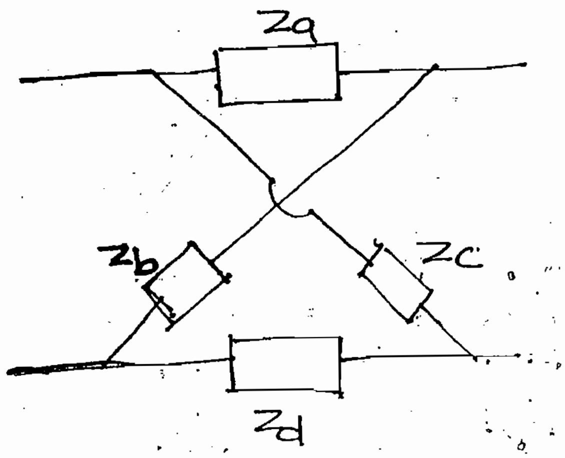
→ Symmetrical T-N/w



→ Unsymmetrical Π N/ω



→ Symmetrical − Π N/ω



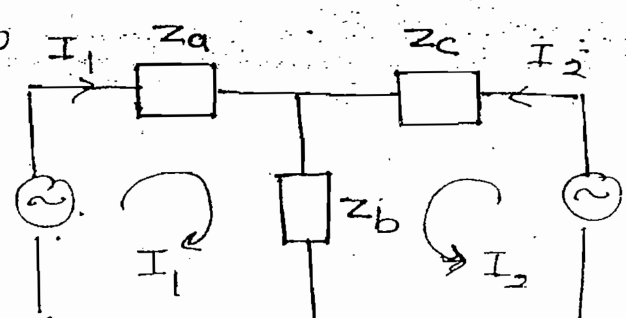
$\left.\begin{array}{l} Z_a = Z_d \\ Z_b = Z_c \end{array}\right] \rightarrow$ Symmetrical Lattice N/ω

$\left.\begin{array}{l} Z_a \neq Z_d \\ Z_b \neq Z_c \end{array}\right] \rightarrow$ Unsymmetrical Lattice N/ω

**

Symmetrical	Reciprocal
$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
$h_{11}h_{22} - h_{12}h_{21} = 1$	$h_{12} = -h_{21}$
$A = D$	$AD - BC = 1$

Z-Parameter → Use T−N/ω



$V_1 = (Z_a + Z_b)I_1 + Z_b I_2$ —(I)

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — (II)

$Z_{11} = Z_a + Z_b$

$Z_{12} = Z_b$

90

$$V_2 = Z_b I_1 + (Z_b + Z_c) I_2 \quad \text{--- (III)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (IV)}$$

$$Z_{21} = Z_b \quad \& \quad Z_{22} = Z_b + Z_c$$

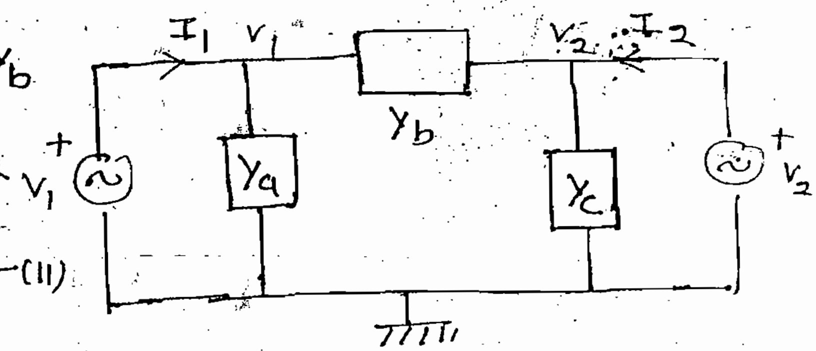
$$Z_{11} = Z_a + Z_b$$

$$Z_{11} = Z_a + Z_{12}$$

**

$$\boxed{\begin{aligned} Z_a &= Z_{11} - Z_{12} \\ Z_c &= Z_{22} - Z_{12} \\ Z_b &= Z_{12} = Z_{21} \end{aligned}}$$

For Y- Parameter → convert complex N/ω into π-N/ω



$$I_1 = V_1 Y_a + (V_1 - V_2) Y_b$$

$$I_1 = (Y_a + Y_b) V_1 - Y_b V_2 \quad \text{--- (I)}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (II)}$$

$$Y_{11} = Y_a + Y_b$$

$$Y_{12} = -Y_b$$

$$I_2 = V_2 Y_c + (V_2 - V_1) Y_b$$

$$\Rightarrow I_2 = -V_1 Y_b + (Y_b + Y_c) V_2 \quad \text{--- (III)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (IV)}$$

$$Y_{21} = -Y_b \quad \& \quad Y_{22} = Y_b + Y_c$$

**

$$\boxed{\begin{aligned} Y_a &= Y_{11} + Y_{12} \\ Y_c &= Y_{22} + Y_{12} \\ Y_b &= -Y_{12} = -Y_{21} \end{aligned}}$$

$Z_a = Z_d$, $Z_b = Z_c$

→ symmetrical lattice N/w

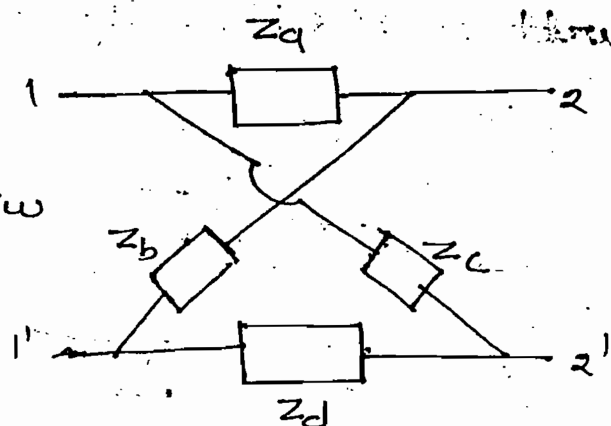


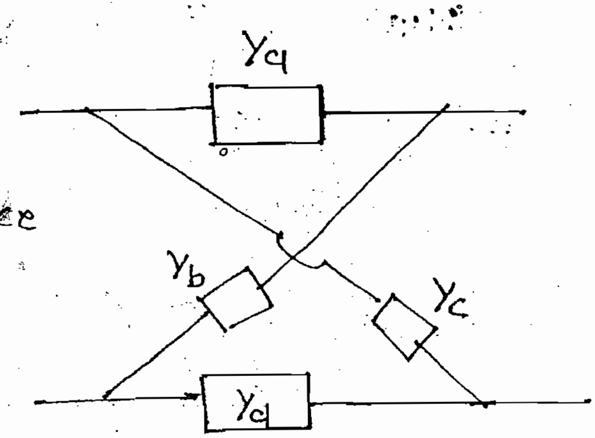
Fig - (1)

$$\boxed{Z_{11} = Z_{22} = \frac{Z_b + Z_a}{2} \qquad Z_{12} = Z_{21} = \frac{Z_b - Z_a}{2}}$$

$$\boxed{Z_b = Z_{11} + Z_{12} \qquad Z_a = Z_{11} - Z_{12}}$$

$Y_a = Y_d$, $Y_b = Y_c$

→ symmetrical lattice N/w

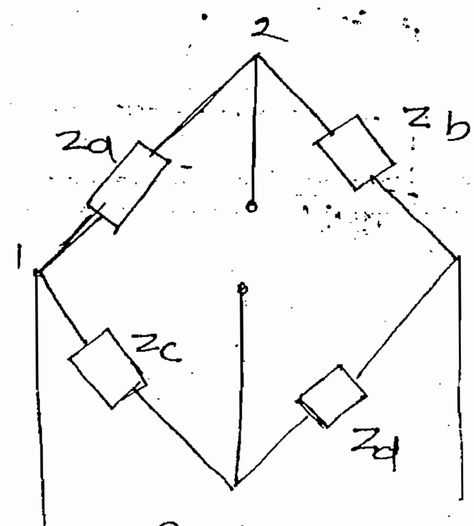


$$\boxed{Y_{11} = Y_{22} = \frac{Y_b + Y_a}{2} \qquad Y_{12} = Y_{21} = \frac{Y_b - Y_a}{2}}$$

$$\boxed{Y_b = Y_{11} + Y_{12} \qquad Y_a = Y_{11} - Y_{12}}$$

→ Fig-(I) & (II) are same.

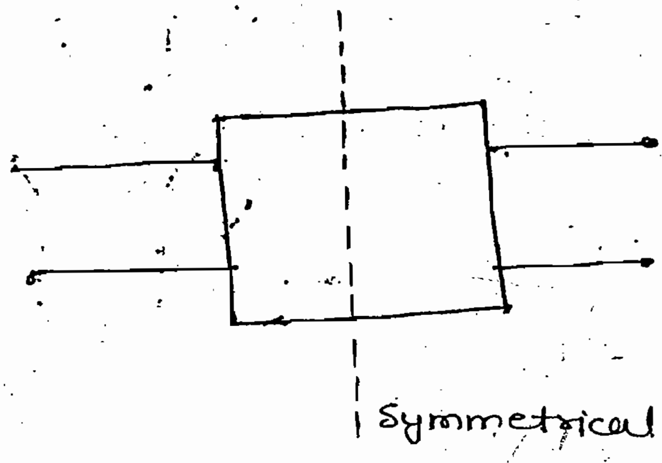
Fig-(II)



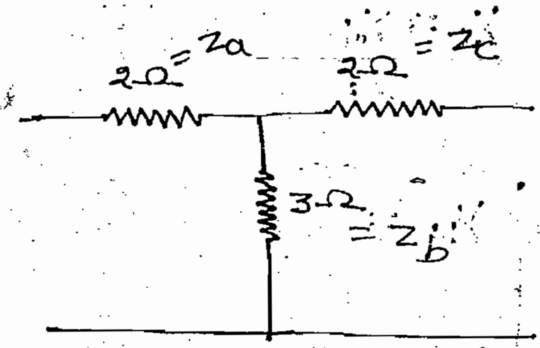
Bridge

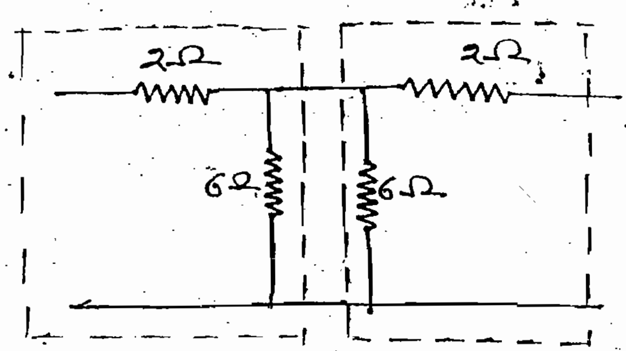
Bartlett's Bi-section theorem :-

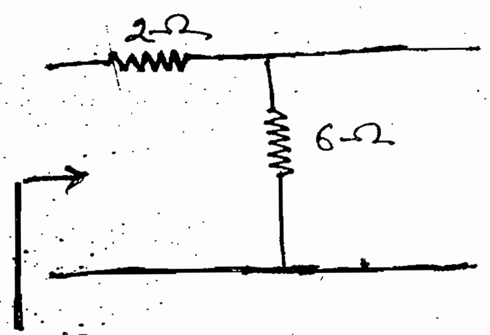
eg :- Filters



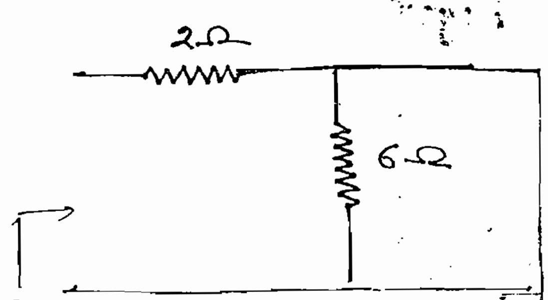
$$\boxed{\begin{aligned} Z_{11} = Z_{22} &= \frac{Z_{OCH} + Z_{SCH}}{2} \\ Z_{12} = Z_{21} &= \frac{Z_{OCH} - Z_{SCH}}{2} \end{aligned}}$$







$Z_{OCH} = 2+6 = 8$



$$z_{11} = z_{22} = \frac{Z_{OCH} + Z_{SCH}}{2} = \frac{8+2}{2} = 5$$

$$z_{12} = z_{21} = \frac{Z_{OCH} - Z_{SCH}}{2} = \frac{8-2}{2} = 3$$

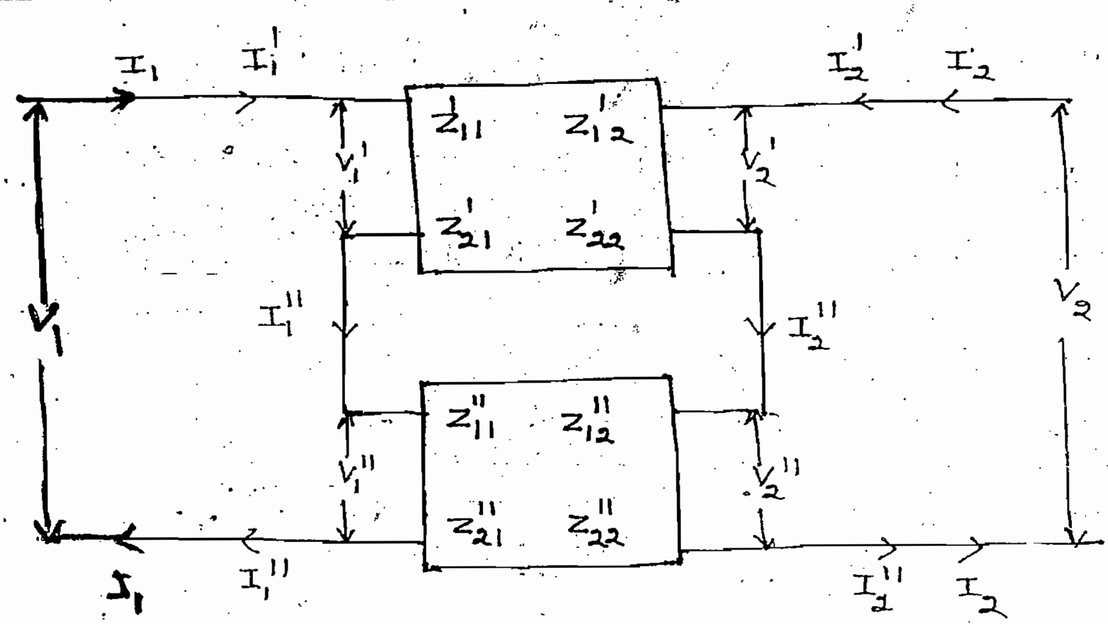
Alternate Way:-

$$z_{11} = Z_a + Z_b = 5$$

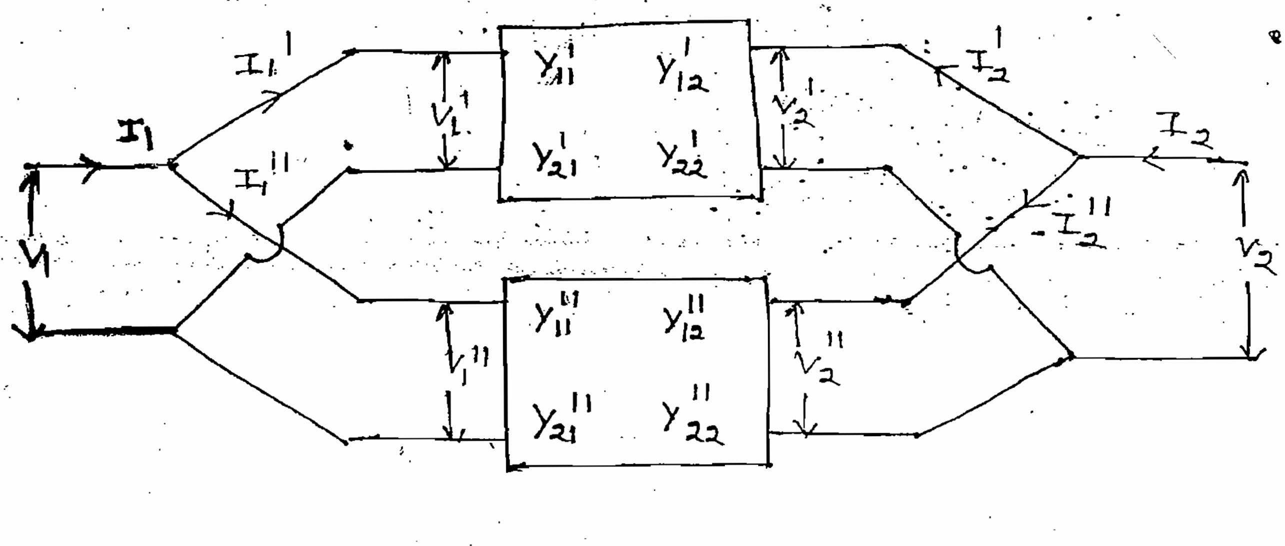
$$z_{22} = Z_b + Z_c = 5$$

$$z_{12} = z_{21} = Z_b = 3$$

Series combination:-



Parallel combination:-



For series combination:—

$$V_1 = V_1' + V_1'' \qquad V_2 = V_2' + V_2''$$

$$I_1 = I_1' = I_1'' \qquad I_2 = I_2' = I_2''$$

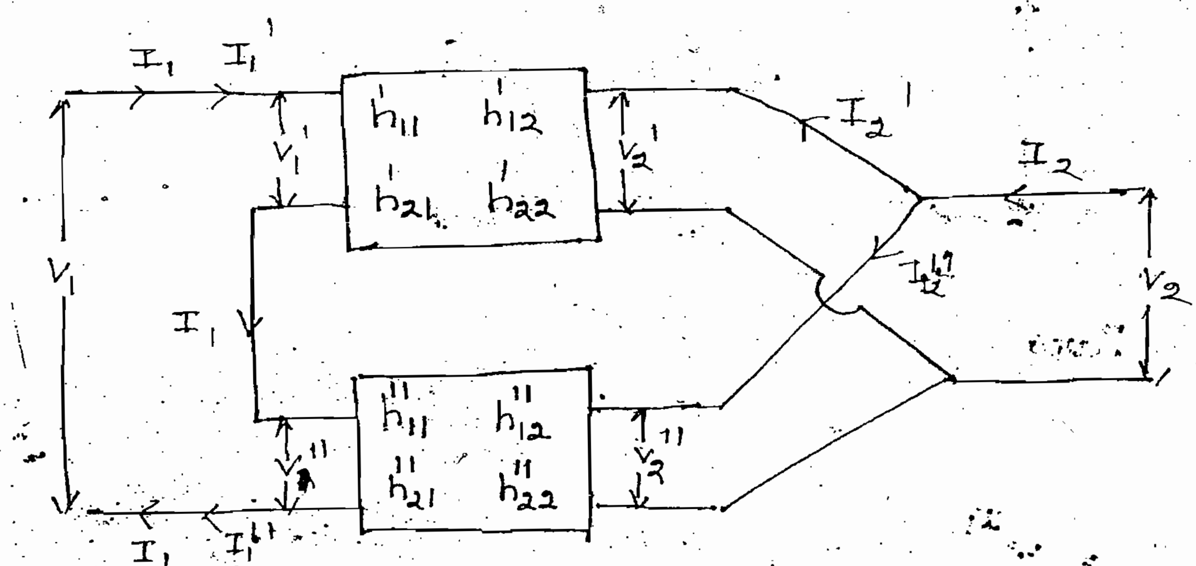
$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11}' & z_{12}' \\ z_{21}' & z_{22}' \end{bmatrix} + \begin{bmatrix} z_{11}'' & z_{12}'' \\ z_{21}'' & z_{22}'' \end{bmatrix}$$

For Parallel combination:—

$$V_1 = V_1' = V_1'' \qquad V_2 = V_2' = V_2''$$

$$I_1 = I_1' + I_1'' \qquad I_2 = I_2' + I_2''$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y_{11}' & y_{12}' \\ y_{21}' & y_{22}' \end{bmatrix} + \begin{bmatrix} y_{11}'' & y_{12}'' \\ y_{21}'' & y_{22}'' \end{bmatrix}$$

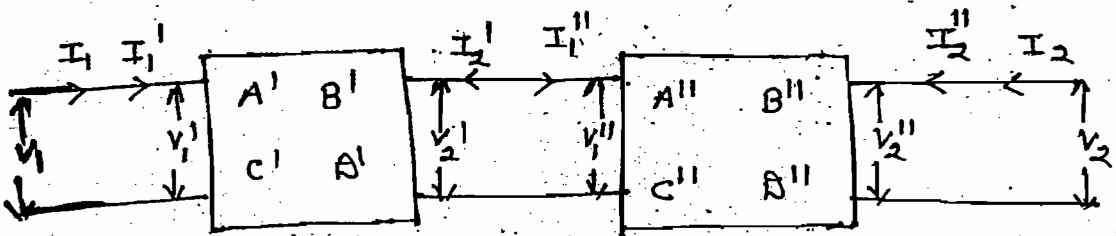


$$V_1 = V_1' + V_1'' \qquad V_2 = V_2' = V_2''$$

$$I_1 = I_1' = I_1'' \qquad I_2 = I_2' + I_2''$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{bmatrix} + \begin{bmatrix} h_{11}'' & h_{12}'' \\ h_{21}'' & h_{22}'' \end{bmatrix}$$

Cascade or Tandum Connection:-

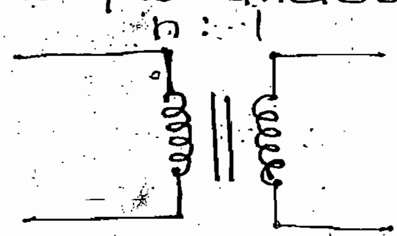


$$\boxed{\begin{matrix} V_1 = V_1' \\ I_1 = I_1' \end{matrix}} \qquad \boxed{\begin{matrix} V_2' = V_1'' \\ I_1'' = -I_2' \end{matrix}} \qquad \boxed{\begin{matrix} V_2 = V_2'' \\ I_2 = I_2'' \end{matrix}}$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}}$$

Ques:- Find Z, Y, ABCD parameters of the N/w shown:-

Ideal t/f



Soln:-

$$A = \frac{V_1}{V_2} = n$$

$$D = \frac{-I_1}{I_2} = +\frac{1}{n}$$

–ve is due to current direction

$$\boxed{\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{n}{1}}$$

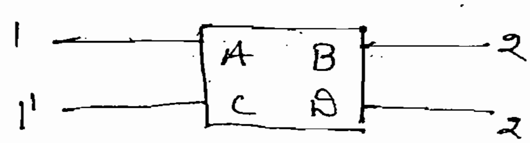
Note:-

In ideal t/f it is not possible to find impedance and admittance values since self and mutual inductance of ideal transformer are ∞

Ques:- Find O.C & S.C impedance w.r.t

(i) $1-1'$

(ii) $2-2'$



Soln:- (I) 1-1' :-

$$Z_{i/p} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$$(Z_{i/p})_{o.c} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{A}{C}\ \Omega$$

$$(Z_{i/p})_{s.c} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{B}{D}\ \Omega$$

(II) 2-2' :-

$$(Z_{o/p})_{oc} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$I_1 = CV_2 - DI_2 = 0$$

$$\Rightarrow \frac{V_2}{I_2} = \frac{D}{C}$$

$$(Z_{o/p})_{o.c} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{D}{C}\ \Omega$$

$$(Z_{o/p})_{s.c} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

$$V_1 = AV_2 - BI_2$$

$$\Rightarrow 0 = AV_2 - BI_2$$

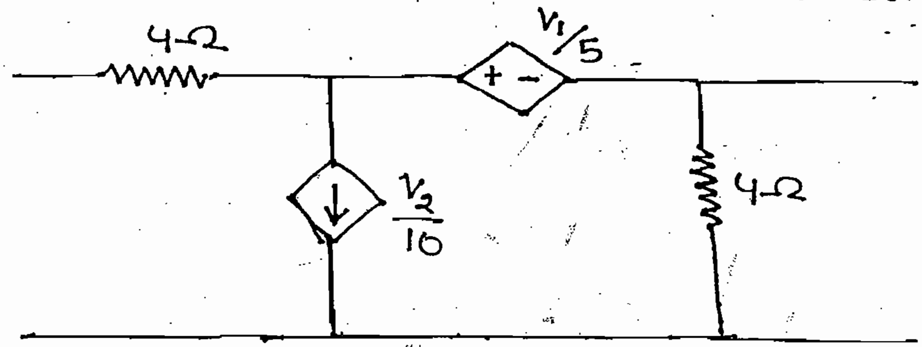
$$\Rightarrow \frac{V_2}{I_2} = \frac{B}{A}$$

$$(Z_{o/p})_{s.c} = \frac{B}{A}\ \Omega$$, Ans.

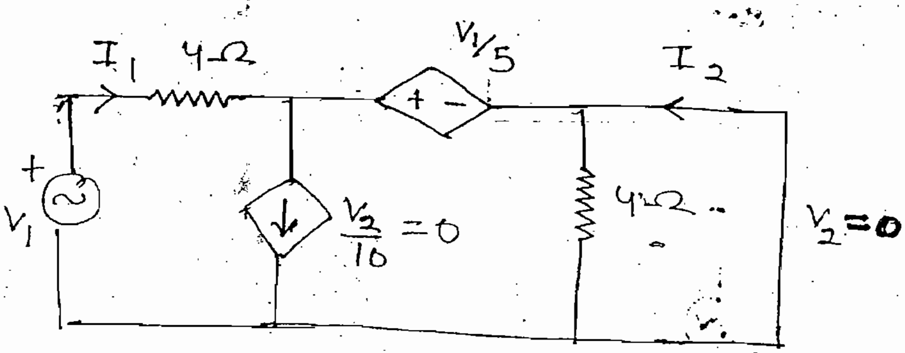
$$B = -\frac{V_1}{I_2}\Bigg|_{V_2=0} \Rightarrow \frac{2s+1}{s^2} = \frac{eq-(1)}{eq-(4)}$$

$$\Rightarrow \boxed{Z(S) = \frac{1}{S}} \quad \text{Ans}$$

Ques:- Find B and D of the circuit shown



Soln:-



$$B = -\frac{V_1}{I_2}\Bigg|_{V_2=0}$$

$$D = \frac{-I_1}{I_2}\Bigg|_{V_2=0}$$

$$= \frac{-(-I_2)}{I_2} = 1$$

$$I_1 = -I_2$$

$$I_1 = \frac{V_1 - \frac{V_1}{5}}{4}$$

$$-I_2 = \frac{4V_1/5}{4}$$

$$\Rightarrow \boxed{B = -\frac{V_1}{I_2} = 5} \quad \text{Ans}$$